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DIAGRAMS FOR USE IN CALCULATION OF INDUCED VELOCITY BY PROPELLERS*

By Matsunosuke IWASAKI

Summary A formula for the calculation of the induced velocity by the vortex systems of propellers and windmills is derived. Diagrams for the calculation of the induced velocity are given.

Introduction There have been many theorys of propellers and windmills since Rankine (1865)[1] and Froude (1878)[2] analysed propellers by momentum theory. Among these theorys, vortex theory seems to have made the most important and significant contribution to the analysis of propellers, because this theory explains very clearly the velocity field caused by propellers. There are two kinds of vortex theory. One is the theory based on the assumption of infinitely many number of blades, and the other is the vortex theory which treats a propeller having finite number of blades. By the papers of Goldstein (1929),[3] Moriya (1933),[4] Kawada (1933),[5] Kondo (1942)[6] and Theodorsen (1944),[7] and by the work made by Lock (1930)[8] on the Application of Goldstein's theory to practical calculation of propeller performance, the latter theory of finite number of blades seems to have been almost completed. On the other hand, though the vortex theory of infinitely large number of blades is considerably simpler than the theory of finite number of blades, remarkable further development of the former beyond the works of Joukowski (1918)[9] and Glauert (1935)[10] has not been made. This seems to be due to two reasons, the first of which is that the theory is not so accurate as the theory of finite number of

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* Presented at the Meeting of the Japan Society of Aeronautical Engineering in Tokai-district, Japan, October 11, 1957.

[1] and [2], Durand, Aerodynamic Theory, Vol. IV, p. 178, Springer, 1935.

[3] Goldstein, S., On the vortex Theory of Screw Propellers, Proceedings of Royal Society of London, Series A, Vol. CXXIII, p. 440, 1929.

[4] Moriya, T., On the induced velocity and characteristics of a propeller, Journal of the Faculty of Engineering, Tokyo Imperial University, Vol. 20, No. 7, 1933.

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[6] Kondo, K., The Potential-theoretical Fundamentals of Aerodynamics concerning the Screw Propeller, Memoirs of the Faculty of Engineering, Kyushu Imperial University, Fukuoka, Japan, Vol. IX, No. 3, 1942.

[7] Theodorsen, T., Theory of Propellers, McGRAW-HILL, 1948.

[8] Lock, C. N. H., The Application of Goldsteins's Airscrew Theory to Design, A.R.C. Rep. and Mem. No. 1377, 1930.

[9] Joukowski, N., Theorie Tourbillonnaire de l'Helice Propulsive, Gauthier-Villars, Paris, 1929.

[10] Durand, *l.c.* ante p. 87.

blades, and the second of which is that there are no complete diagrams of functions which are necessary to apply the theory to practical cases. This theory is, however, considerably simple, and very much instructive in some problems of propellers, ducted propellers and windmills. Therefore if these functions are calculated completely, and illustrated by diagrams for the convenience of interpolation between these calculated values, these curves will be of much use for the analysis of propellers etc.. Accordingly the author tried to obtain the formula of induced velocity by the complete vortex system of propellers having infinitely many number of blades, and to make the diagrams mentioned above. Deriving this formula, the author found that there were three kinds of functions U_r, U_z and U_t which were necessary for the application of this vortex theory of infinitely large number of blades to practical cases. Among these three, the former two corresponded to the functions, which were calculated by Küchemann and Weber[11], by about two hundred and seventy number of values of them respectively in the case of a vortex ring. Consequently the author made use of the Küchemann's results to draw the curves of U_z and U_r. But the number of these calculated values, though they were very precise and almost complete, was insufficient to draw the curves which had sufficient accuracy for the calculation of propellers. Therefore the author calculated still more about three hundred number of them. The latter function U_t was found by the author for the first time. This represents the tangential component of induced velocity. The fuction was also calculated. These three kinds of functions are shown in seventeen figures, so that one may obtain required values of these functions by interpolating between these curves. The values read from these figures will be accurate so far as, at least, two significant figures. The numerical tables of U_r, U_z and U_t are also given in the present report.

Symbols

$\boldsymbol{\omega}$ = unit vector of vorticity, components of which are;
  ω_x of radial direction,
  ω_y of tangential direction,
  ω_z of axial direction.
x, y, z = orthogonal coordinates
r, θ, z = cylindrical coordinates of points. No suffix corresponds to point where induced velocity is culculated, suffix 1 means place where vortex element exists, and suffix 0 indicates propeller surface
$\mathring{i}, \mathring{j}, \boldsymbol{k}$ = unit vectors of x, y, z directions
R = Radius of Propeller
ds = length of vortex element $= dz_1/\omega_z$
$\boldsymbol{v}$ = velocity vector of induced velocity by total vortex system
$\boldsymbol{v}_h$ = velocity vector of induced velocity due to helical vortex cylinder
$\boldsymbol{v}_c$ = velocity vector of induced velocity by central vortex
$\boldsymbol{v}_b$ = velocity vector of induced velocity by bound vortices
V = velocity of oncoming flow
$\Gamma(r_0)$ = circulation around blade element of radius r_0

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[11] Küchemann, D. and Weber, J., Aerodynamics of Propulsion, McGRAW-HILL, 1953.

B = number of blades of propeller

$\gamma = \frac{B\Gamma}{4\pi^2 RV}$, nondimensional representation of Γ

Method of Calculation Because it is the purpose of the present paper, as mentioned above, to obtain the formula of vortex theory of infinitely large number of blades in the form of more practical use than it has been until now, we shall assume infinitely large number of blades. Then the helical vortex sheets of finite number behind propellers may be considered to be coaxial vortex cylinders of infinitely large number, over which helical vortex filaments are uniformly distributed (see Figure 1). The circulation around the vortex sheet is $-d\Gamma B/2\pi$ per unit angle. The generatrices of these cylinders are straight lines, if we neglect the contraction of the wake of a propeller. They are curved, however, if we consider the contraction of the wake. There is another vortex system, which is composed of the central straight vortex $-d\Gamma_0 B$ and the circular vortex sheet which represents the bound vortices of a propeller. On the arc of radius r_0 upon this disk, extended over the angle of unit radian, there is the element of vortex sheet of strengh $B\Gamma(r_0)/2\pi$. These three kinds of vortices compose the complete system of vortex of propellers, windmills and heicopter-rotors in hovering etc..

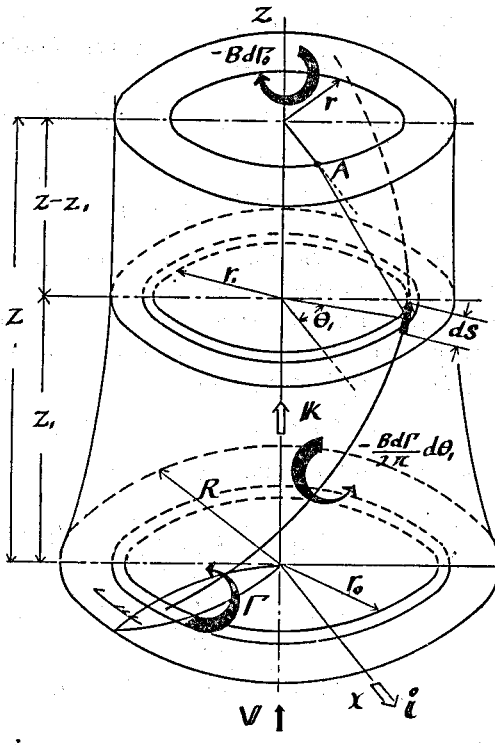

Fig. 1. Wake of propeller

By the negative sign attached to $d\Gamma B/2\pi$, we shall count the trailing vortex, which comes from the place where $\Gamma(r_0)$ is decreasing against r_0, positive.

The Biot-Savart's law in vector notaion is

$$dv_h = \frac{1}{4\pi}\left(-\frac{Bd\Gamma}{2\pi}\right)\frac{[\boldsymbol{\omega}_1\cdot \boldsymbol{l}]}{l^3}d\theta_1 ds \qquad (1)$$

giving the velocity vector dv_h at the point $(r, 0, z)$ induced by a vortex element of length ds at the point (r_1, θ_1, z_1). With the unit vectors $\mathring{\imath}$, $\mathring{\jmath}$, $\boldsymbol{k}$ in the directions of x, y, z axes, $\boldsymbol{\omega}_1$ can be written as

$$\omega_1 = (\omega_x \cos\theta_1 - \omega_y \sin\theta_1)\mathring{\imath} + (\omega_x \sin\theta_1 + \omega_y \cos\theta_1)\mathring{\jmath} + \omega_z \boldsymbol{k}\,. \qquad (2)$$

$\boldsymbol{l}$ is the radius vector from the vortex element to the pivotal point $A(r, 0, z)$:

$$\boldsymbol{l} = (r - r_1 \cos\theta_1)\, \mathring{\boldsymbol{i}} + (-r_1 \sin\theta_1)\, \mathring{\boldsymbol{j}} + (z - z_1)\, \boldsymbol{k} \tag{3}$$

with

$$|\boldsymbol{l}| = l = \sqrt{(r^2 + r_1^2) + (z - z_1)^2 - 2 r r_1 \cos\theta_1}\,. \tag{4}$$

Substituting Equations (2), (3), (4) and the relation of $ds = dz_1/\omega_z$, into Equation (1), and integrating over the space in which vortices exist, we obtain

$$\begin{aligned} \boldsymbol{v}_h &= \frac{1}{4\pi} \int\!\!\int\!\!\int \left(\frac{-B\,d\Gamma}{2\pi}\right) \frac{[\boldsymbol{\omega} \cdot \boldsymbol{l}]}{l}\, ds\, d\theta_1 \\ &= \frac{1}{2} \int\!\!\int\!\!\int \left(\frac{-B\,d\Gamma}{4\pi^2}\right) \Big[\Big\{ (z - z_1) \sin\theta_1 \cdot \mathring{\boldsymbol{i}} - (z - z_1) \cos\theta_1 \cdot \mathring{\boldsymbol{j}} - r \sin\theta_1 \cdot \boldsymbol{k} \Big\} \frac{\omega_x}{\omega_z} + \\ &+ \Big\{ (z - z_1) \cos\theta_1 \cdot \mathring{\boldsymbol{i}} + (z - z_1) \sin\theta_1 \cdot \mathring{\boldsymbol{j}} + (r_1 - r \cos\theta_1)\, \boldsymbol{k} \Big\} \frac{\omega_y}{\omega_z} + \\ &+ \Big\{ r_1 \sin\theta_1 \cdot \mathring{\boldsymbol{i}} + (r - r_1 \cos\theta_1)\, \mathring{\boldsymbol{j}} \Big\} \Big] \frac{dz_1\, d\theta_1}{l^3}\,. \end{aligned} \tag{5}$$

It may be easily seen that the first term expresses the contribution of the radial component of the cylindrical vortex surface, the second corresponds to the effect of the tangential vortex or vortex ring and the third term represents the induced velocity by the cylindrical vortex sheet over which axial straight vortex filaments are uniformly distributed. Now, using nondimensional expressions

$$\boldsymbol{v}_h/V,\ r/r_1,\ z/r_1,\ z_1/r_1,\ z_1/R,\ r_1/R \text{ and } \gamma = B\Gamma/4\pi^2 R V, \tag{6}$$

equation (5) can be written as follows:

$$\begin{aligned} \boldsymbol{v}_h/V = &- \int_{\text{root}}^{\gamma \text{ at tip}} d\gamma \int_0^\infty d\frac{z_1}{R} \int_0^{2\pi} \frac{1}{l_1^3} \left(\frac{R}{r_1}\right)^2 \Big[\frac{1}{2} \Big\{ \left(\frac{z - z_1}{r_1}\right) \sin\theta_1 \cdot \mathring{\boldsymbol{i}} - \left(\frac{z - z_1}{r_1}\right) \cos\theta_1 \cdot \mathring{\boldsymbol{j}} - \\ &- \frac{r}{r_1} \sin\theta_1 \cdot \boldsymbol{k} \Big\} \frac{\omega_x}{\omega_z} + \frac{1}{2} \Big\{ \left(\frac{z - z_1}{r_1}\right) \cos\theta_1 \cdot \mathring{\boldsymbol{i}} + \left(\frac{z - z_1}{r_1}\right) \sin\theta_1 \cdot \mathring{\boldsymbol{j}} + \\ &+ \left(1 - \frac{r}{r_1} \cos\theta_1\right) \cdot \boldsymbol{k} \Big\} \frac{\omega_x}{\omega_z} + \frac{1}{2} \Big\{ \sin\theta_1 \cdot \mathring{\boldsymbol{i}} + \left(\frac{r}{r_1} - \cos\theta_1\right) \cdot \mathring{\boldsymbol{j}} \Big\} \Big] d\theta_1\,, \end{aligned} \tag{7}$$

where

$$l_1 = \left\{ \left(1 + \frac{r}{r_1}\right)^2 + \left(\frac{z - z_1}{r_1}\right)^2 - 2 \frac{r}{r_1} \cos\theta_1 \right\}^{1/2}. \tag{8}$$

When we integrate this equation with respect to θ_1, it is necessary to perform the following three kinds of integrations:

$$\begin{aligned} I_1 &= \int_0^{2\pi} \frac{\sin\theta_1}{l_1^3}\, d\theta_1 \\ I_2 &= \int_0^{2\pi} \frac{\cos\theta_1}{l_1^3}\, d\theta_1 \\ I_3 &= \int_0^{2\pi} \frac{1}{l_1^3}\, d\theta_1\,. \end{aligned} \tag{9}$$

While the first integral is zero, the second and the third integrals can be, after some calculations, written as

$$I_2 = \frac{-2}{\left\{\left(1-\frac{r}{r_1}\right)^2+\left(\frac{z-z_1}{r_1}\right)^2\right\}^{1/2}} \cdot \frac{1}{\left(\frac{r}{r_1}\right)} \left\{K - \frac{\left(1+\frac{r^2}{r_1^2}\right)+\left(\frac{z-z_1}{r_1}\right)^2}{\left(1-\frac{r}{r_1}\right)^2+\left(\frac{z-z_1}{r_1}\right)^2} E\right\} \tag{10}$$

$$I_3 = \frac{4E}{\left\{\left(\frac{r}{r_1}+1\right)^2+\left(\frac{z-z_1}{r_1}\right)^2\right\}^{1/2}\left\{\left(\frac{r}{r_1}-1\right)^2+\left(\frac{z-z_1}{r_1}\right)^2\right\}},$$

where K and E are the complete elliptic integrals of the first and second kinds,

$$K = \int_0^{\pi/2} \frac{d\alpha}{\sqrt{1-k^2\sin^2\alpha}}$$

$$E = \int_0^{\pi/2} \sqrt{1-k^2\sin^2\alpha}\, d\alpha, \tag{11}$$

with modulus

$$k^2 = \frac{4\frac{r}{r_1}}{\left(\frac{r}{r_1}+1\right)^2+\left(\frac{z-z_1}{r_1}\right)^2}. \tag{12}$$

Using Equations (8), (9) and (10), Equation (7) can be rearranged to yield

$$\frac{\boldsymbol{v}_h}{V} = -\int_{\text{root}}^{\gamma \text{ at tip}} d\gamma \int_0^\infty d\left(\frac{z_1}{R}\right)\left(\frac{R}{r_1}\right)^2\left[U_r\frac{\omega_y}{\omega_z}\cdot\mathring{\imath} + \left(U_t - U_r\frac{\omega_x}{\omega_z}\right)\cdot\mathring{\jmath} + U_z\frac{\omega_y}{\omega_z}\cdot k\right]. \tag{13}^*$$

The functions U_r, U_z and U_t in Equation (13) are

$$U_z = \frac{1}{\left(\frac{r}{r_1}+1\right)^2+\left(\frac{z-z_1}{r_1}\right)^2}\left\{K - \frac{\left(\frac{z-z_1}{r_1}\right)^2+\left(\frac{r^2}{r_1^2}-1\right)}{\left(\frac{z-z_1}{r_1}\right)^2+\left(\frac{r}{r_1}-1\right)^2}E\right\}$$

$$U_r = \frac{\frac{-(z-z_1)}{r_1}}{\left(\frac{r}{r_1}\right)\left\{\left(1+\frac{r}{r_1}\right)^2+\left(\frac{z-z_1}{r_1}\right)^2\right\}^{1/2}}\left[K - \left\{1+\frac{2\frac{r}{r_1}}{\left(\frac{z-z_1}{r_1}\right)^2+\left(\frac{r}{r_1}-1\right)^2}\right\}E\right]$$

$$U_t = \frac{r}{r_1}\cdot U_z + \frac{\left(\frac{r}{r_1}-1\right)}{\left(\frac{z-z_1}{r_1}\right)}\cdot U_r. \tag{14}$$

The functions U_z and U_r are the same with $v_{\gamma z}{}^*$ and $v_{\gamma r}{}^*$ of Küchemann[12]

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* Generally speaking r/r_1, z/r_1, z_1/r_1 and r_1/R are the functions of z_1/R_1, for r_1 changes as z_1 changes. Only in the space, between which r_1/R can be considered constant, these variables become independent of the variable z_1/R.

[12] Küchemann, l.c. ante p. 88.

respectively. U_z, U_r and U_t are the functions of r/r_1 and $(z-z_1)/r_1$. When $(z-z_1)/r_1$ is zero, we must use the following formula for U_t:

$$U_{t_{\frac{z-z_1}{r_1}=0}}=\frac{1}{(r/r_1)(1+r/r_1)}\left\{K+E\frac{(r/r_1)^2-1}{(1-r/r_1)^2}\right\}, \tag{15}$$

which can be derived easily by putting U_z and U_r of Equation (14) into the third equation and substituting the value of 0 into $(z-z_1)/r_1$. The induced velocity by the central straight vortex filament of strengh $-Bd\Gamma_0$ is, in nondimensional form,

$$\frac{\boldsymbol{\nu}_c}{V}=-d\gamma_0\frac{\pi}{r/R}\left(1+\frac{z/R}{\{(r/R)^2+(z/R)^2\}^{1/2}}\right)\cdot \mathring{\boldsymbol{j}}\,. \tag{16}$$

And the induced velocity due to the circular vortex disk is

$$\frac{\boldsymbol{\nu}_b}{V}=-\int_0^1 \gamma\cdot\left(\frac{R}{r_0}\right)^2\cdot U_{r_{\frac{z_1}{R}=0}}\,d\left(\frac{r_0}{R}\right). \tag{17}$$

This equation can be obtained, if we put Γ in stead of $-d\Gamma$ in Equation (5), use the following expression for $\boldsymbol{\omega}_1$ and $\boldsymbol{l}$:

$$\boldsymbol{\omega}_1=\omega_x\cos\theta_0\cdot\mathring{\boldsymbol{i}}+\omega_x\sin\theta_0\cdot\mathring{\boldsymbol{j}} \tag{18}$$

$$\boldsymbol{l}\;=(r-r_0\cos\theta_0)\cdot\mathring{\boldsymbol{i}}+(-r_0\sin\theta_0)\cdot\mathring{\boldsymbol{j}}+z\cdot\boldsymbol{k}\,, \tag{19}$$

and perform the same calculation as in the case of $\boldsymbol{\nu}_h/V$.

Therefore $\boldsymbol{\nu}_h/V$, $\boldsymbol{\nu}_c/V$ and $\boldsymbol{\nu}_b/V$ of Equations (13), (16) and (17) being added, the total induced velocity at point A is given by

$$\begin{aligned}\frac{\boldsymbol{\nu}}{V}=\frac{\boldsymbol{\nu}_h+\boldsymbol{\nu}_c+\boldsymbol{\nu}_b}{V}=-\Bigg[&\int_{\text{root}}^{\gamma\text{ at tip}}d\gamma\int_0^\infty d\left(\frac{z_1}{R}\right)\left(\frac{R}{r_1}\right)^2U_r\frac{\omega_y}{\omega_z}\cdot\mathring{\boldsymbol{i}}+\\&+\Bigg(\int_{\text{root}}^{\gamma\text{ at tip}}d\gamma\int_0^\infty d\left(\frac{z_1}{R}\right)\cdot\left(\frac{R}{r_1}\right)^2\left(U_t-\frac{\omega_x}{\omega_z}U_r\right)+\\&+d\gamma_0\frac{\pi}{r/R}\left(1+\frac{z/R}{\{(r/R)^2+(z/R)^2\}^{1/2}}\right)+\\&+\int_0^1\gamma\cdot\left(\frac{R}{r_0}\right)^2U_{r_{\frac{z_1}{R}=0}}\,d\left(\frac{r_0}{R}\right)\Bigg)\cdot\mathring{\boldsymbol{j}}+\\&+\int_{\text{root}}^{\gamma\text{ at tip}}d\gamma\int_0^\infty d\left(\frac{z_1}{R}\right)\left(\frac{R}{r_1}\right)^2U_z\frac{\omega_y}{\omega_z}\cdot\boldsymbol{k}\Bigg].\end{aligned} \tag{20}$$

As mentioned above, U_z and U_r are the same with $v_{\gamma_x}{}^*$ and $v_{\gamma_r}{}^*$ calculated by Küchemann and Weber, which are the induced velocity by a vortex ring. Therefore these represent the induced velocity due to a circular vortex or vortex ring of radius r_1, multiplied by $(2\pi r_1/\text{circulation})$. In the present case this ring is placed on the plane which is perpendicular to z axis at $z=z_1$ with its center on z axis. U_t is $(2\pi r_1/\text{circulation})$ times the induced velocity by the cylindrical vortex of radius r_1, the central section of which is on the plane perpendicular to z axis at z_1 and the element of vortex sheet of which is straight vortex filament of z direction, of unit length. The function U_z and U_t are even functions, and U_r is odd function of $(z-z_1)/r_1$. Therefore these functions vary with $(z-z_1)/r_1$. Consequently there are no change in these functions if z or z_1 changes individually, so long as the value of $(z-z_1)/r_1$ remains unchanged. So that, it is sufficient, for the practical calculation of propellers, to calculate these functions corresponding to various combinations of the values of r/r_1 with positive values of z/r_1, putting $z_1=0$.

Inspecting Equation (20), one will be able to find that, if the values of U_z, U_r and U_t are calculated, and shown by diagrams with the variables of r/r_1 and z/r_1, the integrations of this equation only by z_1 and γ will give the value of v/V. The author has shown by Equation (14) that the third function U_t is composed of U_r and U_z. Therefore we can calculate U_t by this equation, if we know the values of U_z and U_r which correspond to U_t. But, provided the numerical tables and figures of U_t are given in complete form as well as in the cases of U_r and U_z, the calculation of the induced velocity of a propeller will become less laborious than without the figures and table of U_t. According to the considerations mentioned above, the author calculated these three functions in wide ranges of r/r_1 and z/r_1.

U_z and U_r have been calculated by küchemann for about two hundred and seventy combinations of r/r_1 with z/r_1, but this number of calculated values, in spite of their exactness and their being almost complete, is insufficient not only for the calculation of the induced velocity of propellers but also for drawing the diagrams of these values.

Therefore the author calculated still more about three hundred values of U_r and U_z respectively, and over six hundred number of the values of U_t, of which about two hundred and seventy were calculated by using Küchemann's results of calculation.

Some Küchemann's values which had very few number of significant figures were calculated again to increase the number of significant figures, if not so we could not draw smooth curves through these Küchemann's values. In the present calculation the values of K and E which were shown by Equation (11) were read from Hayashi's numerical Table[13] by interpolation. When the value of $1-k^2$ becomes very near zero, K and E were calculated by the following asymptotic formulas:[14]

$$K=A+\frac{A-1}{4}(1-k^2)+\frac{9}{64}\left(A-\frac{7}{6}\right)(1-k^2)^2+\frac{25}{256}\left(A-\frac{37}{30}\right)(1-k^2)^3+\dots$$

$$E=1+\frac{1}{2}\left(A-\frac{1}{2}\right)(1-k^2)+\frac{3}{16}\left(A-\frac{13}{12}\right)(1-k^2)^2+\frac{15}{128}\left(A-\frac{6}{5}\right)(1-k^2)^3+\dots, \tag{21}$$

where

$$A=\log e\frac{4}{\sqrt{1-k^2}}$$

$$1-k^2=\frac{\left(\frac{r}{r_1}-1\right)^2+\left(\frac{z-z_1}{r_1}\right)^2}{\left(\frac{r}{r_1}+1\right)^2+\left(\frac{z-z_1}{r_1}\right)^2}. \tag{22}$$

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[13] Hayashi, K., Advanced Functional Table, Iwanami, Tokyo, 1941, (in Japanese).

[14] Jahnke, E. and Emde, F., Tables of Functions, p. 145, 1933.

The calculated values of U_z, U_r, and U_t are tabulated in Tables 1.1, ..., 3.13,† and shown by diagrams in Figures 2.1, ..., 4.5. These three kinds of curves are composed with six or five sheets of diagrams respectively. These values decrease rapidly as r/r_1 and z/r_1 increase, but the magnitude of $U \cdot d(z_1/R)$ in Equation (20) does not become small so rapidly, even if r/r_1 and z/r_1 become considerably large. Therfore it is necessary to draw curves, from which we can read U_r etc. with considerable number of significant figures in wide ranges of r/r_1 and z/r_1. To obtain such diagrams as these, it is necessary to divide each kind of curves into several groups and to draw each group in separate diagrams respectively, so that each of them may show the values of U_r, U_z and U_t with considerable accuracy in restricted range of variables r/r_1 and z/r_1. Accordingly in each kind of figures, the length of abscisse for unit value of r/r_1 decreases as the number of figure increases, while the length of ordinate corresponding to unit values of U_r, U_z and U_t increases. The curve having the same value of z/r_1 is drawn in different successive sheets repeatedly to assure the continuity of the diagrams of U_r, U_z and U_t respectively. In these diagrams thus obtained one will be able to obtain required values of U_r, U_z and U_t with, at least, two exact significant figures throughout the ranges of $0 < r/r_1 < 40$ and $0 < z/r_1 < 40$. The numerical tables in the present paper will be useful, when one wants to make diagrams of U_r, U_z and U_t anew, or when one tries to calculate a function, which contains U_r etc. in it, with considerable number of significant figures. The curves drawn with the numerical values of Tables 1.1, ..., 3.13 are very smooth. Therefore these diagrams and tables will have sufficient accuracy for practical use.

The analysis of propellers and windmills with these diagrams are now being under way by the author. The results of calculation will be reported in future.

Conclusions A formula for the calculation of the induced velocity by the compete system of vortex system behind propellers is derived on the assumption of infinitely large number of propeller blades. U_r, U_z and U_t, which are necessary for the application of this theory to practical cases, are calculated and shown by diagram of seventy sheets. The numerical tables of these values are also given.

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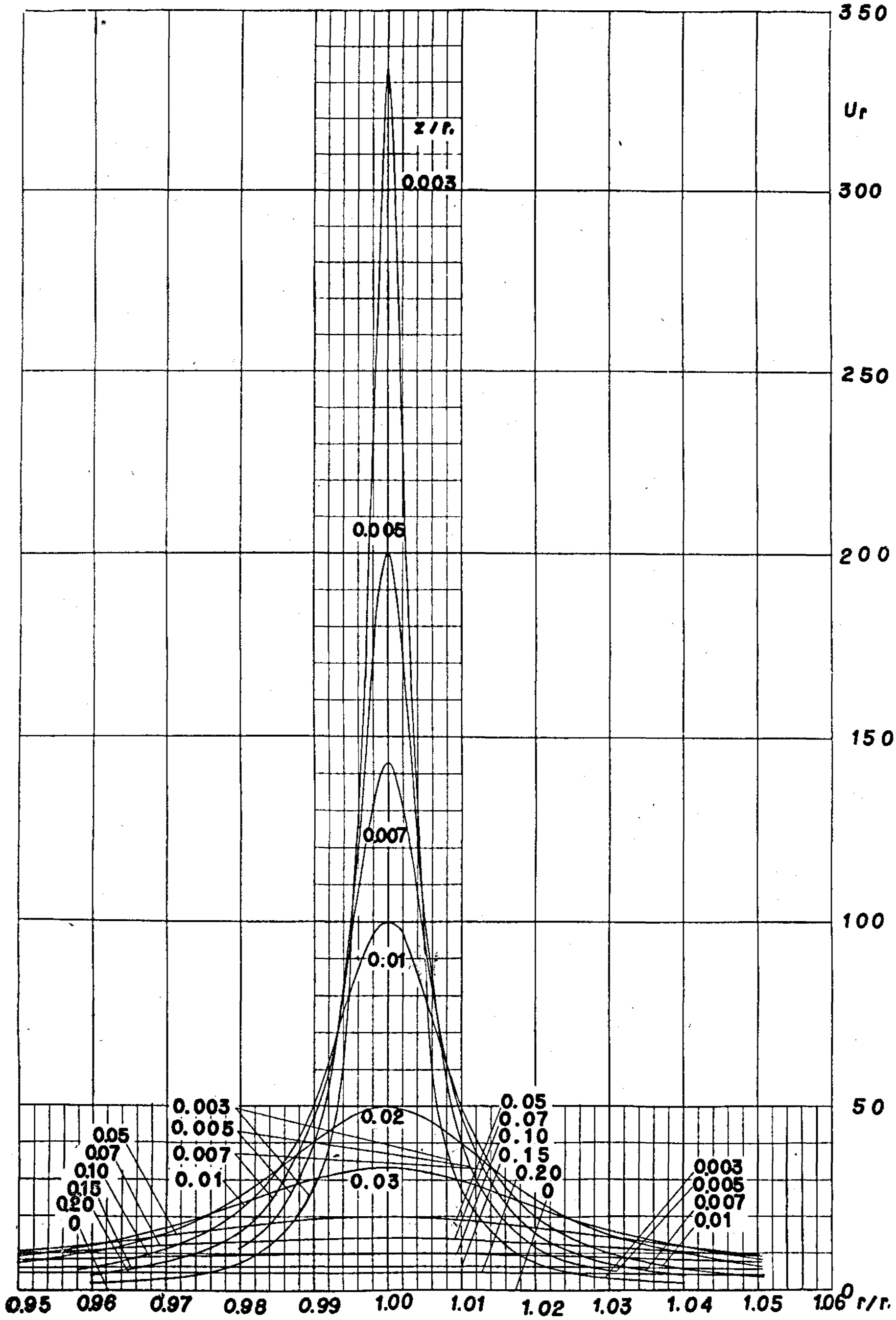
† In these tables, the numerical values calculated by the author are described with three or four significant figures, so that one may obtian smooth curves through every calculated point.


350
Ur
300
250
200
150
100
50
0
z/r
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0.005
0.007
0.01
0.02
0.03
0.05
0.07
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0.95
0.96
0.97
0.98
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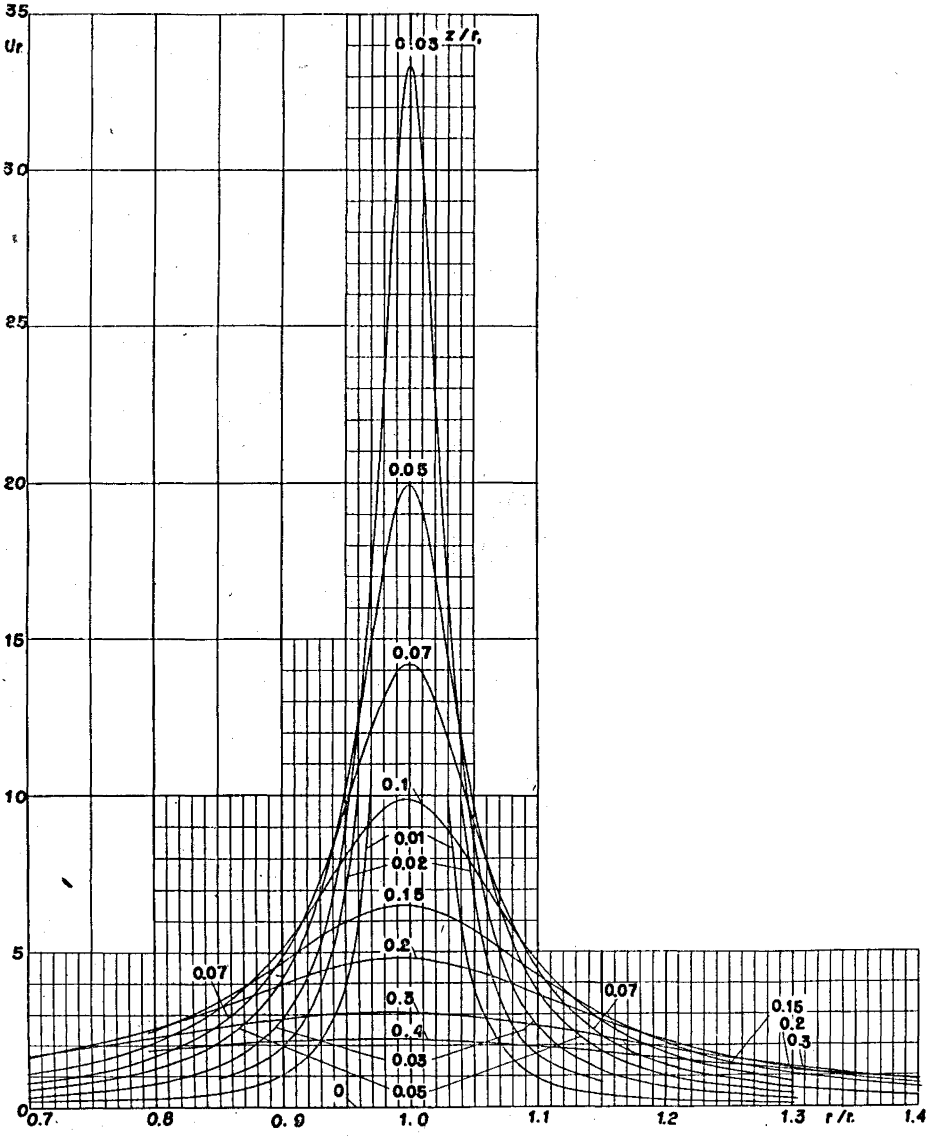
1.02
1.03
1.04
1.05
1.06
r/r

Fig. 2.1. U_r


35
Ur
30
25
20
15
10
5
0
0.7
0.8
0.9
1.0
1.1
1.2
1.3
r/r.
1.4
z/r.
0.03
0.05
0.07
0.1
0.01
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0.15
0.2
0.3
0.4
0.03
0.05
0
0.07
0.07
0.15
0.2
0.3

Fig. 2.2. U_r

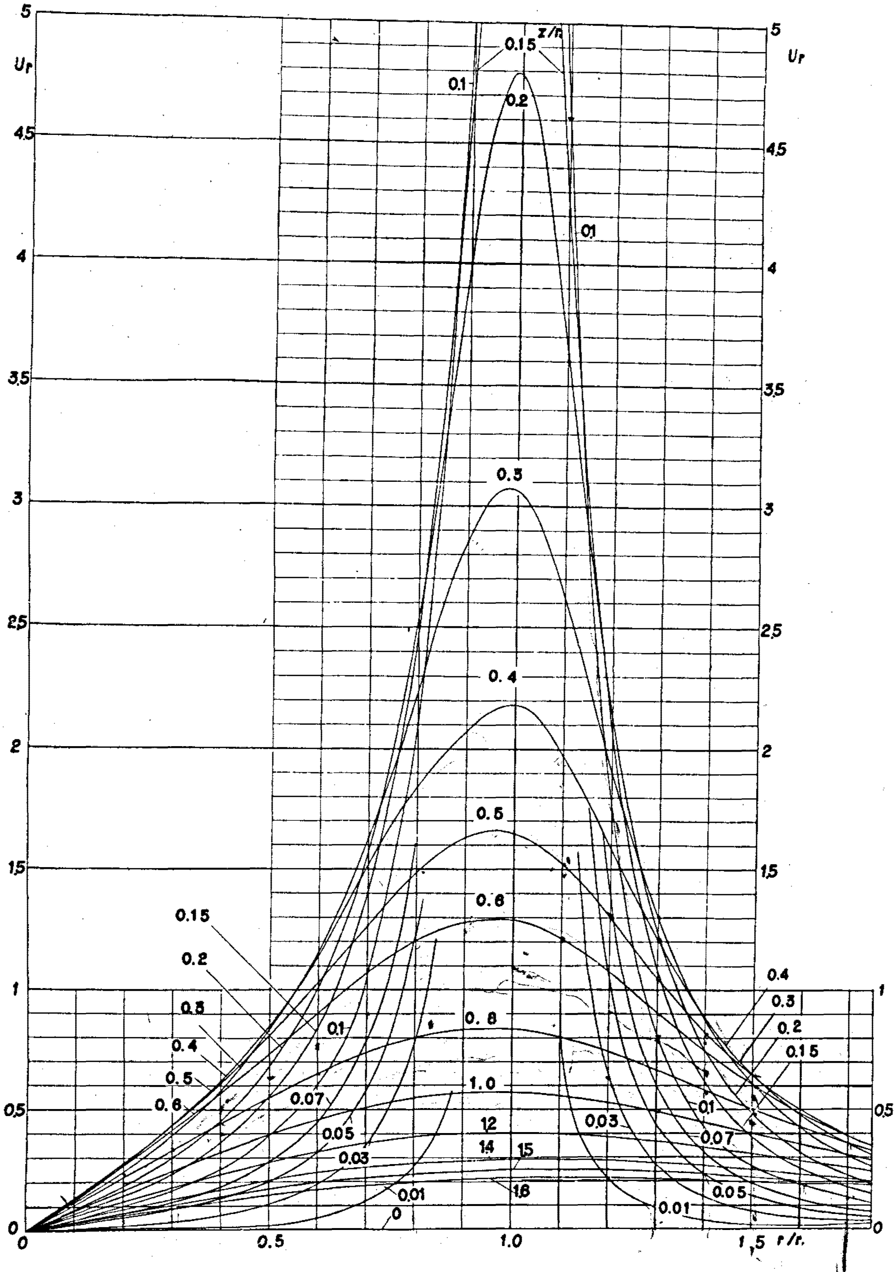

5
U_r
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
z/r_0
0.15
0.1
0.2
0.1
0.3
0.4
0.5
0.6
0.8
1.0
1.2
1.4
1.5
1.6
0.15
0.2
0.3
0.4
0.5
0.6
0.1
0.07
0.05
0.03
0.01
0
0.03
0.01
0.05
0.07
0.1
0.4
0.3
0.2
0.15
0
0.5
1.0
1.5
r/r_0

Fig. 2.3. U_r

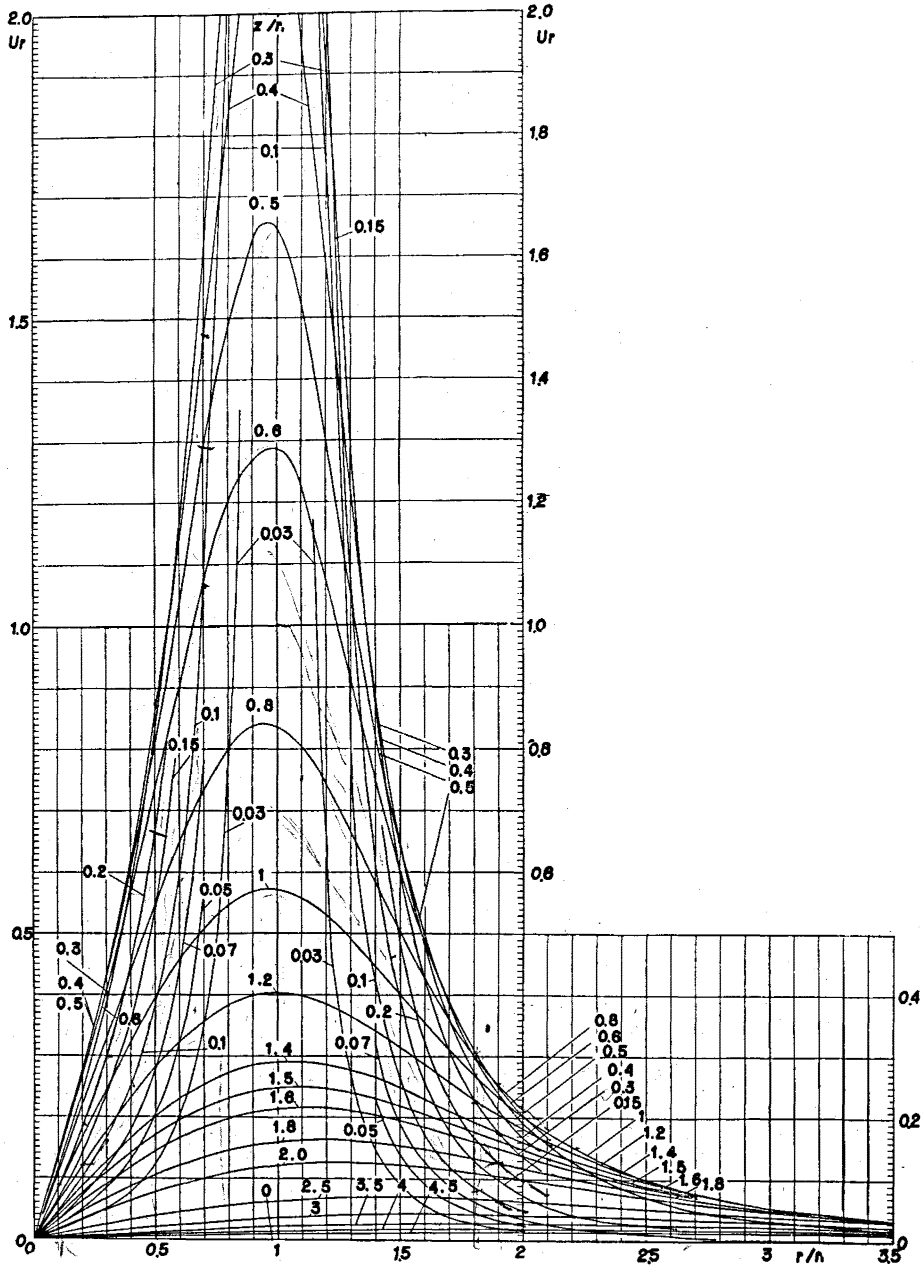

2.0
Ur
1.5
1.0
0.5
0
z / r.
0.3
0.4
0.1
0.5
0.15
0.6
0.03
0.8
0.1
0.15
0.03
0.2
0.05
1
0.3
0.4
0.5
0.6
0.07
0.03
0.1
1.2
0.2
0.1
1.4
0.07
1.5
1.6
1.8
0.05
2.0
0
2.5
3.5
4
4.5
3
0.3
0.4
0.5
0.8
0.6
0.5
0.4
0.3
0.15
1
1.2
1.4
1.5
1.6
1.8
2.0
Ur
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.5
1
1.5
2
2.5
3
r/r
3.5

Fig. 2.4. U_r

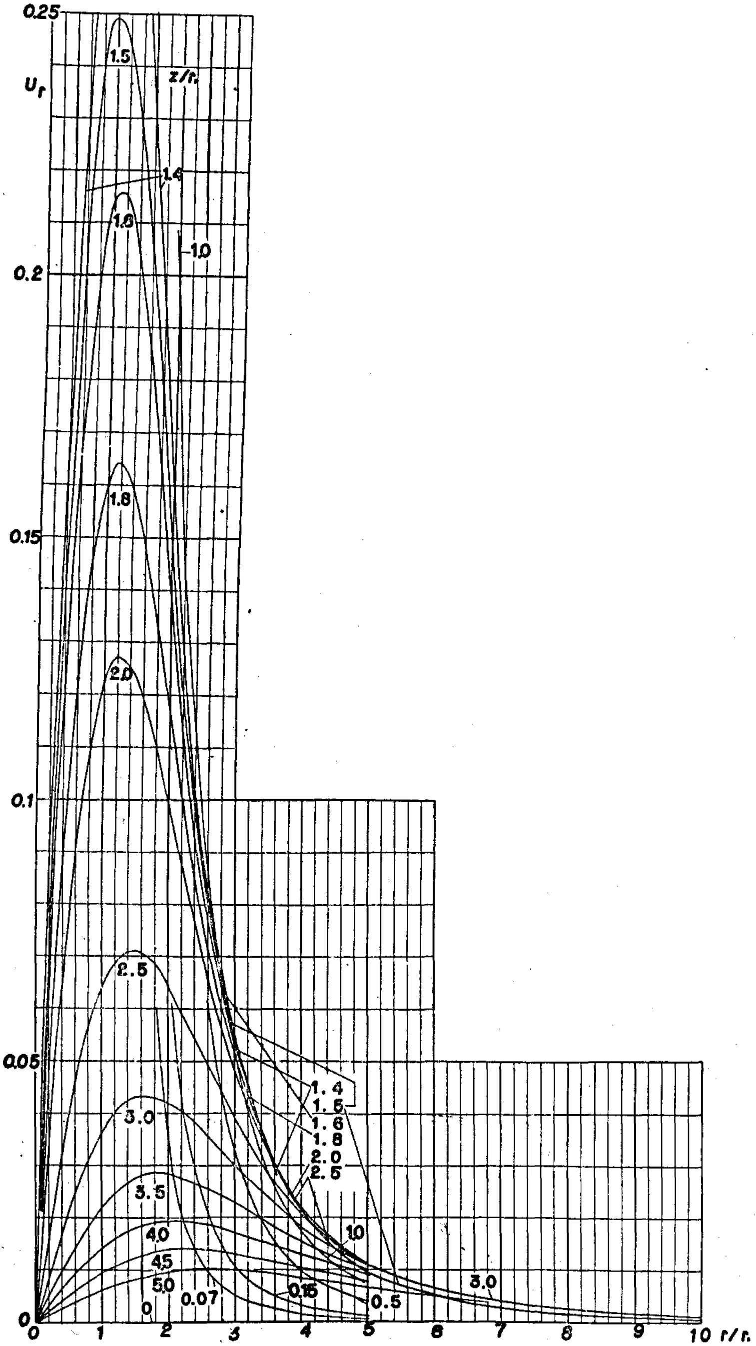

0.25
U_r
z/r.
1.5
1.4
1.6
1.0
0.2
1.8
0.15
2.0
0.1
2.5
0.05
3.0
3.5
4.0
4.5
5.0
0
0.07
1. 4
1. 5
1. 6
1. 8
2. 0
2. 5
1.0
0.15
0.5
3.0
0
0
1
2
3
4
5
6
7
8
9
10 r/r.

Fig. 2.5. U_r

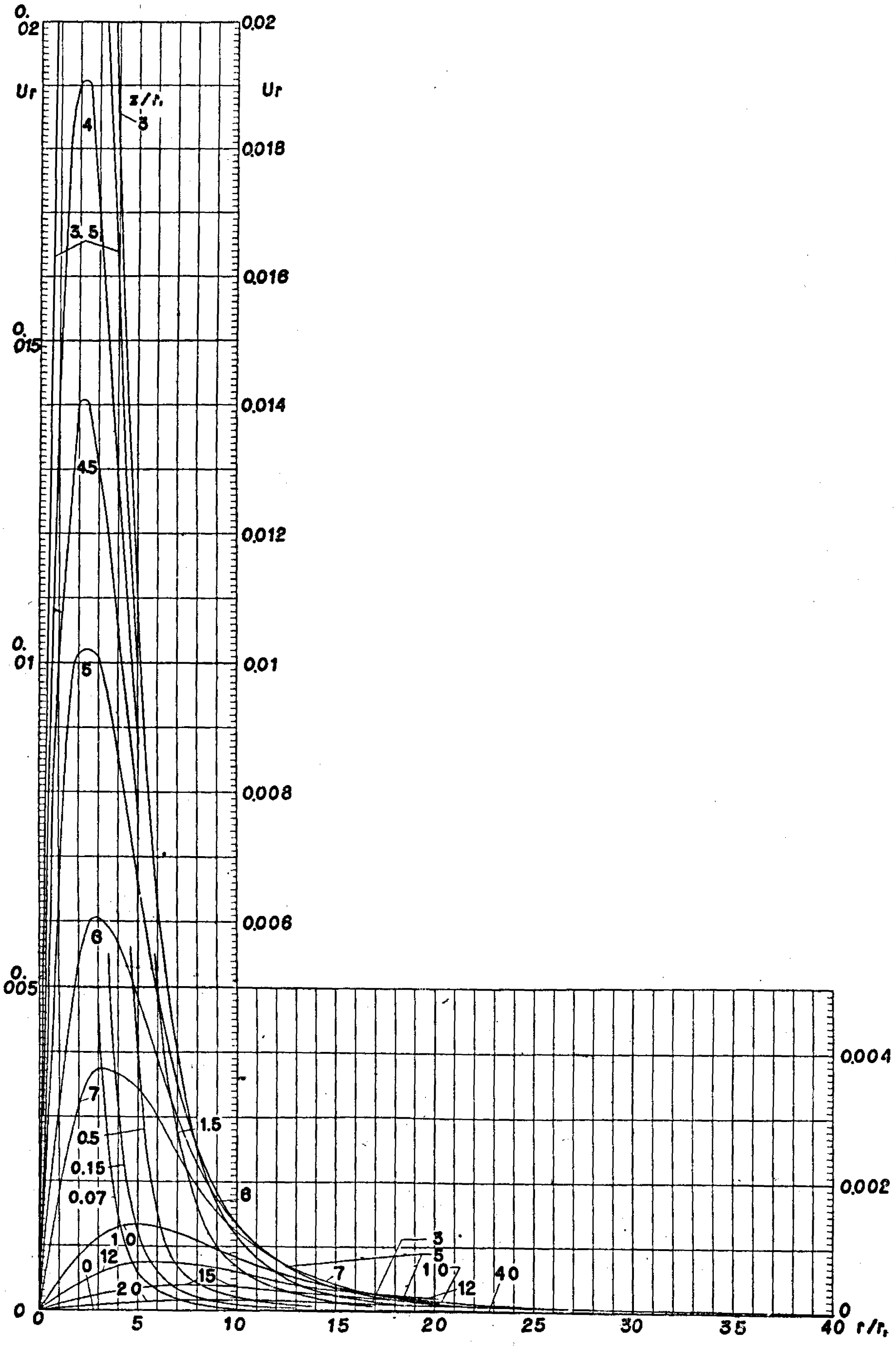

0.02
Ur
z/r₁
4
3
0.02
Ur
0.018
3.5
0.016
0.015
0.014
4.5
0.012
0.01
5
0.01
0.008
6
0.006
0.005
0.004
7
0.5
1.5
0.15
0.07
8
0.002
1.0
3
5
0
12
7
10
20
15
12
40
0
0
5
10
15
20
25
30
35
40
r/r₁

Fig. 2.6. U_r

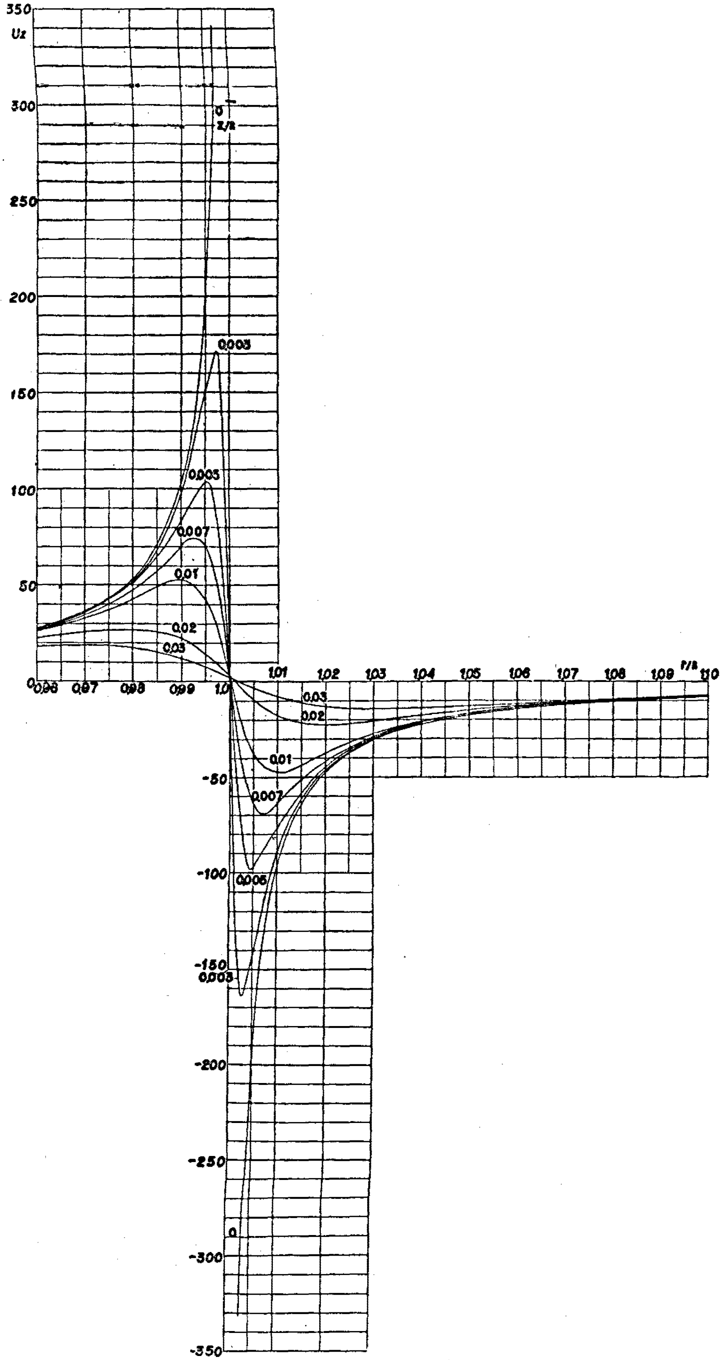

350
Uz
300
250
200
150
100
50
0
0,96
0,97
0,98
0,99
1,0
1,01
1,02
1,03
1,04
1,05
1,06
1,07
1,08
1,09
r/R
1,10
0
z/R
0,003
0,005
0,007
0,01
0,02
0,03
-50
-100
-150
-200
-250
-300
-350

Fig. 3. 1. U_z

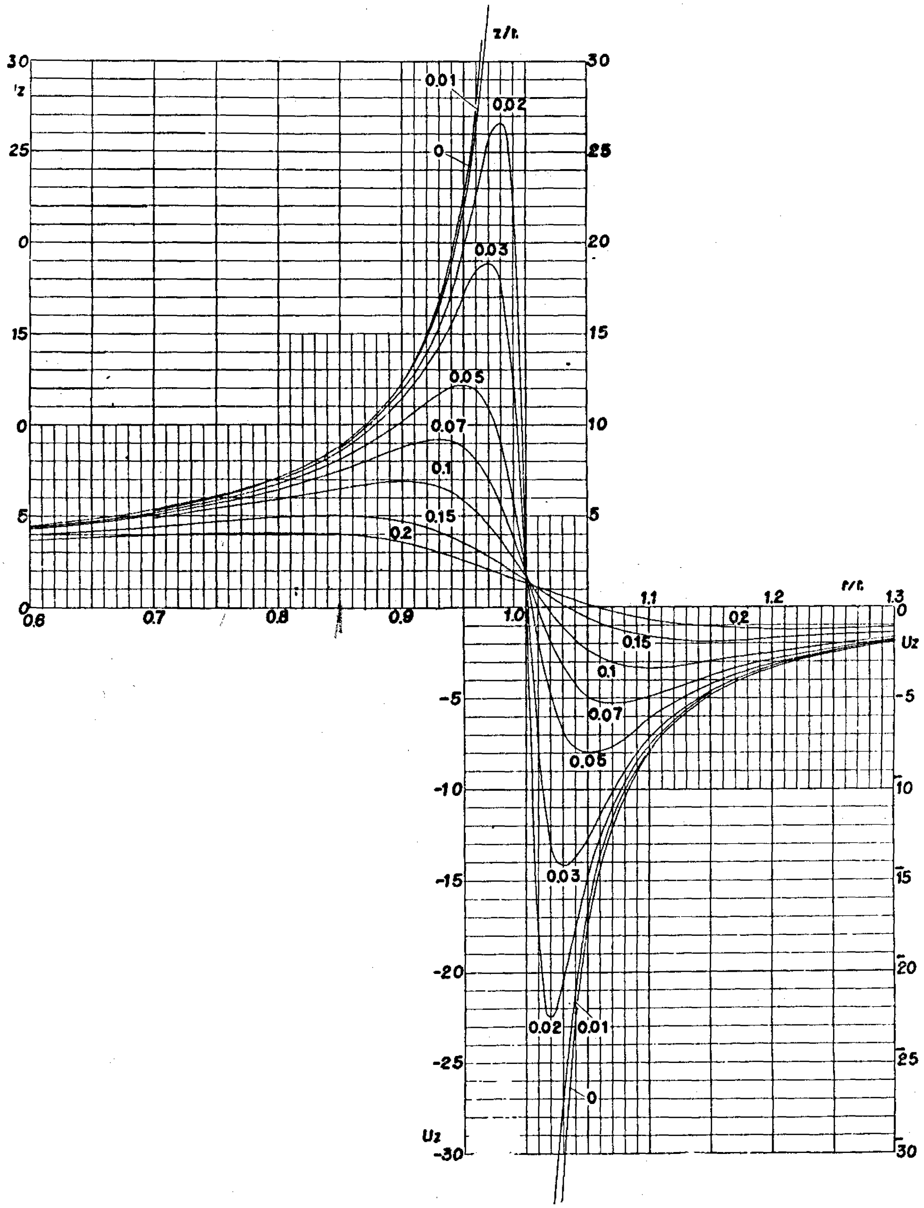

z/r₀
30
'z
25
0
15
0
5
0
0.6
0.7
0.8
0.9
1.0
1.1
1.2
r/r₀
1.3
30
25
20
15
10
5
0
Uz
-5
-10
-15
-20
-25
Uz
-30
10
15
20
25
30
0.01
0.02
0
0.03
0.05
0.07
0.1
0.15
0.2
0.2
0.15
0.1
0.07
0.05
0.03
0.02
0.01
0

Fig. 3. 2. U_z

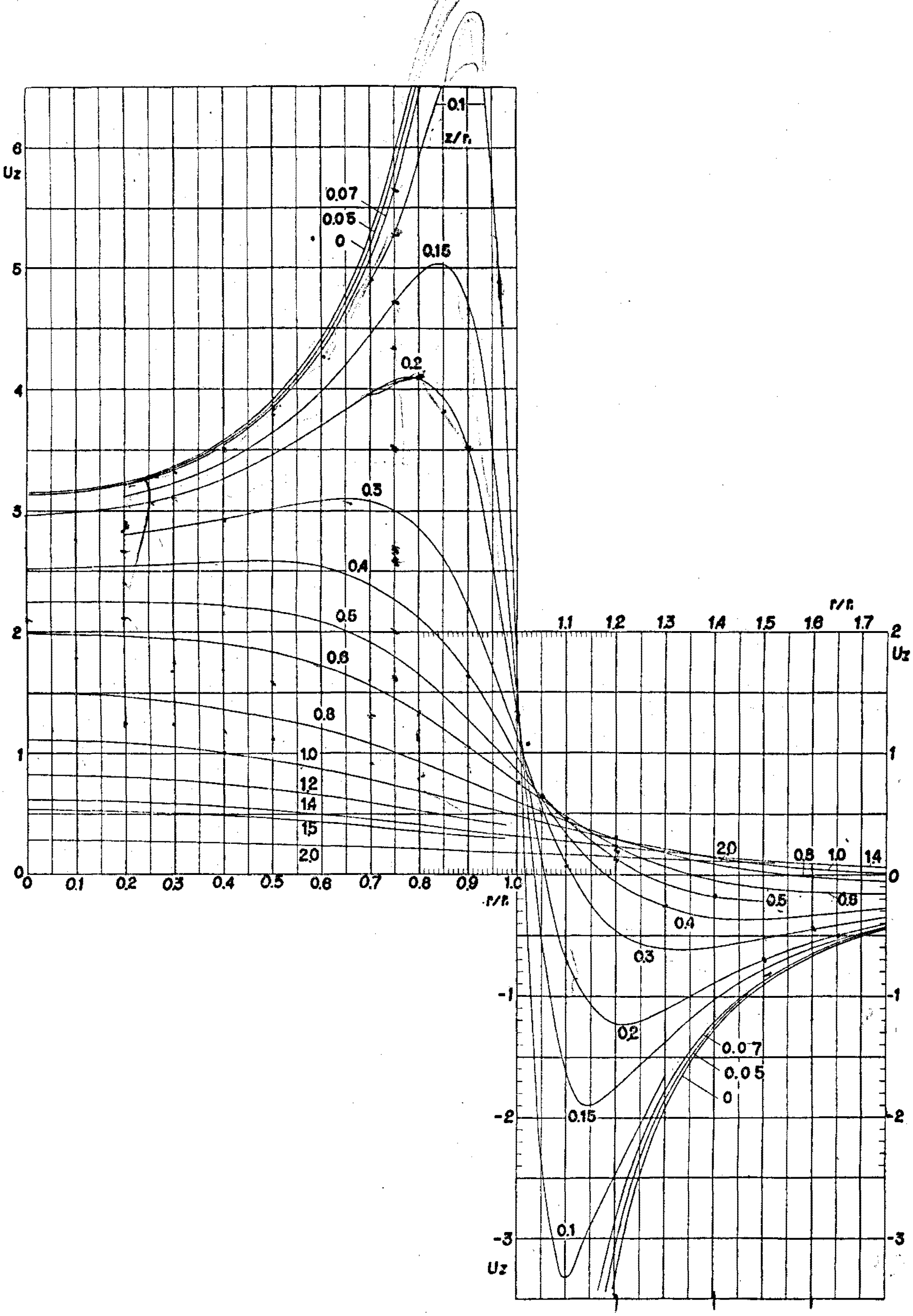

Uz
z/r₀
0.1
0.07
0.05
0
0.15
0.2
0.3
0.4
0.5
0.6
0.8
1.0
1.2
1.4
1.5
2.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
r/r₀
1.1
1.2
1.3
1.4
1.5
1.6
1.7
6
5
4
3
2
1
-1
-2
-3
0.07
0.05
0
0.15
0.2
0.3
0.4
0.5
0.8
1.0
1.4
2.0
Uz

Fig. 3. 3. *U*$_z$

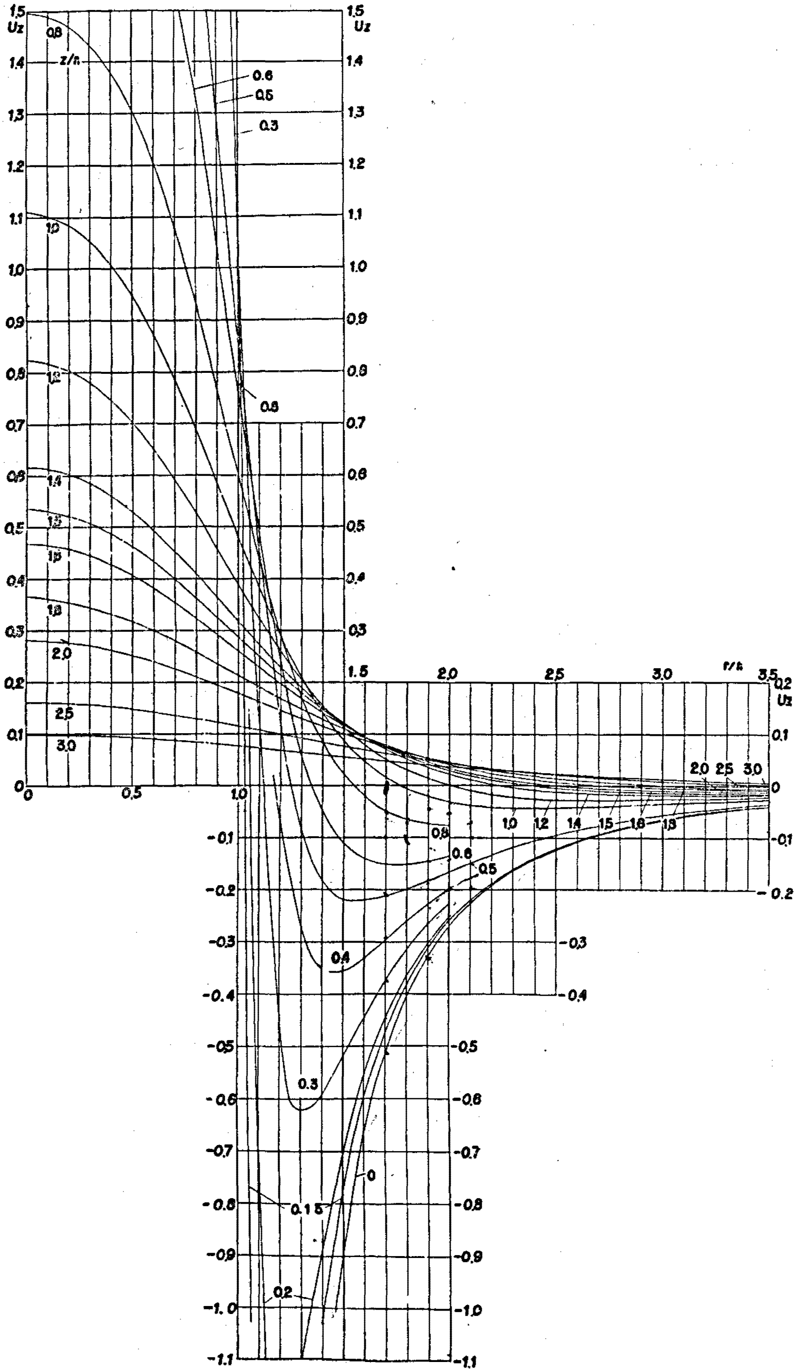

1.5
U_z
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1.0
-1.1
z/h
0.8
1.0
1.2
1.4
1.5
1.6
1.8
2.0
2.5
3.0
0.6
0.5
0.3
0.4
0.15
0.2
0
r/h
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5

Fig. 3.4. U_z

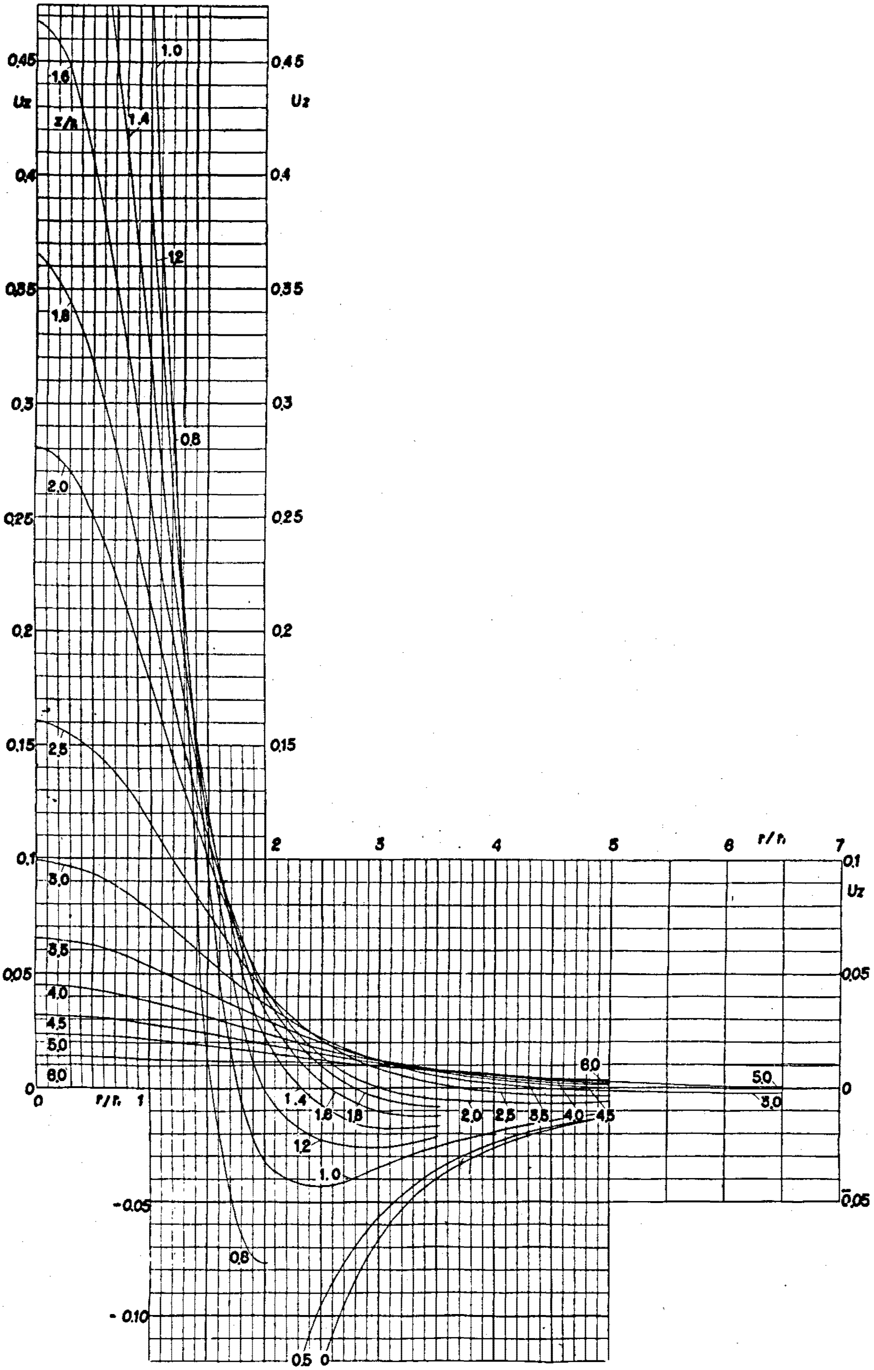

0.45
Uz
z/r₁
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.10
0
r/r₁
1
2
3
4
5
6
7
1.6
1.0
1.4
1.2
1.8
0.8
2.0
2.5
3.0
3.5
4.0
4.5
5.0
6.0
0.5
0

Fig. 3. 5. U_z

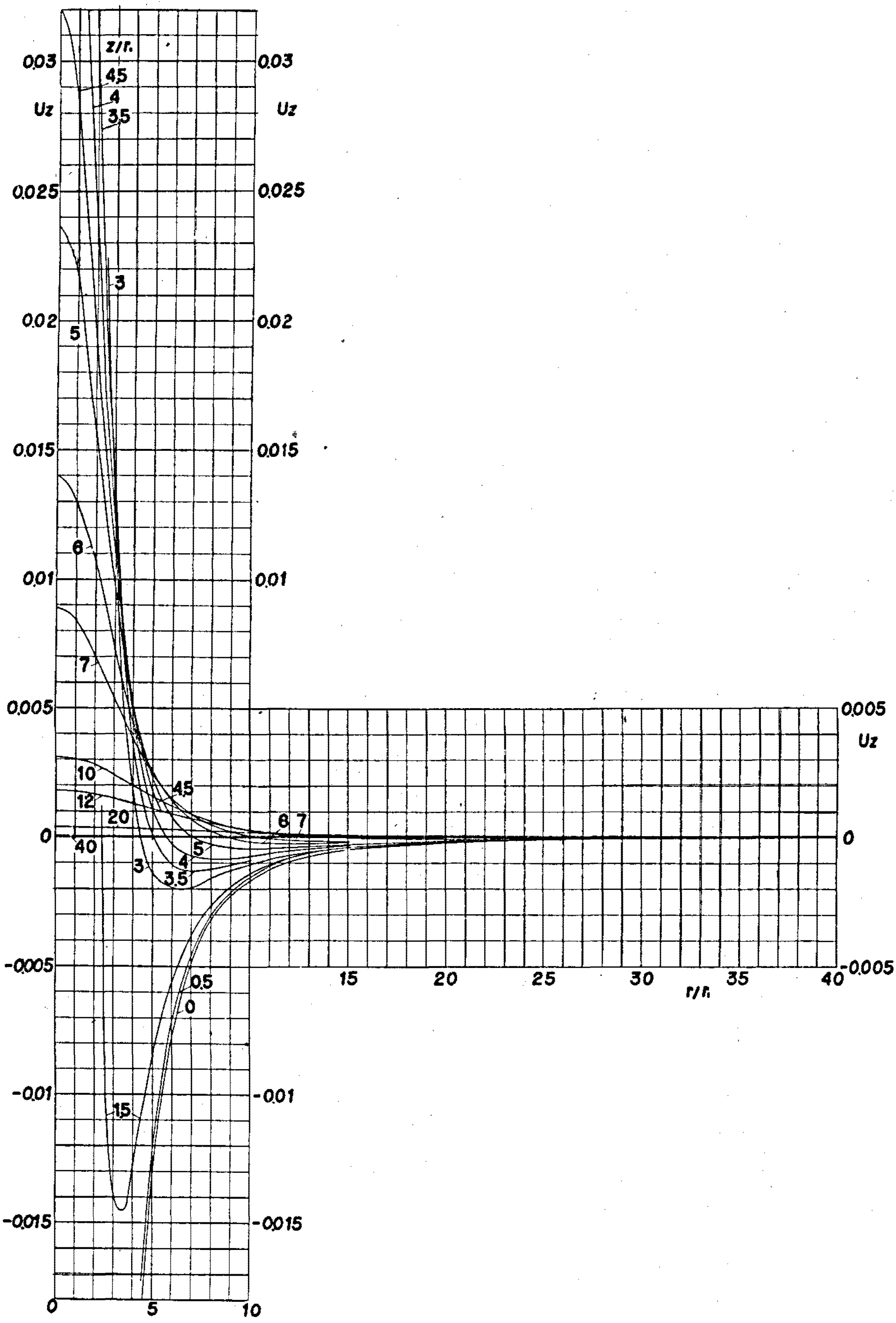

z/r₀
45
4
3.5
3
5
6
7
10
12
20
40
45
6
7
5
4
3
3.5
0.5
0
15
0.03
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
Uz
0.005
0
-0.005
0
5
10
15
20
25
30
35
40
r/r₀

Fig. 3.6. U_z

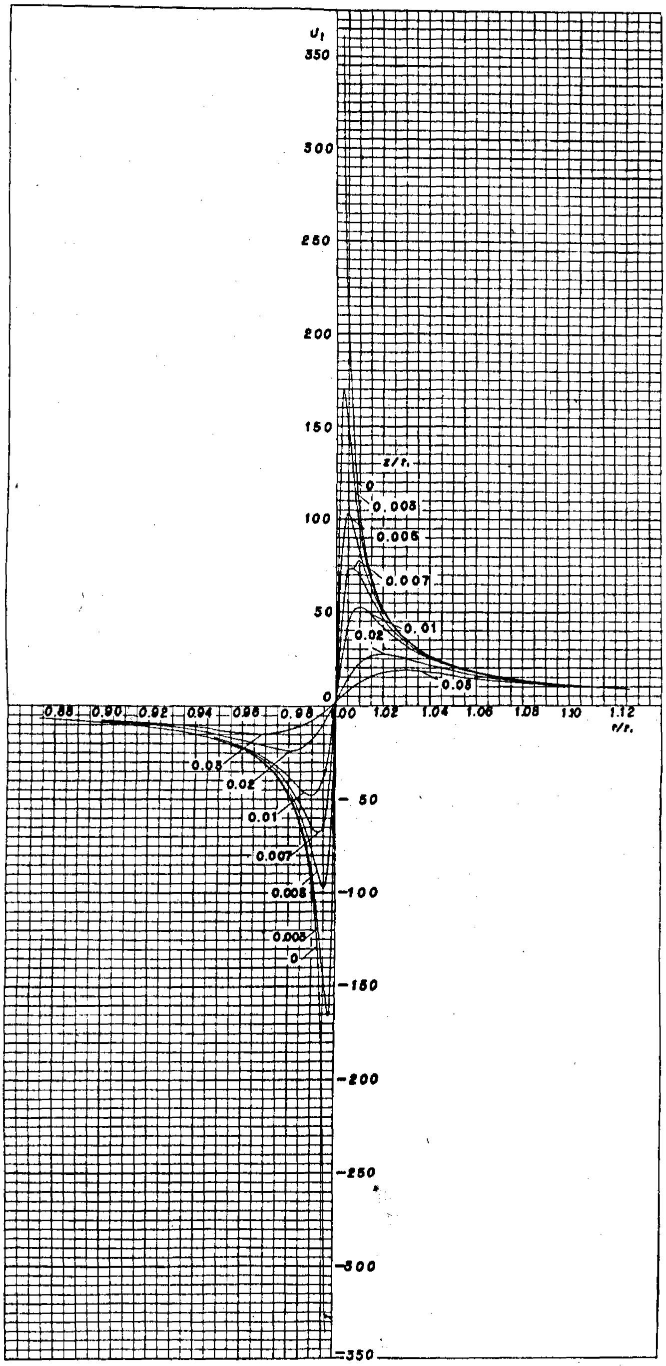

U_t
350
300
250
200
150
100
50
0
− 50
−100
−150
−200
−250
−300
−350
0.88
0.90
0.92
0.94
0.96
0.98
1.00
1.02
1.04
1.06
1.08
1.10
1.12
r/r₁
z/r₁
0
0.003
0.005
0.007
0.01
0.02
0.05

Fig. 4. 1. U_t

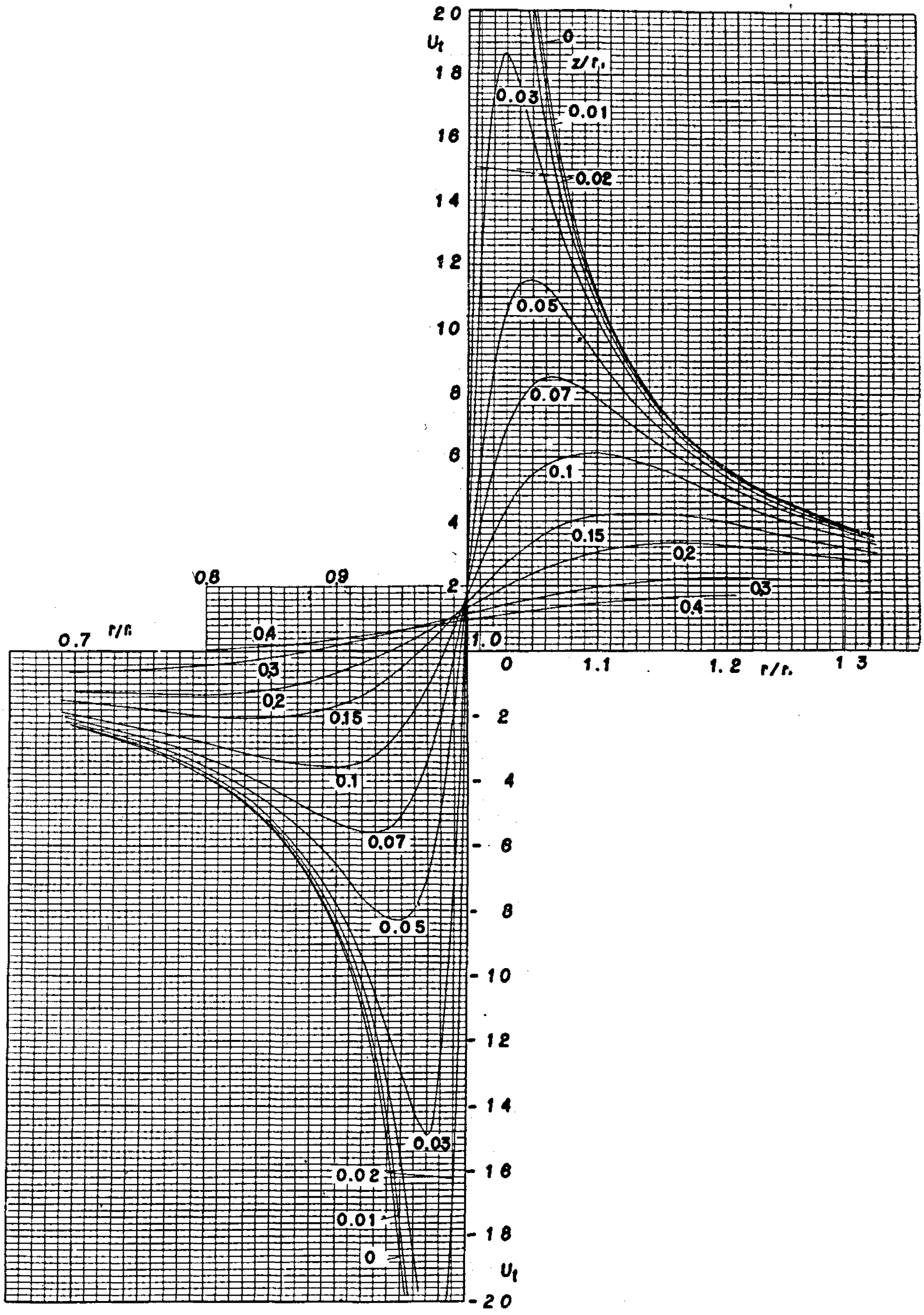

20
U_t
18
16
14
12
10
8
6
4
2
1.0
1.1
1.2
1.3
r/r₁
z/r₁
0
0.01
0.02
0.03
0.05
0.07
0.1
0.15
0.2
0.3
0.4
0.7
0.8
0.9
r/r₁
0.4
0.3
0.2
0.15
0.1
0.07
0.05
0.03
0.02
0.01
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
U_t
-20

Fig. 4.2. U_t

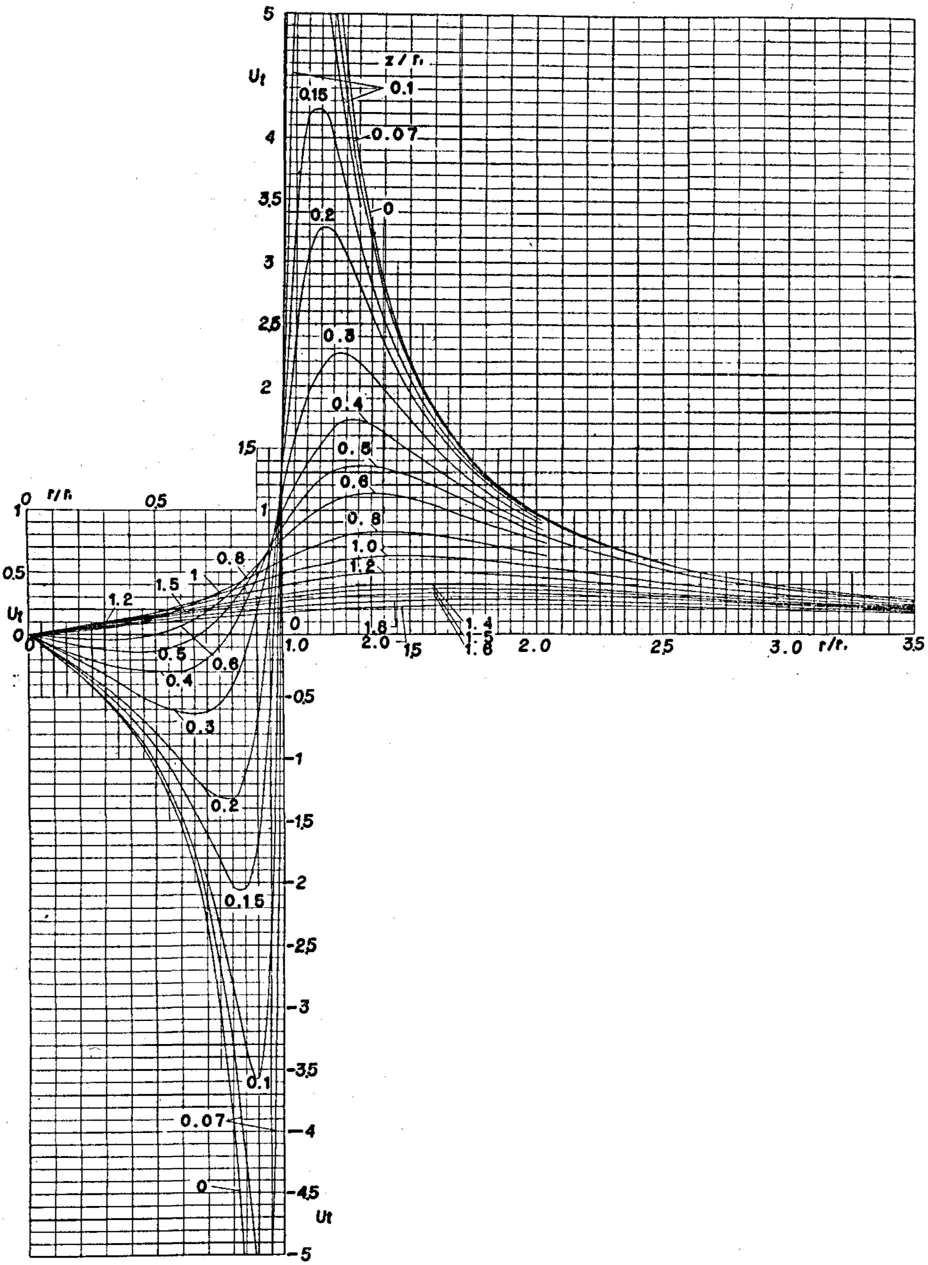

5
Ut
z / r₁
0.15
0.1
4
0.07
3.5
0.2
0
3
2.5
0.3
2
0.4
1.5
0.5
0.6
0 r/r₁ 0.5
1
0.8
1.0
1.2
0.5
1.5
1
0.8
1.2
Ut
0
0
1.0
1.8
2.0
1.5
1.4
1.5
1.6
2.0
2.5
3.0 r/r₁
3.5
0.5
0.4
0.6
0.3
-0.5
-1
0.2
-1.5
-2
0.15
-2.5
-3
-3.5
0.1
0.07
-4
0
-4.5
Ut
-5

Fig. 4.3. U_t

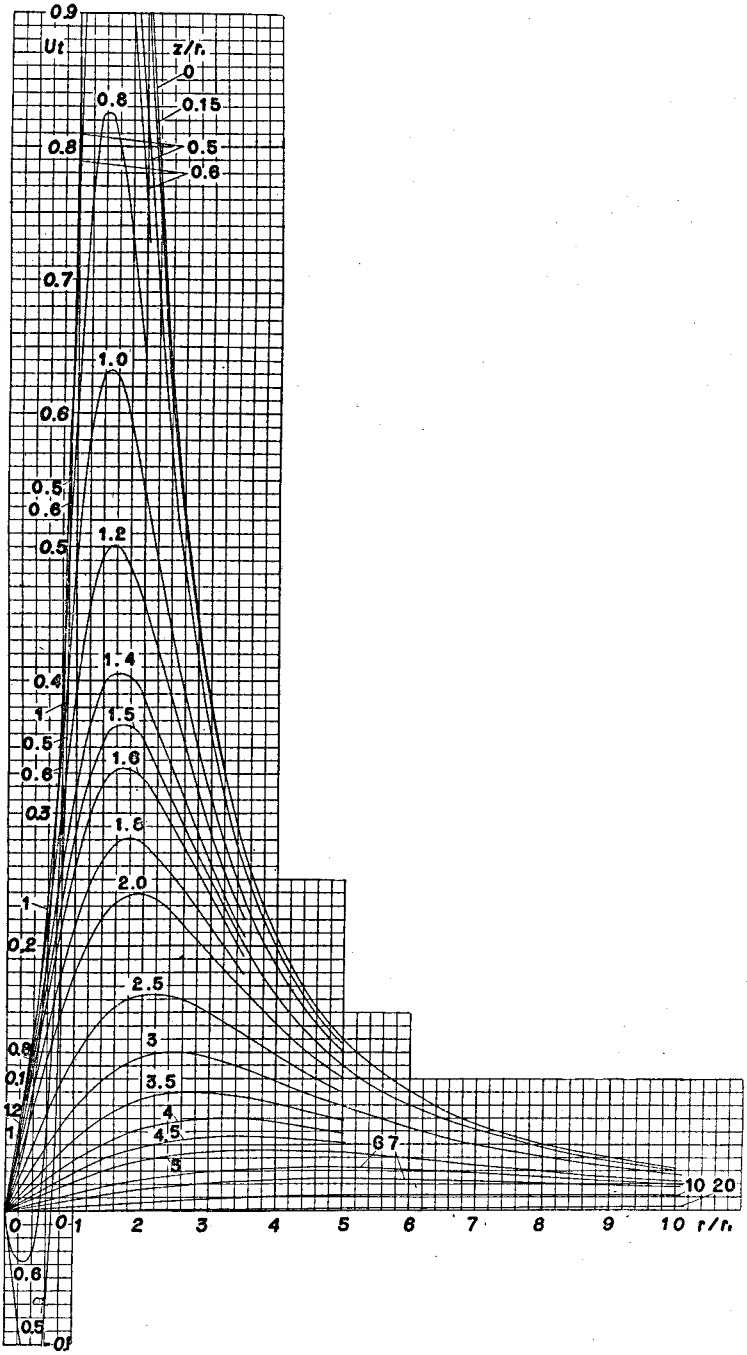
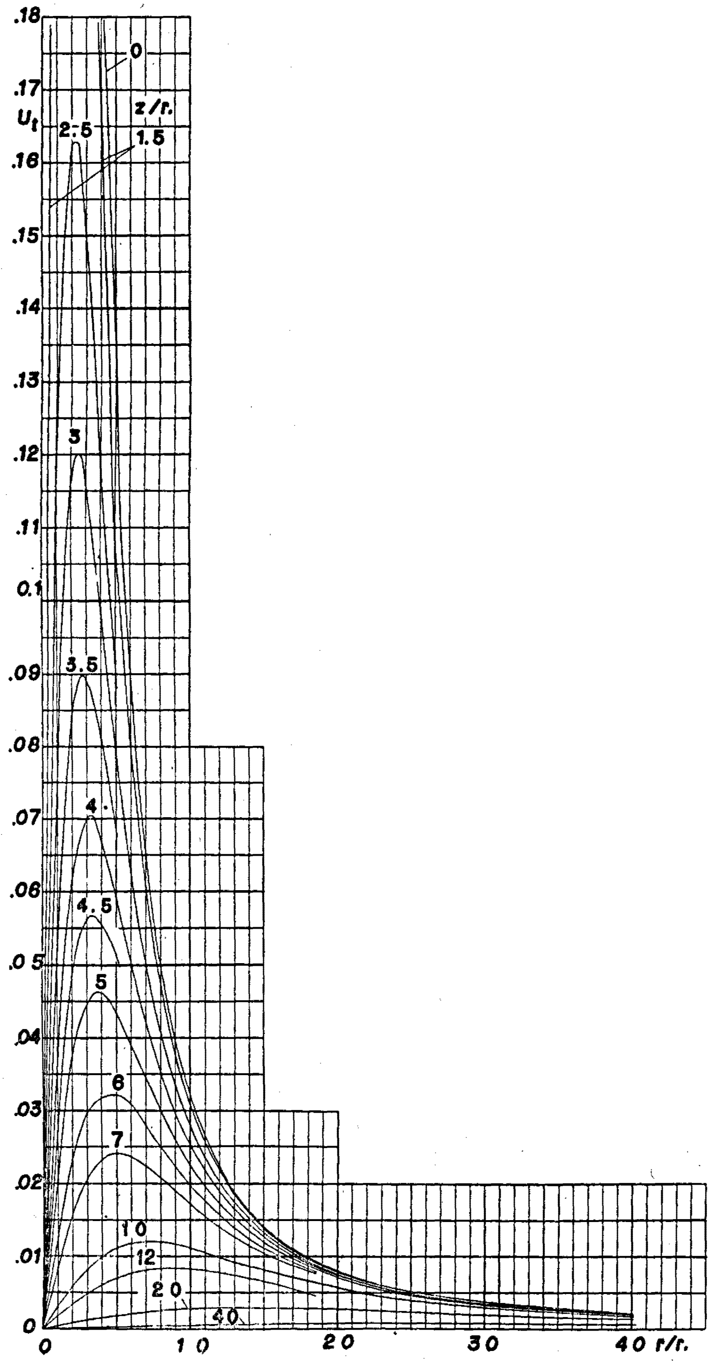

Ut
z/r.
0
0.15
0.5
0.6
0.8
1.0
1.2
1.4
1.5
1.6
1.8
2.0
2.5
3
3.5
4
4.5
5
6 7
10 20
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
2
3
4
5
6
7
8
9
10
r/r.

Fig. 4.4. U_t



.18
.17
U_t
.16
.15
.14
.13
.12
.11
0.1
.09
.08
.07
.06
.05
.04
.03
.02
.01
0
0
10
20
30
40 r/r.
z/r.
0
1.5
2.5
3
3.5
4
4.5
5
6
7
10
12
20
40

Fig. 4. 5. U_t

Table 1.1. U_r

r/r_1 z/r_1	0	0.2	0.4	0.5	0.6	0.7	0.8	0.825	0.85	0.875	0.9
0.0	0	0 *	0 *	0 *	0 *	0 *	0 *	0	0	0	0 *
0.01	0					0.118*	0.265*		0.465		1.028*
0.02	0								0.918		1.997
0.03	0					0.352	0.777		1.347		2.856
0.05	0	0.0505	0.130	0.200	0.323	0.575*	1.245*		2.098		4.142*
0.07	0	0.0703	0.180		0.445	0.783	1.645		2.672	3.546	4.846
0.10	0					1.056*	2.099*		3.200		5.133*
0.15	0	0.143	0.361		0.846		2.487		3.422		4.674
0.20	0 *	0.183*	0.452*		1.012*	1.577*	2.547*				3.982*
0.30	0	0.241	0.579		1.160	1.633*	2.248*				2.858*
0.40	0 *	0.272*	0.619*		1.137*		1.841*				
0.50	0 *	0.278	0.609	0.808*	1.032		1.486				1.630
0.60	0 *	0.268*	0.565*		0.900*		1.202*				
0.80	0 *	0.220*	0.441*		0.649*		0.801*				
1.0	0 *	0.165*	0.323*	0.394*	0.458*		0.547*				
1.2	0 *	0.120*	0.231*		0.323*		0.383*				

* Calculated by Küchemann and Weber, *l.c.* ante p. 88, by courtesy of these authors and the publisher of this book.

Table 1.2 U_r

r/r_1 z/r_1	0	0.2	0.4	0.5	0.6	0.7	0.8	0.825	0.85	0.875	0.9
0.0	0	0 *	0 *	0 *	0 *	0 *	0 *	0	0	0	0 *
1.4	0 *	0.086*	0.165*		0.230*		0.273*				
1.5	0 *			0.170*							
1.6	0 *	0.062*	0.119*		0.166*		0.198*				
1.8	0 *	0.045*	0.087*		0.121*		0.146*				
2.0	0 *	0.033*	0.064*	0.078*	0.090*		0.109*				
2.5	0 *			0.039*							
3.0	0 *			0.021*							
3.5	0 *			0.012*							
4.0	0 *			0.008*							
4.5	0 *			0.005*							
5.0	0			0.00334			0.00517				
6.0	0			0.00167			0.00261				
7.0	0			0.000923			0.00145				
8.5	0			0.000431			0.000632				
10.0	0			0.000227							

Table 1.3. U_r

r/r_1 z/r_1	0	0.2	0.4	0.5	0.6	0.7	0.8	0.825	0.85	0.875	0.9
0.0	0	0 *	0 *	0 *	0 *	0 *	0 *	0	0	0	0 *
12.0	0			0.000112			0.000176				
15.0	0			0.0000438			0.0000732				
20.0	0			0.0000139							
40.0	0			0							

Table 1.4. U_r

r/r_1 z/r_1	0	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	0.995	0.997
0.0	0	0	0	0	0 *	0	0	0	0 *	0	0
0.003	0					1.898	3.345	7.403		88 45	166.9
0.005	0							11.87	40.19	100.1	147.3
0.007	0					4.320		15.73	47.20	94.82	120.9
0.01	0				3.929*	5.986	10.14	20.18	50.23	80.18	
0.02	0				7.042	10.17	15.59	25.22	40.16	47.14	
0.03	0				9.000	12.20	16.87	23.26	30.10	32.46	
0.05	0				10.18 *	12.37	14.85	17.34	19.25	19.77	19.88
0.07	0		7.304		9.601		12.15		13.97		
0.10	0				8.070*	8.661	9.167	9.577	9.816	9.856	
0.15	0				5.970	6.168	6.326	6.438	6.492	6.499	
0.20	0 *				4.602*				4 797*		
0.30	0				3.038*				3.070*		
0.40	0 *										
0.50	0 *				1.657				1.652		

Table 1.5. U_r

r/r_1 z/r_1	0	0.998	0.999	0.9995	1.00	1.0005	1.001	1.002	1.003	1.005	1.01
0.0	0	0	0	0	0 *	0	0	0	0	0	0 *
0.003	0	231.0	300.1	324.4	333.3	324.2	299.8	230.5	166.4	87.97	27.38
0.005	0	172.6	192.4		200.0		192.2	172.2	146.8	99.74	39.79
0.007	0		140.1		142.8		139.9		120.5	94.34	46.73
0.01	0	96.23			99.98			96.23		79.78	49.73
0.02	0				49.96					46.90	39.76
0.03	0				33.28					32.30	29.80
0.05	0				19.92				19.82	19.67	19.06
0.07	0				14.18						13.83
0.10	0				9.867					9.818	9.721
0.15	0				6.490					6.468	6.430
0.20	0 *				4.787*						4.753*
0.30	0				3.061*						3.044*
0.40	0 *				2.182*						
0.50	0 *				1.647*						1.640
0.60	0 *				1.287*						

Table 1.6. U_r

z/r_1 \ r/r_1	0	1.00	1.003	1.005	1.01	1.02	1.03	1.04	1.05	1.06	1.07
0.0	0	0 *	0	0	0 *	0	0	0	0 *	0	0
0.8	0 *	0.836*									
1.0	0 *	0.572*									
1.2	0 *	0.403*									
1.4	0 *	0.291*									
1.5	0 *	0.249*									
1.6	0 *	0.214*									
1.8	0 *	0.160*									
2.0	0 *	0.122*									
2.5	0 *	0.064*									
3.0	0 *	0.0365									
3.5	0 *	0.022*									
4.0	0 *	0.014*									
4.5	0 *	0.010*									
5.0	0	0.00626									
6.0	0	0.00319									

Table 1.7. U_r

z/r_1 \ r/r_1	0	1.00	1.003	1.005	1.01	1.02	1.03	1.04	1.05	1.06	1.07
0.0	0	0 *	0	0	0 *	0	0	0	0 *	0	0
7.0	0	0.00178									
8.5	0	0.000844									
10.0	0	0.000448									
12.0	0	0.000220									
15.0	0	0.0000902									
20.0	0	0.0000289									
40.0	0	0.00000200									

Table 1.8. U_r

z/r_1 \ r/r_1	0	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.10	1.125	1.15	1.175
0.0	0	0	0	0	0 *	0	0	0	0 *	0	0	0
0.003	0	7.257	3.247	1.824								
0.005	0	11.64	5.318	3.009								
0.007	0	15.42		4.152								
0.01	0	19.78	9.836	5.752	3.739*				0.932*		0.403	
0.02	0	24.72	15.13	9.775	6.701				1.810		0.795	
0.03	0	22.79	16.38	11.72	8.567				2.590		1.166	
0.05	0	17.00	14.42	11.89	9.691*				3.757*		1.810	
0.07	0		11.79		9.141		6.820		4.402	3.144	2.316	
0.10	0	9.392	8.914	8.330	7.687*				4.658*		2.770	
0.15	0	6.315	6.147	5.938	5.693				4.255		2.978	
0.20	0				4.395*				3.633*			
0.30	0				2.911*				2.625*			
0.50	0				1.599				1.523			

Table 1.9. U_r

z/r_1 \ r/r_1	0	1.2	1.3	1.4	1.5	1.6	1.8	2.0	2.5	3.0	3.4	3.5
0.0	0	0 *	0 *	0 *	0 *	0 *	0 *	0 *	0 *	0 *	0	0 *
0.01	0	0.219*	0.091*									
0.03	0	0.645	0.270									
0.05	0	1.033*	0.442*	0.234	0.140	0.0911	0.0449	0.0253				
0.07	0	1.363	0.602	0.323		0.127	0.0625	0.0353	0.0117	0.00505	0.00292	
0.1	0	1.745*	0.814*									
0.15	0	2.070	1.077	0.619	0.387	0.257	0.130	0.0739	0.0248	0.0108		0.00550
0.2	0 *	2.135*	1.229*	0.746*		0.325*	0.168	0.096*				
0.3	0	1.908*	1.295*	0.874		0.426	0.231	0.136				
0.4	0 *	1.586*		0.881*		0.479*	0.275*	0.168*				
0.5	0 *				0.640*			0.191*	0.072*	0.033*		0.017*
0.6	0 *	1.073*		0.748*		0.484*	0.312*	0.205*				
0.8	0 *	0.744*		0.584*		0.426*	0.302*	0.213*				
1.0	0 *	0.530*		0.448*	0.400*	0.354*	0.270*	0.202*	0.099*	0.052*		0.029*
1.2	0 *	0.386*		0.342*		0.287*	0.231*	0.182*		0.0544		0.0317
1.4	0 *	0.286*		0.266*		0.230*	0.194*	0.160*		0.0549		0.0332

Table 1.10. U_r

z/r_1 \ r/r_1	0	1.2	1.3	1.4	1.5	1.6	1.8	2.0	2.5	3.0	3.5
0.0	0	0 *	0 *	0 *	0 *	0 *	0 *	0 *	0 *	0 *	0 *
1.5	0 *				0.219*			0.148*	0.090*	0.0544	0.0335
1.6	0 *	0.215*		0.204*		0.184*	0.161*	0.137*		0.0536	0.0337
1.8	0 *	0.164*		0.159*		0.148*	0.133*	0.117*		0.0513	0.0334
2.0	0 *	0.127*		0.125*	0.123*	0.119*	0.110*	0.099*	0.071*	0.048*	0.033*
2.5	0 *				0.071*			0.064*	0.052*	0.0390	0.0285
3.0	0 *				0.043*			0.042*	0.037*	0.030*	0.024 *
3.5	0 *				0.0273			0.0284	0.026*	0.023*	0.019 *
4.0	0 *				0.018*			0.019*	0.019*	0.017*	0.015 *
4.5	0 *				0.012*			0.014*	0.014*	0.013*	0.0120
5.0	0				0.00844				0.0102	0.0101	0.00946
6.0	0				0.00442					0.00602	
7.0	0				0.00252					0.00374	
8.5	0				0.00122					0.00197	
10.0	0				0.000653					0.00112	
12.0	0				0.000323					0.000577	

Table 1.11. U_r

z/r_1 \ r/r_1	0	1.2	1.3	1.4	1.5	1.6	1.8	2.0	2.5	3.0	3.5
0.0	0	0 *	0 *	0 *	0 *	0 *	0 *	0 *	0 *	0 *	0 *
15.0	0				0.000134					0.000251	
20.0	0				0.0000430					0.0000833	
40.0	0				0.00000466					0.00000564	

Table 1. 12. U_r

z/r_1 \ r_1r_1	0	4.0	4.5	5.0	6.5	8.0	10	15	20	30	40
0.0	0	0 *	0 *	0 *	0	0	0	0	0	0	0
0.05	0										
0.07	0	0.00145	0.000884	0.000569			0.0000333		0.00000207		0.000000129
0.15	0	0.00310	0.00189	0.00122			0.0000720		0.00000444		0.000000276
0.2	0 *										
0.4	0 *										
0.5	0 *	0.00990	0.006 *	0.004 *	0.00136	0.000587	0.000239		0.00000148		0.000000921
0.6	0 *										
0.8	0 *										
1.0	0 *	0.0174	0.0110	0.00731							
1.2	0 *										
1.4	0 *										
1.5	0 *	0.0214	0.0141	0.00945	0.00361	0.00163	0.000680	0.000137	0.0000438	0.00000869	0.00000275
1.6	0 *										
1.8	0 *										
2.0	0 *	0.022 *	0.0154	0.0109	0.00435	0.00203	0.000869				

Table 1. 13. U_r

z/r_1 \ r/r_1	0	4.0	4.5	5.0	6.5	7.5	8.0	10	15	20	30	40
0	0	0 *	0 *	0 *	0	0	0	0	0	0	0	0
2.5	0 *	0.0207	0.0151	0.0111	0.00480		0.00233	0.00103				
3.0	0 *	0.018*	0.0140	0.0107	0.00499		0.00253	0.00116	0.000255	0.0000839	0.0000171	0.00000545
3.5	0 *	0.0155	0.0124	0.00985								
4.0	0 *	0.0129	0.0107	0.00879	0.00478		0.00267	0.00131	0.000316	0.000107		
4.5	0 *	0.0105	0.00907	0.00768								
5.0	0 *	0.00858	0.0076	0.00662	0.00414		0.00254	0.00136	0.000360	0.000127	0.0000272	0.00000886
6.0	0			0.00481		0.00260		0.00132	0.000387	0.000143		0.0000105
7.0	0			0.00345		0.00217		0.00122	0.000400	0.000155		0.0000120
8.5	0			0.00212		0.00160		0.00103	0.000396	0.000166		
10	0			0.00133	0.00126		0.00109	0.000832	0.000372	0.000169	0.0000448	0.0000158
12	0			0.000753		0.000743		0.000606	0.000324	0.0000979		
15	0			0.000355		0.000397		0.000370	0.000247	0.000145		
20	0			0.00126	0.000163		0.000162	0.000168	0.000145	0.000104	0.0000464	0.0000211
40	0			0.00000890				0.0000158		0.0000211		0.0000130

Table 2.1. U_z

r/r_1 z/r_1	0	0.2	0.4	0.5	0.6	0.7	0.8	0.825	0.85	0.875	0.9
0.0	3.142*	3.240*	3.586*	3.913*	4.432*	5.316*	7.091*	7.847	8.851	10.25	12.33*
0.01						5.312*	7.076*		8.820		12.23*
0.02									8.719		11.92
0.03						5.273	6.961		8.562		11.44
0.05	3.130	3.226	3.563	3.880	4.376	5.199*	6.743*		8.098		10.17*
0.07		3.213	3.543		4.323	5.092	6.445		7.501	8.131	8.751
0.10						4.881*	5.904*		6.520		6.873*
0.15		3.120	3.396		3.980		4.941		5.034		4.711
0.20	2.962*	3.033*	3.263*		3.695*	3.961*	4.081*				3.546*
0.30		2.805	2.934		3.087	3.079*	2.854*				2.206*
0.40	2.515*	2.534*	2.572*		2.535*		2.115*				
0.50	2.248*	2.246	2.219	2.173*	2.080		1.648				1.281
0.60	1.981*	1.965*	1.899*		1.719*		1.333*				
0.80	1.496*	1.469*	1.380*		1.206*		0.934*				
1.0	1.111*	1.086*	1.008*	0.948*	0.874*		0.690*				
1.2	0.824*	0.804*	0.746*		0.650*		0.527*				

* Calculated by Küchemann and Weber, *l.c.* ante p. 88

Table 2.2. U_z

r/r_1 z/r_1	0	0.2	0.4	0.5	0.6	0.7	0.8	0.825	0.85	0.875	0.9
0.0	3.142 *	3.240*	3.586*	3.913*	4.432*	5.316*	7.091*	7.847	8.851	10.25	12.33*
1.4	0.617 *	0.603*	0.560*		0.494*		0.410*				
1.5	0.536 *			0.463*							
1.6	0.468 *	0.458*	0.428*		0.382*		0.324*				
1.8	0.366 *	0.353*	0.332*		0.300		0.259*				
2.0	0.281 *	0.276*	0.261*	0.251*	0.239*		0.210*				
2.5	0.161 *			0.148*							
3.0	0.099 *			0.093*							
3.5	0.065 *			0.062*							
4.0	0.045 *			0.043*							
4.5	0.032 *			0.031*							
5.0	0.0237			0.0231			0.0221				
6.0	0.0140			0.0137			0.0133				
7.0	0.00889			0.00876			0.00857				
8.5	0.00501			0.00496			0.00488				
10.0	0.00310			0.00307							

Table 2.3. U_z

r/r_1 z/r_1	0	0.2	0.4	0.5	0.6	0.7	0.8	0.825	0.85	8.85	0.9
0.0	3.142*	3.240*	3.586*	3.913*	4.432*	5.316*	7.091*	7.847	8.851	10.25	12.33*
12.0	0.00180			0.00179			0.00178				
15.0	0.000 925			0.000 922			0.000 917				
20.0	0.000 391			0.000 390							
40.0	0.0000 490			0.0000 404							

Table 2.4. U_z

z/r_1 \ r/r_1	0	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	0.995	0.997
0.0	3.142*	14.92	16.76	19.21	22.62*	27.71	36.18	53.04	103.4*	203.7	337.3
0.003						27.57	35.84	51.92		150.6	170.2
0.005								50.05	83.21	103.3	91.49
0.007						26.95		47.49	70.21	70.66	54.79
0.01					21.82*	26.20	32.77	42.88	52.93	42.90	
0.02					19.75	22.56	25.67	26.60	22.56	14.28	
0.03					17.12	18.42	18.91	17.77	12.33	7.711	
0.05	3.130				12.18*	11.92	10.95	8.990	5.905	4.025	3.237
0.07			9.171		8.749		7.110		3.884		
0.10					5.783	5.210	4.484	3.640	2.690	2.190	
0.15					3.548	3.192	2.799	2.379	1.934	1.709	
0.20	2.962*				2.562*				1.593*		
0.30					1.700*				1.242*		
0.40	2.515*										
0.50	2.248*				1.071				0.898		

Table 2.5. U_z

z/r_1 \ r/r_1	0	0.998	0.999	0.9995	1.00	1.0005	1.001	1.002	1.003	1.005	1.01
0.0	3.142*								-329.4	-196.3	-96.68*
0.003		157.4	103.5	57.51	3.444	-50.60	-96.53	-149.3	-163.2	-143.6	-88.49
0.005		72.19	41.66		3.189		-35.27	- 65.76	- 85.00	- 96.75	-76.83
0.007			23.03		3.021		-16.98		- 48.67	- 64.49	-63.56
0.01		22.09			2.842			- 16.38		- 37.12	-47.70
0.02					2.496					- 9.263	-17.48
0.03					2.293					- 3.113	- 7.697
0.05	3.130				2.037				0.838	0.0529	- 1.812
0.07					1.868						- 0.138
0.10					1.688					1.185	0.691
0.15					1.483					1.257	1.033
0.20	2.962*				1.336*						1.080*
0.30					1.126*						1.010*
0.40	2.515*				0.974*						
0.50	2.248*				0.854*						0.811
0.60	1.981*				0.756*						

Table 2.6. U_z

r/r_1 z/r_1	0	1.0	1.003	1.005	1.01	1.02	1.03	1.04	1.05	1.06	1.07
0.0	3.142*		−329.4	−196.3	−96.68*	−47.04	−30.59	−22.42	−17.54*	−14.28	−12.01
0.8	1.496*	0.600*									
1.0	1.111*	0.482*									
1.2	0.824*	0.390*									
1.4	0.617*	0.318*									
1.5	0.536*	0.288*									
1.6	0.468*	0.260*									
1.8	0.366*	0.215*									
2.0	0.281*	0.178*									
2.5	0.161*	0.115*									
3.0	0.099*	0.0767									
3.5	0.065*	0.053*									
4.0	0.045*	0.038*									
4.5	0.032*	0.028*									
5.0	0.0237	0.0213									
6.0	0.0140	0.0129									

Table 2.7. U_z

r/r_1 z/r_1	0	1.0	1.003	1.005	1.01	1.02	1.03	1.04	1.05	1.06	1.07
0.0	3.142*		-329.4	-196.3	-96.68*	-47.04	-30.59	-22.42	-17.54*	- 14.28	-12.01
7.0	0.00889	0.00838									
8.5	0.00501	0.00481									
10.0	0.00310	0.00301									
12.0	0.00180	0.00176									
15.0	0.000925	0.000913									
20.0	0.000391	0.000388									
40.0	0.0000490	0.0000489									

Table 2.8. U_z

r/r_1 z/r_1	0	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.10	1.125	1.15	1.175
0.0	3.142*	-47.04	-30.59	-22.42	-17.54*	-14.28	-12.01	-10.31	-7.934*	-6.065	-4.840	-3.979
0.003		-45.96	-30.27	-22.28								
0.005		-44.15	-29.71	-22.04								
0.007		-41.67		-21.70								
0.01		-37.19	-27.33	-20.99	-16.80*				-7.842*		-4.812	
0.02		-22.46	-20.58	-17.57	-14.91				-7.576		-4.735	
0.03		-13.05	-14.18	-13.71	-12.72				-7.167		-4.609	
0.05	3.130	- 4.852	- 6.768	- 7.698	-7.948*				-6.079*		-4.217	
0.07			- 3.255		-4.878		-5.275		-4.892	-4.327	-3.773	
0.10		- 0.244	- 1.073	- 1.766	-2.315*				-3.327*		-2.896	
0.15		0.598	0.187	- 0.192	-0.531				-1.611		-1.891	
0.20	2.962*				0.140*				-0.687*			
0.30					0.566*				0.098*			
0.40	2.515*											
0.50	2.248*				0.644				0.449			

Table 2. 9. U_z

z/r_1 \ r/r_1	0	1.2	1.3	1.4	1.5	1.6	1.8	2.0	2.5	3.0	3.4	3.5
0.0	3.142*	−3.345*	−1.924*	−1.263*	−0.895*	−0.666*	−0.407*	−0.271*	−0.122	−0.066*		−0.040*
0.01		−3.335*	−1.921*									
0.03		−3.252	−1.897									
0.05	3.130	−3.090*	−1.851*	−1.234	−0.880	−0.658	−0.404	−0.271				
0.07		−2.863	−1.784	−1.207		−0.651	−0.401	−0.268	−0.122	−0.0654	−0.0441	
0.10		−2.482*	−1.655*									
0.15		−1.749	−1.387	−1.030	−0.777	−0.600	−0.381	−0.259	−0.119	−0.0655		−0.0399
0.20	2.962*	−1.219*	−1.103*	−0.887*		−0.554*	−0.363*	−0.250*				
0.30		−0.452*	−0.612*	−0.595		−0.445	−0.315	−0.227				
0.40	2.515*	−0.061*		−0.351*		−0.331*	−0.260	−0.198*				
0.50	2.248*				−0.217*			−0.167*	−0.095*	−0.057*		−0.036*
0.60	1.981*	0.233*		−0.048*		−0.141*	−0.151*	0.135*				
0.80	1.496*	0.295*		0.089*		−0.020*	−0.065*	−0.077*				
1.00	1.111*	0.289*		0.142*	0.088*	0.047*	−0.006*	−0.032*	−0.043*	−0.035*		−0.026*
1.20	0.824*	0.262*		0.156*		0.080*	0.030*	0.001*		−0.0260		−0.0213
1.40	0.617*	0.229*		0.153*		0.093*	0.051*	0.022*		−0.0178		−0.0167

Table 2. 10. U_z

z/r_1 \ r/r_1	0	1.2	1.3	1.4	1.5	1.6	1.8	2.0	2.5	3.0	3.5
0.0	3.142*	−3.345*	−1.924*	−1.263*	−0.895*	−0.666*	−0.407*	−0.271*	−0.122*	−0.066*	−0.040*
1.5	0.536*				0.120*			0.029*	−0.005*	−0.0140	−0.0145
1.6	0.468*	0.198*		0.142*		0.096*	0.060*	0.034*		−0.0105	−0.0123
1.8	0.366*	0.170*		0.128*		0.093*	0.064*	0.041*		−0.00447	−0.00837
2.0	0.281*	0.145*		0.114*	0.100*	0.087*	0.063*	0.045*	0.014*	0 *	−0.005*
2.5	0.161*				0.076*			0.043*	0.021*	0.00829	0.00157
3.0	0.099*				0.056*			0.037*	0.022*	0.012*	0.005*
3.5	0.065*				0.0417			0.030*	0.020*	0.012*	0.007*
4.0	0.045*				0.031*			0.024*	0.017*	0.012*	0.008*
4.5	0 032*				0.024*			0.019*	0.015*	0.011*	0.00756
5.0	0.0237				0.0186				0.0124	0.00956	0.00709
6.0	0.0140				0.0118					0.00722	
7.0	0.00889				0.00780					0.00540	
8.5	0.00501				0.00458					0.00354	
10.0	0.00310				0.00290					0.00240	
12.0	0.00180				0.00172					0.00150	

Table 2. 11. U_z

z/r_1 \ r/r_1	0	1.2	1.3	1.4	1.5	1.6	1.8	2.0	2.5	3.0	3.5
0.0	3.142*	−3.345*	−1.924*	−1.263*	−0.895*	−0.666*	−0.407*	−0.271*	−0.122*	−0.066*	−0.040*
15.0	0.000925				0.000898					0.000822	
20.0	0.000391				0.000385					0.000366	
40.0	0.0000490				0.0000574					0.0000482	

Table 2.12. U_z

z/r_1 \ r/r_1	0	4.0	4.5	5.0	6.5	8.0	10	15	20	30	40
0.0	3.142*	-0.026*	-0.018*	-0.013*	-0.00607	-0.00312		-0.000468		-0.0000583	
0.05	3.130										
0.07		0.0263	-0.0182	-0.0131			-0.00154		-0.000195		-0.0000246
0.15		0.0262	-0.0181	-0.0131			-0.00159		-0.000197		-0.0000246
0.2	2.962*										
0.4	2.515*										
0.5	2.248*	0.0241	-0.017*	-0.0125	-0.00571	-0.00307	-0.00157		-0.000196		-0.0000245
0.6	1.981*										
0.8	1.496*										
1.0	1.111*	-0.0191	-0.0142	-0.0108							
1.2	0.824*										
1.4	0.617*										
1.5	0.536*	-0.0126	-0.0104	-0.00842	-0.00456	-0.00265	-0.00143	-0.000447	-0.000192	-0.0000557	-0.0000244
1.6	0.468*										
1.8	0.366*										

Table 2.13. U_z

z/r_1 \ r/r_1	0	4.0	4.5	5.0	6.5	7.5	8.0	10	15	20	30	40
0.0	3.142*	-0.026*	-0.018*	-0.013*	-0.00607		-0.00312		-0.000468		-0.0000583	
2.0	0.281*	-0.006*	-0.00642	-0.00582	-0.00372		-0.00233	-0.00132				
2.5	0.161*	-0.00165	-0.00300	-0.00342	-0.00284		-0.00196	-0.00119				
3.0	0.099*	0.002*	-0.000380	-0.00141	-0.00199		-0.00158	-0.00104	-0.000390	-0.000178	-0.0000514	-0.0000239
3.5	0.065*	0.00362	0.00145	0.000126								
4.0	0.045*	0.00466	0.00258	0.00121	-0.000550		-0.000861	-0.000735	-0.000338	-0.000164		
4.5	0.032*	0.00507	0.00323	0.00191								
5.0	0.0237	0.00508	0.00352	0.00233	0.000381		-0.000273	-0.000444	-0.000278	-0.000148	-0.0000514	-0.0000229
6.0	0.0140			0.00259		0.000327		-0.000197	-0.000218	-0.000130		-0.0000222
7.0	0.00889			0.00245		0.000594		-0.00000491	-0.000160	-0.000111		-0.0000214
8.5	0.00501			0.00201		0.000749		0.000185	-0.0000823	-0.0000828		
10.0	0.00310			0.00157	0.00103		0.000624	0.000281	-0.0000198	-0.0000561	-0.0000348	-0.0000184
12.0	0.00180			0.00111		0.000642		0.000294	0.0000400	0.0000253		
15.0	0.000925			0.000673		0.000466		0.000289	0.0000827	0.00000825		
20.0	0.000391			0.000326	0.000284		0.000249	0.000197	0.0000926	0.0000348	-0.00000255	-0.00000704
40.0	0.0000490			0.0000468				0.0000408		0.0000246		0.00000433

Table 3. 1. U_t

z/r_1 \ r/r_1	0	0.2	0.4	0.5	0.6	0.7	0.8	0.825	0.85	0.875	0.9
0.0	0	-0.329	-0.764	-1.083	-1.550	-2.321	-3.875	-4.547	-5.449	-6.718	-8.636
0.01	0**					-2.300**	-3.879**		-5.419		-8.520**
0.02	0								-5.329		-8.242
0.03	0					-2.286	-3.759		-5.183		-7.791
0.05	0	-0.325	-0.752	-1.062	-1.508	-2.226**	-3.570**		-4.759		-6.587**
0.07	0	-0.321	-0.740		-1.472	-2.138	-3.305		-4.218	-4.759	-5.277
0.10	0					-1.969**	-2.833**		-3.339		-3.567**
0.15	0	-0.295	-0.664		-1.221		-2.016		-2.051		-1.681
0.20	0**	-0.272**	-0.593**		-1.021**	-1.249**	-1.320**				-0.673**
0.30	0	-0.210	-0.447		-0.621	-0.621**	-0.414**				0.175**
0.40	0**	-0.173**	-0.271		-0.298**		0.035**				
0.50	0**	-0.0853	-0.135	-0.126**	-0.0732		0.249				0.533
0.60	0**	-0.036**	-0.031		0.071**		0.345**				
0.80	0**	-0.030**	0.089**		0.204**		0.387**				
1.0	0**	-0.059**	0.132**	-0.179**	0.231**		0.355**				
1.2	0**	-0.065**	0.137**		0.218**		0.307**				

** Calculated with Küchemann and Weber's results of U_r and U_z

Table 3. 2. U_t

z/r_1 \ r/r_1	0	0.2	0.4	0.5	0.6	0.7	0.8	0.825	0.85	0.875	0.9
0.0	0	-0.329	-0.764	-1.083	-1.550	-2.321	-3.875	-4.547	-5.449	-6.718	-8.636
1.4	0 **	0.062**	0.125**		0.191**		0.258**				
1.5	0 **			0.147**							
1.6	0 **	0.054**	0.109**		0.163**		0.215**				
1.8	0 **	0.047**	0.092**		0.137**		0.178**				
2.0	0 **	0.039**	0.078**	0.096**	0.115**		0.148**				
2.5	0 **			0.062**							
3.0	0 **			0.041**							
3.5	0 **			0.028**							
4.0	0 **			0.020**							
4.5	0 **			0.015**							
5.0	0 **			0.0110			0.0173				
6.0	0			0.00664			0.0105				
7.0	0			0.00428			0.00677				
8.5	0			0.00244			0.00288				
10.0	0			0.00152							

Table 3. 3. U_t

z/r_1 \ r/r_1	0	0.2	0.4	0.5	0.6	0.7	0.8	0.825	0.85	0.875	0.9
0.0	0	-0.329	-0.764	-1.083	-1.550	-2.321	-3.875	-4.547	-5.449	-6.718	-8.636
12.0	0			0.000888			0.00142				
15.0	0			0.000459			0.000732				
20.0	0			0.000190							
40.0	0			0.0000200							

Table 3.4. U_t

z/r_1 \ r/r_1	0	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	0.995	0.997
0.0	0	−11.04	−12.78	−15.10	−18.36	−23.26	−31.47	−47.95	−97.63	−197.3	−330.4
0.003	0					−23.12	−31.14	−46.85		−144.3	−163.6
0.005	0							−45.00	−77.58	−96.90	− 85.27
0.007	0					−22.51		−42.47	−64.68	−64.80	− 48.81
0.01	0 **				−17.58**	−21.78	−28.11	−37.91	−47.55**	−37.30	
0.02	0				−15.57	−18.22	−21.15	−23.87	−17.63	−9.299	
0.03	0				−12.99	−14.19	−14.89	−12.29	−7.757	−3.120	
0.05	0				−8.280**	−7.952	−6.929	−4.920	−1.816**	0.0609	0.845**
0.07	0		−5.567		−5.061		−3.344		−0.126		
0.10	0				−2.374**	−1.788	−1.068	−0.225	0.710**	1.196	
0.15	0				−0.510	−0.160	0.223	0.633	1.054	1.268	
0.20	0 **				0.191**				1.100**		
0.30	0				0.628**				1.026**		
0.40	0 **										
0.50	0 **				0.694				0.823		

Table 3.5. U_t

z/r_1 \ r/r_1	0	0.998	0.999	0.9995	1.00	1.0005	1.001	1.002	1.003	1.005	1.01
0.0	0								336.3	202.7	102.3
0.003	0	−150.6	−96.60	−50.63	3.444	57.48	103.4	158.0	169.7	149.6	94.05
0.005	0	− 65.88	−35.30		3.189		41.62	72.03	91.20	102.8	82.36
0.007	0		−17.00		3.021		23.00		58.28	70.30	69.99
0.01	0 **	− 16.41			2.842**			22.11		42.67	52.39**
0.02	0				2.496					14.20	22.31
0.03	0				2.293					7.639	12.19
0.05	0				2.037**				3.223	3.998	5.831
0.07	0				1.868					3.832	
0.10	0				1.688**					2.176	2.652**
0.15	0				1.483					1.695	1.906
0.20	0 **				1.336**						1.569**
0.30	0				1.126**						1.224**
0.40	0 **				0.974**						
0.50	0 **				0.854**						0.886
0.60	0 **				0.756**						

Table 3. 6. U_t

z/r_1 \ r/r_1	0	1.0	1.003	1.005	1.01	1.02	1.03	1.04	1.05	1.06	1.07
0.0	0		336.3	202.7	102.3	51.95	35.06	26.57	21.44	17.98	15.54
0.8	0 **	0.600**									
1.0	0 **	0.482**									
1.2	0 **	0.390**									
1.4	0 **	0.318**									
1.5	0 **	0.288**									
1.6	0 **	0.260**									
1.8	0 **	0.215**									
2.0	0 **	0.178**									
2.5	0 **	0.115**									
3.0	0 **	0.0767									
3.5	0 **	0.053**									
4.0	0 **	0.038**									
4.5	0 **	0.028**									
5.0	0 **	0.0213									
6.0	0	0.0129									

Table 3. 7. U_t

z/r_1 \ r/r_1	0.0	1.0	1.003	1.005	1.01	1.02	1.03	1.04	1.05	1.06	1.07
0.0	0		336.3	202.7	102.3	51.95	35.06	26.57	21.44	17.98	15.54
7.0	0	0.00838									
8.5	0	0.00481									
10.0	0	0.00301									
12.0	0	0.00176									
15.0	0	0.000913									
20.0	0	0.000388									
40.0	0	6.0000489									

Table 3. 8. U_t

z/r_1 \ r/r_1	0	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.10	1.125	1.15	1.175
0.0	0	51.95	35.06	26.57	21.44	17.98	15.54	13.68	11.05	8.923	7.486	6.447
0.003	0	50.85	34.73	26.43								
0.005	0	49.02	34.16	26.19								
0.007	0	46.51		25.83								
0.01	0 **	41.37	31.75	25.11	20.68**				10.94**		7.458	
0.02	0	27.23	24.87	21.61	18.69				10.67		7.374	
0.03	0	17.38	18.64	17.63	15.92				10.24		7.239	
0.05	0	8.783	10.59	11.39	11.52**				9.092**		6.821	
0.07	0		6.908		8.262	8.474			7.825	7.063	6.332	
0.10	0	3.545	4.323	4.960	5.448**				6.122**		5.602	
0.15	0	2.311	2.688	3.030	3.333				4.184		4.229	
0.2	0 **				2.399**				3.059**			
0.3	0				1.589**				1.945**			
0.4	0 **											
0.5	0 **				1.004				1.134			

Table 3.9. U_t

r/r_1 z/r_1	0	1.2	1.3	1.4	1.5	1.6	1.8	2.0	2.5	3.0	3.4	3.5
0.0	0	5.659	3.777	2.803	2.205	1.800	1.287	0.978	0.575	0.382		0.273
0.01	0 **	5.634**	3.782**									
0.03	0	5.552	3.745									
0.05	0 **	5.382**	3.693**	2.768	2.187	1.790	1.283	0.976				
0.07	0	5.129	3.616	2.735		1.780	1.279	0.974	0.573	0.381	0.206	
0.10	0	4.700**	3.465**									
0.15	0	3.973	3.150	2.520	2.056	1.713	1.251	0.960	0.569	0.380		0.273
0.20	0 **	3.234**	2.806**	2.339**		1.649**	1.228**	0.940**				
0.30	0	2.256**	2.183**	1.964		1.502	1.147	0.910				
0.40	0 **	1.672		1.623**		1.339**	1.072**	0.864**				
0.50	0 **				1.275**			0.812**	0.519**	0.357**		0.257**
0.60	0 **	1.067**		1.130**		1.033**	0.893**	0.755**				
0.80	0 **	0.763**		0.825**		0.799**	0.729**	0.645**				
1.00	0 **	0.580**		0.629**	0.632**	0.627**	0.594**	0.542**	0.412**	0.311**		0.235**
1.20	0 **	0.456**		0.492**		0.501**	0.485**	0.457**		0.284		0.222
1.40	0 **	0.365**		0.397**		0.405**	0.402**	0.387**		0.260		0.208

Table 3.10. U_t

r/r_1 z/r_1	0	1.2	1.3	1.4	1.5	1.6	1.8	2.0	2.5	3.0	3.5
0.0	0	5.659	3.777	2.803	2.205	1.800	1.287	0.978	0.575	0.382	0.273
1.5	0 **				0.363**			0.354**	0.303**	0.248	0.201
1.6	0 **	0.297**		0.321**		0.333**	0.333**	0.325**		0.237	0.193
1.8	0 **	0.244**		0.264**		0.277**	0.281**	0.277**		0.214	0.179
2.0	0 **	0.202**		0.220**	0.227**	0.232**	0.237**	0.239**	0.221**	0.192**	0.168**
2.5	0 **				0.150**			0.162**	0.162**	0.150	0.134
3.0	0 **				0.102**			0.116**	0.120**	0.116**	0.108**
3.5	0 **				0.0722			0.0843	0.089**	0.089**	0.086**
4.0	0 **				0.052**			0.062**	0.067**	0.070**	0.070**
4.5	0 **				0.040**			0.047**	0.054**	0.056**	0.0565
5.0	0 **				0.0300			0.038**	0.0418	0.0448	0.0461
6.0	0				0.0186					0.0297	
7.0	0				0.0122					0.0205	
8.5	0				0.00705					0.0125	
10.0	0				0.00443					0.00808	
12.0	0				0.00261					0.00489	

Table 3.11. U_t

r/r_1 z/r_1	0	1.2	1.3	1.4	1.5	1.6	1.8	2.0	2.5	3.0	3.5
0.0	0	5.659	3.777	2.803	2.205	1.800	1.287	0.978	0.575	0.382	0.273
15.0	0				0.00136					0.00260	
20.0	0				0.000580					0.00113	
40.0	0				0.0000861					0.000146	

Table 3. 12. U_t

z/r_1 \ r/r_1	0	4.0	4.5	5.0	6.5	8.0	10	15	20	30	40
0.0	0	0.206	0.161	0.130	0.0758	0.0497	0.0317	0.0140	0.00787	0.00349	0.00196
0.01	0 **										
0.05	0										
0.07	0	0.206	0.161	0.130			0.0316		0.00787		0.00196
0.15	0	0.206	0.161	0.129			0.0316		0.00787		0.00196
0.2	0 **										
0.4	0 **										
0.5	0 **	0.201	0.155**	0.127**	0.0750	0.0494	0.0315		0.00783		0.00196
0.6	0 **										
0.8	0 **										
1.0	0 **	0.185	0.148	0.121							
1.2	0 **										
1.4	0 **										
1.5	0 **	0.164	0.135	0.109	0.0697	0.0471	0.0306	0.0138	0.00780	0.00348	0.00196
1.6	0 **										
1.8	0 **										

Table 3. 13. U_t

z/r_1 \ r/r_1	0	4.0	4.5	5.0	6.5	7.5	8.0	10	15	20	30	40
0.0	0	0.206	0.161	0.130	0.0758		0.0497	0.0317	0.0140	0.00787	0.00349	0.00196
2.0	0 **	0.141**	0.119	0.101	0.0655		0.0452	0.0298				
2.5	0 **	0.118	0.103	0.0899	0.0608		0.0429	0.0288				
3.0	0 **	0.098**	0.0880	0.0786	0.0557		0.0405	0.0277	0.0132	0.00761	0.00344	0.00195
3.5	0 **	0.0810	0.0747	0.0681								
4.0	0 **	0.0669	0.0631	0.0588	0.0457		0.0351	0.0252	0.0170	0.00742		
4.5	0 **	0.0554	0.0533	0.0515								
5.0	0 **	0.0461	0.0451	0.0434	0.0367		0.0298	0.0225	0.0119	0.00718	0.00335	0.00192
6.0	0			0.0322		0.0264		0.0198	0.0112	0.00691		0.00190
7.0	0			0.0241		0.0216		0.0172	0.0104	0.00661		0.00188
8.5	0			0.0160		0.0162		0.0138	0.00924	0.00612		
10.0	0			0.0110	0.0119		0.0119	0.0110	0,00803	0.00562	0.00294	0.00179
12.0	0			0.00704		0.00830		0.00793	0.00666	0.00376		
15.0	0			0.00393		0.00498		0.00533	0.00492	0.00402		
20.0	0			0.00178	0.00218		0.00250	0.00280	0.00301	0.00277	0.00199	0.00140
40.0	0			0.000239				0.000447		0.000701		0.000694