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ON THE VIRTUAL INERTIA COEFFICIENTS FOR THE VERTICAL VIBRATION OF SHIPS*

By Toyoji KUMAI**

Abstract. An investigation is made of the virtual inertia coefficients for the vertical vibration of ships by means of the strip method, taking the effects of the virtual rotatory inertia of the water and the three-dimensional motion into account. For estimating the three-dimensional correction factors of the virtual inertia coefficients for the vertical vibration of ships, experimental studies on the vibration of cylindrical solid models with several kinds of sectional forms are carried out and their results are taken into account. The analyses of the three-dimensional motion of the circular and elliptic cylinders on the water, and the experimental studies of the measurements of the virtual inertia coefficients for the vibrations of the solid beams and the tanker model are presented in Appendices I and II respectively in the present paper.

Introduction

The virtual inertia coefficient for the vertical vibration of ships, under existing circumstances, is estimated by means of the strip method for the case of translational oscillation, by employing the correction factor obtained from the theoretical results of the inertia coefficient for the flexural vibration of an ellipsoid of revolution [1], [2], [3] and [4].

On the other hand, the virtual inertias of the water due to the motions of ships, including flexural vibration, can be measured by model experiments [5].

In the test results of the virtual inertia coefficient measured on a model of a tanker recently studied by the author and the results calculated by the strip method with Lamb's correction [7] in the heaving motion agreed with the experimental one, but the inertia coefficient for flexural vibration of ships calculated by means of the existing method differed from that measured by the present experiment.

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* Read in Fukuoka at the Spring Meeting of the Japan Society of Naval Architects on May 1959 (in Japanese). Published on "European Shipbuilding", Oslo. No. 2, Vol. VIII, 1959.

** Member of the institute.

[1] F.M. Lewis: "The Inertia of the Water Surrounding Ships." Trans. S.N.A.M.E., 1929.
[2] J. Lockwood Taylor: "Hydrodynamical Inertia Coefficients." Phil. Mag., 1929.
[3] C.W. Prohaska: "Lodrette Skipssvingninger med to Knuder." Copenhagen, 1941.
[4] C.W. Prohaska: "Vibrations Verticales du Navire." A.T.M.A., 1947.
[5] T. Terada: "On the Vibration of a Bar Floating on a Liquid Surface." Proc. Tokyo Math. Phyis. Soc., 1906.

Two aspects of the discrepancies are considered that the one is the discrepancy between the two results of the inertia coefficients measured and calculated by the strip method, the other is the tendency towards an increase of the virtual inertia coefficients with the consequence that the number of nodes of the vibration increases in the present experiment.

The present paper shows the investigations into the above two aspects of the virtual inertia coefficients for the vertical vibration of ships by means of the strip method including rotatory energy of the hull vibration and the corresponding added rotatory energy of the water entrained, and also of the correction factors of the three-dimensional motion, such as the flexural vibration of cylinders with various sections obtained by model experiments.

1. Virtual Inertia Coefficient for the Vibration of a Ship

(a) To measure the natural frequencies of a ship in the air and on the water, the virtual inertia coefficient for the vibration of the ship may be obained by the following formula:

$$\tau = \left(\frac{f_a}{f_w}\right)^2 \cdot \left(\frac{C_w}{C_a}\right)^2 - 1, \qquad (1)$$

where, τ = virtual inertia coefficient

$$f_a = C_a \sqrt{\frac{g E I}{\varDelta L^3}}, \qquad (\mathrm{i})$$

$$f_w = C_w \sqrt{\frac{g E I}{\varDelta L^3 (1+\tau)}}, \qquad (\mathrm{ii})$$

f_a, f_w natural frequencies in air and on water respectively

C_a, C_w eigen-values in air and on water respectively

EI, $\varDelta$, L flexural rigidity, displacement and length of ship respectively.

Since the distribution of the mass of the vibrating ship in the air is different from that on the water, the respective eigen-values C_a and C_w are not accounted as of equal value. The relation between the eigen-value and the typical distribution of the mass of the vibrating beam in two-node mode has already been obtained by Dr. Prohaska [4] and the author [6]. The value C_w/C_a for the given type ship in two-node vertical vibration is estimated accordingly. The value of C_w is in most cases slightly higher than that of C_a in a ship, but $C_w/C_a=1$ in the vibration of a homogeneous cylindrical beam, so that the virtual inertia coefficient of the flexural vibration of the cylindrical beam is obtained by the following simple formula:

$$\tau = \left(\frac{f_a}{f_w}\right)^2 - 1. \qquad (2)$$

It is impossible to measure the natural frequency of a ship in air, but if we wish to carry out the vibration tests on a ship model, the approximate value of the virtual inertia coefficient is easily obtained provided that the distributions of

[6] T. Kumai: "The Effect of Loading Conditions on the Natural Frequency of Hull Vibration." European Shipbuilding No. 2, Vol. VI, 1957.

the flexural rigidity and the mass of the ship model correspond to those of the actual ship.

(b) In the calculation of the virtual inertia coefficients for the vibration of ships, the velocity distribution along the ship length due to vibration should be taken into account as well as the distribution of the virtual mass considered. We should bear in mind that the rotatory energy due to the mass of a hull should be allowed for in the higher mode of the ship vibration, and we must also take into account the corresponding virtual rotatory inertia of the entrained water produced by the rotation of a hull of unit length as in the pitching motion of a ship. The virtual inertia coefficient for the ship vibration is calculated as the ratio of kinetic energy of the entrained water and that of the hull. This is therefore expressed in the following form:

$$\tau=\Theta_1\cdot\Theta_2\,\frac{\sum\limits_0^L\left\{m_e y^2+I_e\left(\frac{dy}{dz}\right)^2\right\}\,\delta z}{\sum\limits_0^L\left\{my^2\;+I\left(\frac{dy}{dz}\right)^2\right\}\,\delta z}\,, \tag{3}$$

where,

τ virtual inertia coefficient for flexural vibration of a ship

m_e, I_e virtual mass and virtual rotatory inertia respectively of the hull section with unit length δz at the length co-ordinate z

m, I mass and rotatory inertia respectively of hull section at the position z

y velocity distribution function or normal mode on water at the position z

Θ_1 correction factor for three-dimensional form

Θ_2 correction factor for three-dimensional motion

2. Three-dimensional Correction Factors

(a) The three-dimensional correction factor Θ_1 in (3) is the factor for the variable section of hull under the vertical translational motion. The value of Θ_1 is therefore taken from the results obtained by Lamb. In the present calculation, L should be defined with the parameter l/L, where l is the length of the parallel body. The Θ_1 is expressed in the following form:

$$\Theta_1=\theta_1\left(1-\frac{l}{L}\right)+\frac{l}{L}, \tag{iii}$$

where θ_1 represents the correction factor obtained by Lamb [7] and has already been verified by Prohaska's experiments on the translational oscillation of a ship-like beam [4]. The coefficient θ_1 is represented by the following empirical expression:

$$\theta_1=1-1.1\left(\frac{B}{L}\right)^{3/2}, \tag{iv}$$

where B is the beam of the ship. Substituting the above relation in (iii), Θ_1 is

[7] H. Lamb: "Hydrodynamics." p. 152, Sixth edition, 1932.

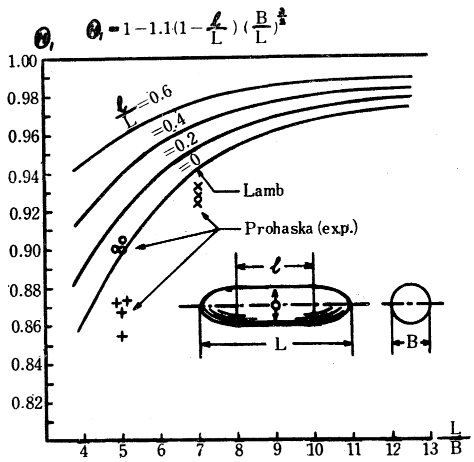


Fig. 1. Correction factors for three-dimensional form.

written as follows:

$$\Theta_1 = 1 - 1.1\left(1 - \frac{l}{L}\right)\left(\frac{B}{L}\right)^{3/2}. \quad (4)$$

Fig. 1 shows the relation between L/B and Θ_1 with the parameter l/L, and also shows the experimental results confirmed by Dr. Prohaska.

(b) With regard to the correction factor Θ_2 in (3), we should remember the investigation of the vibration of a circular cylinder of infinite length in the water which has already been made by Dr. J. Lockwood Taylor [2].

The Θ_2 is presented as the ratio of the virtual inertia coefficients calculated as based on the three-dimensional continuity equation of the fluid and that on the two-dimensional one in the vibration of the above infinite cylinder. In the present calculation, the same consideration as that resulting from Taylor's investigation is applied assuming a velocity distribution of the vibration of a free-free bar with the circular and the elliptic section as shown in Appendix I. The results of the calculation show little difference from those computed by Taylor.

There is a remarkable difference between both results of Θ_2 in the calculations and the experiments, as will be seen in Appendix II. The reason for the discrepancy between these results will be considered as the insufficient satisfaction of the boundary conditions of the liquid at both ends of the cylinder in the calculation. Accordingly, it will be better to employ the value of Θ_2 in the results of experiments than that obtained by the theoretical calculation in the present study. However, since the variations of Θ_2 for a/b in elliptic sections do not show such a conspicuous difference between the theory and the experiments, we may consider that the Θ_2 is assumed to be approximately almost constant along the length of a ship. The determination of the effective value of L/B of a ship for assuming the Θ_2 is also troublesome. However, in the present investigation, the effective length is assumed to be C_wL where C_w is the water plane coefficient and L is the length of a ship. This assumption has been confirmed by the experiments on the models of cylinders of circular, elliptic and rectangular sections, as will be seen in Fig. 2.

For practical use, the values of Θ_2 described above are also represented by the following empirical formulae:

$$\Theta_2 = 1 - 2.7\frac{B}{L_e}, \quad (5)$$

$$L_e = C_wL, \quad (v)$$

where, C_w waterplane coefficient

L length of ship.

The results of experiments and the relation between Θ_2 and L_e/B represented by the above expression are shown in Fig. 2.

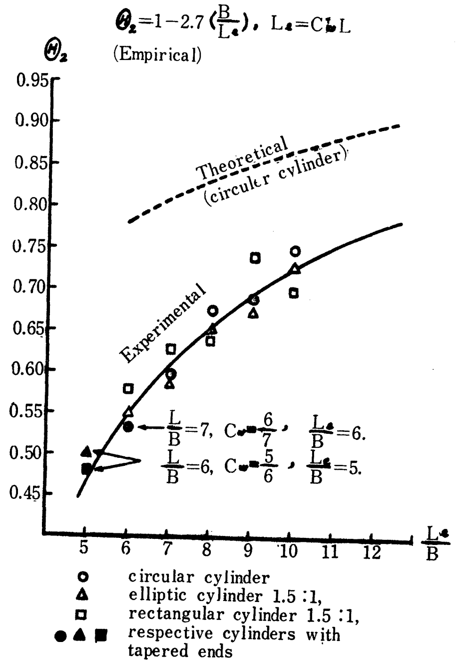


Fig. 2. Correction factors for three-dimensional motion.

3. Virtual Rotatory Inertia of Water

When the energy due to rotatory inertia of the hull of the element length is taken into account in the energy calculation of the ship vibration, the added energy due to virtual rotatory inertia of the entrained water should be allowed for in the hull vibration, which is computed by the same process as that for the pitching motion of a ship.

Putting *dm* for the virtual added mass of the water caused by the vertical motion of the element area of the wetted surface of any sectional strip of the hull element, the rotatory inertia of the mass of the water about the axis of the rotation in the neutral plane of the hull is written in the following form:

$$I_e = \int_s r_1^2 \, dm, \qquad (6)$$

where, I_e virtual rotatory inertia of added mass of water in the element length of the hull

s wetted surface of the hull element

r_1 lever of moment from axis of rotation to the mass element considered.

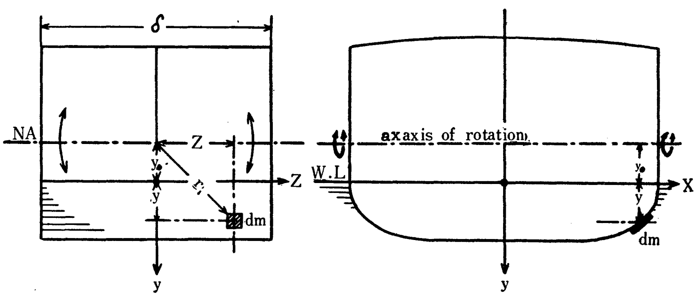


Fig. 3. Explanation of virtual rotatory inertia of entrained water surrounding hull strip.

As will be seen in Fig. 3, r_1^2 is represented by the form:

$$r_1^2 = z^2+(y_0+y)^2 \ , \tag{vi}$$

and dm is computed by means of the calculation of the virtual mass as follows:

$$dm= \rho\left(-\phi\frac{\partial\phi}{\partial\alpha}\right)_{\alpha=0} d\beta\ dz\ , \tag{vii}$$

where

- ρ density of water
- ϕ velocity potential function represented by the hypotrochoidal co-ordinate divided by normal velocity, and the contour $\alpha=0$ shows the underwater sectional form of the hull section under consideration
- dz length element of the hull strip.

Substituting (vi) and (vii) into (6), the added mass moment of inertia of the water due to vertical motion of the hull element shows the following form:

$$I_e=2\rho\int_0^{\delta/2}\int_0^{\pi}\left\{z^2+(y_0+y)^2\right\}\left(-\phi\frac{\partial\phi}{\partial\alpha}\right)_{\alpha=0} d\beta\ dz. \tag{7}$$

As an example, the following relation between x, y and ϕ are shown as the Lewis form of the hull section:

$$\left.\begin{aligned} x&=b_0\{(1+a_1)\cos\beta+a_3\cos 3\beta\},\\ y&=b_0\{(1-a_1)\sin\beta-a_3\sin 3\beta\},\\ \phi&=-b_0\{(1+a_1)e^{-\alpha}\sin\beta+a_3e^{-3\alpha}\sin 3\beta\}.\end{aligned}\right\} \tag{viii}$$

In the above equation, the relation between the sectional form of the actual ships and the Lewis form is obtained provided that the ratio of the half-breadth to draught b/d and the area coefficient σ of an actual ship as the two parameters is given. The calculation formulae giving the constants b_0/b, a_1 and a_3 in the above expressions with given value of b/d and σ were shown in the appendix of the author's previous paper [8].

The values of the virtual mass m_e and the I_e represented by (7) are calculated respectively as follows:

$$m_e=\frac{\rho\pi b_0^2\delta}{2}\left\{(1+a_1)+3a_3^2\right\}. \tag{8}$$

$$\begin{aligned} I_e =\frac{\rho\pi b_0^4\delta}{24}\Bigg[&12\frac{y^2}{b_0^2}\left\{(1+a_1)^2+3a_3^2\right\}\\ &+\frac{192}{\pi}\frac{y_0}{b_0}\Big[(1+a_1)^2\left\{\frac{1}{3}(1-a_1)+\frac{1}{15}a_3\right\}\\ &-\frac{36}{35}a_3^2(1+a_1)-a_3(1-a_1)\left\{\frac{4}{15}(1+a_1)+\frac{27}{35}a_3\right\}\\ &-\frac{1}{3}a_3^3\Big]+3\Big[(1-a_1)^2\left\{3(1+a_1)^2-4a_3(1+a_3)+6a_3^2\right\}\end{aligned}$$

[8] T. Kumai: "Added Mass Moment of Inertia Induced by Torsional Vibration of Ships." European Shipbuilding No. 6, Vol. VII, 1958.

$$+2(1-a_1^2)a_3(1+a_1-8a_3)+a_3^2\Big\{2(1-a_1)^2+9a_3^2\Big\}\Big]\Big]. \qquad (9)$$

As a simple example, calculating the values m_e and I_e of the circular cylinder half-immersed in the water and vibrating with the mode of the sine wave, the virtual inertia coefficient is obtained as the following simple form:

$$\tau=\Theta_2\frac{1+\frac{3}{4}\left(\frac{a}{l}\right)^2(kl)^2}{1+\frac{1}{4}\left(\frac{a}{l}\right)^2(kl)^2}, \qquad (\text{ix})$$

where, a/l ratio of radius of circular section to half length of beam

kl eigen-value of the vibration of beam.

As will be seen in the above example, the higher the order of the eigen-value, the greater the increase in the virtual inertia coefficient. The tendency of the increase or decrease of the virtual inertia coefficient mentioned above depends on the sectional form of the beam and it becomes more noticeably effective when decreasing the value of the length-diameter ratio.

4. Normal Function of the Vibration of a Ship

To the best of the author's knowledge, there exists no illustration of simple expression of the normal function even in the two-node vibration of a ship except that of the shearing vibration [9]. In the present study, for an approximate consideration, the vibration profiles including the higher mode of the vertical vibration of a tanker presented in the previous paper [10] are applied for the computation of the virtual inertia coefficient. Fig. 4 is given as an illustration of the computation.

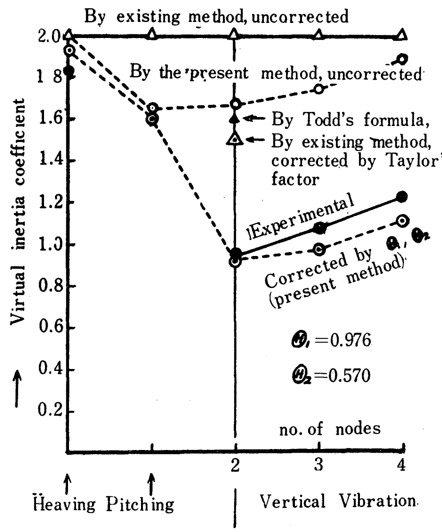


Fig. 4. Virtual inertia coefficients for heaving, pitching and vertical vibration of a tanker.

5. Numerical Example

The numerical calculations of the virtual inertia coefficients for the vertical vibration of a tanker up to four-node mode in ballast condition are carried out according to the present method. The results of the experiments, the former method and the present calculations are compared as shown in Fig. 4. As will be seen in

[9] T. Kumai: "Shearing Vibration of Ships." European Shipbuilding No. 2, Vol. V, 1956.

[10] T. Kumai: "Response of the Higher Modes of the Hull Vibration of a Large Tanker." European Shipbuilding No. 5, Vol. VII, 1958, see also Rep. R.I.A.M. Vol. VI, No. 24, 1958.

the figure, the present calculation is confirmed by the experiments.

Conclusion

A method for the calculation of the virtual inertia coefficient for the vertical vibration of a ship, including the higher mode of the vibration, was proposed. Special consideration was given to the three-dimensional correction factor obtained from model experiments on cylinders with several types of sectional forms, and theoretical investigations into the three-dimensional correction factor of the vibration on circular and elliptic cylinders in the water were carried out. In the calculation of the kinetic energies of the vertical vibration of a ship and the entrained water, the rotatory energy of the hull and the corresponding virtual rotatory energy of the water were taken into account in the present investigation. To confirm present calculation of the virtual inertia coefficient for the vertical vibration of a ship, an experimental study was carried out on a wooden model of a tanker, and the result was found to be in good agreement.

APPENDIX I.

Calculation of Θ_2 for the Vibration of Circular and Elliptical Cylinders

1. Velocity Distribution of the Vibration of a Cylinder:

The velocity distribution of the flexural vibration of a cylinder of length co-ordinate z with the origin at the centre is written as follows:

$$y = A\left\{\begin{matrix}\cos\\ \sin\end{matrix}\; kz + \mu \begin{matrix}\cosh\\ \sinh\end{matrix}\; kz\right\}, \tag{a}$$

where A is a constant with the unit of velocity, and the even and odd function show the symmetric and antisymmetric modes of vibration respectively. Taking the length of the cylinder$=2l$, the coefficient μ and eigen-value kl are obtained to satisfy the boundary conditions of a free-free bar as follows:
for symmetric mode:

$$\mu = \frac{\cos\ kl}{\cosh\ kl} = -\frac{\sin\ kl}{\sinh kl},$$

$$\tan kl = -\tanh kl,$$

for antisymmetric mode:

$$\mu = \frac{\sin\ kl}{\sinh\ kl} = \frac{\cos\ kl}{\cosh\ kl},$$

$$\tan kl = \tanh\ kl.$$

For the sake of simplicity, only the symmetric mode is considered hereafter. The hyperbolic function in the expression (a) is represented by a circular function for convenient use of the potential function. Thence we have,

$$y_1 = A\left\{\cos kz + \mu \sum_{n=2}^{\infty} A_n \cos nkz\right\}, \tag{b}$$

where,

$$\mu A_n = \frac{2(n \sin nkl \cdot \cos kl - \cos nkl \cdot \sin kl)}{kl(n^2+1)} .$$

2. Calculation of Θ_2 for the Vibration of a Circular Cylinder:

The kinetic energy of the entrained water due to the vibration of a circular cylinder half-immersed in the water calculated by the two-dimensional theory is shown by:

$$2T_{\rm II} = \frac{\rho\pi a^2}{2} \int_{-l}^{l} y_1^2 \, dz, \tag{c}$$

where ρ=density of the water, a=radius of the circular section of the cylinder, and the y is given by the expression (a), (b).

The same expression as (c) is obtained for the kinetic energy of the same cylinder vibrating in air, the virtual inertia coefficient therefore yielding to unity in the two-dimensional calculation.

On the other hand, in the three-dimensional analysis, the velocity potential function of the fluid represented by the cylindrical co-ordinate r, θ and z, which satisfy the boundary conditions except for $z = \pm l$, is obtained through the following well-known form:

$$\phi = \sum_{n=1}^{\infty} C_n K_1 (nkr) \cdot \sin\theta \cdot \cos nkz, \tag{d}$$

where K_1 is the second kind of modified Bessel function. The boundary condition at $r=a$ is represented by

$$\left(\frac{\partial \phi}{\partial r}\right)_{r=a} = y_1 \sin\theta. \tag{e}$$

From the expression y_1 shown in (b), the constant C_n in (d) is determined as the following form:

$$C_1 = \frac{A}{ka K_1'(ka)}, \tag{f}$$

$$C_n = \frac{\mu A \cdot A_n}{nkaK_1'(nka)}, \tag{g}$$

where the dash denotes the differentiation with respect to kr or nkr.

By the use of the above relations, the three-dimensional kinetic energy of the entrained water surrounding a circular cylinder vibrating with the mode of the free-free bar is calculated by means of the following formula:

$$2T_{\rm III} = -\rho \int_{-l}^{l} \int_{0}^{\pi} \left(\phi \frac{\partial \phi}{\partial r}\right)_{r=a} a \, d\theta \, dz. \tag{h}$$

We can easily understand that the Θ_2 is obtained from the ratio of $T_{\rm III}$ and $T_{\rm II}$, thus we have:

$$\Theta_2 = \frac{r_1 + \sum_{2}^{\infty} r_n \Phi_n}{1+\Psi_1}, \tag{i}$$

where,

$$r_1 = \frac{K_1\ (ka)}{-ka\ K_1'\ (ka)}\ ,$$

$$r_n = \frac{-K_1\,(nka)}{nka\ K_1'\,(nka)}\ ,$$

$$\Phi_n = \frac{8n^2}{(n^4-1)(n^2+1)} \cdot \frac{(n \sin nkl \cdot \cos kl - \cos nkl \cdot \sin kl)^2}{kl(kl + \sin kl \cdot \cos kl)},$$

$$\Psi_1 = \frac{\cos^2\ kl}{\cosh^2\ kl} \cdot \frac{kl + \sinh kl \cdot \cosh kl}{kl + \sin kl \cdot \cos kl}\ .$$

The numerical results are shown in Table 1. The results are compared with that obtained by Dr. Taylor providing $y=\cos kz$.

Table 1. Calculated Θ_2 in the two-node vibration of a cylinder.

l/a	Θ_2	Θ_2(by Dr. Taylor)
10	0.869	0.890
9	0.853	0.865
8	0.833	0.843
7	0.809	0.820

3. Calculation of Θ_2 for the Vibration of an Elliptic Cylinder:

The virtual mass of an elliptic cylinder due to the translational oscillation in the water obtained by the two-dimensional theory shows the same value as that of a circular cylinder. The fundamental equation of the continuity of the fluid in the three-dimensional calculation is shown by the following form, assuming that $z=\cos nkz$, with a cartesian co-ordinate:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} - n^2 k^2 \phi = 0. \qquad \text{(j)}$$

The above equation is represented by elliptical co-ordinates assuming that $x=h \cosh \xi \cos \eta$, $y=h \sinh \xi \sin \eta$:

$$\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2} - 2n^2\kappa^2 (\cosh 2\xi - \cos 2\eta)\ \phi = 0, \qquad \text{(k)}$$

where,

$$\kappa = k \cdot h/2,$$

$2h$ = distance between foci.

Putting $\phi=\phi_1 \cdot \phi_2$; and $\kappa^2 = q$, the equation (k) separates into the following two types of Mathieu equation:

$$\frac{d^2\phi_1}{d\xi^2} - (a_1 + 2q \cosh 2\xi)\phi_1 = 0, \qquad \text{(l)}$$

$$\frac{d^2\phi_2}{d\eta^2} + (a_1 + 2q \cos 2\eta)\phi_2 = 0. \qquad \text{(m)}$$

The solutions of (j) are therefore obtained as the following Mathieu functions, [11], [12].

[11] M. Ray: "Vibration of an Infinite Elliptic Cylinder in a Viscous Liquid." Z. A. M. M., 1936.

[12] N.W. McLachlan: "Theory and Application of Mathieu Functions." Oxford Press, 1951.

$$\phi = \mathrm{Gek}_1(\xi, -q)\cdot \mathrm{se}_1(\eta, -q) \cos nkz, \qquad 0 \leqq \eta \leqq \pi, \qquad \text{(n)}$$

and also

$$\phi = \mathrm{Fek}_1(\xi, -q)\cdot \mathrm{ce}_1(\eta, -q) \cos nkz, \qquad \frac{\pi}{2} \leqq \eta \leqq \frac{3\pi}{2}. \qquad \text{(p)}$$

Substituting (n) and (p) into (h), the T_{III} is obtained for given values of $\xi = \xi_0$ or the half-beam-draft ratio a/b.

Consequently, the Θ_2 is calculated by the same process as that in the circular cylinder by substituting ζ_1, ζ_n for r_1, r_n respectively in equation (i) ; ζ_1, ζ_n are represented as follows:

$$\zeta_1 = \frac{\mathrm{Gek}_1\ (\xi_0, -q)}{-\mathrm{Gek}_1{}'(\xi_0, -q)},$$

$$\zeta_n = \frac{\mathrm{Gek}_1\ (\xi_0, -q_n)}{-\mathrm{Gek}_1{}'(\xi_0, -q_n)},$$

where, the dash denotes the differentiation with respect to ξ. Table 2 shows some numerical results of Θ_2. As will be seen in Table 2, variation of the value Θ_2 is not so remarkable for the beam-draft ratio in the given length-beam ratio; however, the minimum value of the Θ_2 is found in the circular section.

Table 2. The Θ_2 of the elliptical cylinder in the two-node vibration with various sectional form.

ξ_0	a/b	e_1	Θ_2 (l/a =10)	Θ_2 (l/a =10)
0	∞	1	0.9400	0.9435
0.310	3.333	0.955	0.9242	0.9374
0.805	1.500	0.745	0.8890	0.8945
∞	1.000	0	0.8688	0.8688

APPENDIX II.

Model Experiments

A wooden model of a tanker, length 150 cm, breadth 20 cm, depth 12 cm and with light draught 5.1 cm was prepared in order to obtain the virtual inertia coefficient for the hull vibration. The model was compensated by doubling of the longitudinal bulkheads and giving elevenfold thickness to the transverse bulkheads and also covering with deck plate in order to prevent local vibrations of numerous panels of the model. The structural view of the model without deck plate is shown in Photo. 1. On the vibration test of the above model in the air, the model was supported at about a quarter of the length from the ends. Thick sponge was used for the supports to offer the least constraint to the free vibration of the model. The model was then excited by the reaction of the vibrating cone to the moving coil of a dynamic speaker set on the poop deck, the speaker being operated by the audio-frequency oscillator. The frequency range measured in the present test was

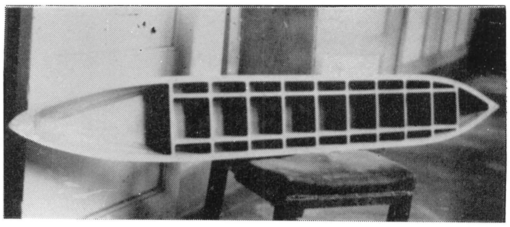


Photo. 1

50 cps to 2,000 cps, the frequency of the vibration being checked by comparison with a ser:es of tuning-forks. The vibration of the model was measured through acceleration by a crystal ear-phone used as a pick-up. The pick-up was set on the forecastle deck to determine the natural frequencies of the model and was set on each station of the hull to measure the normal modes of the vibration of the model. The general view of the model set on the sponge supports is shown in Photo. 2. Next, the model was launched in a tank 3 m long, 1 m wide and 0.7 m deep as shown in Photo. 3. The vibration response of the model was observed by means of an oscilloscope. The natural frequencies of the model in the air and on the water were clearly determined by the oscilloscope observations. The response curves of the vibration of the model in air and on water are shown in Fig. 5, and the modes of two-node vertical vibration of models of an ellipsoid, the tanker and a circular cylinder are shown in Fig. 6.

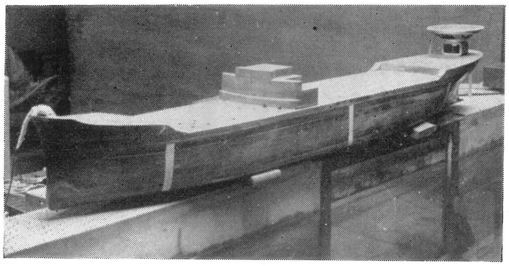


Photo. 2

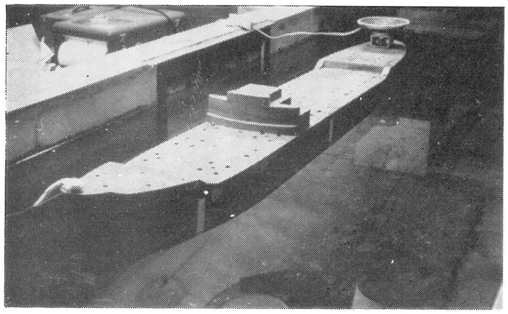
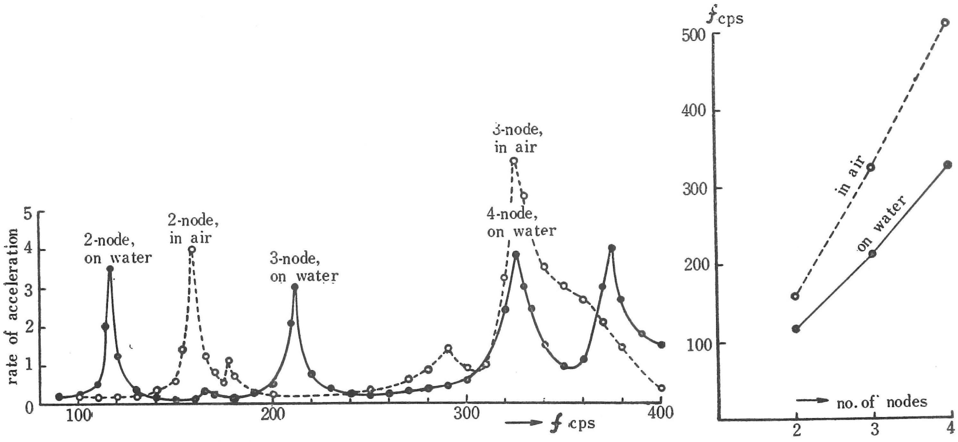


Photo. 3

On the other hand, the natural frequencies of the flexural vibration of sev-



(a) (b)

Fig. 5. Experimental results of vibration measurements on a tanker model.

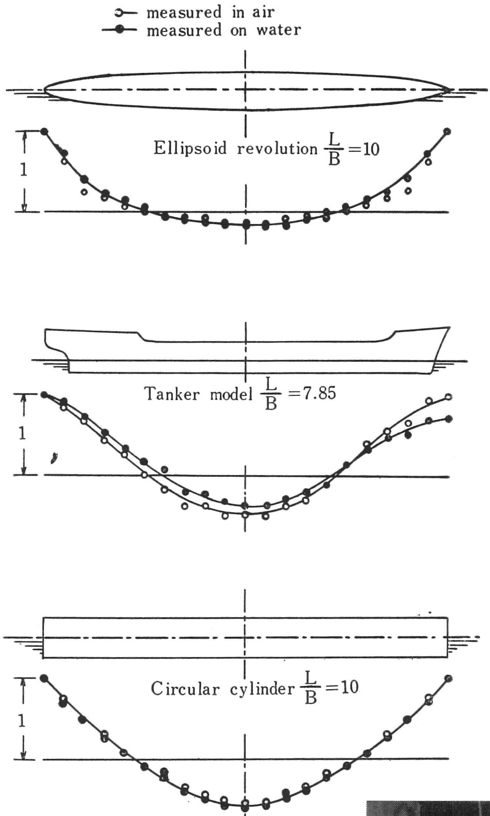


Fig. 6. Two-node vibration profiles of models measured in air and on water.

eral wooden cylindrical models with circular, elliptical and rectangular sections were determined by the same means as above except for the exciting apparatus. The cylinder models were excited by one end of a vibrating light bar set on one end of the model, the other end of the bar being connected to the moving coil excited by the oscillator, as shown in Photo. 4 and Photo. 5.

The results of the experiments of cylinders are shown in Fig. 7 and Fig. 8. As will be seen in the figures, the values of Θ_2 obtained are remarkably low compared with unity. The discrepancy between theory and experiment is

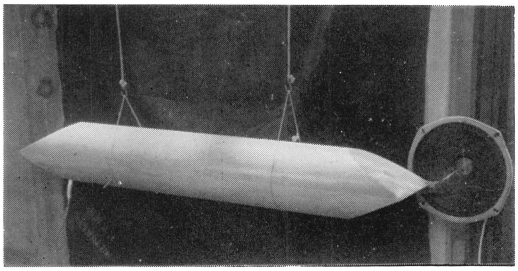


Photo. 4

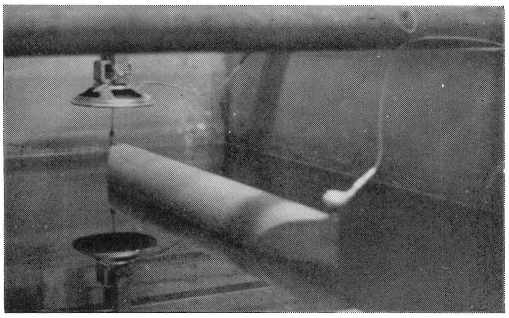


Photo. 5

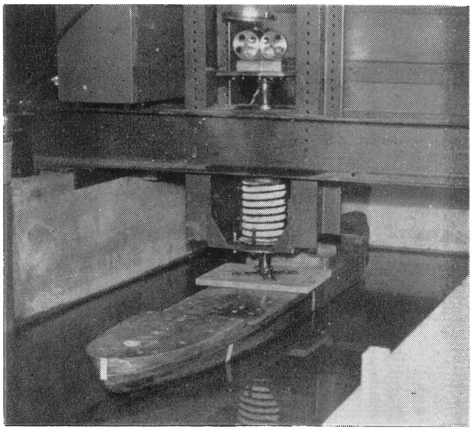


Photo. 6

evident as is also to be seen in Fig. 2.

The apparatus for measuring the virtual inertia coefficient for the heaving oscillation of the ship is shown in Photo. 6. The same principle for obtaining the virtual inertia coefficient as Prohaska's method [4] was used in the present study.

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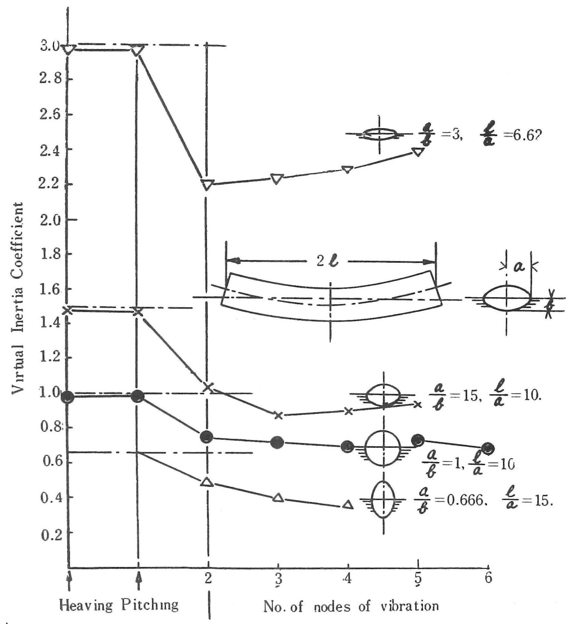

Fig. 7. Virtual inertia coefficients for heaving, pitching and vibration of elliptic cylinder models obtained by experiments.

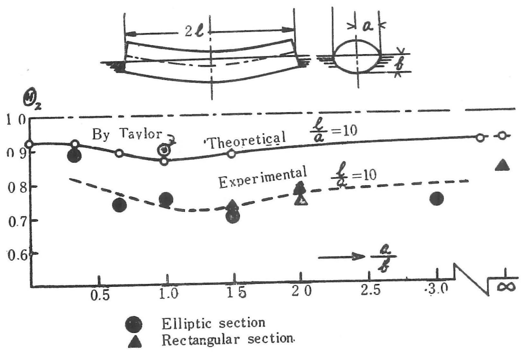

Fig. 8. Theoretical and experimental results of correction factors for three-dimensional motion of circular, elliptic and rectangular cylinders.