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# ADDED MASS MOMENT OF INERTIA INDUCED BY TORSIONAL VIBRATION OF SHIPS*

By Toyoji KUMAI**

**Abstract.** An investigation is made of the added mass moment of inertia induced by the rotational motion of prisms having sections similar to the hull sections on water to obtain the information to estimate the natural frequency of the torsional vibration of ships. Special consideration is paid to the effect of the draught upon the added mass moment of inertia in the present study. Experimental studies using prismatic models with the sectional forms of the main stations of the hull of a tanker are also carried out confirming the theoretical calculation. As a result of the present investigation, a convenient formula for estimation of the added mass moment of inertia induced by the torsional vibration of a tanker, for an example, is deduced for practical use.

## Introduction

The theoretical calculations of the added mass moment of inertia induced by rotation of the prism about its axis through the figure of the section of the ellipse, the section presented by the elliptic hypotrochoidal coordinate and the rectangular section, have already been carried out by H. Lamb [1], Prof. Y. Watanabe [2] and K. Wendel [3] respectively.

With regard to the torsional vibration of the hull, however, the torsional centre does not always coincide with the centre of the double figure of the under-water line of the hull section. Accordingly, the correction for the location of the centre of rotation of the section should be taken into account. R. Brard investigated this correction for the free rolling of the prism with the semi-elliptic section [4].

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** Member of the institute.

[1] H. Lamb: " Hydrodynamics ", 6th ed. p. 88.

[2] Y. Watanabe: " On the Apparent Moment of Inertia of Ship in Free Rolling". Jour. Soc. N.A. Japan, 1933.

[3] K. Wendel: " Hydrodynamischen Massen und Hydrodynamische Massenträgheits momente ". Jahrb. Schiffbautechn. Ges. Vol. 44, 1950.

[4] R. Brard: " Les Effets de l'Eau Entraine sur le Mouvement de roulis en Eau Calme ". Bull. A.T.M.A. No. 43, 1939.

In the present investigation, the two-dimensional calculations of the added mass moment of inertia of the prism are carried out with respect to the sections represented by the Lewis form [5] and the Prohaska form [6] which present the double figure of the underwater line of the hull section with the centre of rotation at any point along the vertical axis of the section. As an application of the calculation, the effect of the draught upon the added mass moment of inertia is investigated. The experimental study is also carried out by use of prismatic models of the sections having the main stations of the hull of a tanker. The distribution of the added mass moment of inertia induced by torsional vibration of a tanker is obtained and by employing the strip method, the computed result is fitted to the empirical formula.

## 1. Two-Dimensional Calculation of Added Mass Moment of Inertia

When the prismatic bar of the symmetric section for the vertical axis rotates about the axis through any point in the vertical axis in or on the perfect fluid, the added water mass moment of inertia induced by the rotation of the prism should be taken into account; this inertia is presented by form [1] :

$$\Delta I = C\rho\pi d^4, \qquad (1)$$

where, $\Delta I$ added mass moment of inertia per unit length
$\rho$ density of fluid
$d$ draught of the section of the prism
$C$ coefficient which depends on the sectional form and on the location of the centre of rotation.

If the prism floats on water like a ship, and also the centre of rotation coincides with that of the double figure of the underwater line section, the added mass moment of inertia is obtained by calculation of the pure rotatory motion. In ordinary cases of rolling or torsional vibration of ships, however, the centre of rotation is not always at the centre of the double figure of the underwater section. The horizontal translational motion [7] is therefore superimposed on the rotation of the prism in ordinary cases. Accordingly, when the added mass moment of inertia is taken into account the result differs from that obtained when only the rotational motion is considered. With regard to the torsional vibration of actual ships, the special case when the two centres coincide almost corresponds to that of the half load condition as shown in Fig. 1 (b). As is seen in the figure, in both conditions of light and full load the effect of the distance between two centres upon the added mass moment of inertia should be taken into account. In the present paper, the analysis of the added mass moment of inertia was carried out

[5] F.M. Lewis: "The Inertia of the Water Surrounding a Vibrating Ship." Trans. S. N. A. M. E., 1929.
[6] C.W. Prohaska: "Vibrations Verticales du Navire." Bull. A. T. M. A., 1947.
[7] L. Landweber and M. C. de Macagno : "Added Mass of Two-dimensional Forms Oscillating in a Free Surface." Jour. of Ship Research, Nov., 1957.

on two types of the section of prism, the Lewis form and the Prohaska form (see Appendix). Examples of the results calculated from the Lewis form are shown in Fig. 2 to Fig. 5. As will be seen from the figures, the effect of distance between two centres ($y_0/d$) upon the coefficient of added mass moment of inertia $C$ obviously depends on the sectional forms and the ratio of half-beam to draft $b/d$.

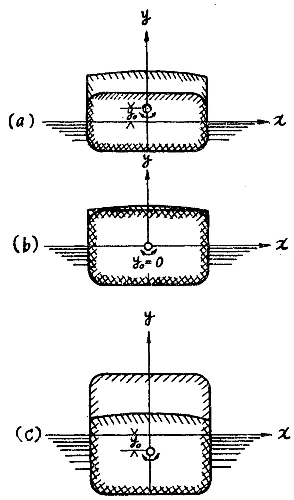


Fig. 1. Locations of the torsional centre of ship section and the centre of the double figure of the underwater line for three loading conditions.

## 2. Model Experiments

Experimental studies of the measurements of the added mass moment of inertia of the prismatic models having the sectional forms of the main stations of a tanker were carried out and comparisons with the calculations made.

The apparatus for measurement of the added inertia is shown in Fig. 6. The main frequencies of the rotational vibration of the model with various centres of rotation in air and on water is compared and the added mass moment of inertia is obtained for a model in a given draught. Table 1 and Fig. 7 show an example of the result of the model experiment and comparison with the calculation. The agreement of both results is fairly good as is seen in the figure.

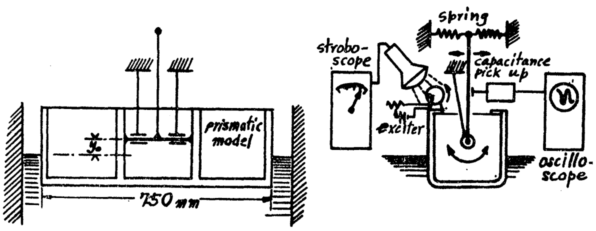


Fig. 6. Measuring apparatus of the frequencies of the rotational vibration of the prismatic model.

## 3. The Distribution of Added Inertia Along the Hull of a Tanker

As an example, the distribution of the added mass moment of inertia along the length of the hull was obtained for a typical tanker in the full and the light load conditions.

The added mass moment of inertia of the main stations of a tanker are cal-

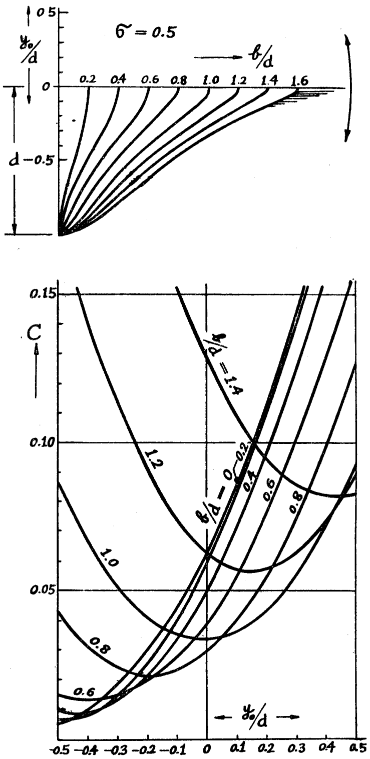


Fig. 2

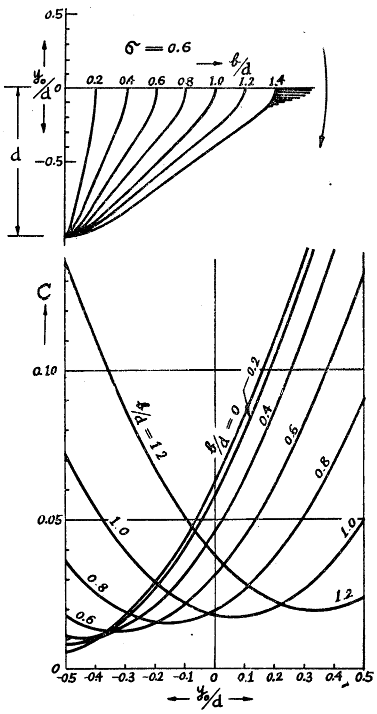


Fig. 3

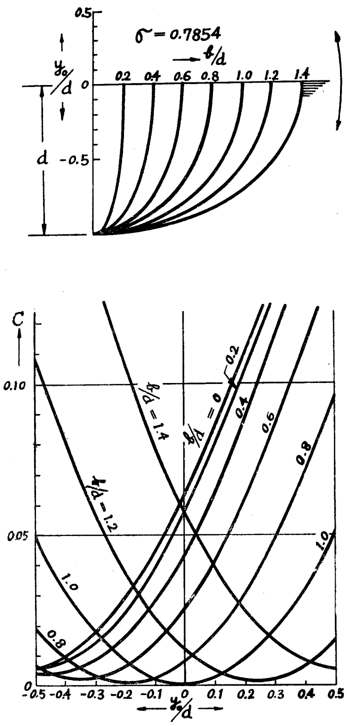


Fig. 4

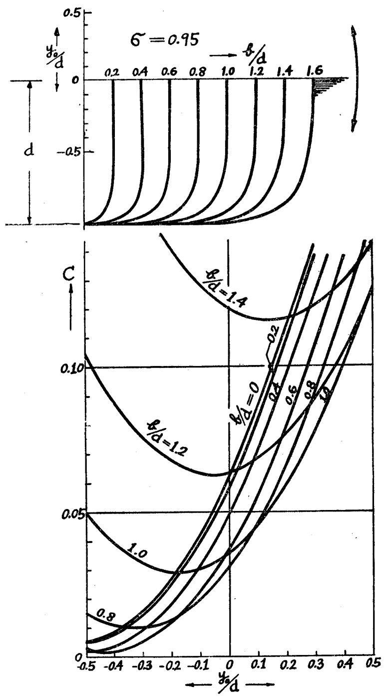


Fig. 5

Fig. 2 to Fig. 5. Calculated values of inertia coefficient $C$ for the distance-draught ratio between two centres $y_0/d$ with the parameter $b/d$ of the sections represented by the Lewis form.

Table 1.

Result of measured frequency and the other data of the prismatic model of the section at the No. 1 station of a tanker.

No. 1 station; $b/d=0.9$, $d=80$ mm, $\sigma=0.67$, $L=750$ mm, $k=10.0$ kg/cm, $\frac{gkl^2}{4\pi^2}=86{,}800$ kg cm²/sec².

| $y_0/d$ | $f_a$ cps | $f_w$ cps | $I$ kg cm² | $\Delta I$ kg cm² | $\Delta I/I$ | $C$ |
|---|---|---|---|---|---|---|
| 0.20 | 24.0 | 20.7 | 151.0 | 50.50 | 0.334 | 0.0520 |
| 0.10 | 25.0 | 22.6 | 138.3 | 31.00 | 0.224 | 0.0319 |
| 0 | 25.7 | 24.0 | 130.9 | 19.60 | 0.149 | 0.0201 |
| −0.10 | 26.6 | 25.4 | 122.3 | 11.80 | 0.0964 | 0.0121 |
| −0.20 | 26.8 | 25.5 | 120.6 | 12.85 | 0.0868 | 0.0132 |
| −0.30 | 26.4 | 24.7 | 124.0 | 17.80 | 0.1430 | 0.0183 |
| −0.40 | 26.2 | 23.7 | 126.0 | 28.60 | 0.2270 | 0.0294 |

culated as in the Lewis form or the Prohaska form taking into account the parameters of the ratio of half beam to draught, the area coefficient and the distance between the centre of the transformed figure and the torsional centre of the corresponding actual ship section. The body plans and the distribution of the inertia coefficients in full and light load conditions are shown in Figs. 8 and 9 respectively. In these figures the coefficients of the added mass moment of inertia are represented by $C$ multiplied by $(2d/B)^4$ for comparison of various loading conditions.

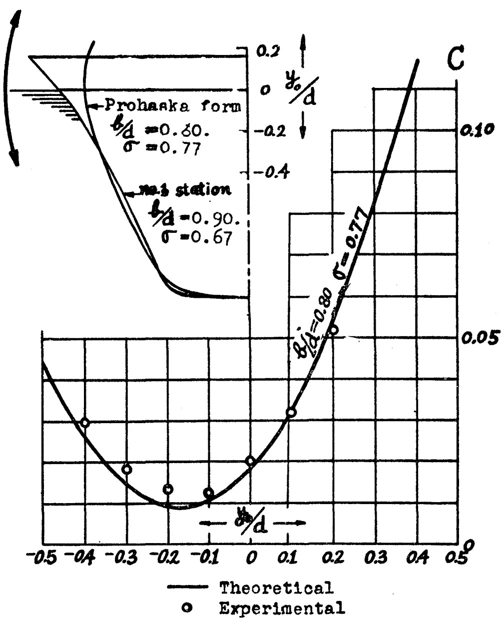


Fig. 7. Comparison of inertia coefficients calculated and measured on the prismatic model of the section at the No. 1 station.

## 4. Empirical Formula

Since we may assume that the modern tankers of 20,000∼40,000 t.d.w. have almost the same type of hull form, the result of the calculation of the present example is applicable to other modern tankers. For the sake of rapid calculation, the empirical formula for estimating the added mass moment of inertia is deduced as follows.

The integration of $C$ values of the main stations of the hull with respect to her length is easily obtained from Fig. 8 or Fig. 9 providing that the effect of the water inertia due to the velocity component of longitudinal direction of a ship is ignored; the added mass moment of inertia of the ship is then presented by:

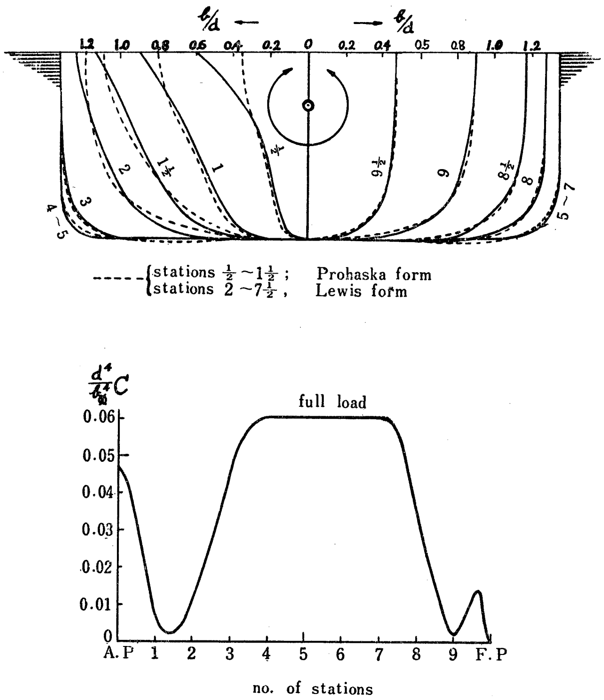


Fig. 8. Body plan of a tanker in full load condition and the distribution of computed inertia coefficients along the ship length in the same condition.

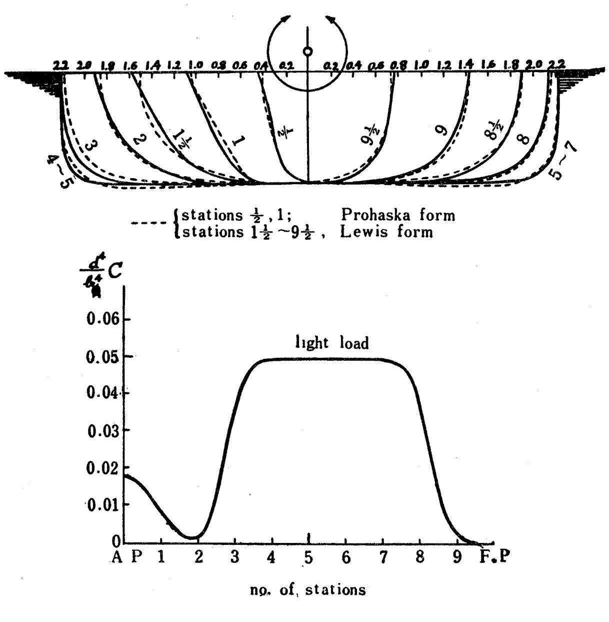


Fig. 9. Body plan of a tanker in light load condition and the distribution of computed inertia coefficients along the ship length in the same condition.

$$\Delta I_s = \rho\pi d^4 L \int_0^1 C d\xi, \tag{2}$$

where, $L$ length of the ship
$\xi$ length co-ordinate.

Putting $k = \left(\frac{2d}{B}\right)^4 \int_0^1 C\, d\xi,$

where, $B$ = breadth of the ship

we have, therefore:

$$\Delta I = k\rho\pi \left(\frac{B}{2}\right)^4 L,$$

where $k$ is the function of the ship's form and the draught ratio and is obtained from the result of the theoretical calculation. As the result, the empirical expression of the added mass moment of inertia in the typical tanker, as an example, is given in the following formula:

$$\Delta I_s = 0.00531\left(1 + 0.365\frac{d}{d_f}\right)B^4\ L \text{ ton } m^2, \tag{3}$$

where, $d_f$ draught at full load in metres
$B$ breadth of the ship in metres.

## 5. Numerical Example

The numerical calculations of the mass moment of inertia $I_s$ in three types of the ballast conditions of a 32,000 t. d. w. tanker with almost the same draught and in the full load conditions were carried out. The ratio of the added mass moment of inertia $\Delta I_s/I_s$ and the frequency ratios of the vibration of the hull on water and in air in the above conditions are approximately obtained as shown in Table 2.

Table 2.

Numerical example of mass moment of inertia, added inertia and frequency ratio on a 32,000 t.d. w. tanker in various loading conditions.

$L\times B\times D, d$; 192 m × 26 m × 13.7 m, 10 m

| conditions | $I_s$ m² | $\Delta I_s$ t m² | $N$ on water / $N$ in air | $N/N_2$ | $d/d_{ull}$ |
|---|---|---|---|---|---|
| 1. light 1, loaded in centre tanks | $0.720\times10^6$ | $0.643\times10^6$ | 0.710 | 1.274 | 0.6 |
| 2. light 2, loaded in centre and wing tanks.............................. | $1.556\times10^6$ | $0.643\times10^6$ | 0.842 | 1.000 | 0.6 |
| 3. light 3, loaded in wing tanks... | $2.416\times10^6$ | $0.643\times10^6$ | 0.890 | 0.844 | 0.6 |
| 4. full load.............................. | $2.834\times10^6$ | $0.723\times10^6$ | 0.893 | 0.786 | 1.0 |

It will be noted from the table, that the critical frequency of the torsional vibration of a tanker varies considerably because various distributions of oil cargo athwartships are made in a light draught. An effective means of avoiding resonant torsional vibration of the hull is to alter the critical frequency by rearranging the load athwartships in the light condition of the modern tankers with longitudinal bulkhead.

## Conclusion

The added water mass moment of inertia induced by torsional vibration of ships, taking into consideration the effect of draught, has been calculated by the strip method, and the numerical results on a modern tanker have been compared with the result of the prismatic model tests. In the results, an empirical formula for estimation of the added inertia of water induced by torsional vibration of a tanker was presented for convenient use in the drawing office.

The author expresses his gratitude to Prof. Y. Watanabe for his valuable advice and criticism in the preparation of this paper.

## Appendix

The added water mass moment of inertia induced by the rotational motion of the prism having section similar to hull sections is calculated by use of the figures represented by the Lewis form and the Prohaska form.

(1) Mathematical Presentation of the Sectional Forms.

i) Lewis form: The parametric equation of this form is presented by [5]:

$$x=b_0\ \{(e^{\alpha}+a_1e^{-\alpha})\cos\beta+a_3e^{-3\alpha}\cos 3\beta\},\qquad \text{(a)}$$
$$y=b_0\ \{(e^{\alpha}-a_1e^{-\alpha})\sin\beta-a_3e^{-3\alpha}\sin 3\beta\},$$

where $\alpha$ takes zero to infinite, $\beta: 0\sim\pm\pi$, $b_0$, $a_1$ and $a_3$ are unknown coefficients.

The double figure of the hull section underwater line is presented taking $\alpha=0$ in (a) as follows:

$$x=b_0\ \{(1+a_1)\cos\beta+a_3\cos 3\beta\},\qquad \text{(b)}$$
$$y=b_0\ \{(1-a_1)\sin\beta-a_3\sin 3\beta\}.$$

The coefficients $a_1$ and $a_3$ are determined for the ratio of draught $d$ and half-beam $b$ of the given section in the following way. Putting $\beta=0$, $\beta=\pi/2$ in (b) respectively, we have:

$$b=b_0\ (1+a_1+a_3),\qquad \text{(c)}$$
$$d=b_0\ (1-a_1+a_3),$$

putting also $d/b=\lambda$, $a_1$ and $a_3$ are represented as follows:

$$a_1=\frac{b}{2b_0}(1-\lambda),\qquad \text{(d)}$$
$$a_3=\frac{b}{2b_0}(1+\lambda)-1.$$

Next, the area of the half section of the figure is shown by:

$$s=\frac{\pi b_0^2}{2}(1-a_1^2-3a_3^2),\qquad \text{(e)}$$

the area coefficient $\sigma$ is presented by:

$$\sigma = \frac{\pi b_0^2}{4\lambda b^2}\,(1-a_1^2-3a_3^2). \tag{f}$$

The unknown coefficient $b_0/b$ is obtained by inserting the relations (d) into (f):

$$b_0/b = \frac{1}{4}\left\{3(1+\lambda)-\sqrt{(1+\lambda)^2+8\lambda(1-4\sigma/\pi)}\right\}. \tag{g}$$

By the use of (g) and (d), the three unknown coefficients $b_0$, $a_1$ and $a_3$ in the equation of the Lewis form are determined for the given values of the draught half-beam ratio $\lambda$ and the area coefficient $\sigma$.

ii) Prohaska form: This form is conveniently used for representation of the section near the stern of the hull of a tanker. The form is expressed by the following equation [6] :

$$x=b_0\,\{(1+a_1)\cos\beta+a_5\cos 5\beta\},$$
$$y=b_0\,\{(1-a_1)\sin\beta-a_5\sin 5\beta\}. \tag{h}$$

The coefficients $b_0$, $a_1$ and $a_5$ are determined as in the following process.

In the first place, $b$ and $d$ are represented by

$$b=b_0\,(1+a_1+a_5),$$
$$d=b_0\,(1-a_1-a_5)\,. \tag{i}$$

From the above two equations, we obtain

$$b_0/b = \frac{1+\lambda}{2},$$
$$a_5 = \frac{1-\lambda}{1+\lambda} - a_1. \tag{j}$$

The sectional area and the area coefficient are respectively shown by:

$$s = \frac{\pi b_0^2}{2}\,(1-a_1^2-5a_5^2),$$
$$\sigma = \frac{\pi b_0^2}{4b^2\lambda}\,(1-a_1^2-5a_5^2). \tag{k}$$

Putting the relation (j) into (k), the coefficient $a_1$ is obtained in the following form:

$$a_1 = \frac{1}{6(1+\lambda)}\left\{5(1-\lambda) - \sqrt{1+\lambda^2+2\lambda(11-48\sigma/\pi)}\right\}. \tag{1}$$

The coefficients $a_1$, $b_0$ and $a_5$ in the equation of the Prohaska form are thence determined by the use of (j) and (1) for given values of $\lambda$ and $\sigma$.

The selection of the form in the above two types of the sectional forms is easily made by means of visual comparison of the form with the body plan of actual ships using the values of $\lambda$ and $\sigma$ as two parameters.

## (2) Calculation of the Added Mass Moment of Inertia of a Prism.

When a prism having the section presented by the transformed figure considered in the above article is rotating on the perfect fluid with free surface at $y=0$ about the axis through any point $y_0$ on the vertical axis of the figure, the normal velocity $v_n$ to the contour of the figure is presented by

$$v_n = h\omega \left\{ x \frac{\partial y}{\partial \alpha} - (y+y_0) \frac{\partial x}{\partial \alpha} \right\}_{\alpha=0}, \tag{m}$$

where, $x, y$ parametric equation which represents the transformed figure

$y_0$ co-ordinate of the centre of rotation

$h\frac{\partial y}{\partial \alpha}$, $h\frac{\partial x}{\partial \alpha}$ direction cosines of the curvilinear co-ordinate

$\omega$ angular velocity of rotation of the prism.

Representing the velocity potential of the fluid by $\phi$, the boundary condition along the contour of the figure is shown by:

$$h\left(\frac{\partial \phi}{\partial \alpha}\right)_{\alpha=0} = v_n. \tag{n}$$

At the free surface, $\phi$ must satisfy the following condition:

$$\text{at } \beta=0,\ \pi; \qquad \omega^2\phi - g\frac{\partial \phi}{\partial y} = 0. \tag{p}$$

Since the first term in (p) is high enough compared with the second term in the vibration problems, the condition is simplified as in the following formula:

$$\beta=0,\ \pi; \qquad \phi=0. \tag{p)'$$

For example, the normal velocity is computed by use of the Lewis formula, as follows:

$$v_n = h\omega b_0^2 [2a_1(1+a_3) \sin 2\beta + 4a_3 \sin 4\beta - \frac{8}{\pi} \frac{y_0}{b_0} \sum_{n=1}^{\infty} n \left\{ \frac{1-a_1}{4n^2-1} - \frac{3a_3}{4n^2-9} \right\} \sin 2n\beta]. \tag{q}$$

In the above expression, the even function with respect to $\beta$ is replaced by the odd function for satisfaction of the condition at the free surface, namely:

$$\cos m\beta = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{4n}{4n^2-m^2} \sin 2n\beta.$$

In the next place, we assume the velocity potential function $\phi$ which satisfies the condition of continuity of the fluid and that of the free surface to be as in the following formula:

$$\phi = -\omega \cdot b_0^2 \sum_{n=1}^{\infty} A_{2n}\, e^{-2n\alpha} \sin 2n\beta, \tag{r}$$

and also

$$\frac{\partial \phi}{\partial x} = \omega \cdot b_0^2 \sum_{n=1}^{\infty} 2nA_{2n} e^{-2n\alpha} \sin 2n\beta,$$

where $A_{2n}$s are unknown constants determined by the boundary condition at the contour of the section of the prism.

In the last process, the kinetic energy of the fluid performing a potential motion outside the contour $\alpha=0$, and rest at infinity, is given by:

$$T = -\frac{\rho}{2} \int_0^{2\pi} \left(\phi \frac{\partial \phi}{\partial \alpha}\right)_{\alpha=0} d\beta \tag{s}$$

or

$$T = \Delta I\omega^2/2.$$

Substituting the expression (r) in (s), the added mass moment of inertia $\Delta I$ is obtained from (s) as in the following formula previously shown in the present paper:

$$\Delta I = C\rho\pi d^4. \tag{1}$$

The inertia coefficient $C$ is calculated for the two types from the formulae:

(i) for the Lewis formula;

$$C = \frac{1}{\lambda^4}(p^2+2q^2) - \frac{8}{\pi\lambda^2}(pK_2+2qK_4)\frac{y_0}{d} + \frac{16}{\pi^2}\sum_{n=0}^{\infty} nK_{2n}^2 \cdot \frac{y_0^2}{d^2}, \tag{t}$$

where, $p = \dfrac{1-\lambda^2}{4}$, $q = \dfrac{b_0}{b}\left(\dfrac{1+\lambda}{2} - \dfrac{b_0}{b}\right)$

$$K_{2n} = \frac{k_1}{4n^2-1} - \frac{k_3}{4n^2-9};$$

$$k_1 = \frac{1}{\lambda}\left(\frac{b_0}{b} - \frac{1-\lambda}{2}\right), \quad k_3 = \frac{3}{\lambda}\left(\frac{1+\lambda}{2} - \frac{b_0}{b}\right).$$

(ii) for the Prohaska formula;

$$C = S^4(a_1^2+2a_1^2a_5^2+3a_5^2) - \frac{8S^2}{\pi}(a_1H_2+2a_1a_5H_4+3a_5H_6)\frac{y_0}{d} + \frac{16}{\pi^2}\sum_{n=1}^{\infty} nH_{2n}^2\frac{y_0^2}{d^2}, \tag{u}$$

where, $S = \dfrac{1+\lambda}{2\lambda}$, $H_{2n} = S\left(\dfrac{1-a_1}{4n^2-1} - \dfrac{5a_5}{4n^2-25}\right)$.

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