ON THE FREE HEAVING OF A CYLINDER FLOATING ON THE SURFACE OF A FLUID

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ON THE FREE HEAVING OF A CYLINDER FLOATING ON THE SURFACE OF A FLUID

1. Free Heaving of the circular Cylinder

2. Bilge-Keel Effect on the Damping Force and Added Mass

By Fukuzô TASAI

Abstract

In the free heaving experiments of the circular cylinder it was found that the decay curve is nearly normalized, and added mass and damping coefficient obtained from the decay slope after one cycle is in good coincidence with the F. Ursell's results. In the case of rectangular cylinder with rounded corner, decay curve shows nonlinear characteristic slightly. In high frequency tests the damping coefficient and added mass were larger than the theoretical values which were obtained for the forced heaving. A rectangular cylinder with bilge-keel has a smaller wave damping than the cylinder without bilge-keel. Therefore, although eddy-making resistance by the bilge-keel is moderately large, the damping coefficient is about the same with the one without bilge-keel at the natural period.

1. Introduction

For the subject of free heaving we refer, first of all, to the papers written by A. Dimpker [1]* and H. Holstein [2].

The former, by means of the free-heaving experiments, measured the damping force and added mass of a circular and a triangular cylinder floating at various draughts. The latter measured, for the rectangular cylinder, the progressive wave height produced by the forced heaving and carried out free heaving test also. They studied two-dimensionally by means of a cylinder having a nearly same length with the breadth of the water tank. In this paper, free heaving experiment was carried out making use of a similar experimental arrangement, and then the damping force and added mass were measured. Moreover we compared the results of experiments with the calculations by F. Ursell [3] and by the Author [4] for forced heaving cylinders.

On the other hand the free heaving of a cylinder floating on the surface of water was not a simple damped oscillation. Free pitching of a ship was studied by P. Golovato [5]. In the next place, experiments was carried out, making use of a rectangular cylinder, for the cases with bilge-keel and without bilge-keel. Then the effect of bilge-keel was investigated.

* Numbers in brackets designate References at the end of this paper.

2. Free Heaving of a circular Cylinder

2.1. Experimental Arrangement and the Model

Experimental arrangement is, as shown in Fig. 1, a similar one with that of Dimpker [1]. The frame S–Q rotates about the Z-Z axis making use of the ball-

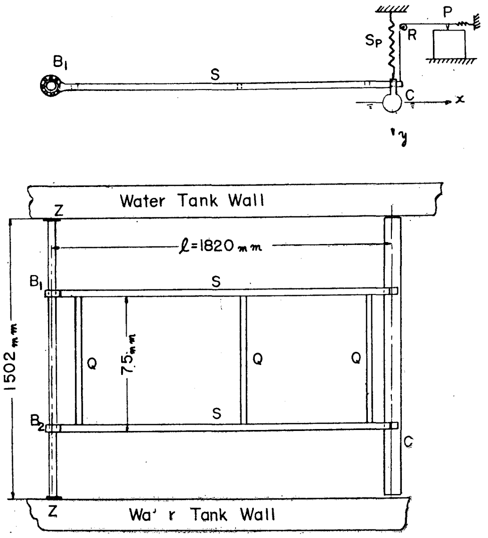

Fig. 1.

bearing B_1, B_2. This frame was made of brass pipe and rotating inertia was made as small as possible. The model of a circular cylinder is fixed under the frame and suspended by the spring Sp. In order to make the model float at the constant draught through the centre of the circle, some ballast-weights are loaded in accordance with the strength of Sp. The wooden model of a circular cylinder have length L of 1495 mm and radius a 50 mm. The clearance between the model and the tank wall was about 3.5 mm respectively.

2.2. Summary of Experiments

Initial displacement of heaving exceeded rarely 30 mm and was smaller than 20 mm in many cases. As this is far small in comparison with the length of arm S, the effect of rotating and horizontal motion of the model is extremely small and may be neglected. Records were obtained making use of the Tungsten wire and pen P, as was done in [6].

A example of the decay curve of the free heaving experiment is shown in Fig. 2. Period of the motion was longer for the first one cycle and hereafter it was damped with a almost constant period. Performing the free heaving test of the total experimental arrangement with the model in the air we obtained the logarithmic decrement δ_0 for each frequency.

Circular Cylinder-Decay Curve $T=0.4950$ sec, $\xi_0=0.8230$

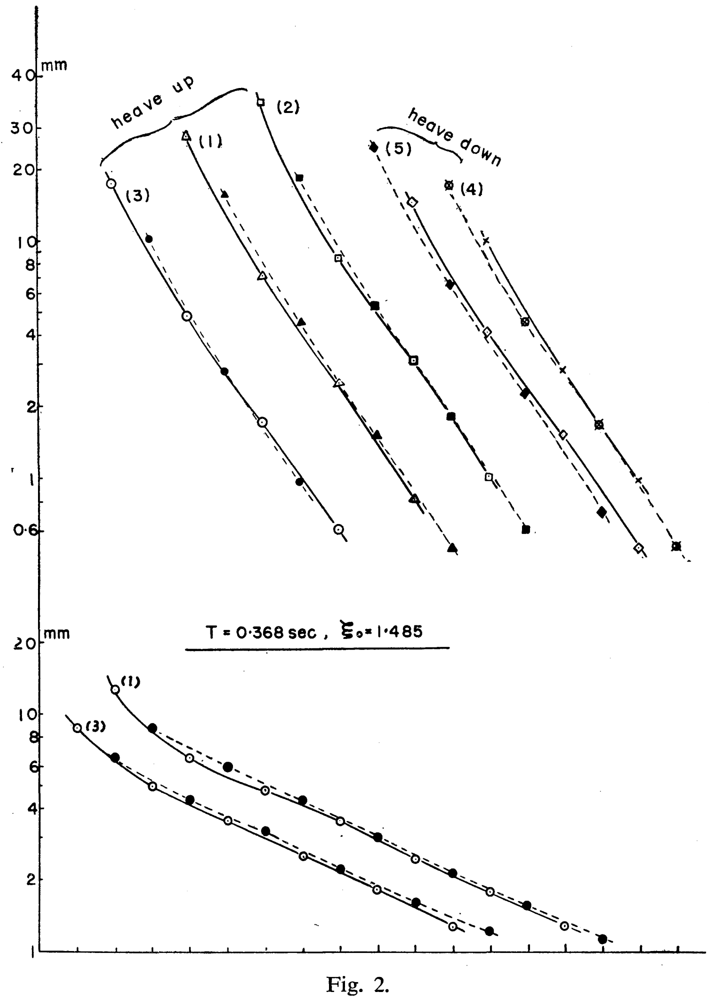

Fig. 2.

Deducting δ_0 from the logarithmic decrement measured for the free heaving test on the free surface, the logarithmic decrement for the free heaving of the model was determined. For a certain period, namely making use of the spring S_p with a certain strength determination of δ was carried out several times and then a mean value, δ_m, was obtained.

2.3. Added Mass for Heaving of a circular Cylinder

As the Frame S–Q rotates, equivalent heaving mass of the Frame is approximately given as the following equation: $M_0 \doteqdot \frac{I}{l^2}$,

where I=rotating mass inertia of the Frame about the axis, now putting

Normalized Decay Curve ξ_0=0.823

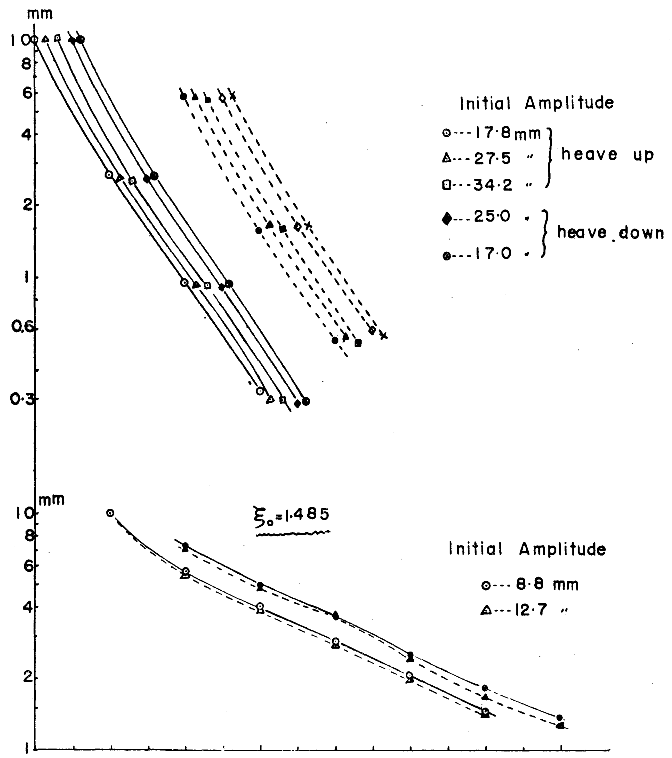

Fig. 3.

M_1=Mass of the circular cylincer and ballast-weights

ΔM=added heaving mass, A_w=water plane area of the cylinder,

k_s=spring constant of S_p

linear free heaving equations can be expressed as follows:

$$(M_0+M_1+\Delta M)\ddot{y}+N\dot{y}+(\rho g A_w+k_s)y=0\,. \tag{1}$$

Then putting

$$M_0+M_1=M, \quad \frac{N}{M+\Delta M}=2h, \quad \nu_0^2=\frac{\rho g A_w+k_s}{M+\Delta M} \tag{2}$$

the equation (1) becomes $\ddot{y}+2h\dot{y}+\nu_0^2 y=0\,.$ (3)

With initial conditions $y=y_0$, $\dot{y}=0$, we obtain

$$y=y_0 e^{-ht}\left[\cos\omega t+\frac{h}{\omega}\sin\omega t\right], \tag{4}$$

where $\omega=\sqrt{\nu_0^2-h^2}\,.$ (5)

With the aid of the period T_1, logarithmic decrement δ_m is expressed as the following equation

$$\delta_m = T_1 h = \frac{2\pi h}{\omega} \,. \tag{6}$$

From equations (2), (5) and (6), we have

$$\Delta M = \frac{\rho g A_w + k_s}{\omega^2 \left(1 + \frac{\delta^2_m}{4\pi^2}\right)} - M \,. \tag{7}$$

In the case of a circular cylinder, as the δ_m in the neighbourhood of the natural period is nearly 1.0 and is considerably small in high frequencies, calculating by means of the next equation

$$\Delta M \doteqdot \frac{\rho g A_w + k_s}{\omega^2} - M \,, \tag{8}$$

its error is about 1.0% at the natural period.

Measuring δ_m and ω by the experiments we can obtain added mass by means of the equation (7).

On the other hand, heaving added mass for a circular cylinder has been given by the following formula:

$$\Delta M = \frac{1}{2} \rho \pi a^2 L K_4 \,.$$

Therefore K_4 can be obtained from experiments. Then we compared the above K_4 with the results by F. Ursell [3] for a forced heaving of a circular cylinder. This is shown in Fig. 4. The agreement between experiments and calculations are generally good, but in the case of $\xi_0 > 2.0$, experiments give a few percent larger value than the F. Ursell's value $\left(\xi_0 = \frac{\omega^2}{g} \frac{a}{2}\right.$: non dimensional heaving parameter$\left.\right)$.

K_4-Circular Cylinder

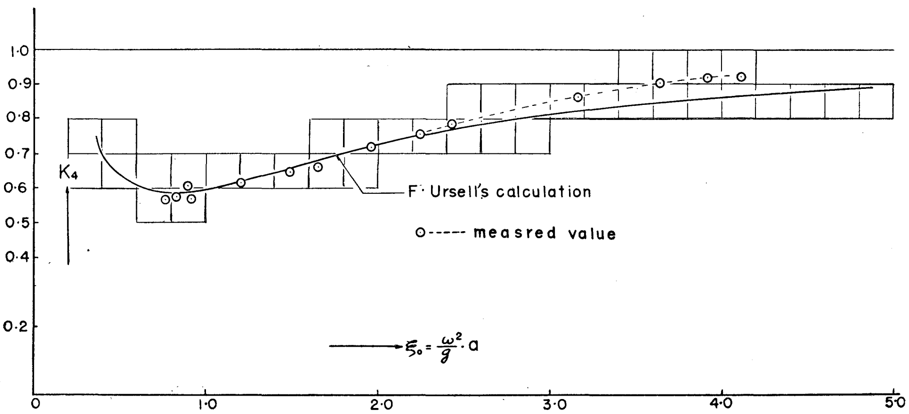

Fig. 4.

2.4. Characteristic of the Decay Curve

In the decay curve in Fig. 2, in the case of $\xi_0=0.823$ the results of experiments have been included, those carried out with positive initial displacement (heave up), and those carried out with negative initial displacement (heave down). Difference of the decay curve was not almost found. Namely decay curve does not depend on the direction of the initial displacement. In both cases $\xi_0=0.823$ and $\xi_0=1.485$, decay curve showed a non-linear form at first one cycle and also the curvature became larger with the increasing ξ_0.

After one cycle it is considered that the slope of the decay curve may be almost constant, as is seen from Fig. 2.

Added mass in Fig. 4 and damping coeffecient in Fig. 5 were obtained by measuring T_1 and δ_m for almost constant decay slope.

P. Golovato [5], making an experiment on the free pitching for a ship-model studied the characteristic of the decay curve qualitatively.

For the results of experiments with various amplitude, each maxima and minima were divided by the initial amplitude. He then showed that these values were normalized without concerning to the value of initial amplitude. The maxima and minima in Fig. 2 were divided by the initial amplitude respectively and were shown in Fig. 3. In the case of $\xi_0=0.823$, each maxima and minima were almost normalized without regard to the initial amplitude. On the other hand for $\xi_0=1.485$ in the case of a large initial amplitude, the values divided by the initial amplitude were a little lower than the case of small initial amplitude. The more ξ_0 increase, the clearer the above tendency becomes.

It was because of the non-linear damping, but in $\xi_0<2.0$ nonlinear characteristic of the decay curve was small. Synthesizing the decay characteristic of a circular cylinder, in the range of $\xi_0<2.0$, we obtained the following observations:

(1) Without regard to the direction of the initial displacement, decay curve has almost a similar form.

(2) After the first on cycle the slope of the decay curve shows almost constant.

(3) In small ξ_0, even though the initial amplitude being changed, the decay curve were almost normalized.

(4) At large ξ_0 the non-linear decay characteristic appeared though it being very small.

The observation (1), (2), (3), are the same with the results by P. Golovato [5].

In his experiments as ξ_0 was comparatively small, it is thought that the non-linear decay characteristic has not appeared.

Thus decay curve has, for the first onecycle, a non-linear form and after that almost takes an linear form. It is very important that except for the extremely large ξ_0, decay curve can be normalized without regard to the initial amplitude.

Quoting the Golovato's saying, for small ξ_0, the fact mentioned above verifies that " the time history of the motion or the time elapsed from release determines the slope of the decay curve rather than the instantaneous values of θ, $\dot{\theta}$

and $\ddot{\theta}$. Thus the waves generated by the body at a previous time significantly affect the motion".

The phenomena that in large ξ_0 the decay curve has, after the first one cycle, a non-linear form will be clearly observed in a cylider with a rectangular section.

2.5. Damping Force

From the equation (2) and (6), a equation for the damping coefficient N are obtained and written as follows:

$$N = 2h(M+\Delta M) = \frac{\delta_m(\rho g A_w + k_s)}{\pi\omega\left(1+\frac{\delta_m^2}{4\pi^2}\right)} \quad . \tag{10}$$

In the case of a small δ_m we may use the equation

$$N \doteqdot \frac{\delta_m(\rho g A_w + k_s)}{\pi\omega} \quad . \tag{11}$$

Making use of the measured ω and δ_m, with the aid of the equation (10) we can obtain N. The results are shown in Fig. 5.

Theoretical damping coefficient was calculated making use of the equation $N = \frac{\rho g^2}{\omega^3} \cdot \bar{A}^2 \cdot L$ assuming that the damping force depends on the energy dissipation of the waves produced by the heaving motion of the model. Namely, this is a Wave Damping obtained with the aid of the Ursell's $\bar{A}$ [3] and is shown with

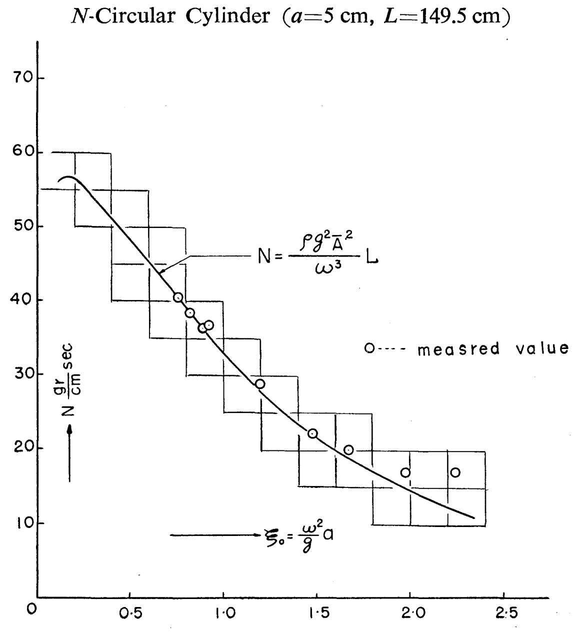

Fig. 5.

a full line in Fig. 5. In the range of $\xi_0<2.0$, the results of experimets and of calculations were in good coincidence. For the case of $\xi_0=2.3$, the experiment was about 50% larger than the calculation. This fact had the same tendency with the Golovato's experiments [7] in which at the higher frequencies the experiments showed the damping being fallen toward zero and then increase with increasing frequency. That may be considered to the action of the Viscous Damping in addition to the Wave Damping.

With increasing frequency, the Viscous Damping becomes larger, as is known from the fact that the normalized characteristic of the decay curve has been lost and at the same time the non-linear characteristic has appeared.

In the range of $\xi_0<2.0$, added mass and damping coefficient obtained from the slope of the deday curve after the first one cycle were in good coincidence with the results by Ursell [3] for the forced heaving.

Now we should like to speak about the Dimpker's experiments. He also measured δ_m and ω, and then indicated the relation of δ_m and ω at the various draughts in a graph. It is clearly seen that with increasing ω, δ_m quickly decreases. As the constant of the spring which corresponds to the δ_m and ω measured, was not explicitly given we could not plot his results in Fig. 5.
Making use of the $æ_h$ (non-dimensional damping coefficient of heave) and $A=\frac{\omega}{\nu_h}$ (See [8]) we think about $\pi æ_h A=\pi\left(\frac{2h}{\nu_h}\right)\left(\frac{\omega}{\nu_h}\right)$.

With the aid of $\frac{2h}{\nu_h^2}=\frac{N}{\rho g A_w}$ we obtain $\pi æ_h A=\frac{\pi N\omega}{\rho g A_w}$.

On the other hand from the equation (11) it will become

$$\delta_m\left(1+\frac{k_s}{\rho g A_w}\right)\doteqdot\frac{\pi N\omega}{\rho g A_w}.$$

Now neglecting the Viscous Damping and putting $N=\frac{\rho g^2}{\omega^3}\cdot\bar{A}^2\cdot L$

we obtain the following equation

$$\delta_m\doteqdot\frac{\pi\bar{A}^2}{2\xi_0(1+æ_s)}. \tag{12}$$

where

$$æ_s=\frac{k_s}{\rho g A_w} \tag{13}$$

Accordingly $\delta_m(1+æ_s)$ obtained by the free heaving test is equal to $\pi æ_h A$. For a cylinder it generally becomes

$$\delta_m(1+æ_s)\doteqdot\pi æ_h A=\frac{\pi\bar{A}^2}{2\xi_0}.$$

3. Belge-Keel Effect on the Damping Force and Added Mass

3.1. Progressive Wave Height produced by the forced Heaving

In the mid part of the ship, sections are full and generally have a bilge keel in order to decrease rolling of the ship. We intend to investigate how the bilge keel works for the heaving and pitching. In the first place we made an experiment to know how the progressive wave height produced by the forced heaving vary with the bilge keel. O. Grim [9], by calculation, indicated that a rectangular cylinder with bilge keel had generally a smaller $\bar{A}$ than that without bilge keel. (Fig. 6). In this case the bilge keel sticked out from the corner of that section. The height of the bilge keel was $d=0.15\frac{B}{2}$.

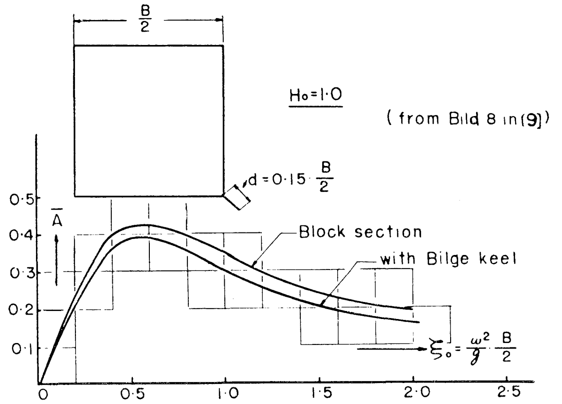

Fig. 6.

To get the effect of the bilge keel for a similar full section with the midship section of a ship we used two models, one was B-model which was used in [6] and the other a fuller $\bar{B}$-model. Their breadth B was 300 mm. Section contour and the appearance of bilge keel are shown in Fig. 7. Draught T, $H_0=\frac{B}{2T}$ and section coefficient σ and other particulars of models are given in the Table.

Bilge keel of B-model

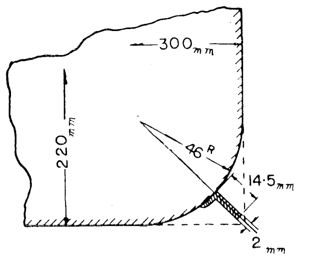

Fig. 7(1).

Bilge keel of $\bar{B}$-model

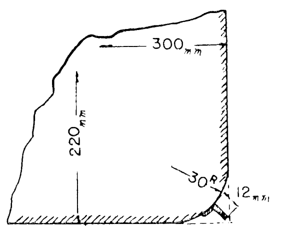

Fig. 7(2).

Table

model		draft: T	$H_0=\frac{B}{2T}$	σ	Bilge keel Height: d	$\frac{d}{B/2}$	d/T
B-model	B_2	120 mm	1.25	0.9750	14.5 mm	0.0967	0.121
	B_4	75 mm	2 0	0.9600			0.193
$\bar{B}$-model	$\bar{B}_2$	120 mm	1.25	0.9892	12.0 mm	0.0800	0 100
	$\bar{B}_4$	75 mm	2.0	0.9828			0.160

By means of the same method with [6] we measured the progressive wave

B_2-Model $H_0=1.25$, $\sigma=0.9750$

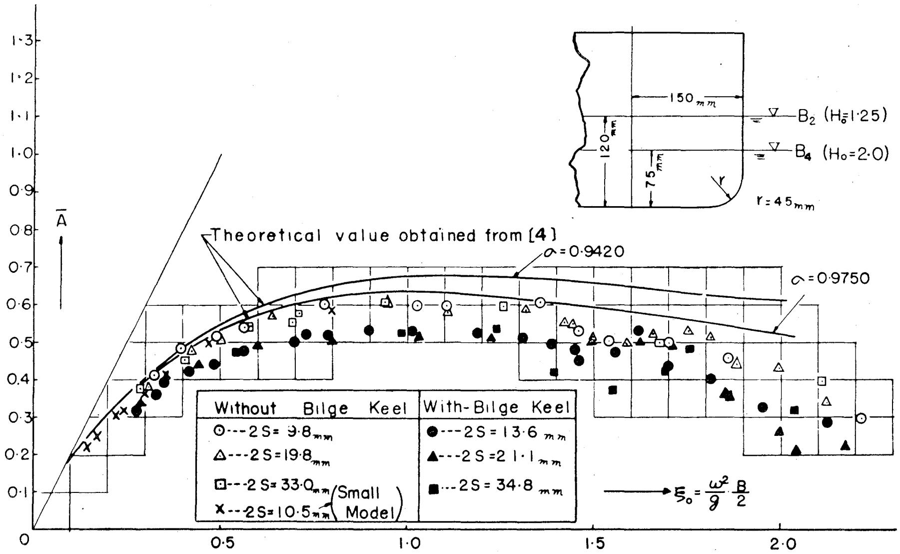

Fig. 8(1).

B_4-Model $H_0=2.0$, $\sigma=0.9600$

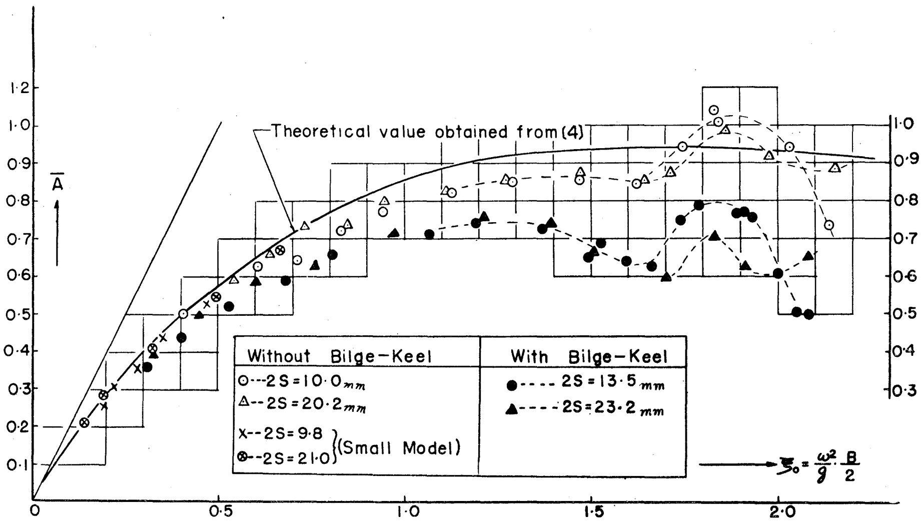

Fig. 8(2).

height. These results are shown in Fig. 8(1), (2) and Fig. 9(1), (2).

As was seen in the Grim's calculations, the amplitude ratio $\bar{A}$ decreased for the case with the bilge keel. At any rate though it is considered that it will be due to interaction between the waves produced by the body of the cylinder and by the bilge keel, from Fig. 8 and Fig. 9, we have

$\bar{B}_2$-Model $H_0=1.25$, $\sigma=0.9892$

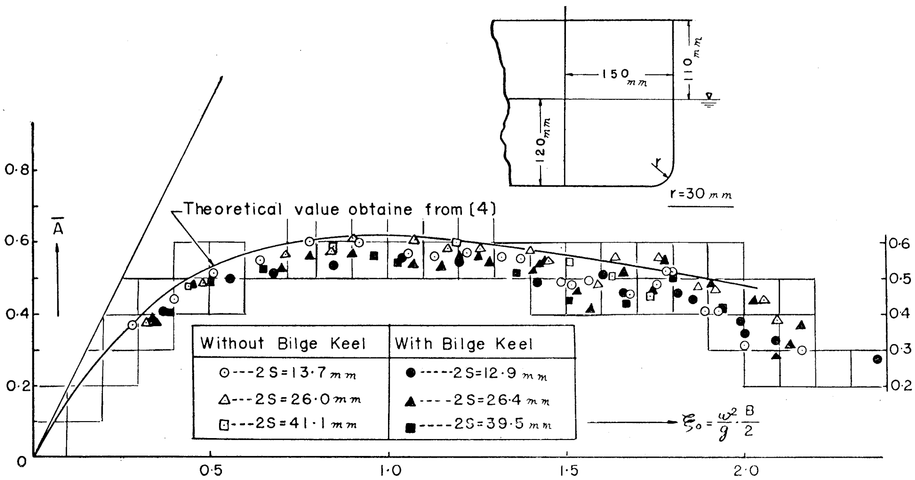

Fig. 9(1).

B_4-Model $H_0=2.0$, $\sigma=0.9828$

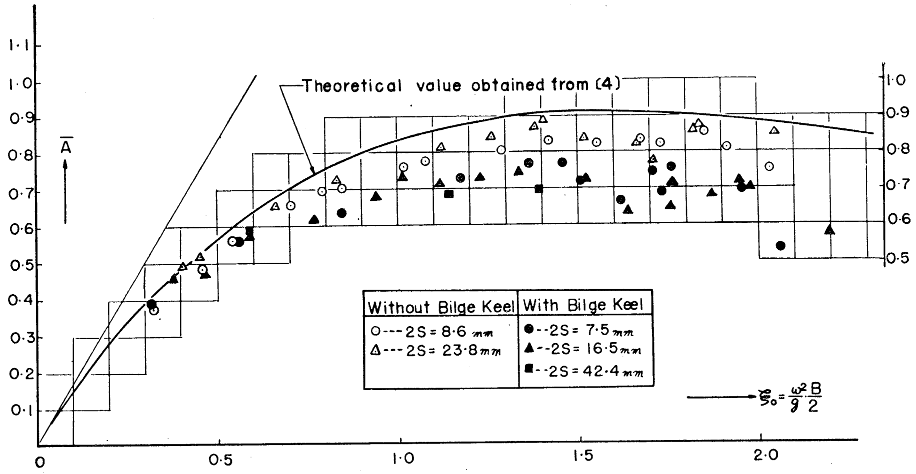

Fig. 9(2).

(1) With increasing bilge keel-height d and decreasing σ, the rate of the reduction of $\bar{A}$ increased.

(2) In the same model, with increasing H_0, namely approaching the bilge keel to the free surface of water, the rate of the reduction of $\bar{A}$ increased. It means reduction of the Wave damping that $\bar{A}$ for the case with bilge keel decreases.

3.2. Added Mass

Free heaving tests were carried out by means of the same method as was done for a circular cylinder. Results of experiments, those for $H_0=1.25$ are shown in Fig. 10(a), (b) and those for $H_0=2.0$ in Fig. 10(c) and (d). Added mass ΔM can be expressed as the following equation

$$\Delta M = \frac{\rho\pi}{2}\left(\frac{B}{2}\right)^2 \cdot C_0 \cdot K_4\ L, \tag{14}$$

where C_0 is a added mass coefficient for the very high frequency $\omega \to \infty$ and is given by F.M. Lewis [10]. By measuring δ_m and ω and making use of the equation (7) and (14) K_4 can be obtained. The following were obtained from the results in Fig. 10. For the model without bilge keel, both experimental results for $H_0=1.25$ and $H_0=2.0$ approached to the author's calculations [4]. Namely it will be found that even for such a full section, by means of the calculation making use of the author's K_4 [4] we can fairly well estimate the value of added mass.

K. Wendel [11] calculated C_0 for the same contour as Grim did.

According to this calculation, C_0 increases with increasing the bilge keel height. It is shown as follows:

Put $$\frac{d}{T} = \frac{\text{bilge keel height}}{\text{draft}}, \qquad K_{c_0} = \frac{C_0 \text{ with bilge keel}}{C_0 \text{ without bilge keel}}.$$

Then

d/T	0.05	0.10	0.25
K_{c_0}	1.066	1.14	1.45 .

In the above calculations H_0 is equal to 1.0 and bilge keel sticked out trom the corner. In our experiments as H_0 and the condition of bilge keel are different from the Wendel's section, it is considered that the increase of C_0 will not become so large as the above table shows.

For the two models the increase of added mass was not so different and was 10–15% on the average.

Though the increase of added mass will be mainly due to the increase of C_0, we could not clear how the effect of the frequency contributes to the increase of the added mass.

3.3. Damping Coeffecient

(a) The cylinder without Bilge Keel

In this case the decay curves were almost normalized and after the first one cycle were approximately expressed as a straight line, as was seen in the experiments for the circular cylinder. In the large ξ_0 (for example $\xi_0>1.54$ for B_2-model) with increasing the initial amplitude the non-linear characteristic of the decay curve, after one cycle, appeared slightly. Results of experiments N in Fig. 10 were obtained making use of the mean value of δ_m for the case whose heaving amplitude was, after first one cycle, about 5 mm–1.0 mm.

Curves given in Fig. 10 are the Wave Damping calculated with the aid of the formula $N=\frac{\rho g^2}{\omega^3}\cdot\bar{A}^2\cdot L$. Full line shows the results obtained making use of the author's $\bar{A}$ [4], dotted line by using the $\bar{A}$ measured for a forced heaving and chain line using the $\bar{A}$ measured for a forced heaving with bilge keel. Though surface friction and eddy-making resistance were actuated, it was found that Vis-

B_2-Cylinder (H_0=1.25, σ=0.9750)
Damping Force

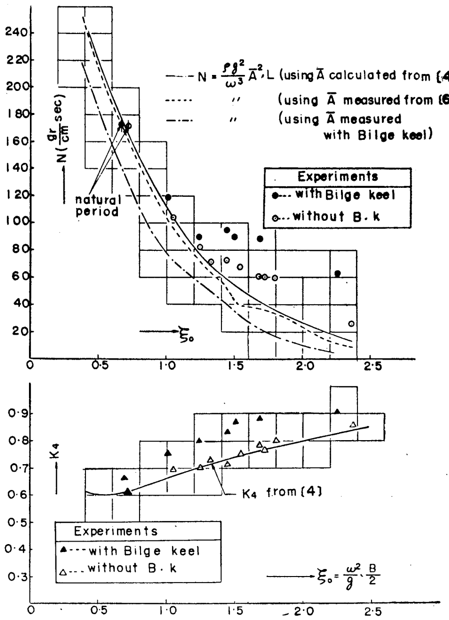

Fig. 10(a).

$\overline{B}_2$-Cylinder (H_0=1.25, σ=0.9892)
Damping Force

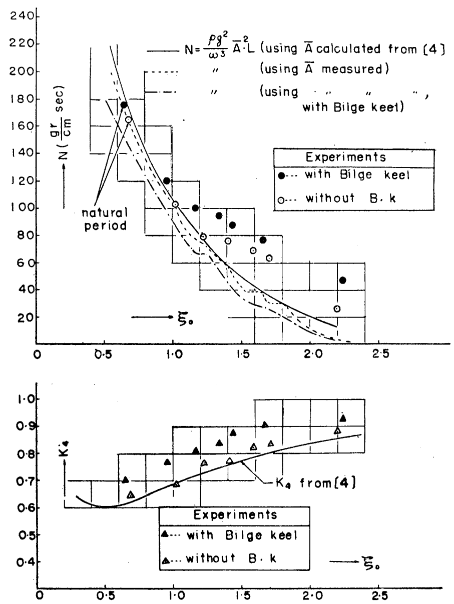

Fig. 10(b).

B_4-Cylinder (H_0=2.0, σ=0.96)
Damping Force

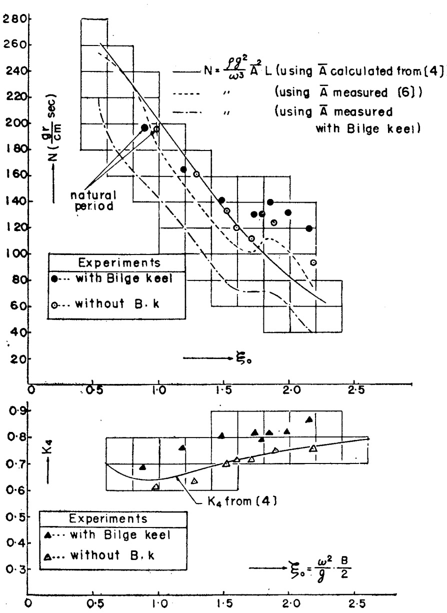

Fig. 10(c).

$\overline{B}_4$-Cylinder (H_0=2.0, σ=0.9828)
Damping Force

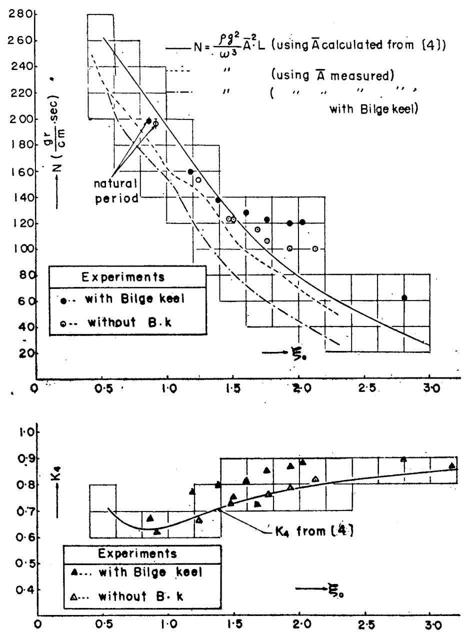

Fig. 10(d).

cous Damping was comparatively small, for $H_0=1.25$ to $\xi_0 \doteqdot 1.3$ and for $H_0=2.0$ to $\xi_0 \doteqdot 1.6$, and then theoretical calculations for Wave Damping fairly agreed with the results of experiments. In high frequencies the results of experiments increased about 50 % for $H_0=2.0$ in $1.3<\xi_0<2.4$ and about 20 % for $H_0=2.0$ in $1.6<\xi_0<2.2$.

(b) The Cylinder with Bilge Keel

In this case the decay curve shows generally non-linear form. Black points in Fig. 10 were obtained making use of the mean value of δ_m which corresponded to the amplituds 5 mm–1 mm. Difference between N for 5 mm amplitude and N for 1 mm amplitude were about 10–20 %.

On $\xi_0>1.1$ for $H_0-1.25$ and in $\xi_0>1.5$ for $H_0=2.0$, damping coefficient increased. With bilge keel Viscous Damping owing to eddy-making phenomena etc. acts strongly with increasing frequency.

On the other hand, for the heaving of a ship, even though the damping force becomes larger considerably in both cases of which the frequency of external force is extremely low and high, magnification factor will hardly altered. It is in the neighbourhood of the natural period that the value of the damping force affects strongly to the magnification factor.

In Fig. 10, at the neighbourhood of the natural period, results of experiments N with bilge-keel are nearly equal to the theoretical values without bilge-keel in case of $H_0=1.25$ and they are smaller than theoretical one in case of $H_0=2.0$.

Though Viscous Damping based on the eddy-making is too great, the extreme decrease of Wave Damping by Bilge Keel might let N take the value mentioned-above.

As the bilge keel is in mid part of the ship, it will have an influence, if it were, on the heaving rather than on the pitching. But as is given in the present work it is found that damping force for heaving in the neighbourhood of the natural period will scarcely increase owing to the action of the bilge keel.

4. Conclusions

From the present work following conclusions were obtained:

(1) In a circular cylinder, for $\xi_0>2.0$, decay curves of free heaving tests are almost normalized. Damping force that was obtained using the logarithmic decrement after the first one cycle was in good coincidence with the Ursell's theoretical Wave Damping. Added mass also, except for high frequencies, agreeds very well.

(2) In the case of a rectangular cyliuder with rounded corner, decay curves are normalized at small ξ_0 but at large ξ_0 it becomes slightly non-linear after one cycle. Added mass due to experiments is comparatively well coincided with the author's theoretical results but damping force increased considerably at large ξ_0.

(3) Progressive wave height produced by the forced heaving of rectangular cylinder decreased owing to the action of the bilge keel.

The higher the bilge keel and H_0 were, and the smaller the section coefficient σ became, the reduction of the amplitude ratio $\bar{A}$ increased.

(4) For a full section, generally, Wave Damping for heaving decreases owing to the action of the bilge keel.

(5) In the cases of full section, $\sigma = 0.96 \sim 0.99$, added mass increased about 10–15% owing to the bilge keel which is sticked out to the corner.

(6) Decay curve of the free heaving of a rectangular cylinder with bilge keel was generally non-linearly. Damping force, in high frequencies, increased largely owing to the eddy-making phenomena.

(7) In the neighbourhood of the natural period of a cylinder with a full section, it will not be expected that the damping force increases by the possession of the bilge keel.

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