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DAMPING FACTORS IN THE HIGHER MODES OF SHIP VIBRATION*

By Toyoji KUMAI

Abstract. An analysis is made of damping factors in the higher modes of ship vibration. The solutions of the fundamental equation of the vibration of the uniform beam taking into account the effects of shear deflection, rotational inertia, internal damping force due to both normal and tangential viscosities of a ship structure and the external damping force are obtained. The damping factors represented by logarithmic decrement in ship vibrations including higher modes are calculated from the solutions under the assumption of some numerical values of the damping coefficients. Some of the damping factors measured from actual ships are compared with those obtained by the present theory. The empirical formula for estimating the damping factors in the higher modes of the hull vibration suggested by Dr. J. Lockwood Taylor is confirmed by the present theory.

Introduction. It is necessary to estimate the damping factors in the higher modes of ship vibrations when estimates of the hydrodynamic forces due to propeller blade and the unbalanced forces of the main engine from the measured response of the hull vibrations are required. The approximate formula for estimation of the damping factors in the higher modes of the hull vibrations has already been shown from the test result on a cargo ship by Dr. Taylor [1].

The theoretical value of the damping factors of vibration of the beam, however, is shown as proportional to N_n provided only the normal viscosity is considered as the internal damping factor of a steel structure, which has been deduced by Dr. Suehiro [2] and Dr. Sezawa [3].

The author mentions, as one of the reasons for the discrepancy between results of measurement and the previous theory mentioned above, failure to take into account the effects of the damping force due to tangential viscosity as well as of the shear deflection and rotational inertia of a ship structure in the higher modes of the flexural vibration.

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* Read in Tokyo at the Autumn Meeting of the Japan Society of Naval Architects on Nov. 16, 1957, (in Japanese). Published on "European Shipbuilding." Oslo, No. 1, Vol. VII, 1958.

[1] J. Lockwood Taylor; "Vibration of Ships" Trans. I.N.A. 1930.

[2] K. Suehiro; "On the Damped Transversal Vibration of Prismatic Bars" Bull. Earthq Res. Inst. Tokyo Imp. Univ. VI. 1929.

[3] K. Sezawa; "Die Wirkung des Enddruckes auf die Biegungsschwingung eines Stabes mit innerer Dampfung" Z.A.M.M. 1932.

The present paper deals with the theoretical investigation into the vibration of uniform beam taking the effects of shear deflection, rotational inertia, damping force due to normal and tangential viscosity and external damping force into account.

The numerical damping factors of a flexural hull vibration including higher modes of the vibration are presented from the theoretical results under some assumptions of the damping coefficients of the above factors. Moreover, the torsional vibration of a hull taking into account the internal damping factor is calculated. The damping coefficient of the torsional vibration measured from an actual ship was obtained and is used for the present calculation of the damping factor due to tangential viscosity in the flexural vibration mentioned above. The previous empirical formula for estimation of the damping factors in the higher modes of the ship vibrations suggested by Dr. Taylor and some results obtained from the measurements on actual ships which have been recently tested are confirmed by the present theory.

1. Theoretical investigation into the internal damping factors of ship structures.

a). Flexural vibration.

The fundamental equation of the flexural damped vibration of the beam considering the shear deflection, rotational inertia, damping factors due to normal and tangential viscosities and the external damping force reads (see Appendix)

$$
\begin{aligned}
& EI\left(1+\alpha\frac{\partial}{\partial t}\right)\left(1+\beta\frac{\partial}{\partial t}\right)\frac{\partial^4 y}{\partial x^4} \\
& -\frac{w}{g}\left\{\frac{EI}{k'AG}\left(1+\alpha\frac{\partial}{\partial t}\right)+r^2\left(1+\beta\frac{\partial}{\partial t}\right)\right\}\frac{\partial^4 y}{\partial x^2\partial t^2} \\
& +\zeta\left\{\left(1+\beta\frac{\partial}{\partial t}\right)\frac{\partial y}{\partial t}-\frac{EI}{k'AG}\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial^3 y}{\partial x^2\partial t}-\frac{wr^2}{gk'AG}\frac{\partial^3 y}{\partial t^3}\right\} \\
& +\frac{w}{g}\left(1+\beta\frac{\partial}{\partial t}\right)\frac{\partial^2 y}{\partial t^2}+\frac{w^2}{g^2}\frac{r^2}{k'AG}\frac{\partial^4 y}{\partial t^4}=0,
\end{aligned}
\tag{1}
$$

with the solution of

$$
\begin{aligned}
y = & (A_1\sin\lambda_1 x + A_2\cos\lambda_1 x + A_3\sinh\lambda_2 x + A_4\cosh\lambda_2 x) \\
& \times (C_1\sin pt + C_2\cos pt)\cdot\exp\cdot(-qt).
\end{aligned}
\tag{2}
$$

The characteristic values of the vibration of the beam are determined by the boundary conditions of the free-free bar, thence, corresponding natural frequency and the damping factor of the vibration may be written

$$
p \doteqdot \sqrt{\frac{gEI}{2wL^2r^2}\left\{a-\sqrt{a^2-4\mu(\lambda_1 L)^4\{1-(\alpha+\beta)q\}+4(\lambda_1 L)^2(1+\mu)g\zeta/w}\right\}}, \tag{3}
$$

$$
q \doteqdot \frac{gEI(\lambda_1 L)^2}{2wL^2r^2}\cdot\frac{\mu\{\alpha(\mu+\nu)+\beta\}}{(1-\mu)^2+\nu\{2(1+\mu)+\nu\}}+g\zeta/w. \tag{4}
$$

The damping factor represented by the logarithmic decrement is easily obtained by use of the above (3) and (4) as the following form;

$$\delta = 2\pi q/p, \tag{5}$$

where,

$$a = \{(1+\mu)(\lambda_1 L)^2 + \mu L^2/r^2\} - 3\{(\alpha + \mu\beta)(\lambda_1 L)^2 + \mu\beta L^2/r^2 + g\zeta/w\}q - (\alpha + \nu\beta)g\zeta/w,$$

$$\nu = \mu \frac{L^2}{r^2(\lambda_1 L)^2},$$

$$\mu = \frac{k' A G r^2}{EI}.$$

The first term of (4) includes the coefficients of normal viscosity ξ and of tangential viscosity η which constitute the internal damping factor of a ship structure and the second term including ζ represents the external damping factor in which the damping force is proportional to the velocity of the vibration of the beam.

b). Torsional vibration.

The fundamental equation of the torsional vibration of a beam taking the internal and external damping force into account is deduced as follows;

$$GJ\frac{\partial^2\varphi}{\partial x^2} + \eta J \frac{\partial^3\varphi}{\partial x^2 \partial t} = \frac{w r_0^2}{g}\frac{\partial^2\varphi}{\partial t^2} + \zeta\frac{\partial\varphi}{\partial t}. \tag{6}$$

The solution of the above equation is obtained as

$$\varphi = (A_5 \sin\lambda x + A_6 \cos\lambda x)(C_3 \sin pt + C_4 \cos pt)e^{-q_t t}. \tag{7}$$

Ignoring the external damping force, p_t and q_t are presented as the following forms,

$$\left.\begin{aligned} p_t &\doteqdot \sqrt{\frac{gGJ(\lambda L)^2}{w r_0^2 L^2}(1-\beta q)}, \\ q_t &\doteqdot \frac{\beta}{2}\frac{gGJ(\lambda L)^2}{w_0 L^2}. \end{aligned}\right\} \tag{8}$$

The logarithmic decrement is expressed by the following simple form;

$$\delta_t = \pi\beta p = \frac{2\pi^2\beta N_t}{60}, \tag{9}$$

where, N_t is the natural frequency of torsional vibration of a ship in cpm. As is seen in the above result, the damping factor presented by logarithmic decrement in the torsional vibration of a beam is proportional to the natural frequencies of the higher modes of the torsional vibration. Accordingly, the coefficient η or β can easily be calculated by (9) if β is obtained from the results of the test of torsional vibration. The damping coefficient β in the above equation is assumed to be the same value as that of the tangential viscosity in the case of the flexural vibration presented by (4) and (5). The numerical example will be shown in the last paragraph of the present paper.

2. The result of the measurement of external damping factor by model tests. In accordance with the result of theoretical investigation which has been carried out by Dr. Sezawa and W. Watanabe [4], the external damping force in a ship vibration may be divided into three sources, namely, (1) the water friction, (2) generation of pressure wave, (3) generation of surface wave. According to the theoretical calculations, the sum of all the external damping factors is negligibly small compared with the structural damping factor except for that due to surface wave in shallow draught. On the other hand, with regard to the experiments on actual ship, the damping factor of the hull vibration measured from a record of the free vibration immediately after the vessel had been launched showed a lower value notwithstanding the shallow draught [1]. The damping factor obtained by this method shows almost the same value as that measured from the record of free vibration immediately after the anchoring test on the trial trip. As a result of the comparison of the measured damping factors mentioned above, it may be supposed that the damping force due to surface wave is not so conspicuous in actual ships as theory would suggest.

In the present paper, as an approximate consideration, the external damping force is assumed to be proportional to the velocity of the vibration of the ship's hull including the higher modes of the vibration. The theoretical calculation of the damping factor made on the above assumption has been shown in the previous paragraph.

As a result of calculation on the above assumption the damping coefficient ζ may be assumed to be constant for similar types of ships.

It is impossible to measure the vibration of actual ships in air, therefore, we can not separate the external damping factor from that obtained on water. In the present investigation, the vibration measurements on two models were carried out in air and on water, and the respective damping factors were measured. As a results of the model tests, the mean value of the external damping coefficient ζ, for example, was determined by use of the second term of (4) as follows;

$$\zeta = 1.10\,\text{gr.\,sec/cm}^2\,. \tag{10}$$

3. The damping coefficient due to tangential viscosity obtained from the results of measurements of torsional vibration on a tanker. A vibration generator survey was recently made on a 32,500 ton dw. tanker at the Harima Shipbuilding and Engineering works, using a 12 ton exciter. The measurements of the torsional vibration were carried out in this experiment with the aid of a mechanical type of accelerograph with two

[4] Sezawa and W. Watanabe; "Damping Forces in Vibration of a Ship" Jour. of Soc. of Naval Architects, Japan, No. 59, 1936.

Table I Damping factors of one-node and two-node torsional vibration of a 32,500 ton dw. tanker in ballast condition.

no. of nodes	N_t (cpm)	δ_t	$\eta/G\ (=\beta)$
1	235	0.0308	0.45×10^{-3} (sec)
2	420	0.0660	

components of vibration direction, vertical and horizontal transverse. The accelerograph was set at the port side deck stringer plate on upper deck near the position of the propeller. Both the vertical and horizontal resonant vibrations were recorded at the same time with the frequencies of 235 cpm and of 420 cpm when the horizontal exciting forces were applied at the poop deck. It will be evident that these types of record are of one-node and two-node torsional vibrations respectively. From their response curve the damping factors in the respective nodes of torsional vibrations are easily obtained as shown in Table I and Fig. 1.

By use of the formula shwon in the previous paragraph, 1. (b), the damping coefficient due to tangential viscosity $\beta\ (=\eta/G)$ is easily calculated and is also shown in Table I.

The coefficient of tangential viscosity which is used in the case of the flexural vibration of the structure may be taken as the same value as that of the torsional vibration.

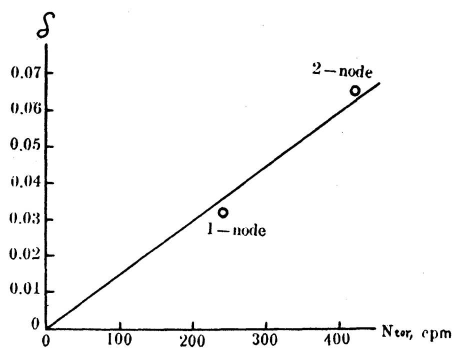
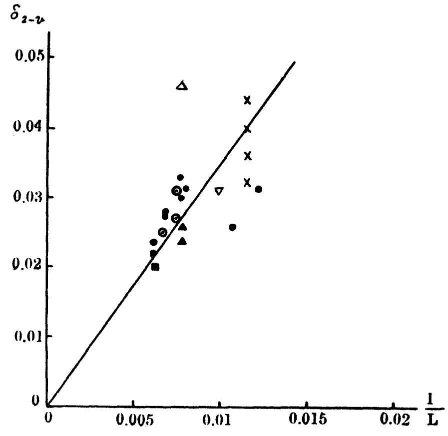


Fig. 1. Measured results of the logarithmic decrements of the torsional vibration of a 32,500 ton dw. tanker in ballast condition.



— empirical formula; $\delta_{2-v} = 3.50/L$, ($L_{\text{in m}}$)
▽ Cargo ship (Riveted), J. L. Taylor
× Torpedo boat, ● Cargo ship, } T. Tomita
△ "Clan Alpine" (Riveted), ▲ "Ocean Vulcan" } A. J. Johnson
⊙ Cargo ship, K. Kitada and Y. Matsumura
■ "Gopher Mariner," R. T. McGoldrick

Fig. 2. Relation between measured results of the logarithmic decrements of the two-node vertical vibration of various ships and their length.

4. Damping factors in vertical vibration already measured on actual ships.

a). Damping factors in 2 node vertical vibration.

The damping factors in 2 node vertical vibration of various ships measured by several investigators are arranged in Fig. 2.

As is seen in the results of the theoretical calculation, the structural damping factors of the two-node vibration of similar ships are inversely proportional to their length provided that the structure, loading condition and ship's shape are similar each other. The same relation also holds for the external damping factor in similar ships. Accordingly, the empirical formula for preliminary estimation of the damping factor in the two-node vertical vibration of cargo ships and tankers of 80 m ~ 200 m in length that in ballast condition is obtained from the data measured by numerous investigators as in the following form;

$$\delta_{2-v}=\frac{C}{L}, \tag{11}$$

where, δ_{2-v}: logarithmic decrement in 2 node vertical vibration of ships with length of 80 m ~ 200 m in ballast condition
C: 3 ~ 4
L: length of ships in meter

b). Damping factors in the higher mode of vertical vibration.

Dr. Taylor suggested that the damping factors in the higher modes of vertical vibration are proportional to the natural frequency N_n by the classical theory, the result measured on an actual ship, however, shows that the damping factors are almost proportional to $N^{0.76}$ as an empirical expression [1].

The results of the measurements of damping factors in the higher modes of vertical vibration of actual ships measured by three authors [1], [5] and [6] are shown in Fig. 3.

As will be seen in these measured results, the damping factors in the higher mode of the vertical vibrations are empirically represented as suggested by Dr. Taylor. The convenient formula is also presented by

$$\delta_{n-v}=\delta_{2-v}(N_n/N_2)^{3/4}, \tag{12}$$

where, δ_{n-v}; logarithmic decrement of n-node vertical vibration in ballast condition

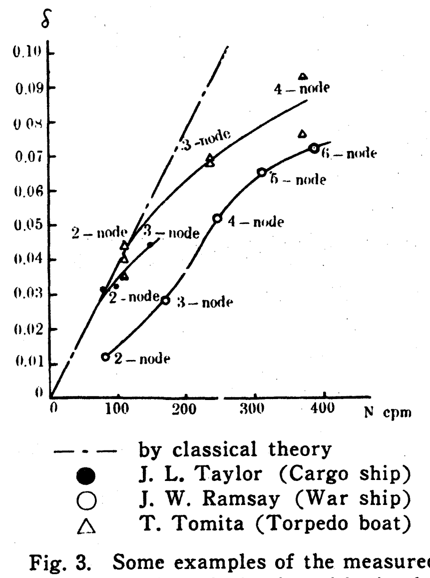


Fig. 3. Some examples of the measured results of the logarithmic decrements in the higher modes of the vertical vibration of ships.

[5] T. Tomita; "Vibration Measurement on a Coast Guard Vessel" (in Japanese) Mitsui Tech. Review, No. 17, 1956.

[6] J. W. Ramsay; "Aspects of Ship Vibration induced by Twin Propellers" Quart Trans. I.N.A. Vol. 98, 1956.

δ_{2-v}; logarithmic decrement of 2-node vertical vibration presented by (11)
N_n, N_2; natural frequencies of n-node and 2-node modes of the hull vibrations respectively

5. Numerical Example. The following numerical data, as an example, were assumed for the calculations of the present theory.

$$L \times B \times D, d; \quad 164\,\text{m} \times 22.4\,\text{m} \times 12.3\,\text{m},\ 4.5\,\text{m}$$
$$I = 63\,\text{m}^4, \quad A = 1.17\,\text{m}^2,$$
$$k' = 0.2, \quad r^2 = 27.4\,\text{m}^2,$$
$$w = 209\,\text{ton/m}, \quad g = 9.8\,\text{m/sec}^2,$$
$$E = 2 \times 10^7\,\text{ton/m}^2, \quad G = 0.8 \times 10^7\,\text{ton/m}^2.$$

In the first stage, the natural frequencies including higher modes of the vertical vibrations are computed by the theoretical calculations presented in the previous paragraph. Since the effect of the damping factor on the natural frequencies is negligibly small, we put this effect out of account for the present computations of the natural frequencies of the hull vibration.

As the second stage, the damping factor of the hull in the two-node vertical vibration δ_{2-v} is estimated by the formula presented by (11).

Next, the damping factor due to tangential viscosity is calculated by the use of the coefficient η which has been obtained from the measurement of torsional vibration of a 32,500 ton dw. tanker. On the other hand, the external damping factor is calculated using the coefficient ζ in (10).

Finally, from the residual $(\delta - \delta_\eta - \delta_\zeta = \delta_\xi)$, the factor due to normal viscosity in the two-node vibration is obtained, which is confirmed by the data already measured by Dr. Suehiro [2]. Consequently, the damping factors in the higher mode of the vertical hull vibration are computed by the present calculation.

The numerical results of computation of the components of the damping factors in the higher symmetric modes of the vertical vibrations which have been computed by the above process are shown in Table II. The comparisons between the theoretical and the empirical results are also shown in Table II and Fig. 4.

Table II The numerical results of the components of logarithmic decrement in the higher symmetric modes of vertical vibration of a ship.

no. of nodes	N_v cpm	δ_ξ	δ_η	δ_ζ	δ	δ_{emp}
2	62.04	0.01813	0.00317	0.00032	0.0216	0.0216
4	173.80	0.02410	0.02110	0.00010	0.0453	0.0467
6	289.30	0.02260	0.04070	0.00006	0 0634	0.0687
8	401.00	0.02180	0.06000	0.00004	0.0818	0.0890

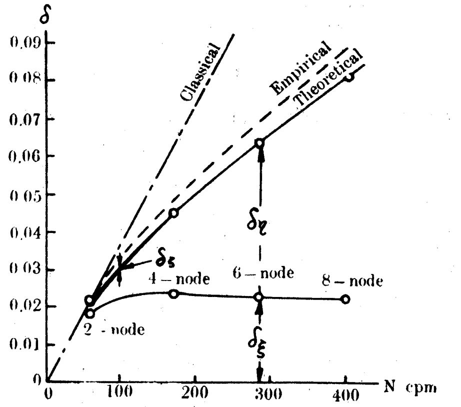


Fig. 4. Theoretical and empirical results of the logarithmic decrements in the higher symmetric modes of the vertical vibration of a hull in ballast condition.

As may be seen from the above table and figure, the theoretical results are found to show good agreement with the empirical ones. Moreover it is clearly shown that the components of the internal damping factor due to normal and tangential viscosities in the higher mode of the vibration of a ship are explained with respect to the natural frequencies by the theoretical results.

Conclusion. The follwoing conclusions may be drawn as a result of the theoretical investigation into the damping factors in the higher modes of ship vibrations under the assumption of the damping coefficients and by use of the results obtained from the tests on a ship and on model experiments.

1. The damping factor due to normal viscosity of ship structure is considered to be almost constant for all the natural frequencies of the vertical vibration of a hull.
2. The damping factor due to tangential viscosity of ship structure is considered to be proportional to the natural frequencies of the vibrations including the higher modes of the hull vibration.
3. The external damping factor is negligible compared with internal damping factor in the large types of cargo ships and tankers.
4. The damping factor in the case of the higher modes of the torsional vibration of a ship is considered to be proportional to the natural frequencies of the vibration.
5. The empirical formulae shown in the present paper may be used for the estimation of the vibratory exciting forces in the higher modes of the vertical hull vibration.

In conclusion, the author wishes to express his thanks to the Mitsubishi Hiroshima, Mitsui Tamano, and Harima Shipbuilding and Engineering Works for permission to publish the numerical data and to carry out the tests on shipboard. He is also grateful to the Science Research Division of the Department of Education whose support made the work possible.

This paper was discussed at the meeting of the Western Section of Ship Structure Committee of Japan Society of Naval Architects.

Appendix

1. Symbols

M	bending moment
Q	shearing force
I	moment of inertia of the section of the beam
A	cross sectional area of the beam
E	Young's modulus
G	shear modulus
$k'AG$	effective shear rigidity of hull section
L	length of the beam
x	length coordinate
y	amplitude of vibration of the beam
ψ	slope due to bending moment
$dy/dx-\psi$	slope due to shear force
ξ, $(\alpha=\xi/E)$	coefficient of normal viscosity
η, $(\beta=\eta/G)$	coefficient of tangential viscosity
ζ, ζ_t	coefficients of external viscosities in flexural and torsional vibrations of the beam respectively
r	radius of gyration of mass moment of inertia
w	weight of beam with load per unit length
t	time coordinate
g	acceleration due to gravity
p, p_t	natural circular frequencies of flexural and torsional vibrations of the beam respectively
q, q_t	damping factors in flexural and torsional vibrations of the beam respectively
δ, δ_t	logarithmic decrements in flexural and torsional vibrations of the beam respectively
$\lambda_1 L$, $\lambda_2 L$; λL	eigen values of flexural and torsional vibrations respectively
GJ	torsional rigidity of the beam
φ	twisting angle
r_0	radius of gyration of polar mass moment of inertia
$A_1A_2\cdots C_1C_2\cdots$	integration constants

2. Deduction of Fundamental Equation of Damped Vibration of Beam taking into account the Effects of Shear Deflection and Rotational Inertia.

The bending moment and the shearing force of the element of the beam in the state of transverse damped vibration due to internal viscosity are presented respectively as the following forms;

$$M=-EI\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial\psi}{\partial x}, \tag{a}$$

$$Q = k'AG\left(1+\beta\frac{\partial}{\partial t}\right)\left(\frac{\partial y}{\partial x}-\psi\right). \tag{b}$$

The equations of the equilibrium of the moment and the vertical force for the inertia force and the external damping force of the element of the beam are respectively written as follows;

$$-\frac{\partial M}{\partial x}dx + Q\,dx = \frac{wr^2}{g}\frac{\partial^2\psi}{\partial t^2}dx\,, \tag{c}$$

$$\frac{\partial Q}{\partial x}dx = \frac{w}{g}\frac{\partial^2 y}{\partial t^2}dx + \zeta\frac{\partial y}{\partial t}dx\,. \tag{d}$$

Substituting from (a) and (b) into (c) and (d), we have

$$EI\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial^2\psi}{\partial x^2} + k'AG\left(1+\beta\frac{\partial}{\partial t}\right)\left(\frac{\partial y}{\partial x}-\psi\right) - \frac{wr^2}{g}\frac{\partial^2\psi}{\partial t^2} = 0\,, \tag{e}$$

$$k'AG\left(1+\beta\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 y}{\partial x^2}-\frac{\partial\psi}{\partial x}\right) - \frac{w}{g}\frac{\partial^2 y}{\partial t^2} - \zeta\frac{\partial y}{\partial t} = 0\,. \tag{f}$$

Eliminating ψ, $\partial\psi/\partial x \cdots$ from (e) and (f), the fundamental equation of the damped vibration of the beam taking into account the effects of the shear deflection and the rotational inertia is obtained as was seen in (1).

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