DAMPING FORCE AND ADDED MASS OF SHIPS HEAVING AND PITCHING (continued)

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DAMPING FORCE AND ADDED MASS OF SHIPS HEAVING AND PITCHING (continued)

By Fukuzô TASAI

Abstract

In this paper the author shows convenient figures of $\bar{A}$ and C_0K_4 which can be used for calculating the damping force, added mass and added moment of inertia of a ship heaving and pitching.

The added mass and damping coefficient of nine ships were calculated by the Strip Method. It was found that the results by the Strip Method give a good approximate value. Three dimensional correction for the damping coefficient of heave was about 20% in the neighbourhood of the natural period. Practical formulæ which give good approximate values of added mass and damping coefficient in the neighbourhood of natural period were obtained.

1. Introduction

M. D. Haskind [1]*, [2], T. Hanaoka [3], [4], H. Maruo [5] and J. N. Newman [6], [7] have dealt with the problem in three dimensions. For the effect of forward motion of a ship, the results of Newman [7] showed a fairly good qualitative agreement with the Golovato's experiments.

These three dimensional calculations, however, have not arrived at any general useful numerical results. On the other hand, two dimensional values for cylinders have been given by F. Ursell [8] and O. Grim [9].

Making use of the "Strip Method" K. Kroukovsky and Jacobs [10] calculated heaving and pitching motions for some widely different ship forms, and these calculations were compared with the results of tank experiments in regular waves. In many cases a reasonable agreement was found between theory and experiment, but in the case of yacht model some significant differences were found. In respect to the evaluation of added mass and damping force, F. Ursell's K_4 and O. Grim's $\bar{A}$ [9] were used. O. Grim [9] had some doubtful results, which was alluded to by the author in [11]. The author exactly calculated the added mass and progressive wave height for Lewis-Form cylinders heaving on the free surface [11], and then compared this theoretical results with the experiments in our water tank [12]. In general, the measured $\bar{A}$ was in good coincidence with the theo-

* Numbers in brackets designate References at the end of this paper.

retical one. In that paper Wedge Effect correction for non wall-sided sections were given. In the next place the free heaving of cylinders and effect of the bilge keel were investigated [13]. In [11] with the aid of the Strip Method making use of the exact values of $\bar{A}$ and K_4, the author calculated the damping force and added mass of the two ships which had been respectively put to test by P. Golovato [14] and J. Gerritsma [15]. The added mass and added moment of inertia gained by Ship Method showed good coincidence with the results of Golovato's and Gerritsma's experiments. In the damping coefficient of pitch also, a good agreement was found between calculation and experiment. But calculated values of damping coefficient for heave were 20 per cent smaller than the experiments at the natural period of heave. Though we should take into consideration the effect of ship speed and three dimensional effects, it is thought that the Strip Method gives a good approximate value than the three dimensional method in generally.

In this paper the author discussed a practical calculating method of Strip Theory making use of the author's $\bar{A}$ and K_4, and for nine ships calculations were carried out.

Then calculated values of the added mass and damping coefficient for pitch and heave were compared with the experiment of J. Gerritsma [16], P. Golovato [14] and S. Motora [17]. Moreover a calculating method of natural period for pitch and heave was given. Finally taking into consideration the three dimensional effect, practical formulæ which give good approximate values of the damping coefficient, added mass and added moment of inertia in the neighbourhood of the natural period of pitch and heave were given.

2. Strip Method

Taking into consideration the hydrodynamic coupling between pitching and heaving, coupled equations may in general be written as follows [10], [18] and [19]:

$$\left.\begin{aligned} a\ddot{y} + b\dot{y} + cy + d\ddot{\theta} + e\dot{\theta} + g\theta &= F_h e^{iwt} \\ A\ddot{\theta} + B\dot{\theta} + C\theta + D\ddot{y} + E\dot{y} + Gy &= M_p e^{iwt}. \end{aligned}\right\} \quad (1)$$

Though inertia term, damping term and coupled term in the above equations can be calculated by the Strip Method, in this paper the coupling effect don't be dealt with. Except the coupled term and changing the symbol, the equation (1) was written as follows:

$$\left.\begin{aligned} (M+M_h)\ddot{y} + N_h\dot{y} + \rho g A_w y &= F_h \cos(\omega t + \varepsilon_h) \\ (J_p+I_p')\ddot{\theta} + N_p\dot{\theta} + \rho g V GM_L \theta &= M_p \cos(\omega t + \varepsilon_\theta), \end{aligned}\right\} \quad (2)$$

where

M = mass of ship, M_h = heaving added mass

J_p = longitudinal mass moment of inertia

I_p' = added mass moment of inertia

N_h = damping coefficient of heave

N_p = damping coefficient of pitch

A_w = water plane area, V = displacement of volume

GM_L = lingitudinal metacentric radius

ω = circular frequency of external force

F_h, M_h = amplitude of external force and moment

In order to calculate the added mass and damping force by Strip Method we must divide the length of ship into several sections. At first two dimensional values are calculated for each sections and the total damping force and added mass of a ship are then obtained by integrating the results throughout the length of a ship.

Two dimensional damping coefficient N and added mass ΔM are expressed as follows:

$$\left.\begin{aligned} N &= \frac{\rho g^2}{\omega^3}\bar{A}^2 \\ \Delta M &= \frac{1}{2}\rho\pi\left(\frac{B}{2}\right)^2\cdot C_0\cdot K_4 \end{aligned}\right\} \qquad (3)$$

For the calculation of N and ΔM it is necessary to know the $\bar{A}$ and C_0K_4. These values are given by the calculations [11] for the Lewis-form sections. Wedge Effect correction and Bilge Keel effect are given in [12] and [13]. From Grim's diagram [9], G. Vossers [20] has given graphs in which $\bar{A}_z$, for four different value of ξ_B (0.25, 0.5, 1.0, 1.5) are expressed as functions of $s=\frac{B}{2d}$ and of the section coefficient β.

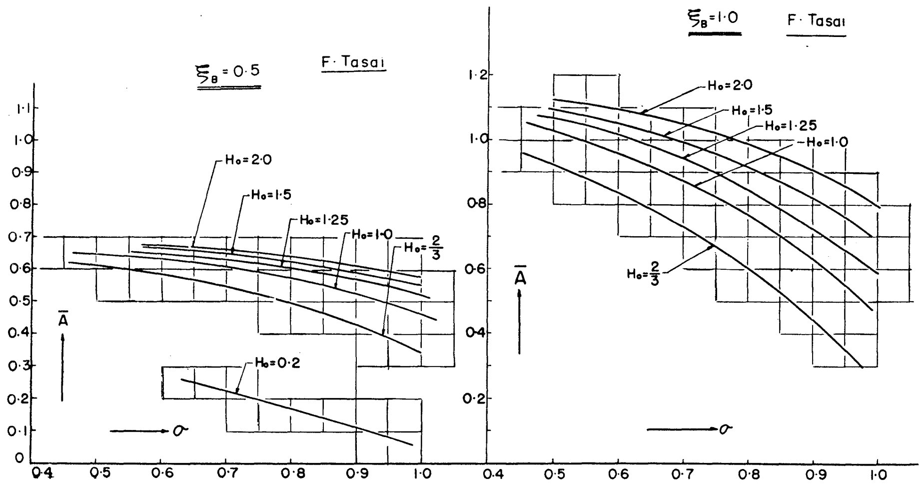

Fig. 1(b).

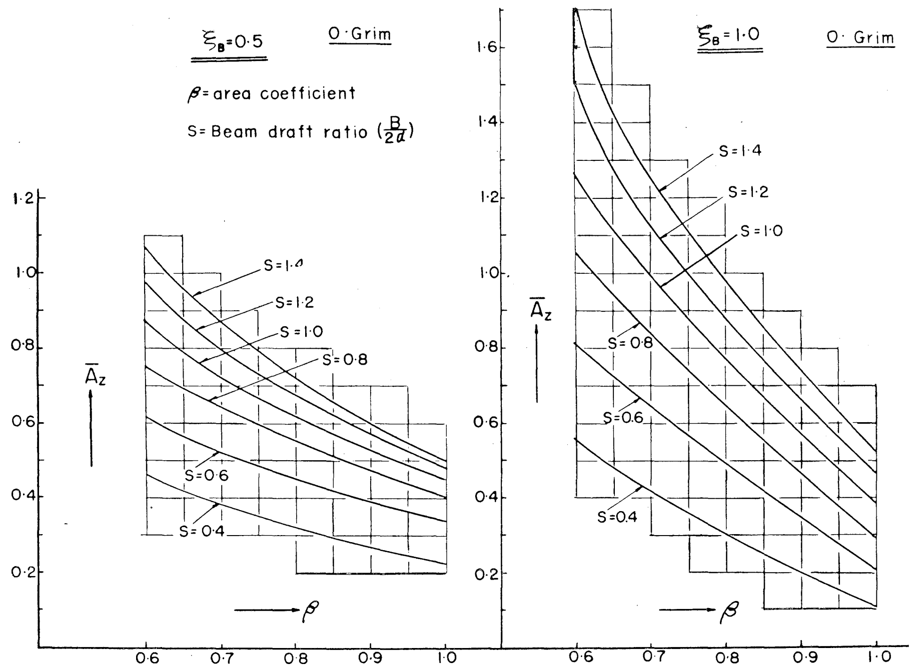

Fig. 1(a).

Graphs for $\xi_B=0.5$ and 1.0 are shown in Fig. 1(a)**. Author's calculations [11] also are shown in Fig. 1(b). These $\bar{A}$ are expressed against the frequency parameter $\xi_B=\frac{\omega^2}{g}\cdot\frac{B}{2}$ in which B is the breadth of the cylinder on the free surface. At each sections of the ship B has different value respectively. Therefore on a circular-frequency ω, ξ_B is not the same throughout the length of a ship,

On the other hand when a ship floats at a even keel, as the draught of a ship d has almost constant value over the length, it is convenient to express $\bar{A}$ against a parameter $\xi_d=\frac{\omega^2}{g}\cdot d$. Since it is sufficient to calculate for several ξ_d, actually, the values of $\bar{A}$ for five different values of ξ_d (0.25, 0.5, 1.0, 1.5 and 2.0) are calculated from [11] as functions of $H_0=\frac{B}{2d}$ and of the section coefficient σ (Figs. 2(a)–(e)). For a very high frequency ω it is possible to calculate C_0 by means of the F. M. Lewis' formula given in [21]. These are shown in Fig. 3. Moreover C_0K_4 also are expressed against the parameter ξ_d (Figs. 4(a)–(e)).

When a ship has a large trim, as the draught d and ξ_d vary at each sec-

** Very recently O. Grim published new graphs.
It is said that his new calculations are in good coincidence with the author's. As I have not received his new paper until writing this paper, the comparison of his results [9] and the author's calculations was shown.

tion calculating procedure is same whichever parameter we take.

A example of the calculation making use of the Figs. 2–4 is briefly expressed as follows:

The cargo ship "T" has displacement of 18,000t and sea speed of 18 kts. Properties of the ship are given Table 1. The beam draft ratio at midship, $H_0^*=\frac{B^*}{2d}$, equals 1.0725. Therefore in addition to the midship section we choose six sections of which H_0 are 1.0, 2/3 and 0.2 respectively. Sectional coefficient σ and the distance from the midship of these sections are obtained from the Lines. The amplitude ratio $\bar{A}$ and C_0K_4 are, for each section, read as a function of σ and H_0 for values of ξ_a=0.25, 0.5, 1.0 and 1.5 from Fig. 2 and 4.

Substituting these values into equation (3) N and ΔM for each section are obtained, and therefore M_h, N_h, I_p' and N_p may be calculated from the next equations.

$$\left.\begin{aligned} M_h &= \int_{-L/2}^{L/2} \Delta M dx, \quad I_p' = \int_{-L/2}^{L/2} \Delta M\cdot x^2 dx \\ N_h &= \int_{-L/2}^{L/2} N dx, \quad N_p' = \int_{-L/2}^{L/2} N\cdot x^2 dx. \end{aligned}\right\} \tag{4}$$

Practically the procedure is as follows:

Values of the sectional added mass ΔM for heaving, damping coefficient N, pitching added mass moment of inertia $\Delta M\cdot x^2$ and damping coefficient $N\cdot x^2$ at each section are written graphically along the length of the ship. With the aid of Simpson's numerical integration (with 10 sections or 20 sections), then, total heaving added mass M_h etc. were obtained.

Following non-dimensional symbols are used.

$$\left.\begin{aligned} N_h' &= \frac{\sqrt{N_h g L}}{\triangle}, \quad N_p' = \frac{N_p \sqrt{gL}}{\triangle\cdot L^2} \\ K_h &= \frac{M_h}{\triangle/g}, \quad K_p = \frac{I_p'}{(0.25L)^2 M} \text{ or } K_y'/L. \end{aligned}\right\} \tag{5}$$

where L=length of ship, $\triangle$=displacement of ship

$$\left.\begin{aligned} K_y' = \text{added radius of gyration for pitching} = \sqrt{\frac{I_p'\cdot g}{\triangle}} \end{aligned}\right\} \tag{6}$$

The results concerning the "T" ship are shown in Fig. 5(a) and (b). K_p value doesn't show a large variation in the range of $0.25<\xi_a<2.0$, but K_h, N_h' and N_p' varies considerably with ξ_a.

3. Values of N_h' etc. at the Natural Period

By exciting the model in stillwater, S. Motora [17] measured N_h', N_p', K_h and K_p with 10 models of which parent model is Kunikawa-Maru (145.0 m × 19.5 m × 12.2 m × 8.03 m). Then N_h' etc. at the natural period were shown as functions of C_b, L/B^* and d/B^*. He, moreover, calculated heaving and pitching motions making use of these values for natural period and comparing it with the results making use of N_h' etc. for each frequency he showed that so much error had not been found.

With the aid of this approximate method roughly estimation of heaving and pitching motions of various ships can be done. In order to know N_h' etc. at the natural period of heaving or pitching motion with the aid of the Strip Method, following method were adopted in this paper.

The natural period are obtained by solving the equation of the free oscillation.

For heaving, the well-known equation is as follows:

$$(M+M_h)\ddot{y}+N_h\dot{y}+\rho g A_w y=0. \tag{7}$$

Putting $\quad \nu^2=\dfrac{\rho g A_w}{M+M_h}\,,\quad 2h=\dfrac{N_h}{M+M_h}\,,\quad \delta=\dfrac{2\pi h}{\omega}$

natural circular frequency ω can be calculated by the next equation

$$\omega^2=\frac{1}{\left(1+\dfrac{\delta^2}{4\pi^2}\right)}\cdot\frac{\rho g A_w}{(M+M_h)}\,. \tag{8}$$

As the value of δ is about 1.0 at the neighbourgood of the natural period, so that approximately we may use the followinn formula.

$$\omega \doteqdot \nu=\sqrt{\frac{\rho g A_w}{M+M_h}}\,. \tag{8)$'$$

A_w and $M+M_h$ are expressed as follows making use of the C_b and water plane coefficient C_w. $\quad A_w=LB^*C_w$

$$M+M_h=\frac{\triangle}{g}(1+K_h)=\rho C_b LB^* d(1+K_h).$$

From (8)′ and the above equation we obtain

$$\omega^2=\frac{C_w\cdot g}{C_b\cdot d(1+K_h)}$$

and therefore

$$\frac{\omega^2}{g}\cdot d\equiv\xi_d=\frac{C_w}{C_b(1+K_h)} \tag{9}$$

(9) is the equation which is obtained from the definition of the natural period. As C_w, C_b and d are known, we can indicate K_h as a function of ξ_d. On "T" ship this is shown in dotted line (Fig. 5(a)).

From a point of intersection of the above dotted line and the K_h curve which were calculated with the aid of the Strip Method, we can obtain $\xi_{dh}=0.72$ and $K_h=0.68$. From $\xi_{dh}=0.72$ we obtain circular frequency ω and consequently natural heaving period T_h.

Therefore N_h' at the natural heaving period is determined. This is shown in Fig. 5 with double circle. For pitching also the same calculation can be carried out. Though mass moment inertia J_p varies with the distribution of weight, we assume, as a mean value, that the radius of gyration approximately equals to

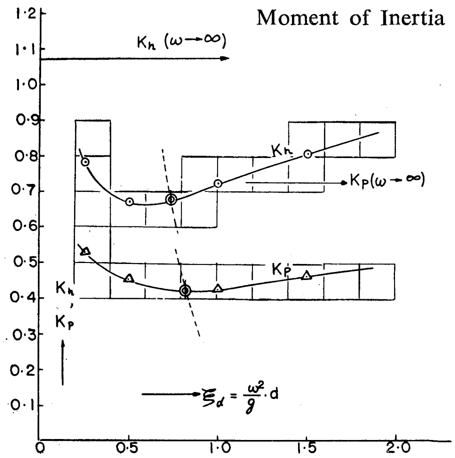

Fig. 5(a).

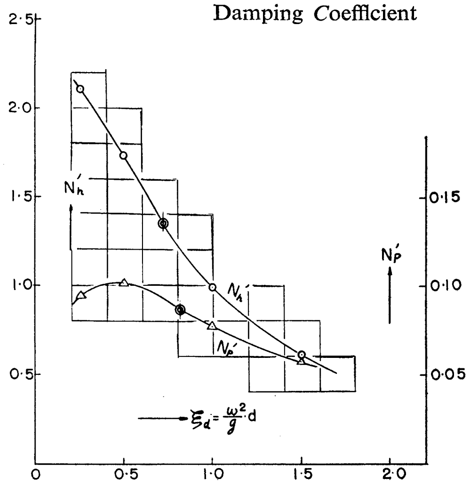

Fig. 5(b).

$L/4$, that is to say

$$J_p=(0.25L)^2M. \tag{10}$$

With the approximation $GM_L \doteqdot BM_L = \frac{I_w}{V}$ one finds the following equation for pitching,

$$\xi_{dp}=\frac{I_w}{(0.25L)^2C_b\cdot LB^*(1+K_p)}\ . \tag{11}$$

where I_w = longitudinal water plane inertia.

For "T" ship it becomes $\xi_{dp}=0.82$ and $K_p=0.42$. From these values the natural pitching period K_p and damping coefficient N_p' can be calculated. Putting now $\xi_d=\tan\theta$, K_p and K_h have been indicated as a function of θ (Fig. 5(c)). From Fig. 5(c) added mass and moment of inertia are, for a certain frequency parameter, easily obtained. By the above mentiond method we calculated N_h', N_p', K_p and K_h, as a function of ξ_d, for nine ships indicated in Table 1, and the values for the natural period were also obtained. For "C" ships, which is a oil tanker, we also calculated the added mass for the case with bilge keel making use of the experiments [12]. Owing to bilge keel, the increase of the added mass was few percent. For all nine ships ξ_{dp} was larger than ξ_{dh}, such as $\omega_p>\omega_h$, and then natural period of heaving was larger than the natural period of pitching ($T_h<T_p$). Difference between T_h and T_p was 7–10% in case of $C_b \doteqdot 0.8$ and about 2% in $C_b \doteqdot 0.60$. The fuller a ship is, the larger the difference becomes.

This was a similar tendency with the experiments by J. Gerritsma [16] (C_b =0.8, 0.7, 0.6) and also the value of difference was nearly the same.

In the next place, values of N_h' etc. for the natural period were shown in Table II and III, in which C_p and $C_{\otimes}$ are prismatic coefficient and midship sec-

Table I.

Ship	$L \times B^* \times d$	$\triangle$	H_0^*	C_b	C_p	$C_{\boxtimes}$	C_w	L/B^*	d/B^*	$H_0^* \cdot C_w$	$H_0^* \cdot C_p^2$	$\left(\frac{H_0^* \cdot C_w}{C_p}\right)$	$\frac{C_w \sqrt{H_0^*}}{C_p}$
"*T*"	m m m 145.0×19.4×9.044	K.T 17,950	1.0725	0.686	0.699	0.978	0.810	7.474	0.4662	0.869	0.523	1.239	1.190
"*C*"	m m m 211.86×31.7×11.265	K.T 61,038	1.407	0.786	0.791	0.994	0.860	6.683	0.3553	1.210	0.880	1.530	1.290
Series 60													
C_b = 0.60	400′×53.33′×21.33′	L.T 7,807	1.25	0.60	0.614	0.977	0.706	7.5	0.4	0.8825	0.471	1.437	1.280
C_b = 0.65	400′×55.17′×22.07′	L.T 9,051	1.25	0.65	0.661	0.982	0.746	7.25	0.4	0.9325	0.546	1.409	1.260
C_b = 0.70	400′×57.14′×22.86′	L.T 10,456	1.25	0.70	0.710	0.986	0.785	7.0	0.4	0.9813	0.630	1.382	1.236
C_b = 0.75	400′×59.26′×23.70′	L.T 12,048	1.25	0.75	0.758	0.990	0.827	6.75	0.4	1.034	0.718	1.365	1.221
C_b = 0.80	400′×61.54′×24.62′	L.T 13,859	1.25	0.80	0.805	0.994	0.871	6.50	0.4	1.089	0.810	1.352	1.209
P. Golovato's model	136″×16″×6.4″	lbs 317	1.25	0.64	0.665	0.9621	0.667	8.5	0.4	0.83	0.553	1.250	1.120
"*S*" Trawler	m m m 54.0×9.432 {4.083 da, 3.333 dm, 2.583 df}	K.T 1,000	1.415	0.584	0.648	0.901	0.761	5.72	0.353	1.077	0.595	1.660	1.394

Taable II

Ship		K_h				K_p	K_y'/L		
		Strip	S. Motora	Gerritsma	Golovato	Strip	Strip	S. Motora	Gerritsma
"T"		0.68	0.72			0.42	0.162	0.162	
"C"		{0.89 {0.94 (B.K)	0.94			0.73	0.214	0.213	
Todd Series 60	C_b=0.60	0.73	0.77	0.75		0.40	0.157	0.145	0.157
	C_b=0.65	0.75	0.80			0.438	0.165	0.162	
	C_b=0.70	0.775	0.83	0.84~0.75		0.505	0.178	0.179	0.18~0.17
	C_b=0.75	0.81	0.86			0.583	0.191	0.195	
	C_b=0.80	0.84	0.89	0.88~0.84		0.700	0.207	0.197	0.21~0.18
Golovato's model		0.67	0.77		0.68				
"S"		0.88	0.89			0.518	0.180	0.175	

Table III.

Sihp		N_h'				N_p'		
		Strip	S. Motora	Gerritsma	Golovato	Strip	S. Motora	Gerritsma
"T"		1.36	1.40			0.087	0.090	
"C"		1.84	1.88			0.124	0.110	
Todd Series 60	C_b=0.60	2.0	2.20	2.4		0.095	0.093	0.094
	C_b=0.65	1.75	1.90			0.098	0.098	
	C_b=0.70	1.63	1.75	2.1~2.0		0.095	0.098	0.085~0.10
	C_b=0.75	1.55	1.60			0.102	0.098	
	C_b=0 80	1.54	1.45	1.90		0.100	0.0995	0.095~0.11
Golovato's model		1.40	2.35		1.60 (mean value)			
"S"		2.30	2.20			0.120	0.105	

tion coefficient respectively. The results of experiments have been included in Tables I–III, those obtained with the three Series-60 models having been taken from a publication by Gerritsma [16], those for a mathematical model from a publication by Golovato [14] and those with ten models from a publication by S. Motora [17]. Since the results of the experiments by Gerritsma and Golovato varied as the forward speed of ships, we adopted a mean value respectively.

First for the K_h, agreement between the calculations by the Strip Method and the three experiments were good, but S. Motora's values were generally a little

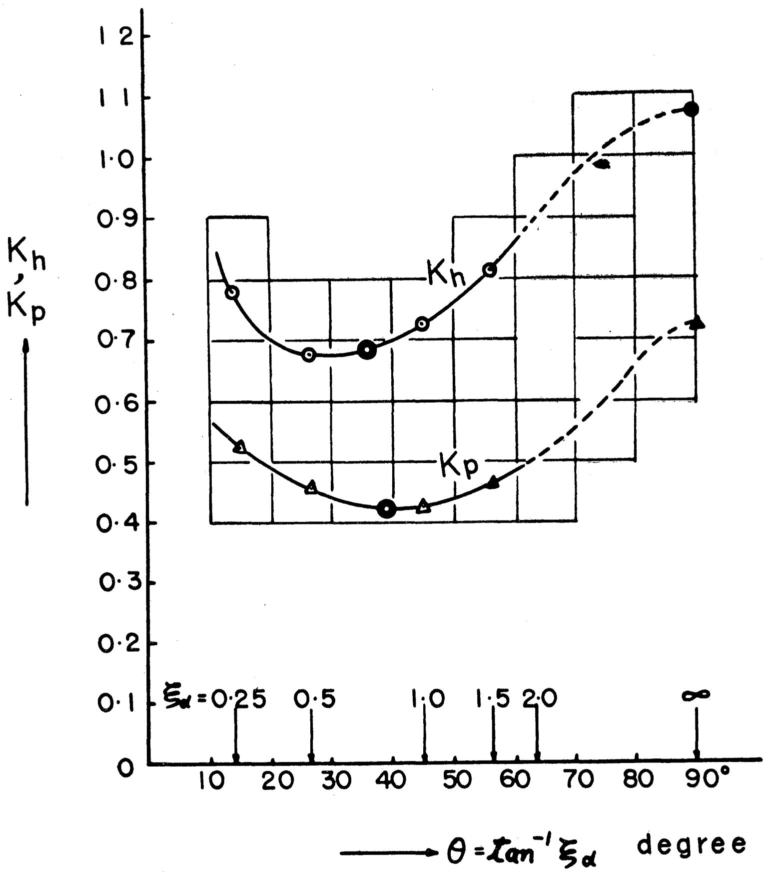

Fig. 5(c).

larger and especially its error was large for the Golovato's model. For a certain ship which has considerably different form from the model used in S. Motora's experiments, it is supposed that his chart should be used with attention. Generally speaking, it was found that K_h by the Strip Method gave a good approximate value, as can be seen in Table II. In the next place, for the pitching, S. Motora adopted the ratio K_y'/L instead of added mass moment of inetria coefficient K_p.

From equations (6) and (10) we obtain

$$K_y'/L=0.25\sqrt{K_p}\,. \qquad (12)$$

Therefore, calculating K_p by the Strip Method we can obtain K_y'/L by means of the equation (12). The values by the Strip Method, by the S. Motora's chart and results of experiments by J. Gerritsma were all in good coincidence. For N_h', the values by the Strip Method was smaller than the results of experiments. The difference between the values by the Strip Method and by the Motora's chart was few percent, but Gerritsma's values were about 20 % larger than the calculated values. Motora's chart has given a very large value for the Golovato's model.

Finally for N_p', the values by the Strip Method, by Motora's chart and by Gerritsma's experiments were in good coincidence, as was seen in the case of K_p.

4. Three Dimensional Effect

In the neighbourhood of the natural period of heave, effect of forward speed of ship has not been so large, as was seen from the Golovato's and Gerritsma's experiments. Then it is considered to be due to the three dimensional wave pattern around the ship that N_h' obtained by the experiments were 15–20 % larger than the values by the Strip Method.

As for the three dimensional effect, calculations by T. H. Havelock [22] and G. Vossers [23] had been carried out. The former calculation was made for a spheroid with $L/B=8$, the latter was for a thin Ship with $L/B=6$, 7 and 8. These are shown in Fig. 6. In this figure E_H/E_{HS} and N_H/N_{HS} are the ratio of the damping for heaving according to a three dimensional theory to the damping according to a two dimensional Strip Theory. E_P/E_{PS} and N_P/N_{PS} are also for pitching.

Both ships have $H_0^*=1.0$, but the tendency of the ratio, E_H/E_{HS} etc., are considerably different. Namely, on the ξ_L which Vossers' curve gives maximum values ($N_H/N_{HS}\doteqdot 1.2$ and $N_P/N_{PS}\doteqdot 1.4$), Havelock's curve has $E_H/E_{HS}\doteqdot E_P/E_{PS}\doteqdot$ 1.0. The ξ_L at which Havelock's result gives a maximum value is larger than the Vossers', and also maximum value of N_H/N_{HS} by Vossers is about 20 % larger than the one by Havelock.

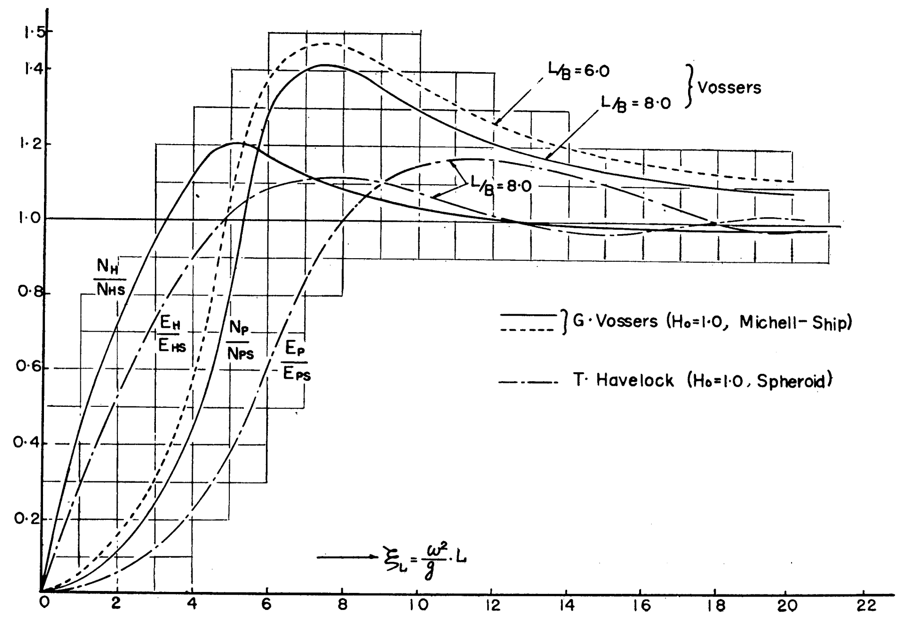

Fig. 6.

It will be found that with the same H_0^*, owing to the difference of the form, three dimensional effect differs considerably. In table III Motora's N_h' are not the results for the model of the ship for which calculations by the Strip Method has been done. Gerritsma's and Golovato's N_h' are the results of experiments for the same model with the calculation. From N_{HS}, N_{PS} obtained by the Strip Method and N_H, N_P by Golovato's and Gerritsma's experiments, we calculated the three dimensional effect. A mean value was adopted for the value of experiment. In Fig. 7 N_P/N_{PS} and N_H/N_{HS} are given as a function of the dimensionless frequency parameter $\xi_L = \frac{\omega^2}{g} \cdot L$. The black points in Fig. 7 corresponds to the one for natural period. It is clear that these curves are much different from the Havelock's and Vossers' curves.

Three Dimensional Effect

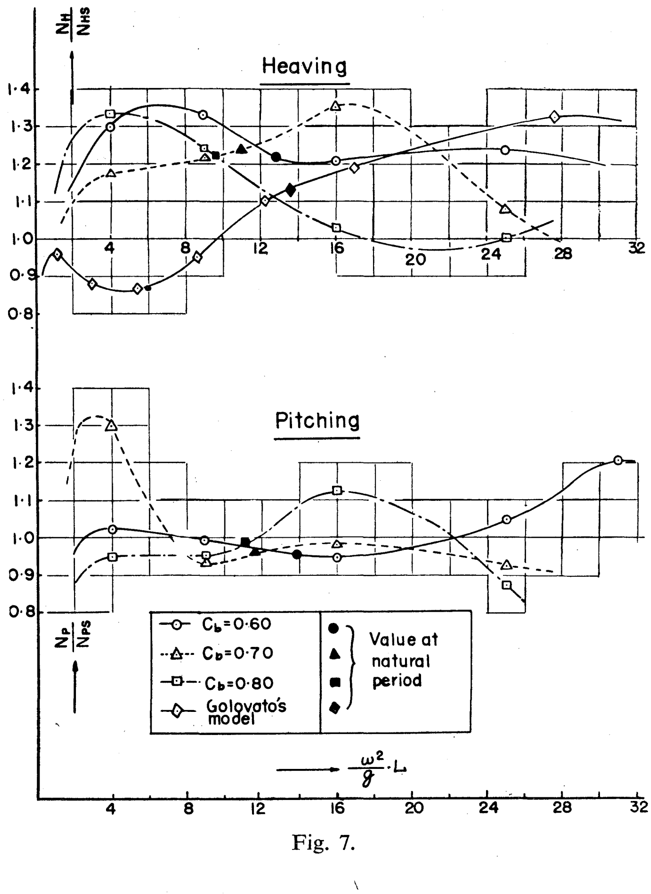

Fig. 7.

Results which are found in Fig. 7 are as follows:

(1) In the case of $C_b=0.60$ and $C_b=0.80$, ξ_L for maximum N_H/N_{HS} is smaller than ξ_L for maximum N_P/N_{PS}. This tendency is the same with Havelock's and Vossers' results. But for the case of $C_b=0.70$ it is adverse.

(2) ξ_L for the natural period of eight ships were shown in Table IV. At these ξ_L, 8–15, N_H/N_{HS} was larger than N_p/N_{PS}.

(3) N_P/N_{PS} generally showed a smaller variation than N_H/N_{HS}. This shows an adverse tendency with Havelock's and Vossers' results.

(4) In the neighbourhood of the natural period N_P/N_{SP} is nearly 1.0 and N_H/N_{HS} is 1.15–1.25.

Table IV.

Ship	Heaving		Pitching		L/d
	ξ_{dh}	ξ_{Lh}	ξ_{dp}	ξ_{Lp}	
"T"	0.720	11.37	0.820	13.14	16.03
"C"	0.540	10.16	0.583	10.96	18.806
C_b=0.60	0.680	12.75	0.740	13.88	18.75
C_b=0.65	0.660	11.96	0.690	12.50	18.12
C_b=0.70	0.630	11.03	0.680	11.90	17.50
C_b=0.75	0.615	10.38	0.660	11.14	16.88
C_b=0.80	0.590	9.59	0.683	11.11	16.26
Golovato's model	0.640	13.60			21.25

On the other hand, for the added mass of heaving and added mass moment of inertia of pitching, except for small ω, three-dimensional effect is small.

5. Practical Formula

In order to estimate the increase of resistance, bending moment and sea-keeping qualities among waves, we must know the ship motion of pitch and heave. N_h', N_p', K_h and K_p used in the calculation of the heaving and pitching motion of ships, were approximately obtained by the Strip Method. When we estimate the ship motion in the first approximation making use of the N_h' etc. for the natural period, though these coefficients depend on the frequency of external force, it is convenient to express these in brief formula.

In the first place we take up the K_h.

$\varDelta M$ for each section is given by the equation (3) and is modified into the following form,

$$\varDelta M=\frac{1}{4}\rho\pi H_0^*\left(\frac{C_0K_4}{\sigma}\right)\left(\frac{B}{B^*}\right)B\cdot d\cdot\sigma\,.$$

Therefore K_h is expressed as follows:

$$K_h=\frac{M_h}{M}=\frac{\frac{1}{4}\rho\pi H_0^*\int^L\left(\frac{C_0K_4}{\sigma}\right)\left(\frac{B}{B^*}\right)B\cdot d\cdot\sigma}{\rho\int^L B\cdot d\cdot\sigma dx\,.}$$

As can be seen from Fig. 4, $\frac{C_0 K_4}{\sigma}$ does not vary, for $H_0=1.0\sim1.5$, in the range of $\xi_a=0.5\sim1.0$ but it has almost constant value.

Therefore it will be found that

$$K_h \propto H_0{}^* \frac{\int^L \left(\frac{B}{B^*}\right) B\ d\cdot\sigma dx}{\int^L B\cdot d\ \sigma dx}$$

Then it is considered that the K_h is roughly proportional to $H_0{}^*C_w$. The results of calculation in Table III was plotted as a function of $H_0{}^*C_w$ and shown in Fig. 8. Taking a mean line so as to $K_h\to0$ with $H_0{}^*\to0$, K_h is approximately expressed by following formula,

$$K_h=0.8H_0{}^*C_w. \tag{13}$$

In the next place, with the aid of the Strip Method we can calculate K_p by the following equation

$$K_p=\frac{\int^L \frac{1}{2}\rho\pi\left(\frac{B}{2}\right)^2\cdot C_0\cdot K_4\cdot x^2\cdot dx}{(0.25L)^2M}.$$

Considering similary as the case of K_h, it will be assumed that K_p depends

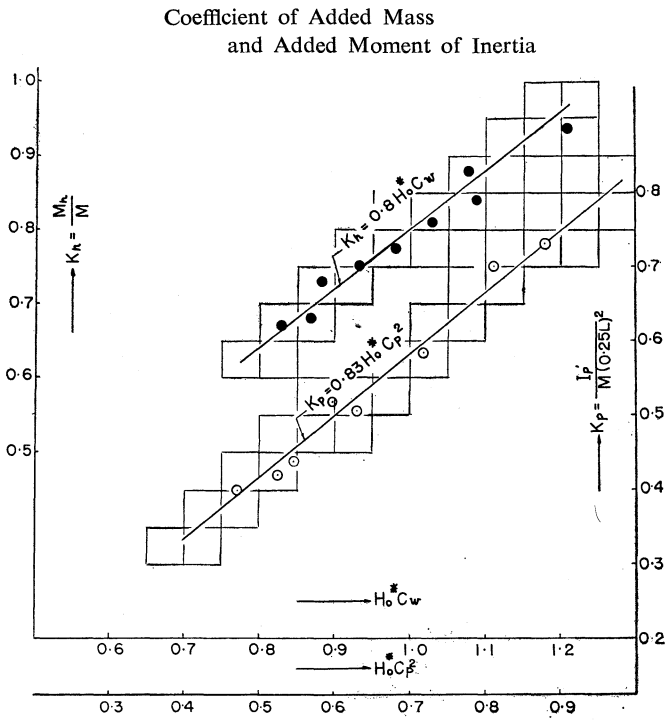

Fig. 8.

on C_p. Then making use of a parameter, $H_0{}^*C_p{}^2$ we obtain

$$K_p = 0.83 H_0{}^* C_p{}^2 \,. \tag{14}$$

Added moment of inertia at the natural period for pitching may be approximately estimated by the above formula, as is shown in Fig. 8.

The equation in calculating N_p' is

$$N_p' = \frac{\sqrt{gL}}{\triangle L^2} \int^L \frac{\rho g^2}{\omega^3} \cdot \overline{A}^2 \cdot x^2 dx \,.$$

From the Motora's chart [17] it is seen that N_p' does not so much depend on C_b and the effect of B^*/L is small except for small B^*/L. As a first approximation, drawing a mean line for the calculations and results of experiments in Table III we have the following formula

$$N_p' = 0.08 H_0{}^* \,. \tag{15}$$

This is shown in Fig. 9.

Damping Coefficient of Pitching

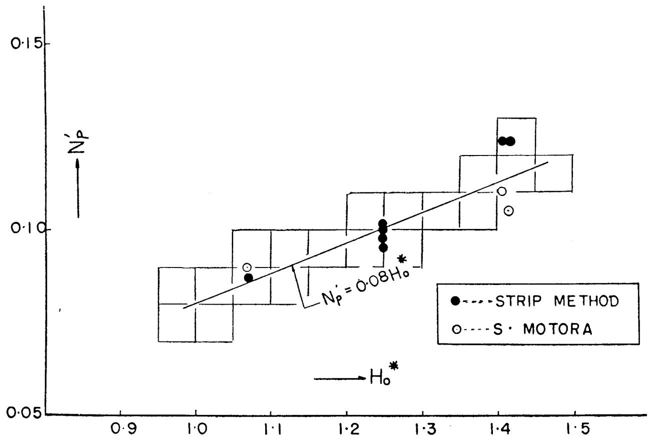

Fig. 9.

Finally N_h' can be calculated, with the aid of the Strip Method, by the following equation

$$N_h' = \frac{\sqrt{gL}}{\triangle} \int^L \frac{\rho g^2}{\omega^3} \cdot \overline{A}^2 \cdot dx \,.$$

It seems that N_h' depends on $H_0{}^*$, C_w, C_b and C_p.

Now making use of a parameter $\left(\frac{C_w\sqrt{H_0{}^*}}{C_p}\right)$ the results of calculations in Table III were shown in Fig. 10 with the black points.

Golovato's model is a mathematical ship-form and is slightly different from the practical ship-form. From the above point of view, except for the Golovato's result we take a mean of the calculations, and then it is expressed by the following equation,

$$N_{hs}'=4.5\left(\frac{C_w\sqrt{H_0^*}}{C_p}\right)-3.9\,.$$

As was found in the section 5, the value of N_h' is larger than N_{hs}' owing to the three dimensional effect. From the comparison between Golovato's, Gerritsma's N_h' and N_{hs}' by the Strip Method, assuming that N_h' is about 20 % larger than N_{hs}' we do not make a large error. Consequently taking into consideration of 20% correction for the three dimensional effect we had, for N_h' at the natural period, the following formula

$$N_h' \doteqdot 1.20N_{hs}'=5.4\left(\frac{C_w\sqrt{H_0^*}}{C_p}\right)-4.7 \qquad (16)$$

This is shown with a full line in Fig. 10.

Damping Coefficient of Heaving

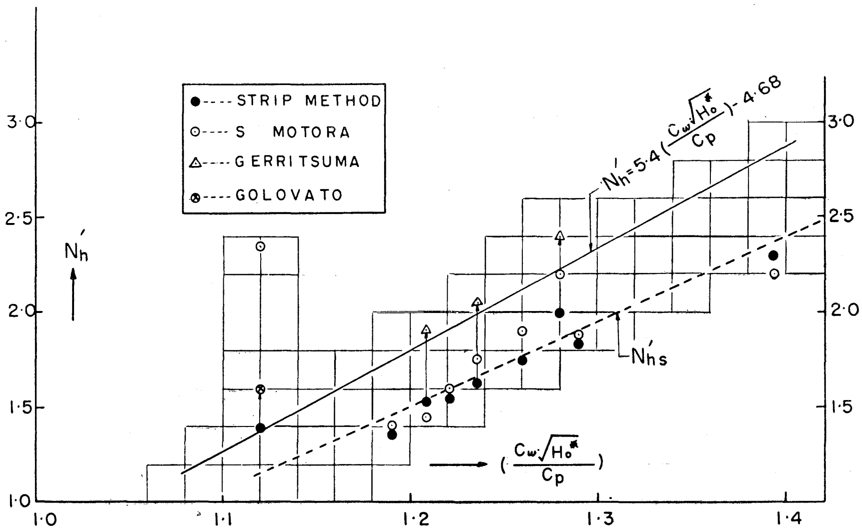

Fig. 10.

For the Golovato's model, the value by the formula (16) is coincidence with the value by the Strip Method. This value corresponds to the experiment of which Froude number is about 0.27, and therefore we take about 15% under value in low-speed of this model.

These practical formula, (13), (14), (15) and (16), give the value for the natural period. However, it may not be suitable that these formula were applied to a full ship with a large L/B^* and a ship of particular form and draft-condition.

6. $æ_h$ and $æ_p$

The heaving equation (2) has the solution:

$$y=y_0\cos(\omega t+\varepsilon)\ ,$$

where

$$y_0=\frac{1}{\sqrt{(1-\Lambda^2)^2+æ_h^2\Lambda^2}}\cdot\frac{F_h}{\rho g A_w}$$

$$\Lambda=\frac{\omega}{\nu_h}\ ,\quad \nu_h^2=\frac{\rho g A_w}{M+M_h}\ ,\quad 2h=\frac{N_h}{M+M_h}\ . \tag{17}$$

Therefore we obtain $æ_h=\dfrac{2h}{\nu_h}=\sqrt{\dfrac{N_h^2}{(M+M_h)\rho g A_w}}$. (18)

With the aid of N_h', $æ_h$ can be written as follows:

$$æ_h=N_h'\times\gamma_h\ , \tag{19}$$

where

$$\gamma_h=\sqrt{\left(\frac{d}{L}\right)\left(\frac{C_b}{C_w}\right)\frac{1}{1+K_h}}\ . \tag{20}$$

For the pitching, with the approximation $GM_L \doteqdot BM_L=\dfrac{I_w}{V}$ in the equation (2), we obtain, for $æ_p$, the following formula

$$æ_p=\sqrt{\frac{N_p^2}{(J_p+I_p')\rho g I_w}}\ . \tag{21}$$

Put

$$\left.\begin{aligned}J_p&=C(0.25L)^2M\\ I_w&=C_L B^* L^3\ .\end{aligned}\right\} \tag{22}$$

With the aid of $I_p'=(0.25L)^2MK_p$ making use of N_p', $æ_p$ will then become

$$æ_p=N_p'\times\gamma_p\ , \tag{23}$$

where

$$\gamma_p=4\sqrt{\left(\frac{d}{L}\right)\left(\frac{C_b}{C_L}\right)\frac{1}{(C+K_p)}}\ . \tag{24}$$

Putting $C=1.0$, for the nine ships, we calculated the $æ_h$ and $æ_p$ at natural period.

In the above calculation we used N_h' by the formula (16) and N_p' by the Strip Method. These results are shown in Table V and Fig. 11.

It was plotted against the $\dfrac{C_w\sqrt{H_0^*}}{C_p}$ for $æ_h$ and $\left(\dfrac{d}{L}\right)^2\cdot\left(\dfrac{C_b}{C_L}\right)^2 H_0^{*4}$ for $æ_p$. As is shown in the Table V, $æ_h > æ_p$ for every ship.

K. Kroukovsky has given the damping coefficient κ of Rolling for several ships in Table II–1, Chapter II of his MONOGRAPH. The mean value of κ for ships with bilge keel was 0.082.

Making use of the symbol $æ_r$ for the non-dimensional damping coefficient of rolling like $æ_h$ and $æ_p$, it becomes $æ_r=2\kappa$.

Accordingly the mean value of $æ_r$ given by K. Kroukovsky is resulted 0.164. It will be then found that the non-dimensional damping coefficient for heaving and pitching is considerably larger than the one for rolling.

Table V.

	$\frac{C_w\sqrt{H_0^*}}{C_p}$	$æ_h$	$\left(\frac{d}{L}\right)^2\left(\frac{C_b}{C_L}\right)^2 H_0^{*4}$	$æ_p$
"*T*"	1.190	0.315	0.970	0.270
"*C*"	1.290	0.376	2.24	0.328
C_b=0.60	1.280	0.361	1.65	0.291
C_b=0.65	1.260	0.352	1.92	0.308
C_b=0.70	1.236	0.338	1.92	0.292
C_b=0.75	1.221	0.329	2.0	0.310
C_b=0.80	1.209	0.326	1.74	0.282
Golovato's	1.120	0.235		
"*S*"	1.394	0.478	3.19	0.368

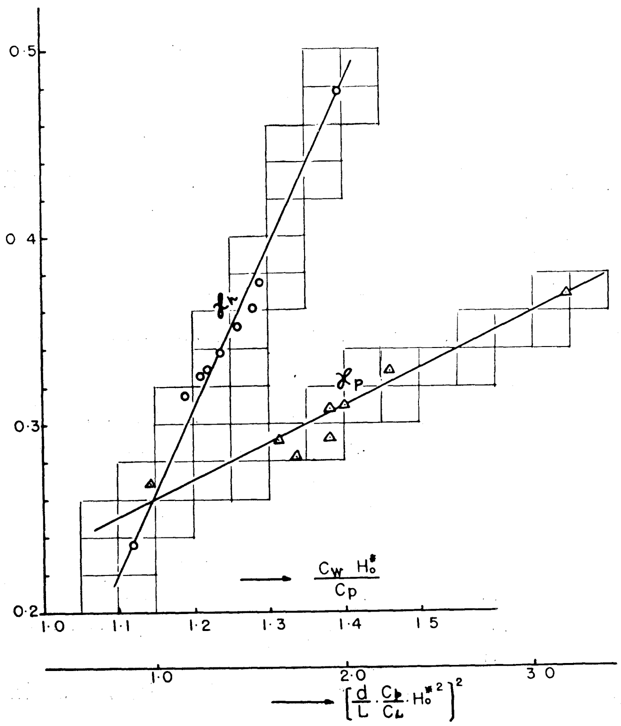

Fig. 11.

7. Conclusions

As mentioned above, we showed conventional figures for $\breve{A}$ and C_0K_4 and calculated K_h, K_p, N_h' and N_p' by the Strip Method for nine ships, which have different form. Then comparison between these calculation and experiments was done. As a practical method taking into consideration the three dimensional effect we obtained a practical formula for the natural period. From the present work, general appearances of the natural period of heave and pitch, damping force, added mass for heaving and added mass moment of inertia for pitching were roughly found.

That is summarized as:

(1) K_h, K_p, N_h' and N_p' calculated by the Strip Method give reasonable and good approximate values.

(2) Three-dimensional effect for the damping force is extremely different from the Havelock's and Vossers' results. In the neighbourhood of the natural period, this effect for N_p' is generally small and N_h' is about 20 % larger than N_{hs}' by the Strip Method.

(3) $$K_h=0.8H_0{}^*C_w\,,\qquad K_p=0.83H_0{}^*C_p{}^2$$

$$N_p'=0.08H_0{}^* \quad \text{and} \quad N_h'=5.4\left(\frac{C_w\sqrt{H_0{}^*}}{C_p}\right)-4.7$$

These practical formulæ may be used for usual merchant ships.

Of course these give the values for the natural period of heaving and pitching.

(4) Another nondimensional damping coefficient $æ_h$ and $æ_p$ is considerably larger than $æ_r$ for rolling.

* * * * *

Acknowledgment

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APPENDIX—Natural period, T_h and T_p

For heaving, from the equation (9) putting $\tau_h = T_h\sqrt{\frac{g}{d}}$ (i)

we obtain the following formula

$$\tau_h = 2\pi\sqrt{\frac{C_b(1+K_h)}{C_w}}\ . \qquad \text{(ii)}$$

Making use of the equation (13) for K_h, τ_h will become

$$\tau_h = 2\pi\sqrt{\frac{C_b}{C_w} + 0.4\frac{B^*}{d}C_b}\ . \qquad \text{(iii)}$$

For pitching with the aid of the Bauer's formula which is

$$C_L = \frac{(5.55C_w+1)^3}{3450}$$

$\tau_p = T_p\sqrt{\frac{g}{d}}$ can be expressed as follows:

$$\tau_p = \frac{\pi}{2}\sqrt{\frac{(C+K_p)C_L}{C_b}} \quad . \tag{iv}$$

In the case of $C=1.0$, the radius of gyration of the mass moment of inertia for pitching is 0.25L.

Making use of the approximate formula (14) τ_p will become

$$\tau_p = \frac{\pi}{2}\sqrt{\frac{\left(C+0.83\frac{B^*}{2d}\right)C_b\times 3450}{(5.55C_w+1)^3}} \quad . \tag{v}$$

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$\xi_d=0.25$

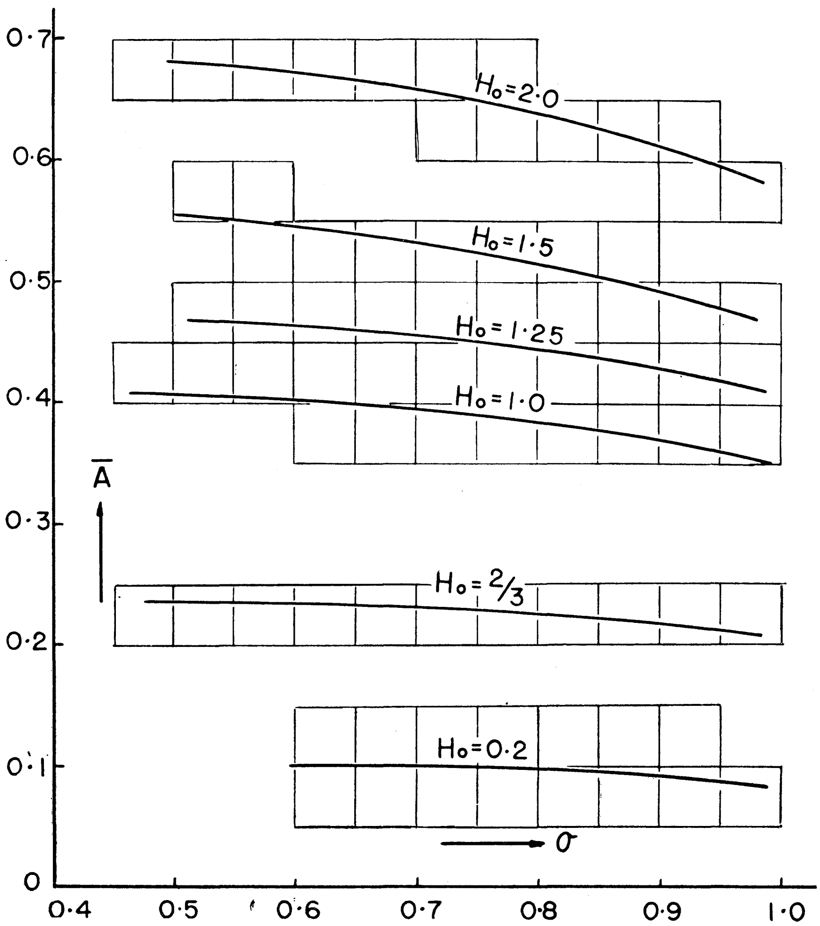

Fig. 2(a).

$\xi_a = 0.5$

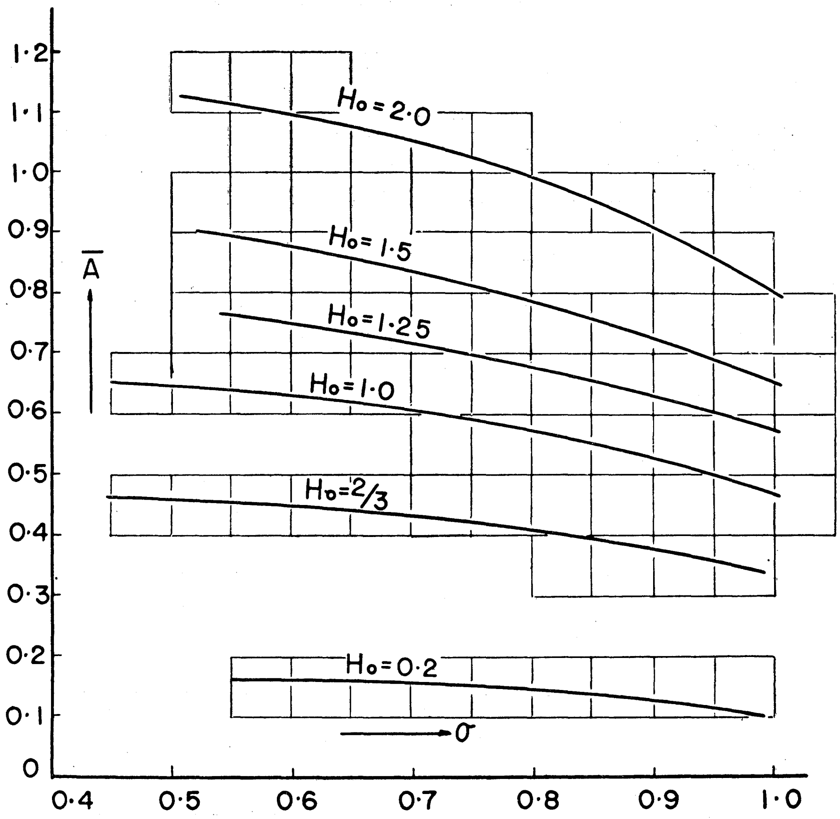
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Fig. 2(b).

ξ_d=1.0

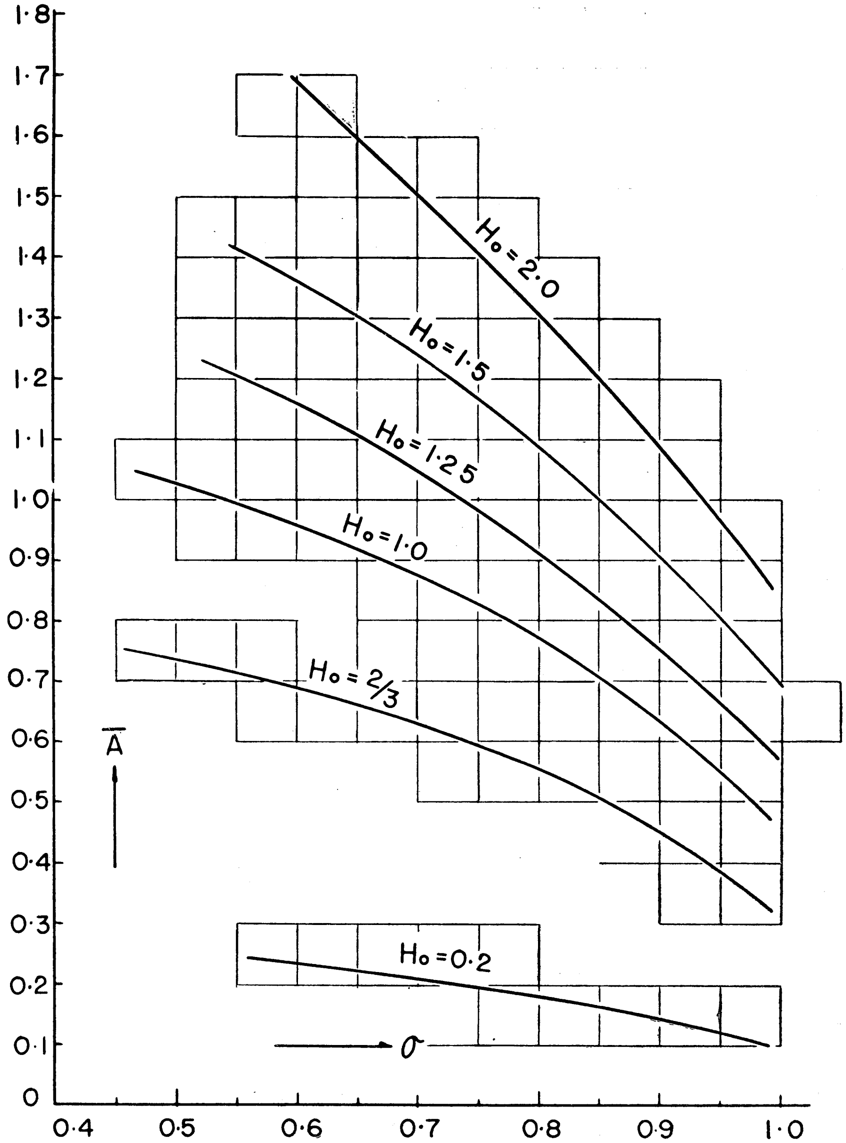
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Fig. 2(c).

$\xi_d = 1.5$

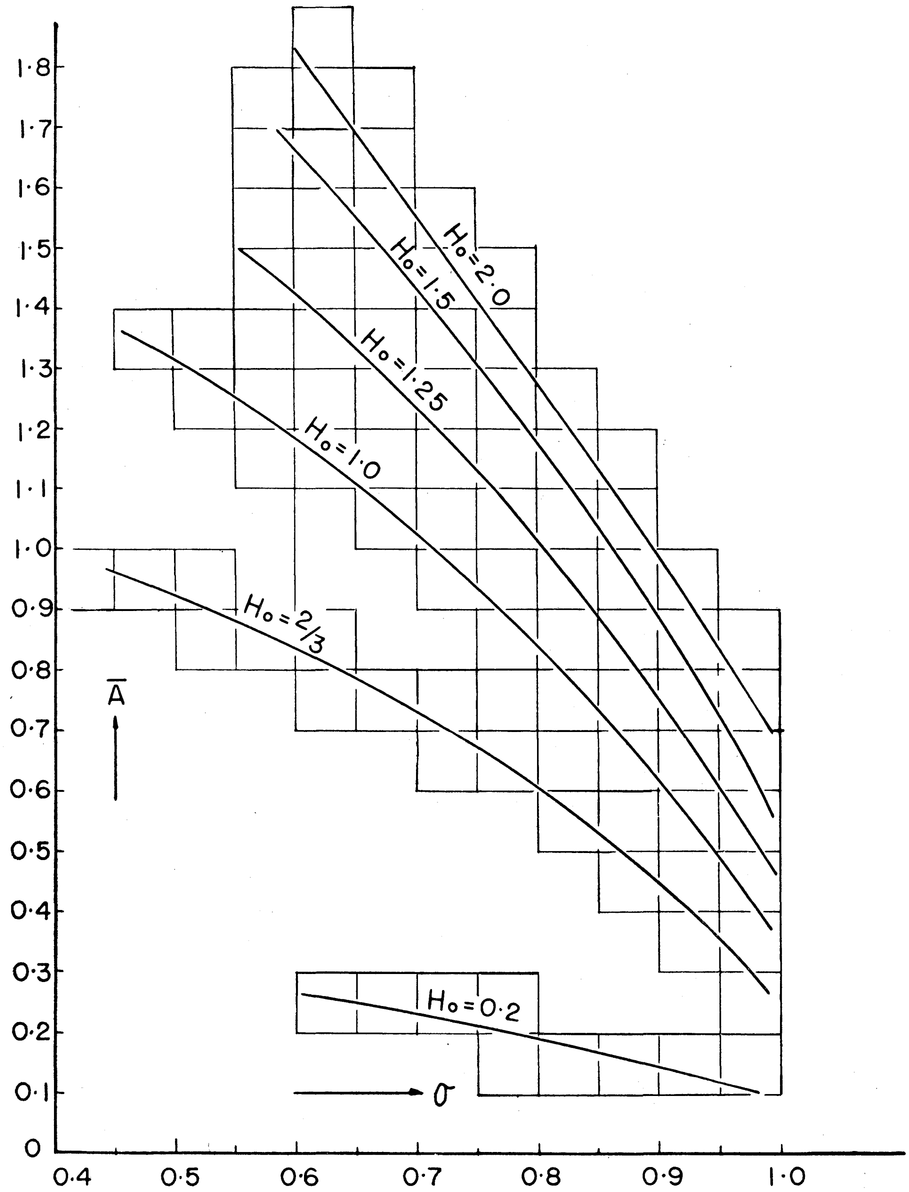
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Fig. 2(d).

$\xi a = 2.0$

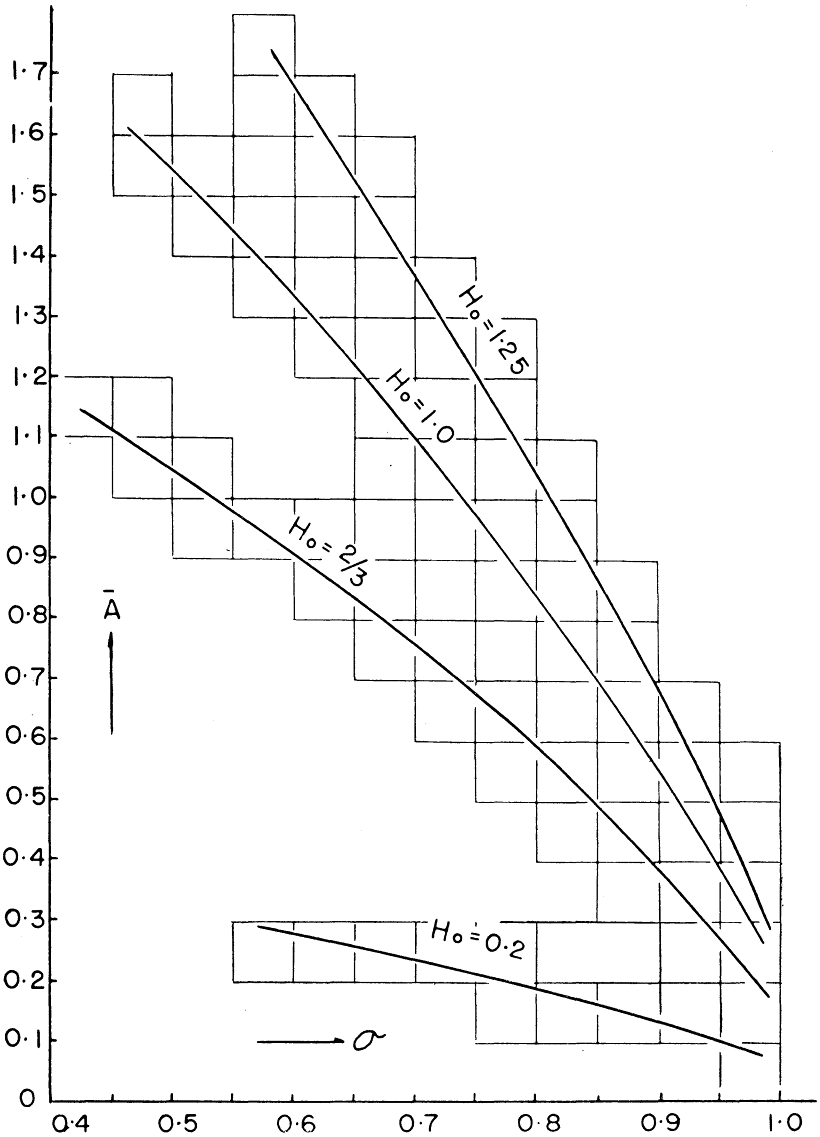
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Fig. 2(e).

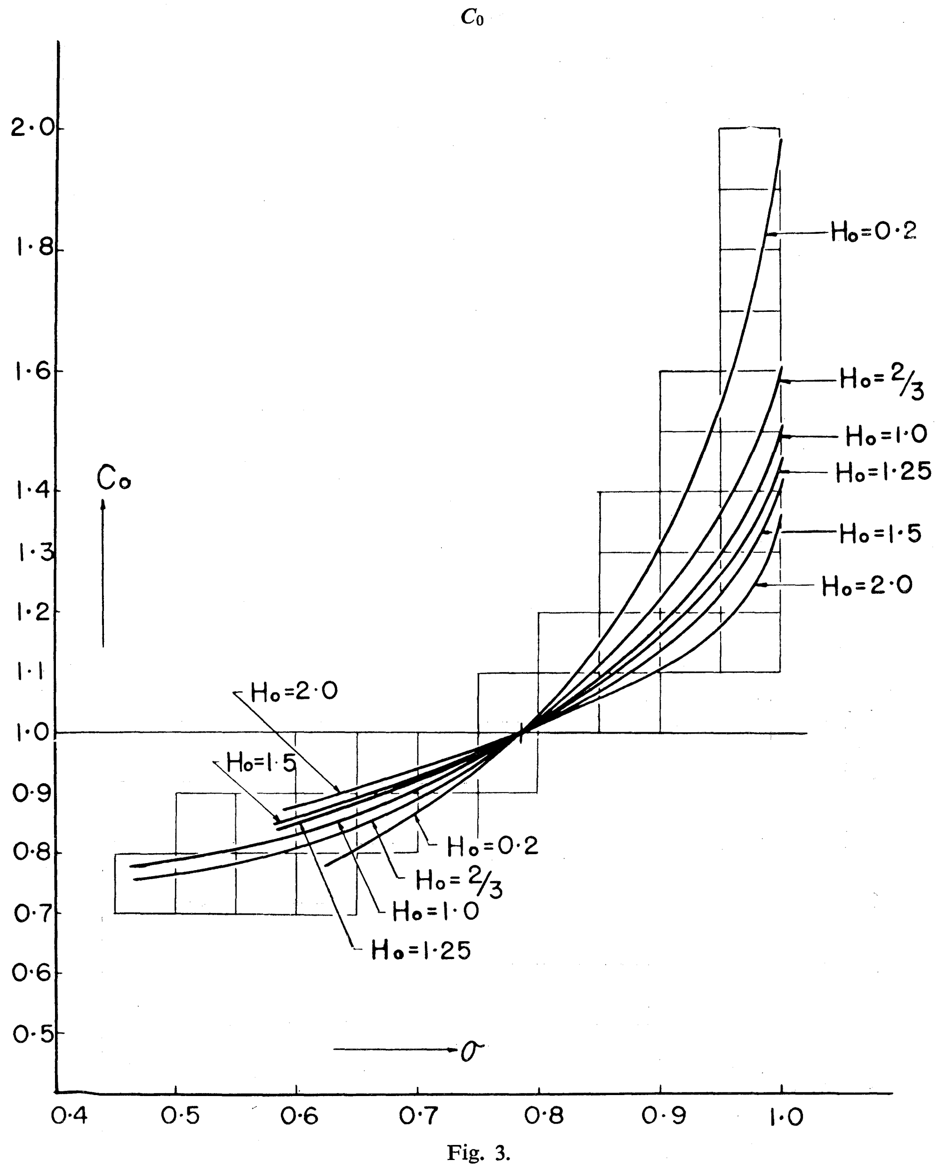
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Fig. 3.

$\xi_a=0.25$

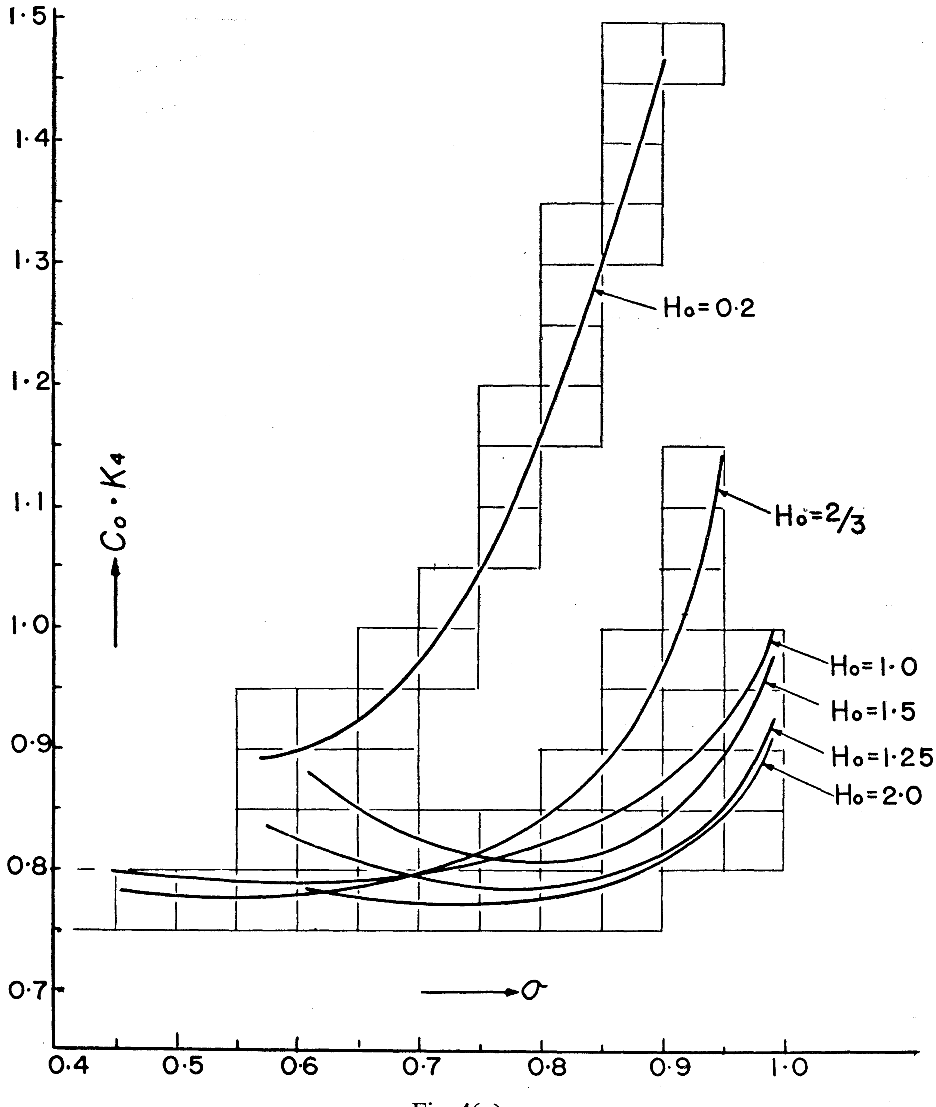

Fig. 4(a).

$\xi_d = 0.5$

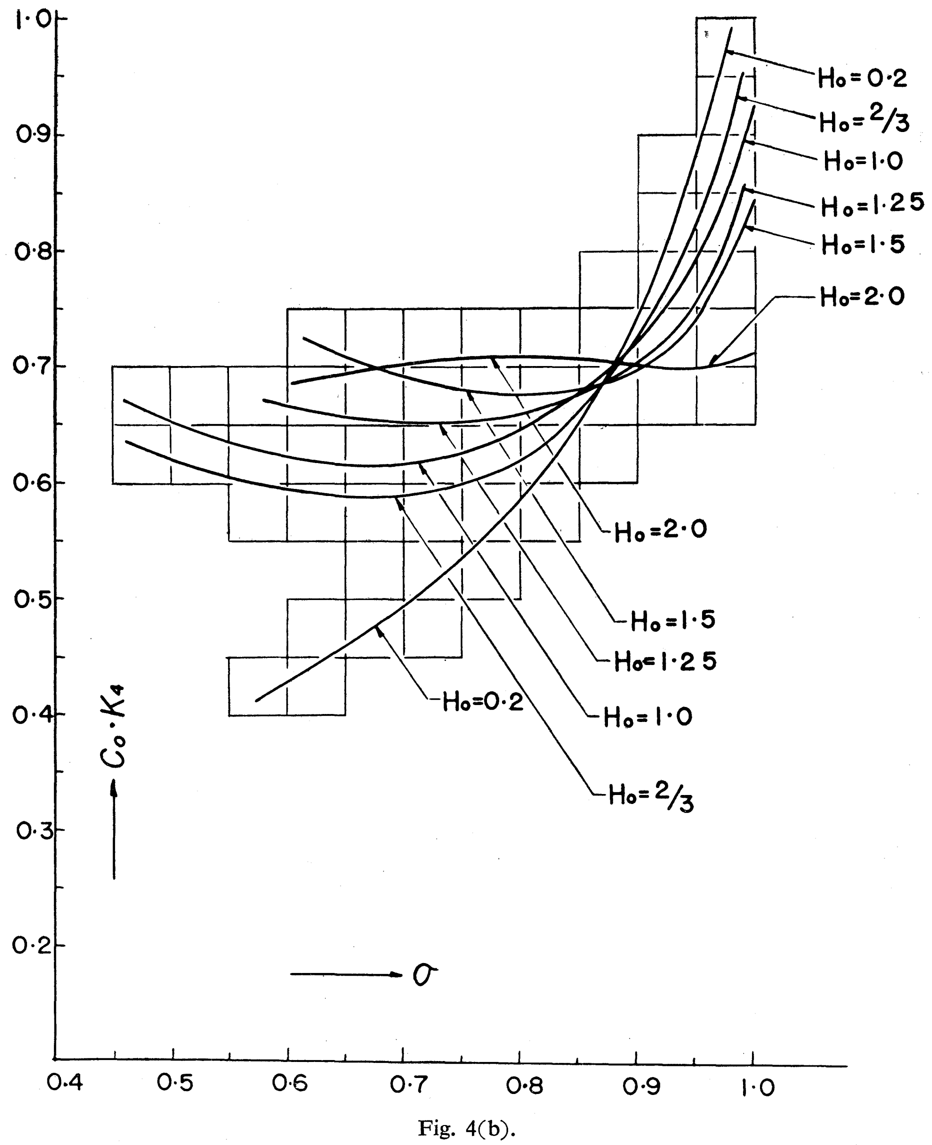
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Fig. 4(b).

$\xi_d = 1.0$

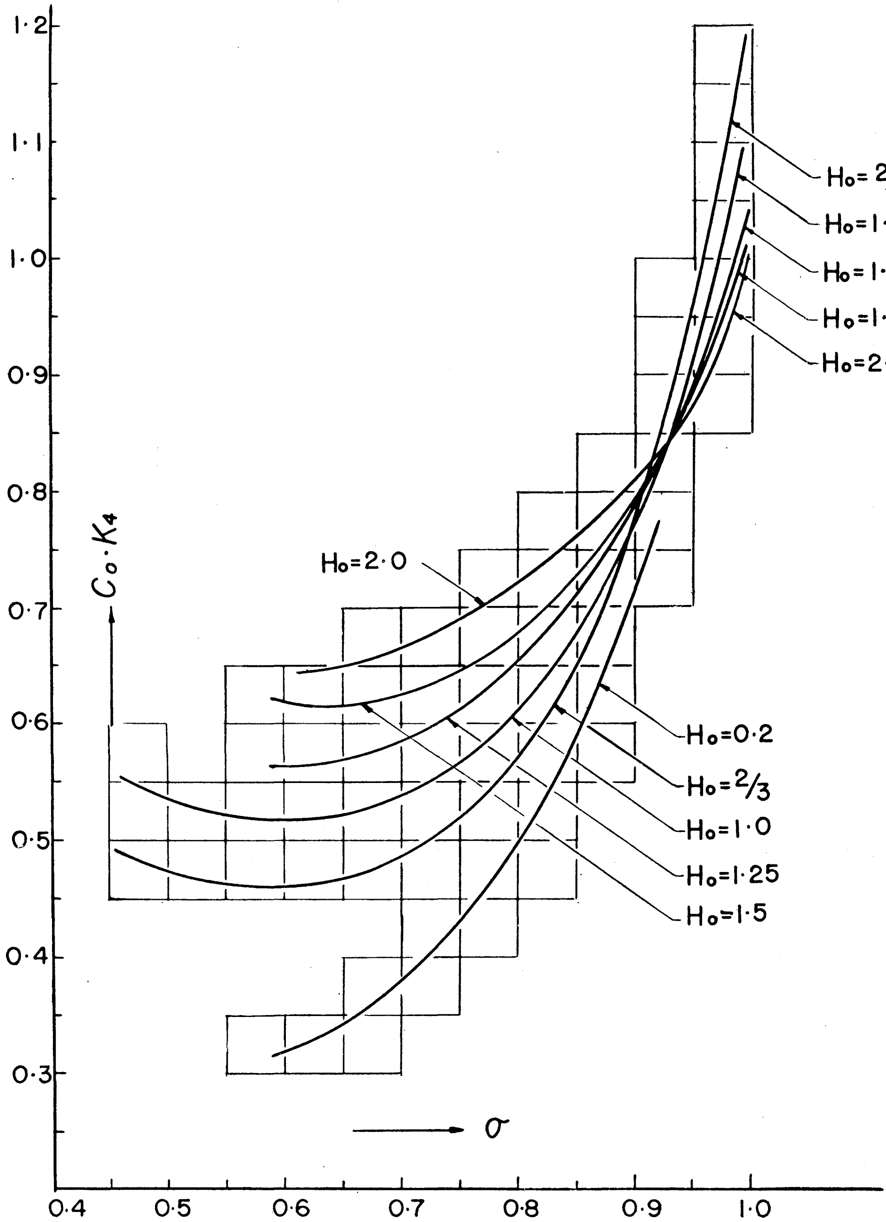
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Fig. 4(c).

$\xi_d = 1.5$

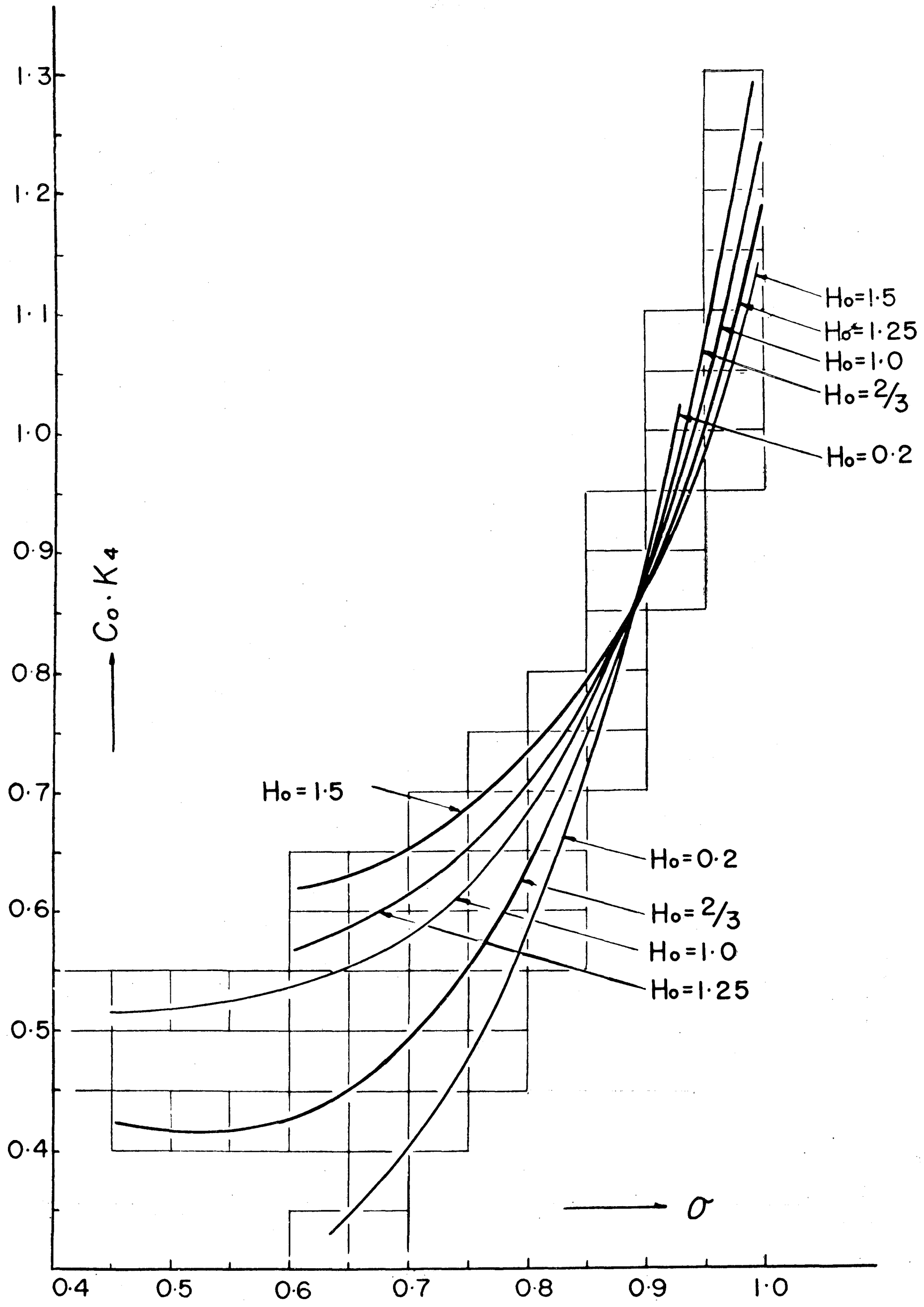
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Fig. 4(d).

$\xi_d = 2.0$

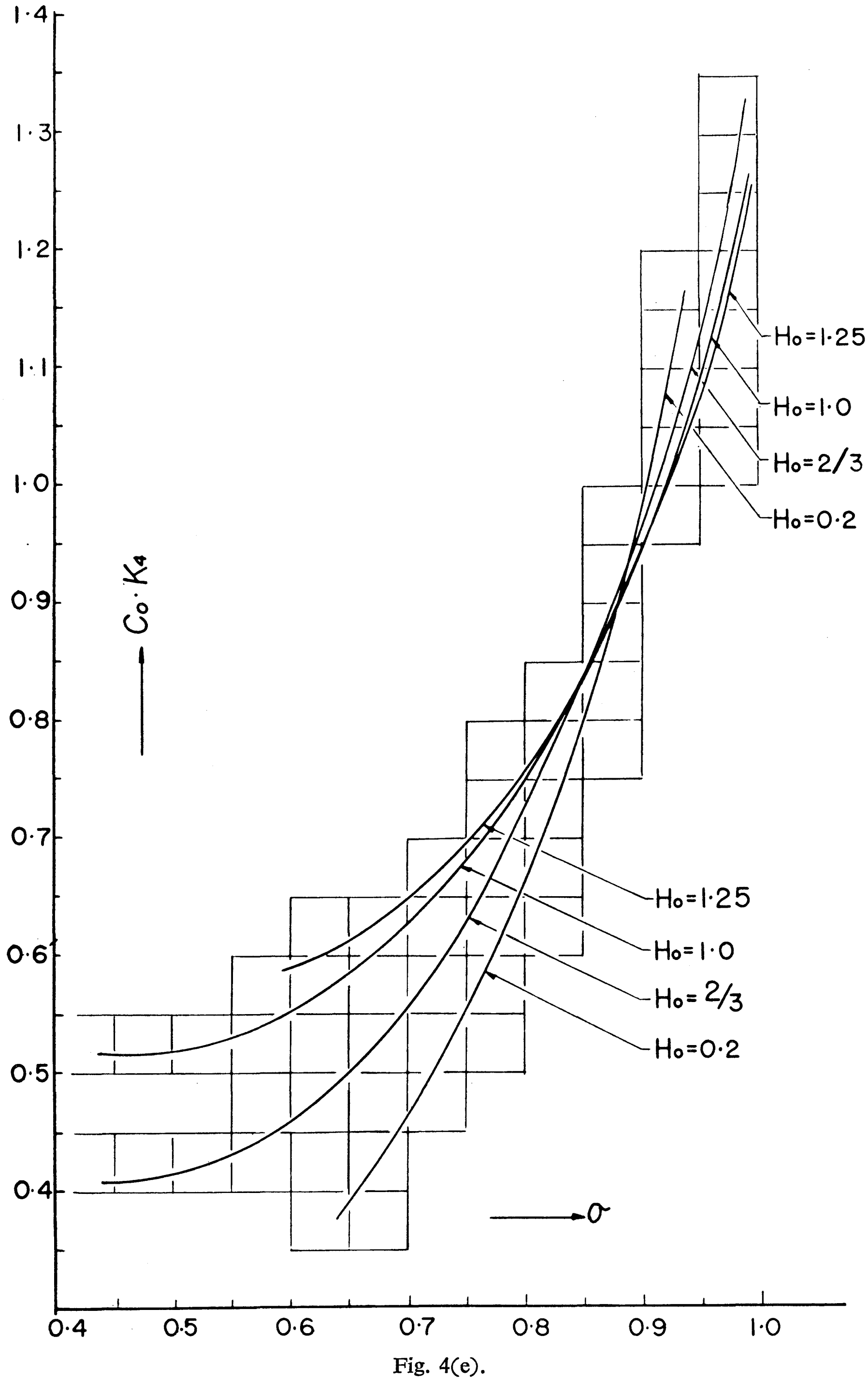

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Fig. 4(e).