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By Yasutoshi SENOO and Taijiro KASAI

ABSTRUCT

A similarity law is proposed and verified using a pump operating at several speeds of rotation. The work on the liquid by the rotor and the work on the gas by the liquid are theoretically calculated. The comparison of the calculated work to the ideal work shows some of the causes of energy dissipation which are feature of the pump. At a low speed and at a high compression ratio, the pump does not operate properly. The mechanism of abnormal operation is explained and the critical condition was measured for a pump with three different types of discharge ports.

§ 1. Introduction.

Wet vacuum pumps are widely used to handle wet air in the paper industry, to blow chlorine, and to prime centrifugal and axial pumps. In spite of the popularity and large capacity—up to 200 horse powers —, the maximum efficiency hardly reaches 50%, a level which is unbelievably low for this size fluid machine. Since liquid and gas coexist and influence each other in a wet vacuum pump, no theory or similarity law for a machine which handles only gas or liquid is applicable, and consequently the mechanism of pumping action has not yet been thoroughly clarified. A series of experiments was done by the authors to find basic factors which crucially affect the performance.

§2. Apparatus and experimental method.

The main dimensions of the pump used for the experiment are: diameter of rotor $2R=412$ mm, outer width of rotor 204 mm, inner width of rotor $B=182$ mm, number of blades 24, the arc-centers of the casing lie a distance $e=40$ mm from the minor axis. The suction and discharge ports are located on both side-covers as shown in Fig. 1.

The speed of rotation ω, input power w, rate of air flow at suction condition q, suction pressure and temperature p_s, T_s, discharge pressure and temperature p_d, T_d, and atmospheric pressure and temperature were measured together with the static pressure on the wall at eight positions shown in Fig. 1. For measurement of the air flow rate, a two-inch pipe with an orifice and a three-inch pipe with an orifice were connected to the suction pipe of the pump in parallel, and either one of them was used depending upon the rate of flow; the pressure difference across the orifice was measured with either a mercury manometer or a water manometer, depending upon the magnitude of the difference.

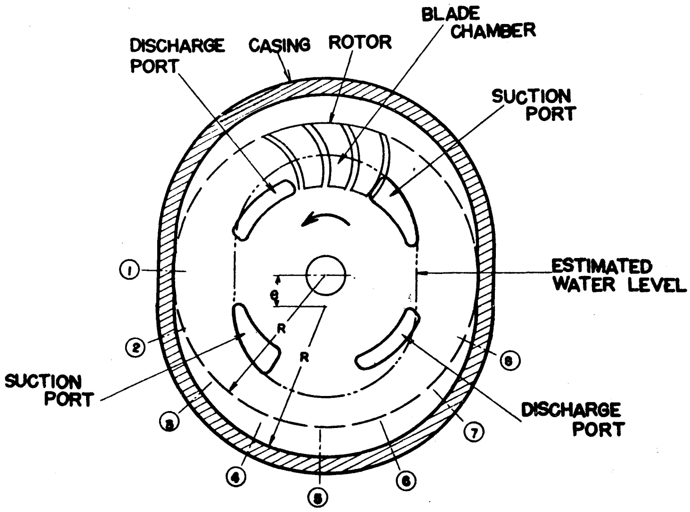

Fig. 1. Cross-section of pump

Since the performance of a pump depends on the amount of fluid circulating in the casing, the amount of fluid in the pump should be kept constant. If the amount of water supplied to the pump is a little more than the amount carried out by the discharged air, the excess water is discharged through the discharge pipe and the amount of fluid in the pump is kept constant. In this experiment the rate of water flow was adjusted to approximately 5 l/min so that a filament of water fell from the discharge pipe.

§ 3. Similarity Law.

When the rotor revolves, the fluid in the casing is pushed back toward the elliptic casing by centrifugal force and seals the periphery of the rotor as shown in Fig. 1, so that the space enclosed by the water level and two adjacent blades (named blade chamber) is air tight; the pressure in a blade chamber varies with the volume, or the radial height of the water level. That is, the volume of the blade chamber is maximum near the major axis, and air sucked in the chamber at this location is compressed as the rotor revolves and finally is discharged through the port located near the minor axis. The mechanism of compression of this pump is identical to that of reciprocating compressors except that in the former, the suction and discharge ports do not open in an optimum manner unless the pump operates at the rated compression ratio. In general the characteristics of this pump depend upon the motion of the water level, which is in turn affected by the operating conditions, e. g. compression ratio, suction pressure and peripheral velocity.

In order that the flow in a model pump is similar to the prototype pump, many conditions must be satisfied. However, the effect of Reynolds number is not significant and the gravitational force is negligible compared to the centrifugal force on the water; so the important conditions for similarity are represented by the following three conditions; where ρ_L and ρ_G are density of liquid and gas respectively: (1) compression ratio is identical for the two pumps, (2) concerning liquid flow, $[p_s/\rho_L U^2]_M = [p_s/\rho_L U^2]_P$ that is, depending upon the peripheral velocity ratio either the suction pressure is adjusted or a liquid of different density is chosen for the model, (3) concerning gas flow, $[p_s/\rho_G U^2]_M = [p_s/\rho_G U^2]_P$ that is, depending upon the peripheral velocity ratio and the suction pressure either the gas temperature is adjusted or a gas of different density is chosen for the model.

In general, pressure drop due to gas-flow is small compared to pressure drop due to liquid-flow. Therefore unless the gas flows through a narrow passage or a lot of gas leaks internally, the performance of a pump is not much effected by the gas flow. That is, if the third requirement is not significant the operating condition at 690 rpm, $p_s=0.905\ \mathrm{kg/cm^2}$ absolute, $p_d/p_s=1.8$ is similar to the condition at 550 rpm, $p_s=0.575\ \mathrm{kg/cm^2}$ absolute, $p_d/p_s=1.8$ because they satisfy the first

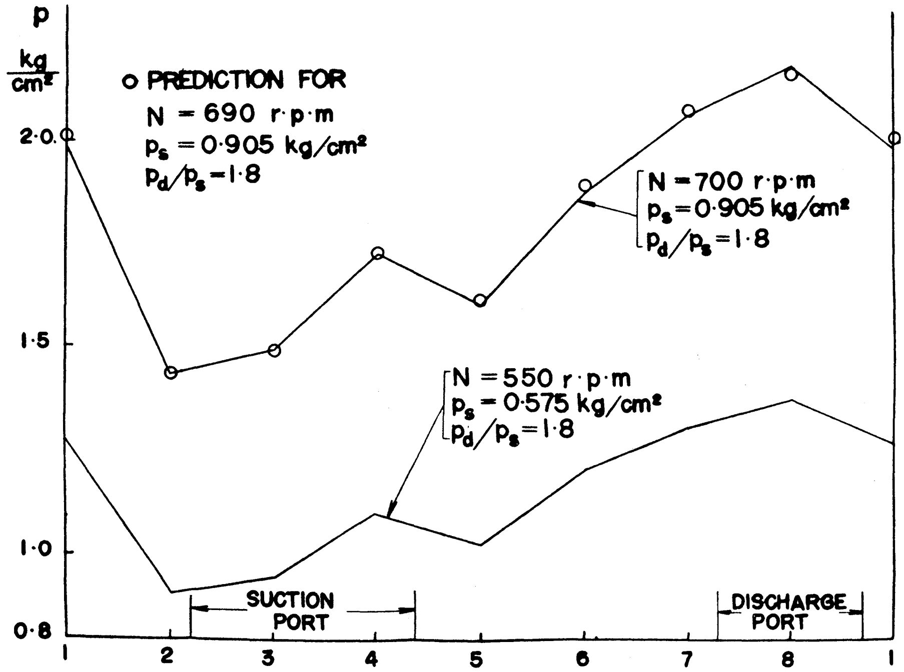

Fig. 2. Pressure distribution around casing

and the second requirements for similarity. The pressure distribution around the casing at 690 rpm, $p_s=0.905\ \mathrm{kg/cm^2}$, $p_d/p_s=1.8$ was predicted from the experimental pressure distribution at 550 rpm, $p_s=0.575\ \mathrm{kg/cm^2}$, $p_d/p_s=1.8$ and is shown in Fig. 2 as circles, while the experimental pressure distributions at 550 rpm, $p_s=0.575\ \mathrm{kg/cm^2}$, $p_d/p_s=1.8$ and at 700 rpm, $p_s=0.905\ \mathrm{kg/cm^2}$, $p_d/p_s=1.8$ are shown as the two full lines. The good agreement of the predicted values to the experimental values at 700 rpm, $p_s=0.905$ kg/cm^2 verifies the insignificance of the third condition of similarity.

Since the performance of a pump is mainly decided by liquid flow rather than gas flow in the pump, the following dimensionless parameters are proposed:

suction pressure: $p_s/(\rho_L U^2/2)=P_s$

discharge pressure: p_d/p_s

rate of flow: $q\,/\,2UeB=Q$

input power: $w\,/\,(\rho_L U^3 e B)=W$

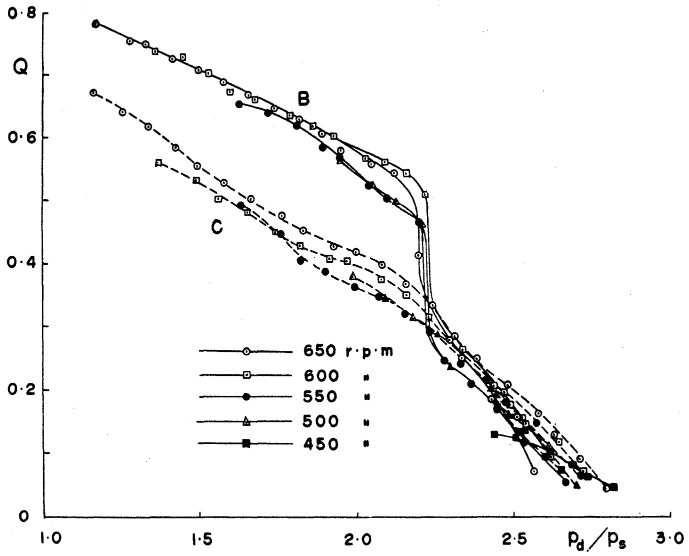

Fig. 3. Flow rate vs. compression ratio

Fig. 3 shows the relationship between the dimensionless gas flow rate and the compression ratio at $P_s=0.905$ for two pumps B and C. For each of them, the relationship is given by a group of curves which slightly depend on the speed of rotation. For a given compression ratio, a slightly larger value of Q is obtained at a higher speed. This is due to the fact that denser air was used at a higher speed than air at a lower speed, thus the experiment did not satisfy the similarity condition with respect to air and internal leakage was less significant at a higher speed.

Fig. 4 shows the relationship between dimensionless input power and compression ratio of the two pumps. It is recognized that the input power increases in proportion to the compression ratio. There is no radical change in input power near $p_d/p_s=2.2$ although Q decreases radically there. For a given compression ratio, a smaller dimensionless input power was observed at a higher speed, this is partly due to the mechanical friction.

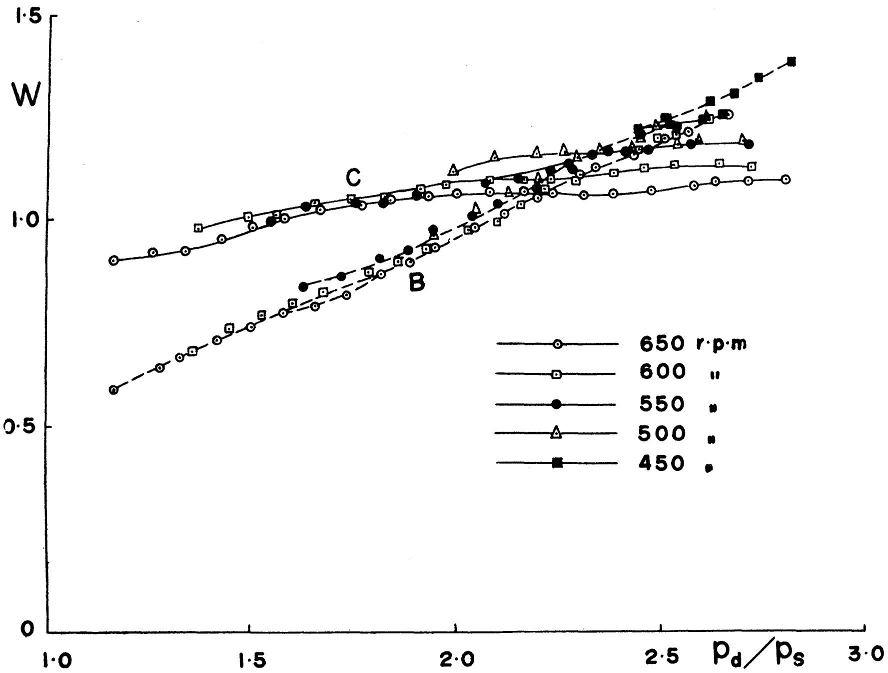

Fig. 4. Input power vs. compression ratio

§ 4. **Gas Flow Rate.**

Since a certain amount of liquid is circulating in the casing, the rate of liquid flowing through a meridian is identical to the rate through any other meridian. When liquid flows through the meridian of the minor axis the entire liquid is in the rotor, but in another meridian a certain amount of liquid flows in the tip space of crescent shape and consequently the volume of the blade chamber increases. Thus, the volume of gas flowing through the meridian is larger than that through the minor axis by the rate of liquid flowing in the tip space $c'\delta R\omega$, where δ is the depth of tip space and c' is a flow coefficient. As a result, the increment of gas volume enclosed by two meridian planes of angle $d\theta$ apart is $c'\delta Rd\theta$; that is, the volume of gas per angle $d\theta$ is

$$v\, d\theta = v_G d\theta + Rc'\delta\, d\theta, \qquad (1)$$

where v_G is the volume of gas per unit angle at the minor axis. As a result, the suction capacity of the pump is decided by the rate of liquid flowing through the tip space at the meridian where the suction port closes and by the amount of gas v_c left in blade chamber per unit angle when the discharge port closes.

Since a part of the liquid overflows through the discharge port, the liquid level or the amount of gas left in the blade chamber v_c is approximately kept constant and independent of the operating condition. The flow rate of liquid in the tip space varies depending on the operating condition, e. g. on the discharge pressure, but unless the flow rate varies in the meridian where the suction port closes the suction capacity is not affected and the performance of the pump is similar to that of a reciprocating compressor. That is, the gas flow rate at $p_d/p_s=1$ is equal to the rate of liquid flowing through the tip space in the meridian of the suction port closing, and the gas flow rate decreases by $v_G[(p_d/p_s)^{1/n}-1]$ as pressure ratio increases, where n is a polytropic coefficient of gas expanding in the blade chamber. The thick full line in Fig. 5 shows this relationship; six operating conditions,

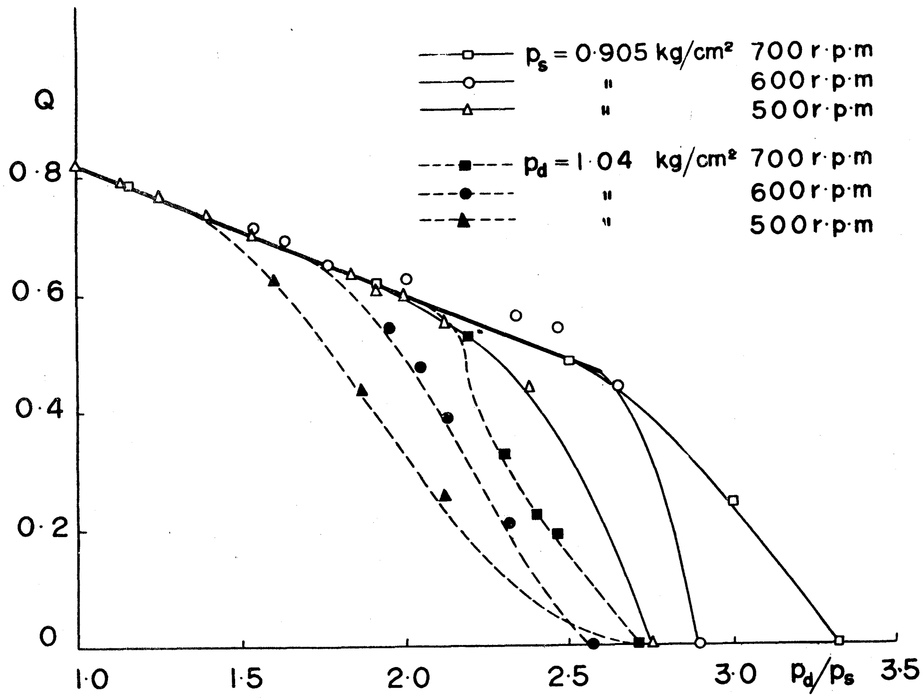

Fig. 5. Flow rate vs. compression ratio

which are not similar with each other, are represented by this single curve at a low compression ratio range. As the pressure ratio increases the six performance curves deviate from the thick full line one after another in the order of larger value of P_s, and at the highest compression ratio no experimental data remain on the thick line. The three fine full lines represent the operating conditions with a constant suction pressure $p_s=0.905$ kg/cm^2, while the three fine broken lines represent the operating conditions with a constant discharge pressure, $p_d=1.05$ kg/cm^2.

§ 5. Work done on Gas by Liquid.

The valves of a reciprocating compressor are operated by springs and are opened and closed at the ideal time. Since the locations of the suction and discharge ports of a wet vacuum pump are not adjustable, however, the ports open and close at unproper times unless the pump is operated at the rated condition; consequently additional work is required to drive the pump.

For the sake of simplicity, it is assumed that the liquid level at the suction and discharge ports is not effected by operating condition. In such cases the mechanism of gas compression is identical to that of a sliding-vane-type rotary compressor. Fig. 6A shows the indicator diagram of a pump which was designed for compression ratio of two but is operating at a compression ratio of four. When the compression ratio is two, the work done on the gas by the pump is the area

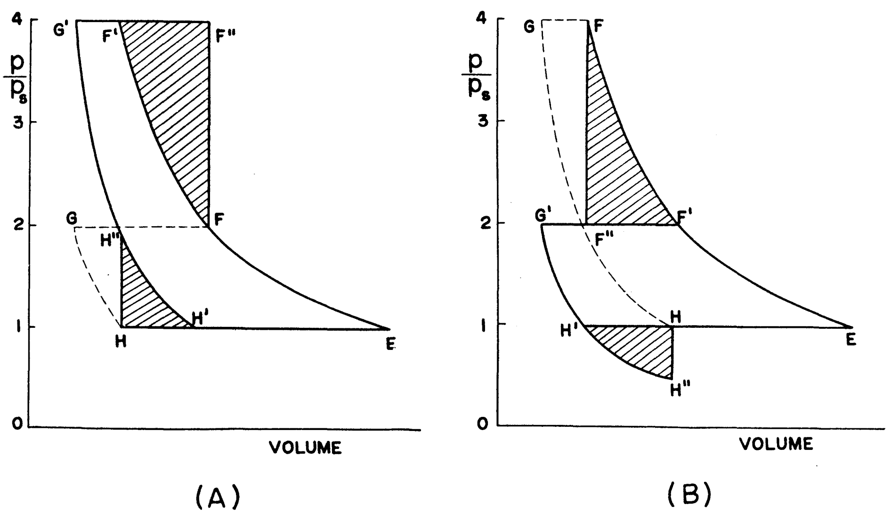

Fig. 6. Indicator diagram

EFGHE per cycle. If the pump is used at a compression ratio of four and the suction and discharge ports are opened and closed at the ideal time, the work done on gas is *EF'G'H'E*. As a matter of fact, however, the discharge port is opened when the pressure ratio becomes two, at *F* in the diagram, and the high discharge pressure *F''* works on the liquid surface in the blade chamber, or on the piston. Since then, the water level in the blade chamber moves radially inward against the discharge pressure until the discharge port is closed, and successively experiences the conditions *FF''F'G'*, that is, the water level does additional work shown by the shaded area *FF''F'F*. Similarly, during the expansion process the suction port is opened before the pressure in the blade chamber decreases down to the suction pressure and consequently a portion of the energy of the gas in the blade chamber is wasted without being conveyed to liquid and the loss is shown by the shaded area *H''HH'H''*.

Fig. 6B is an indicator diagram of a high pressure pump $p_d/p_s = 4$ being used for a low pressure $p_d/p_s=2$. Since the suction and discharge ports do not open early enough, additional work shown by the shaded areas $F'FF''F'$ and $H'H''HH'$ is done on gas in comparison to the ideal case. In either case, the additional work done on gas is simply consumed as mixing loss and is not converted to effective energy at all.

The work done on the gas per cycle per blade chamber is given by the equation

$$w=v_E\left[p_s\left(\frac{1}{n-1}\frac{1}{V_F^{n-1}}-\frac{n}{n-1}+V_H\right)\right.$$
$$\left.+p_d\left(\frac{1}{n-1}\frac{V_G^n}{V_H^{n-1}}-\frac{n}{n-1}V_G+V_F\right)\right] \tag{2}$$

where n is a polytropic coefficient, v_E is the volume of blade chamber when the suction port is closed, V_F, V_G and V_H are the ratios of the volume of the chamber when the discharge port opens, closes and the suction port opens to the volume v_E. If V_F is large and V_H is small the first term is negative and the second term is positive, and consequently the work on gas increases in proportion to the increase of p_d and to the decrease of p_s. Fig. 6A is an example of this kind of work. On the other hand if V_F is small and V_H is large the first term is opsitive and the second term is negative, and consequently the work on gas decreases in proportion to the increase of p_d and to the decrease of p_s. This kind of relationship was calculated with Equation (2) for six compressors of different rated compression ratios. In the calculation it was assumed that $V_G=0.10$, p_s is a constant and $n=1.25$. The results are shown in Fig. 7. The number on each straight line shows the rated compression ratio of the pump. The envelope of these straight lines shows the ideal relationship of work and compression ratio which can be achieved by an ideal reciprocating compressor.

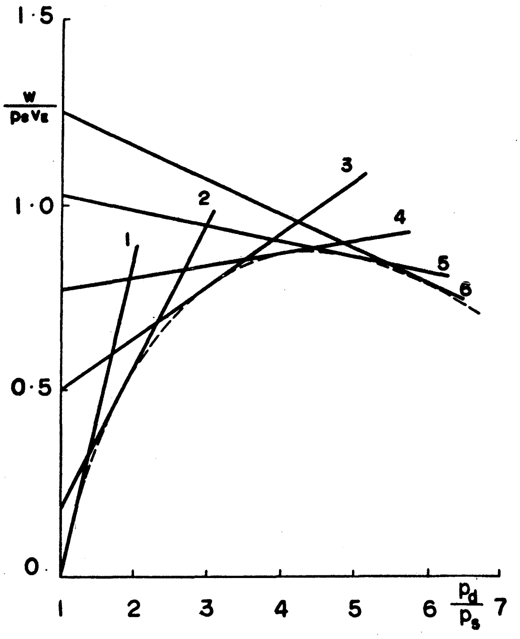

Fig. 7. Theoretical work on gas vs. compression ratio.

Fig. 4 shows the relationship between work and compression ratio for two pumps B and C. Both of the relationships are straight lines and, as the above theoretical calculation shows, the gradient is steep for the pump B which is designed for a lower pressure ratio.

In the above calculation it was assumed that the ports opened and closed independently of the operating condition and at a not-rated compression ratio the pressure in the blade chamber changes discontinuously at the discharge port. However, since the liquid-flow in the tip space may be decelerated or accelerated by a steep unfavorable or favorable pressure gradient at the discharge port, the volume and the pressure in the blade chamber may become close to the discharge pressure before the chamber is connected to the discharge port. If this is the case, the additional work is not as great as the above theory shows and the relationship between work and compression ratio is convex, i.e. between the straight line and the ideal envelope in Fig. 7. As a matter of fact, the relationships in Fig. 4 are almost straight lines. It is suspected that the above mentioned self-adjusting tendency due to steep pressure gradient is not significant.

§ 6. Work done on Liquid by the Rotor.

The gas in the rotor is compressed by the liquid surface, but no work is done directly on gas by the rotor because the velocity of gas relative to the rotor is small. That is, all the work supplied to the rotor is conveyed to the liquid, and a part of it is used to compress the gas as explained in the last sections.

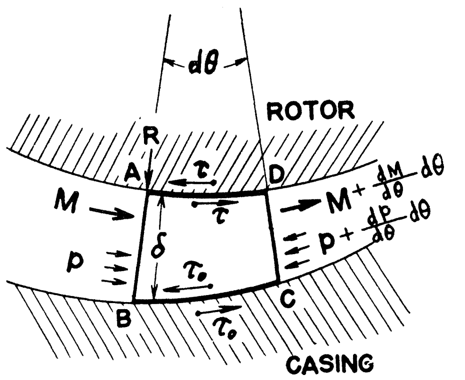

Fig. 8. Balance of forces in circumferential direction.

For simplicity's sake, it is assumed that the number of blades is infinity, so that the circumferential force on the blades of rotor is represented by tangential force τ around the rotor. If δ/R is small, the balance of tangential force on the control volume ABCD in Fig. 8 is

$$\tau=\tau_0+(\delta/R)\,(dp/d\theta)+(dM/Rd\theta) \qquad (3)$$

where M is the tangential momentum passing the tip space. The work of the rotor is

$$w=R^2\omega\int_0^{2\pi}\tau d\theta=R^2\omega\int_0^{2\pi}\left\{\tau_0+(\delta/R)\,(dp/d\theta)\right\}d\theta \qquad (4)$$

because the integral of the tangential-momentum change around the rotor is zero. In general, the tangential force τ_0 on the stationary wall depends on the velocity profile of the flow through the tip space, but it is assumed here that $\tau_0=0$ and the contribution of τ_0 will be considered later. The static pressure around the rotor is the sum of gas pressure in the blade chamber and of radial pressure difference due to the centrifugal force on liquid in the rotor. However, it is assumed here that the peripheral pressure around the rotor is equal to the gas pressure in the blade chamber, because the work absorbed by the liquid as radial displacement against the centrifugal force during compression process will be recovered by the rotor during expansion process.

Since the volume of blade chamber is related to the liquid flow in the tip space, the gas pressure around the rotor is obtainable from the geometry of the casing and the local flow coefficient $c'=\alpha c$, where c is the velocity coefficient at the meridian where the suction port is closed. The work on the liquid by the rotor is estimated under the assumption that the liquid level is not changed by the operating condition and that the gas changes state polytropically. Elimination of τ and p using Equations (3), (1), and pv^n=constant changes Equation (4) to

$$w=R^2\omega\int_0^{2\pi}\tau\, d\theta$$

$$=2R\omega p_s\left[\frac{(\alpha\delta)_F}{\alpha_1}\left(\frac{1}{V_F}\right)^n-\frac{\delta_E}{\alpha_1}+\frac{V_F^{1-n}-1}{(n-1)\alpha_2}\left(\frac{v_E V_G}{cR}+\delta_E\right)+\delta_F\left\{\frac{p_d}{p_s}-\left(\frac{1}{V_F}\right)^n\right\}\right]$$

$$+2R\omega p_d\left[\frac{(\alpha\delta)_H}{\alpha_3}\left(\frac{V_G}{V_H}\right)^n+\frac{v_E V_G}{(n-1)\alpha_4 cR}\left\{\left(\frac{V_G}{V_H}\right)^{n-1}-1\right\}+\delta_H\left\{\frac{p_s}{p_d}-\left(\frac{V_G}{V_H}\right)^n\right\}\right] \quad (5)$$

where α_1, α_2, α_3 and α_4 are respectively a type of weighted average value of α in the integration range. The equation shows that input power increases and efficiency decreases as c decreases provided v_E, V_F, V_G, V_H and p_s, p_d do not vary, or the work exerted on the gas remains constant. If the liquid in the tip space moves at the peripheral velocity of rotor, the coefficients α and c are unity and Equation (5) can be reduced to Equation (2).

In the above equation, it was assumed that the friction force on the wall τ_0 was zero. However, τ_0 is not zero and it can be estimated from disc friction force on the rotor if the pressure is uniform around the rotor. As the discharge pressure increases, liquid near the wall in the tip space is retarded and consequently τ_0 decreases. According to a periphery pump theory† τ_0 decreases by about 10~13% of the force which supports the pressure gradient in the tip space. It is concluded, therefore, that the work to drive a pump is the sum of the disc-friction loss at zero pressure rise and 0.87~0.9 of the work of Equation (5).

Disc-friction loss varies in proportion to the fifth power of pump dimension and to the cube of rotational speed, while effective work varies in proportion to the cube of pump dimension and to the rotational speed if suction pressure and compression ratio remain constant. Consequently efficiency of a pump increases as the peripheral velocity of rotor decreases. However, it will be explained in next section that there is a lower limit for rotational speed below which the pump

† Y. Senoo: Researches on peripheral pump. Reports of Research Institute for Applied Mechanics, Kyushu University Vol. 3, No. 10, 1954, pp. 53~113.

does not operate properly.

From the above considerations several factors were found which reduce the efficiency of a pump. They are: (1) polytropic coefficient is smaller than the adiabatic coefficient. (2) since the suction and discharge ports do not open at proper time at off-rated conditions, additional work, shown by shaded areas in Fig. 6, is required. (3) since the velocity of liquid in the tip space is smaller than the peripheral velocity of rotor, the work exerted on liquid by the rotor is larger than the work exerted on gas by the liquid, the difference being consumed in the liquid. (4) work due to friction force on the casing. (5) internal leakage of gas and pressure loss due to gas flow. (6) mechanical loss. Because of these losses, it is not easy to get a high efficiency even at the rated condition.

§ 7. Mechanism of Abnormal Operation.

The performance curve in Fig 3 shows a radical decrease in gas flow at a pressure ratio higher than 2.2. A pump should not be used at such a condition, so the limit of proper operation is studied here. Since the gas flow rate is simply decided by the volume of the blade chamber at the suction port, the flow rate—compression ratio performance is similar to that of a reciprocating compressor if the liquid flow near the suction port does not change depending upon the operating condition.

In general, the liquid flow in the tip space is retarded by pressure gradient

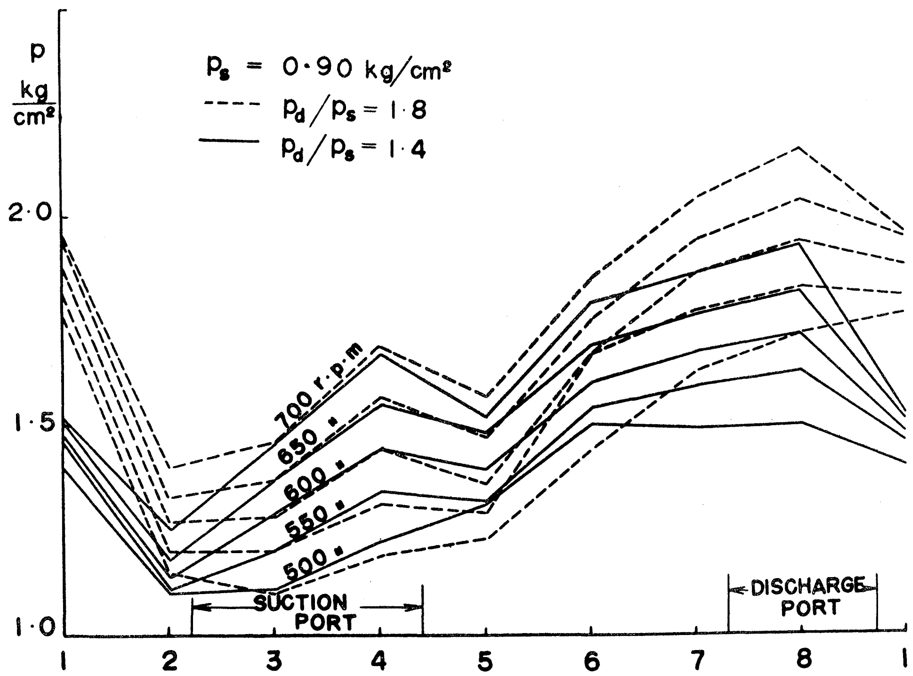

Fig. 9. Pressure distribution around casing.

and consequently the volume of the blade chamber decreases. Such an effect occurs at first near the exit port where a steep pressure gradient exists and the effect spreads upstream as the discharge pressure increases. If the effect reaches the suction port, the suction capacity of the pump decreases suddenly, but there is no basic change in liquid flow in the casing at that condition. Therefore, the input power—compression ratio performance does not show a peculiarity at the critical condition.

The above hypothesis is supported by pressure distribution around the pump casing. The lines in Fig. 9 show the pressure distribution for different compression ratios and for different speeds of rotation, while the suction pressure p_s is held constant, at 0.90 kg/cm^2 absolute. When the compression ratio changes from 1.4 to 1.8, the pressure at stations No. 3, No. 4, and No. 5 remains unchanged at 650 rpm, but at 600 rpm the variation of discharge pressure influences the pressure at No. 5 and at 550 rpm the variation influences the pressure at No. 4. Since the suction port closes between No. 4 and No. 5, it is expected that the flow rate—compression ratio performance curve at 650 rpm is normal at $p_d/p_s=1.4$ and 1.8, and that the flow rate should be less than the normal value at $p_d/p_s=1.8$ if the pump revolves at a speed less than 550 rpm. This expectation is supported by

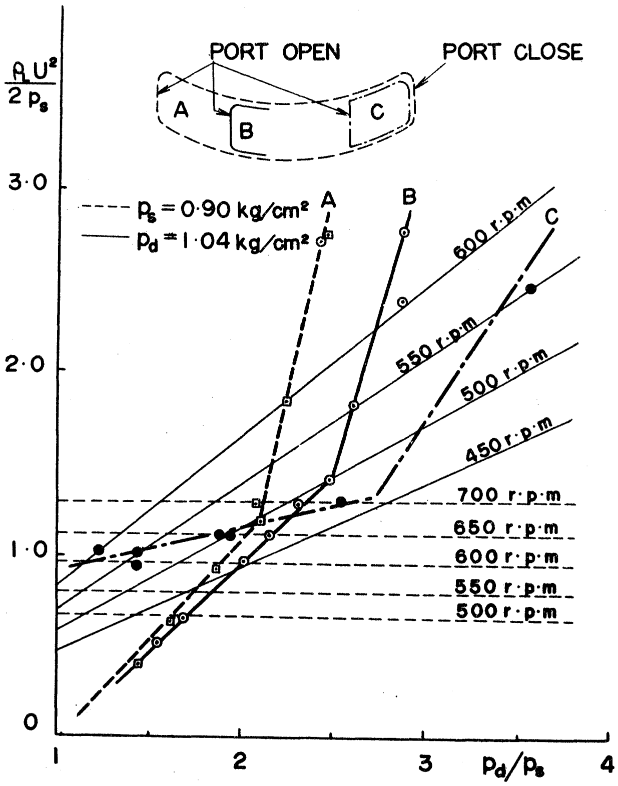

Fig. 10. Critical lines.

the performance curve in Fig. 5.

Fig. 10 is a graph of the critical compression ratio and the dimensionless dynamic pressure of peripheral velocity. The three lines A, B, and C show the relationship for the three pumps with the respective discharge ports A, B and C shown in the figure; the area to the left of each of these lines is composed of points each of which represents a normal operating condition.

If the pump A, which is designed for a low compression ratio, is used at a compression ratio of unity, no pressure gradient exists between the suction port and the discharge port. Therefore, however small the peripheral velocity may be, the operating condition should be normal, i.e. the critical line should pass through the origin. If the pump C, which is designed for a high compression ratio, is used at compression ratio of unity however, a high pressure region exists between the suction port and the discharge port because the tip space becomes narrower toward the discharge port. In order that a flow occurs against the pressure gradient, a certain peripheral velocity is necessary for the rotor. That is, the critical line passes through a non zero point on the ordinate axis.

If the suction pressure and the discharge pressure are reduced by the same amount at a constant peripheral velocity, the compression ratio increases and the variation of condition is represented by a line through the origin in Fig. 10; a point close to the origin corresponds to a condition with a large p_s. For a sufficiently large value of p_s, p_d/p_s is smaller than the rated value and a pressure higher than the discharge pressure occurs between the suction port and the discharge port; this condition deteriorates the tip space flow at the suction port, unless the rotor runs at a sufficiently high speed, consequently suction capacity decreases, and an abnormal performance appears. As p_s decreases and the value p_d/p_s becomes close to the rated value, the pressure difference decreases between the suction pressure and the maximum pressure in the tip space. That is, the operating condition is closer to the normal condition than the operating condition with a higher suction pressure. Consequently, if p_d/p_s is smaller than the rated compression ratio, the critical line in Fig. 10 is not so steep as the straight line which passes through the condition and the origin.

When p_d/p_s becomes the rated condition by reducing the suction pressure p_s, the pressure in the tip space increases monotonically from the suction port to the discharge port. If p_s decreases further, p_d/p_s becomes larger than the rated compression ratio and the region near the suction port does not contribute much to the pressure rise unless the liquid flow is deteriorated in this region; that is, the flow coefficient is reduced near the suction port. If the region covers the suction port the suction capacity decreases. That is, at a compression ratio range higher than the rated condition, the critical line is steeper than the line which passes through the origin and the given condition, and consequently the critical line changes slope at the rated compression ratio.

The shape of the critical line depends not only on the locations of the suction and discharge ports but also on the ratio of the tip space dimension to the diameter of rotor, on the hub tip ratio of the rotor and on several other as yet unexplored factors.

When this pump is used as a gas compressor, the operating condition changes on broken lines in Fig. 10 parallel to the abscissa depending upon the speed of rotation, and the pump does not operate properly at a compression ratio higher than the critical line. When the pump is used as a vacuum pump, the operating condition changes on a straight line rising toward right, fine full lines in the figure, depending upon the speed of rotation. A similar trend, rising toward right, is observed in the critical line shown as a thick broken line or a thick full line. Therefore, if the pump is used as a vacuum pump, by increasing the speed of rotation slightly the critical compression ratio increases considerably. The operating condition of the pump C, designed for a high compression ratio, crosses the critical line twice at a certain speed range. For example, at 500 rpm the pump operates properly only at the compression ratio between 2.0 and 3.0.

§ 8. Conclusion.

In order to make clear the mechanism of a wet type vacuum pump, a series of experiments were performed in the light of a simplified theory, and the following results were obtained.

(1) A fluid mechanically similar condition exists between two geometrically similar pumps when they are working at an identical compression ratio and in addition the suction pressure is proportional to the dynamic pressure of liquid moving at the peripheral velocity of the rotor.

(2) When a pump operates properly, the gas flow rate—compression ratio performance has a similar character to that of a reciprocating compressor. That is, a high suction vacuum is not obtainable unless the volume of blade chamber is very small at the discharge port closing. Therefore, suction and discharge ports on the hub are recommended rather than on the side walls for high suction vacuum pumps.

(3) The rate of gas flow decreases discontinuously at the critical operating condition but no radical change occurs for the input power; consequently, the efficiency decreases beyond this condition.

(4) There is a linear relationship between compression ratio and input power, and the increment of input power per increment of compression ratio is larger for a pump designed for a low compression ratio than for a pump of high compression ratio.

(5) The work on the gas varies in proportion to the peripheral velocity and the liquid friction work varies in proportion to the cube of the peripheral velocity. Therefore, the highest efficiency is achieved when a pump is used near the critical line.

(6) The work on the gas is proportional to the rate of liquid flow at the tip space $c\delta U$ and the input power is porportional to δU; therefore a high efficiency is achieved by increasing the velocity coefficient c.

This experiment was conducted at the Hydraulic Laboratory of Kyushu University by Messrs. Tanaka, Nakayama, Arakawa, Aono, Yoshihara, Nishitani, Muta and Motomatsu. Mr. Barry S. Seidel of the Massachusetts Institute of Technology aided by editing the paper. The authors express their thanks to these people and to the staff of the laboratory who helped them.

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