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SOME CORRECTION FACTORS FOR THE VIRTUAL INERTIA COEFFICIENT FOR THE HORIZONTAL VIBRATIONS OF A SHIP*

By Toyoji KUMAI**

Abstract The present paper shows the three-dimensional correction factors for the virtual inertia coefficient and the correction for the rotatory inertia of the virtual added mass of water for horizontal vibration of a ship, including higher modes. Special consideration is given to the three-dimensional correction factors obtained from the theoretical study of the three-dimensional calculation of the virtual mass of water on a half-immersed circular cylinder of finite length which vibrates on water. In calculating the kinetic energies for the horizontal vibration of the ship and the entrained water, the rotatory energy of the hull and the corresponding virtual rotatory energy of the water are taken into account. In addition the experimental determination of the virtual mass has been carried out on a wooden model of a tanker and the theoertical results are confirmed.

Introduction

The method for calculating the two-dimensional added mass of entrained water for the horizontal vibration of a ship has already been carried out by J. Lockwood Taylor [1], [2] and L. Landweber [3]. Taylor also showed the correction factors for the three-dimensional flow of the water by use of the results of the theoretical calculation of the virtual mass of water surrounding an infinitely long circular cylinder vibrating on water. The discrepancy between the foregoing calculation and the model experiments is considerable. The cause of the discrepancy is expected to be that the boundary conditions at both ends of the cylinder of the finite length differ from those at the ends of a cylinder cut out from an infinitely long cylinder.

On the other hand, in the calculation of the kinetic energies of the water surrounding a vibraing ship, the rotatory energy of the water should be taken into account. It is noted that the effect of the rotatory energy of the water on the

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[1] J. Lockwood Taylor: "Hydrodynamical Inertia Coefficients", Phil. Mag., 1929.

[2] J. Lockwood Taylor: "Vibration of Ships", T. I. N. A., 1930.

[3] L. Landweber and M. C. de Macagno; "Added Mass of Two-dimensional Forms Oscillating in a Free Surface", Jour. of Ship Research, Nov. 1957.

virtual inertia coefficient becomes conspicuously in the higher modes of the flexural vibration of a ship. In the present paper, theoretical studies are given of the three-dimensional virtual mass of water and of the rotatory energy of water when a cylinder of finite length vibrates in the horizontal direction on water. With regard to the correction factors for horizontal vibrations of ship, the caluculation of the above correction factors of a ship-like beam are shown in the modes up to four-node mode of the horizontal vibration, and the results are compared with that of ship model experiments.

1. Calculation of the Virtual Ineria Coefficient for Horizontal Vibrations of a Ship

The two-dimensional virtual inertia coefficient in the flexural vibration of a ship is computed by means of a strip method as follows:

$$\tau_{II}=\frac{\int_0^L m_e x^2 dz}{\int_0^L m x^2 dz}, \qquad \cdots\cdots\cdots\cdots(1)$$

where,

τ_{II} two-dimensional virtual inertia coefficient
m_e two-dimensional added mass of water per unit length
m mass of ship per unit length
x normal function of vibration of ship
L length of ship
z length co-ordinate

It should be noted that the effect of the number of nodes of the flexural vibration on the virtual inertia coefficient appears even in the two-dimensional calculation as is seen in (1), since the normal function of the vibration of a ship should be taken into account.

The special consideration of the corrections to the two-dimensional result mentioned above is given as follows:

$$\tau=\Theta_1\cdot\Theta_2\cdot\mu\cdot\tau_{II}, \qquad \cdots\cdots\cdots\cdots(2)$$

where

Θ_1 factor for three-dimensional form
Θ_2 factor for three-dimensional motion
μ factor for the effect of rotatory energy of water surrounding the ship

The correction factors Θ_1, Θ_2 and μ in (2) are treated in the following sections.

2. Correction Factor Θ_1

The three-dimensional correction factor for the form of the water plane in the horizontal vibration of a ship is given as the ratio of the added mass moment of inertia of the water of a water plane lamina of a ship to that of a rectangular water plane lamina of the same length. The horizontal rotational motion is performed about the centre of the water plane lamina. If we assume the form of the water plane to be a narrow elliptical one, we can easily obtain the virtual mass mement of inertia of the water surrounding the lamina of half-length l and half-breadth b as follows:

$$I_{bl}=\frac{\rho\pi}{8}(l^2-b^2)^2, \tag{a}$$

where ρ is the density of the water. When b approaches zero in (a), the boundary of the above lamina becomes a straight line of half length l. The added mass moment of inertia of water is then presented by

$$I_{0l}=\frac{\rho\pi}{8}l^4. \tag{b}$$

The effect of the form of the water plane compared with a corresponding straight boundary such as the water plane of a cylinder is obtained by the ratio I_{bl} to I_{0l}, namely:

$$\Theta_1=\left\{1-\left(\frac{B}{L}\right)^2\right\}^2, \qquad \cdots\cdots\cdots\cdots(3)$$

where L and B denote the length and breadth of a ship respectively. As an approximate value of the three-dimensional correction factor for the form of water plane, the Θ_1 presented in (3) may conveniently be used. The value of Θ_1 versus L/B is shown in Fig. 1.

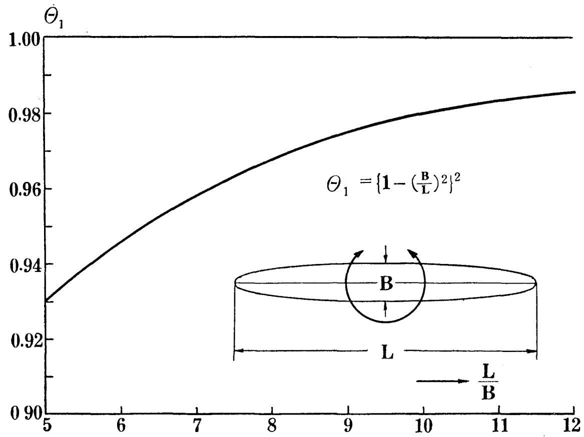

Fig. 1 Three-dimensional form factor for horizontal rotation of elliptical cylinder

3. Correction Factor Θ_2

The three-dimensional correction factor for the vibratory motion is defined by the ratio of the three-dimensional virtual mass to the two-dimensional one in the horizontal flexural vibration of a half-immersed circular cylinder of finite length. In the present paper, special consideration is given to satisfy the end conditions of the velocity potential function at both ends of the cylinder of finite length.

Let us consider a cylindrical co-ordinate system r, θ and z with the origin at the left end of the cylinder of length $2l$ and radius a. If the cylinder vibrates in the horizontal direction on water with draught a, the normal velocity to the wetted surface of the cylinder denoted V_n is obtained assuming a shear-flexural mode of vibration as follows:

$$V_n = U\cos\theta\cdot\cos\frac{s\pi z}{2l}, \tag{c}$$

where,

U unknown coefficient including a unit of velocity

s number of nodes of vibration of cylinder

The above functions, representing the normal velocity, are expanded to the following Fourier series with respect to θ and z:

$$V_n = U\sum_m^\infty\sum_n^\infty \frac{4}{\pi}\left(\frac{2n}{4n^2-1}\right)\sin 2n\theta\cdot\frac{4}{\pi}\left(\frac{m}{m^2-s^2}\right)\sin m\zeta, \tag{d}$$

wehre,

$$\zeta = \frac{\pi z}{2l}$$

$m=1, 3, 5, \cdots\cdots$ for $s=0, 2, 4, 6\cdots\cdots$ nodes

$m=2, 4, 6, \cdots\cdots$ for $s=3, 5, 7$ $\cdots\cdots$ nodes

$n=1, 2, 3\cdots\cdots$

As a next step, the solution of the Laplace equation of the potential function, Φ, which satisfies the boundary conditions:

i $\Phi=0$ when $r\to\infty$,

ii $\Phi=0$ when $\theta=0$ and π,

iii $\Phi=0$ when $z=0$ and $2l$,

are written by the use of a cylindrical function as follows:

$$\Phi = \sum_m^\infty\sum_n^\infty A_{mn}\cdot K_{2n}(k_m r)\cdot\sin 2n\theta\cdot\sin m\zeta, \tag{e}$$

where, K_{2n} is K-Bessel function of $2n$ order, $k_m = m\pi/2l$.

A further condition not mentioned above is

$$\text{iv}\quad \frac{\partial\Phi}{\partial r} = V_n \qquad \text{when } r=a$$

From this condition, the unknown coefficient A_{mn} in (e) is determined by use of (d):

$$A_{mn} = U\cdot\frac{4}{\pi}\left(\frac{2n}{4n^2-1}\right)\cdot K'_{2n}(k_m a)\cdot\frac{4}{\pi}\left(\frac{m}{m^2-s^2}\right), \tag{f}$$

where dash denotes the derivative of K_{2n} with respect to r. The total kinetic energy of the water surrounding the vibrating cylinder under consideration is computed from the above potential function satisfying the boundary conditions, by the following integral:

$$2T = -\rho\int_0^\pi\int_0^{2l}\Phi\left(\frac{\partial\Phi}{\partial r}\right)_{r=a}\cdot a d\theta\cdot dz. \tag{g}$$

On the other hand, the result of the two-dimensional calculation of the kinetic energy of the water surrounding a vibrating circular cylinder is given as follows:

$$2T_{\text{II}} = \frac{2\rho a^2}{\pi}\int_0^{2l}\cos^2\frac{s\pi z}{2l}\cdot dz. \tag{h}$$

In the results, the three-dimensional correction factor considered is obtained by taking the ratio of T by T_{II}; this is given by

$$\Theta_2=\frac{16}{\pi^2}\sum_m^\infty\sum_n^\infty\frac{\left(\frac{4n}{4n^2-1}\right)^2\left(\frac{m}{m^2-s^2}\right)^2}{2n+k_m a\,\frac{K_{2n-1}(k_m a)}{K_{2n}(k_m a)}}\ . \tag{4}$$

Fig. 2 shows the calculated value of Θ_2 obtained by (4) versus L/d, where $L=2l$, $d=a$. The results obtained from an infinitely long circular cylinder studied by J. L. Taylor are shown in the same figure by dotted lines. The discrepancy between Taylor's results and that of the present study is clearly seen in the figure. The cause of difference in the results is that of satisfaction or non-satisfaction of the boundary condition of the potential function at both ends of the cylinder of finite length. The three-dimensional correction factor presented in Fig. 2 may be used approximately for the horizontal vibration of a ship together with the other correction factors given in the present paper.

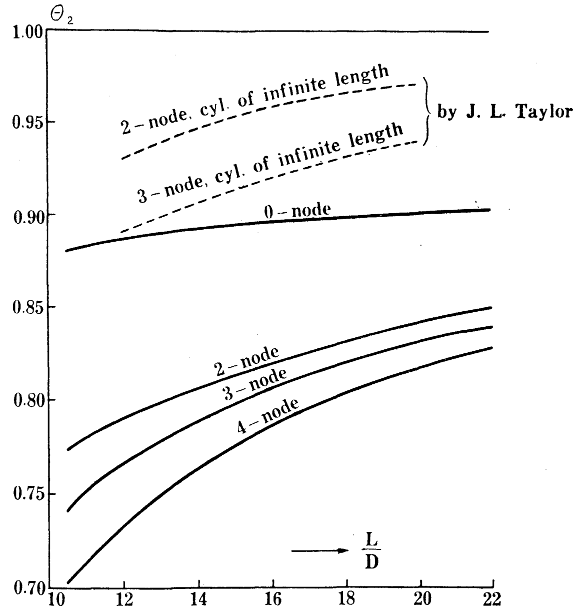

Fig. 2 Three dimensional correction factor for virtual mass of the horizontal vibration of a cylinder of finite length

4. Correction Factor for Rotatory Energy of Added Mass of Water, μ

It should be noted that the rotatory energy of the entrained water is taken into account in the calculation of the kinetic energy of the water surrounding a vibrating ship except in the torsional vibration of the ship. It will readily be appreciated that the added mass moment of inertia should be considered in the motions of pitching and yawing of a ship. In the flexural vibration of a ship, the mass moment of inertia of entrained water per strip at any section of the hull under the water-line should be computed in the first stage. In the horizontal transverse vibration of a ship, the mass moment of inertia of the water surrouding a unit length strip is written as follows:

$$I_e=\int_s (x_1^2+z_1^2)\,dm, \tag{i}$$

where dm is the mass of entrained water of the element area on the wetted surface of the strip, x_1 and z_1 are the co-ordinates at the surface of the strip in the transverse and the longitudinal directions respectively and s denotes the wetted surface of the strip. The mass element of the entrained water is represented by potential function along a contour of the strip section, and the moment of inertia is then expressed by

$$I_e=-\rho\int_0^{\pi} x_1^2\left(\Phi\frac{\partial\Phi}{\partial\alpha}\right)_{\alpha=0} d\beta, \tag{j}$$

provided that the potential function which satisfies the boundary conditions of the horizontal vibration of the strip is represented by the curvilinear co-ordinates α and β, for instance in the Lewis form. Putting b_0, a_1, and a_3, the coefficients for the section of the Lewis form [4], the added mass moment of inertia for a prism having Lewis form under the action of horizontal rotation of a strip becomes

$$I_e=\frac{8\rho b^4{}_0}{\pi}\left[\left\{(1+a_1)^2+a_3^2\right\}\sum_{n=1}^{\infty} n\left(\frac{1-a_1}{4n^2-1}-\frac{3a_3}{4n^2-9}\right)^2-a_3(1+a_1)\left\{\frac{1-a_1}{3}+\frac{3a_3}{5}\right\}^2\right]. \tag{k}$$

By use of I_e metioned above, the correction factor for the rotatory energy of the entrained water is obtained by the following calculation:

$$\mu=\frac{1+\dfrac{\int_0^L I_e\left(\dfrac{dx}{dz}\right)^2 dz}{\int_0^L m_e x^2 dz}}{1+\dfrac{\int_0^L I\left(\dfrac{dx}{dz}\right)^2 dz}{\int_0^L m x^2 dz}}, \tag{5}$$

where,

I, I_e mass moment of inertia of strip and of entrained water respectively

$\dfrac{dx}{dz}$ the slope of normal function of the vibration of a ship

If the normal function is assumed to be of the sinusoidal form and the sections of a prism under the water-line are of half elliptical form with half-beam b and half-length l, the μ is shown as follows:

$$\mu=1+0.250\left(\frac{s\pi b}{l}\right)^2, \tag{1}$$

where s; the number of nodes of horizontal vibration.

If the form of the sections is replaced by a rectangular one with half-beam of b, the μ is presented by

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[4] T. Kumai; "Added Mass Moment of Inertia Induced by Torsional Vibration of Ships". European Shipbuilding. Vol. Vll, No. 6, 1958.

$$\mu=1+0.666\left(\frac{s\pi b}{l}\right)^2. \tag{m}$$

In an actual ship, the water plane has variable breadth along her length. For example, if the form of water plane is assumed to be sinusoidal and all the sections are represented by semi-elliptical forms, the value of μ for such a ship-like beam is analytically obtained provided that the hull form is symmetric about the water plane, as follows:

$$\mu=1+0.092\left(\frac{s\pi b}{l}\right)^2. \tag{n}$$

Comparing (n) with (1), the equivalent breadth of the ship-like beam to the corresponding prism is approximately obtained as

$$b_e=0.606b. \tag{p}$$

The water plane coefficient of the above ship-like beam is $C_w=0.636$. The same consideration as to the equivalent breadth is performed for the beam with rectangular sections and with 20% taper at both ends, the result is obtained as

$$b_e=0.749b. \tag{q}$$

The water plane coefficient of such a tapered beam is $C_w=0.80$. With regard to the case of a prism, $b_e=b$ and $C_w=1$. Finally the ratio b/l of a ship-like beam is defined as b_e/l where b_e is obtained approximately

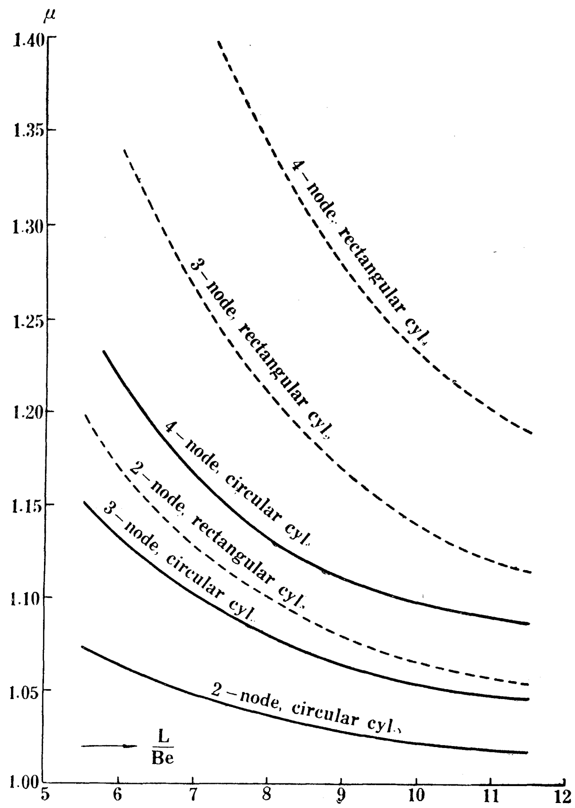


Fig. 3 Vritual mass moment of inertia coefficients of various cylinders

$$b_e = C_w b$$
$$\text{or} \quad B_e = C_w B \qquad \text{(r)}$$

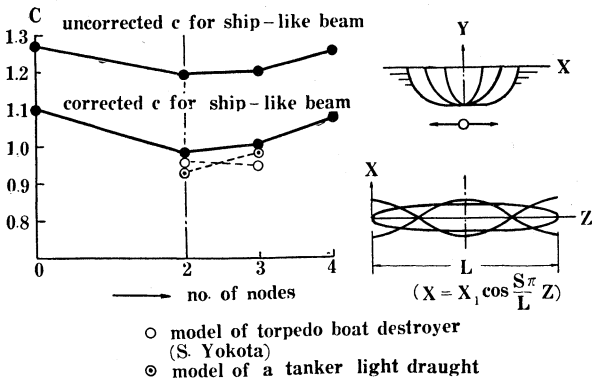
Fig. 3 shows the value of μ for the soild prisms with elliptical and rectangular sections versus L/B_e. The approximate value of the correction factor for the rotatory energy of the entrained water will be estimated by use of the curves presented in Fig. 3.

5. Estimate of the Inertia Coefficient for Horizontal Vibrations of a Ship

The added mass of water per unit length of a ship in the horizontal vibration is proportional to the square of the draught and the corresponding displacement is proportional to the sectinoal area of the hull under the waterline, and consequently the inertia coefficient is proportional to the ratio of draught to beam, namely,

$$\tau = c\frac{d}{B} \qquad \cdots\cdots\cdots\cdots(6)$$

The coefficient c is determined by employing the three-dimensional correction factors described above with the two-dimensional results obtained by means of the strip method. As an example, c is obtained as 0.978 for the two-node vibration of the ship-like beam previously mentioned. The results of Dr. S. Yokota's experiment [5] using the ship model of a destroyer gives $c=0.95$. As another



no. of nodes	C_b	c_{II}	Θ_1	Θ_2	μ	c
0		1.272		0.902	1.000	1.100
2		1.192		0.842	1.018	0.978
3	0.636	1.200	0.958	0.832	1.046	1.002
4		1.250		0.818	1.086	1.065

Ship-like beam with half elliptical sections and sinusoidal water plane. $L/B=7$, $L/d=20$, $L/B_e=11.5$

Fig. 4 c-values obtained by theory and experiments

[5] S. Yokota; "On Vibration of Steamer". Jour. College of Eng. Tokyo Imp. Univ. Vol. V, No. 1, 1910.

example, a wooden tanker model was tested for determination of virtual mass in the horizontal vibration of the hull. The result was $c=0.93$ for the two-node mode of vibration. As will be seen in Fig. 4, the value of c at three-node mode of vibration is a little higher than that of the two-node mode for a full ship; the value, however, is a little lower than that of the two-node mode for a fine ship such as a torpedo boat destroyer. The reason for this fact is explained as a difference in effect of rotatory energy on the virtual inertia coefficient for the horizontal vibration of the ship. The estimated c-values for the horizontal vibrations, including higher modes, are given in Fig. 4.

Conclusion

A broad conclusion which may be drawn from the study is that the three-dimensional correction factors obtained for the virtual inertia coefficients of the horizontal vibrations from zero-node to four-node modes of a ship are about 15% lower than those obtained by J. L. Taylor. In addition, the effect of the rotatory energy of the added mass of water on the virtual inertia coefficient was accounted for. The calculations of the inertia coefficients of the horizontal vibration, including higher modes, were carried out taking into account the above correction factors for a ship-like beam as example. The calculated values were compared with those determined by the experiments on a model of a torpedo boat destroyer carried out by Dr. S. Yokota and that on a model of a tanker. The results obtained from the experiments comfirmed the theoretical considerations for the three-dimensional corrections and the rotatory energy effect.

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