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## ON THE EFFECTIVENESS OF PANTING STRINGERS AND WEB FRAMES OF A SHIP

(2nd Report)

#### By Jiro Suhara\*

Introduction. In the first report [I] (1) the author studied the "Effectiveness of Panting Stringers and Web Frames of A Ship" and showed that the stiffness of the stringers with web frames whose sizes were determined by the Rules of The Classification Societies are not strong enough to support side frames. Recently, however, strong stringers with web frames are often adopted, which is known as "Web and Stringer System." This paper gives the result of an analysis made to the structure of panting zone of an 8,750 G.T. cargo ship which has been strengthened in such a way as stated above. Beside a exact analysis on the above problem, the author has undertaken a simpler kind of calculation on the preceding problem. As the result of the comparison made to both the methods, i.e., a precise analytical method and a convenient calculation, the latter has been found to be appropriate as far as it is concerned with the "Web and Stringer system."

Assumptions and Procedures of Analysis. In this report, the uniform pressure of 1.0 kg/cm<sup>2</sup> is assumed to be applicable to the whole surface of shell plating in panting zone as the severest loading condition.\*\*

The structure between two transverse bulkheads in panting zone is consisted of two panting stringers and three web frames with side frames and shell plating as shown in Fig. 1. For simplicity, upper and lower ends of frames and web frames connecting the points of stringers to the bulkheads are assumed to be all clamped.

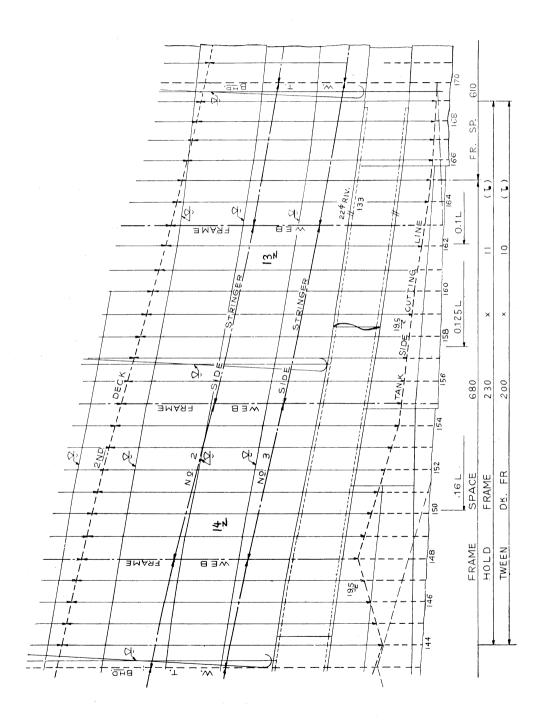
As the first step, an analysis is given to a slightly different structure which has symmetrical arrangement of web frames and stringers in longitudinal direction and side frames of equal size, as shown in Fig. 2.

At this stage, the thickness of shell plating was assumed to be equal at all parts to 13.5 mm, the average value of real plating. However, as the sizes of the webs of upper and lower stringers are varied respectively, moments of inertia were treated separately to each span and were applied the value obtained by calculating separately each of upper and lower stringers.

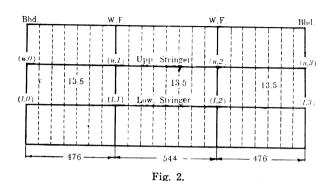
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<sup>(1)</sup> Numbers in parentheses [ ] refer to Bibliography at the end of the paper.

<sup>\*\*</sup> The reason for adopting this condition is given in the 1st report.



In the second stage, the result of the above calculation was compared to what was obtained by evaluating the moments of inertia of both upper and lower stringers on the basis of an assumption that either of them is equivalent to the average of its real value.



And in the 3rd stage, calculation was made by assuming that the value of the moments of inertia of both upper and lower stringers equal to that which is given by

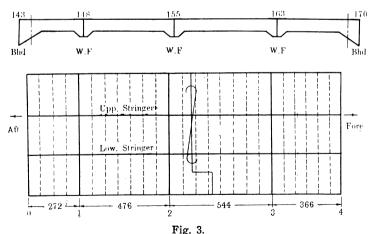
$$1/I = (1/2)\{(1/I_u) + (1/I_l)\}, \tag{1}$$

where it is provided  $I_u$  and  $I_t$  are respectively the actual values of the moments of inertia of upper and lower stringers.

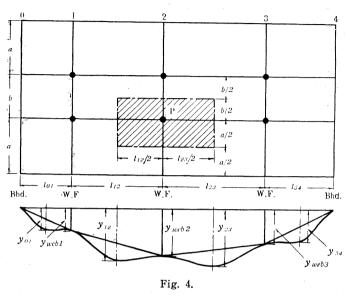
The results obtained on the basis of this assumption fairly coincide with what were derived from the separate treatment of the moments of inertia of upper and lower stringers, as stated in the first step calculation.

Therefore, in fourth step, the same assumptions as adopted in the third step are used only on the treatment of the moments of inertia of stringers, but actual arrangement of web frames and stringers are analyzed as shown

in Fig. 3, where the thickness of shell plating is assumed to be taken 13 mm and 14 mm for the aft and the fore part of Fr. No. 155 respectively, and moments of inertia of side frames are separately calculated for each web frame spacing.



In the last stage, a simple calculation was performed on the basis of an assumption that web frames are applied the concentrated loads at all crossing points of stringers and them, and the magnitudes of the loads are obtained as the total sum of uniform pressure of 1.0 kg/cm² applied upon the part enclosed by the straight lines dividing equally the adjoining spans of



web frames and stringers (for the crossing points *P* in Fig. 4, as example, the part stated above is indicated by shading).

In that case, the deflections of the web frames at the crossing points of them and stringers can be easily obtained by the elementary calculations. The base lines of deflection curves of

stringers are obtained by connecting rectilinearly the deflected crossing points of web frames and stringers. And superposing the deflection curves of stringers clamped at both ends of each web frame span of stringers, approximate deflection curves for stringers may be obtained, where the stringers are assumed to be applied to the uniform external pressure 1.0 kg/cm² between the part divided by straight lines through the middle of the consecutive spans of web frames.

Through all stages of analysis, the calculations of the effective breadth of shell plating in the process of the evaluation of the moments of inertia of all stiffening members are carried out on the basis of Dr. C. H. A. Schade's theory that treats of the cases where all the ends of stiffening members are clamped. And all the points at which stringers are connected to bulkheads have been assumed to be clamped, too.

By using accurate or approximate deflection curves of stringers obtained by the above step, it is possible to estimate the supporting force of stringer to any side frame at the connecting point between them.

Comparing the bending moments in side frames due to external pressure applied to one frame spacing and supporting forces from stringers to the bending moments due to external pressure only, the effectiveness of panting stringers and web frames to any side frame is made clear.

Analysis for the Structure with Symmetrically Arranged Stringers in Longitudinal Direction. For the structure shown in Fig. 2, the analysis is performed by using the following data.

#### Stringers:

	Location of Span	(01) and (23)	(12)
	Length of Span	$l_{01} = l_{23} = 476 \mathrm{cm}$	$l_{12} = 544 \text{ cm}$
Mts. of	Upp. Stringer	$I_{u  01} = 11.690 \times 10^4  \mathrm{cm}^4$	$I_{u12}=12.098\times 10^4\mathrm{cm}^4$
Inertia	Low. Stringer	$I_{l  01} = 14.281 \times 10^4  \mathrm{cm}^4$	$I_{l12} = 14.834 \times 10^4 \mathrm{cm}^4$
Ratio of Span		$ ho \equiv l_{12}/l_{01} = 1.143$	
Ratio of Mts. of Inertia		$r_{01} \equiv I_{l \ 01}/I_{u \ 01} = 1.222$	$r_{12} \equiv I_{l  12}/I_{u  12} = 1.226$

#### Web Frames:

Full length  $l_{web} = 3 \times 205 \text{ cm} = 615 \text{ cm}$ 

Spans of web frames a = b = 205 cm

Moment of Inertia.  $I_{web} = 33.276 \times 10^4 \text{ cm}^4$ 

Torsional rigidities of web frames are neglected, as they affect slightly the stringers.

#### Side Frames:

Full length.  $l_{fr} = 615 \, \mathrm{cm}$ Spans of side frames  $a = b = 205 \, \mathrm{cm}$ Moment of inertia.  $I_{fr} \equiv 0.5666 \times 10^4 \, \mathrm{cm}^4$ Frame spacing  $s = 68.0 \, \mathrm{cm}$ 

Parameters determined from relative flexual rigidities between side frames and stringers are

for (01) span,  $\alpha = \alpha_{01} = (3l_{01}/l_{fr})(I_{fr}l_{fr}/2I_{01}s)^{1/4}$ for (12) span,  $\alpha = \alpha_{12} = (3l_{12}/l_{fr})(I_{fr}l_{fr}/2l_{12}s)^{1/4}$ 

#### Coefficients in Slope Deflection Equations for Stringers.

$$A(\alpha) = 2\alpha \left( \sinh 2\alpha - \sin 2\alpha \right) / \left( \cosh 2\alpha + \cos 2\alpha - 2 \right)$$
 etc.

For brevity we put  $A(\alpha) = A(\alpha_{01}) \equiv A_{01}$ ,  $B(\alpha) = B(\alpha_{12}) \equiv B_{12}$  etc.

General formulae based on the assumption  $I_{u01} = I_{l01} \equiv I_{01}$  and  $I_{u12} = I_{l12} \equiv I_{12}$ , are given in the 1st report [1].\*\*

Numerical results derived from the assumptions  $I = (1/2)(I_u + I_l)$  and  $(1/I) = (1/2)\{(1/I_u) + (1/I_l)\}$  for each web frame spacing respectively, are tabulated in Table I. (See page 80)

## Analysis under Separate Treatment of the moment of Inertia of Upper and Lower Stringer.

#### Equation of Stringers at any Sapan.

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* Other coefficients are given in the 1st report. p. 72. (12),[1]  
** In the 1st report, following misprints should be corrected: page 71, Fig. 4 for "=x/l_{st}" read "\xi = x/l_{st}" page 73, line 21, for "\eta \{1 - h_1(\xi, \alpha) - h_2(\xi, \alpha)\}" read "\eta \{1 - h_1(\xi, \alpha) - h_3(\xi, \alpha)\}," page 74, Table 1. line 4, for "Int½ Pl. 0.5" read "Int½ Pl. 9.5" page 75, line 11, for "-\{\rho^3 G_{10} + (G_{12} - H_{12})\overline{\mu}\}\varphi" read "-\{\rho^3 G_{10} + (G_{12} - H_{12}) + \overline{\mu}\}\varphi" page 75, line 14, for "\overline{\eta}" read "\overline{\mu}". page 76, line 33, for "(fre)_{R_1=0}" read (M_{fre})_{R_1=0}"
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Taking  $\xi$ -axis along stringer from each left end of span, the equations of flexure of stringers are

$$\frac{\lambda_{uu}(d^{4}y_{u}/d\xi^{4}) + \lambda_{ul}(d^{4}y_{l}/d\xi^{4}) + y_{u} = \eta_{u}}{\lambda_{lu}(d^{4}y_{u}/d\xi^{4}) + \lambda_{ll}(d^{4}y_{l}/d\xi^{4}) + y_{l} = \eta_{l}}$$
(2)

where  $y_u$  and  $y_l$  are respectively the deflections of upper and lower stringers at any section of stringers, and are also the deflections of side frames at the crossing points of stringers due to external pressure in the case where stiffening effects due to stringer are neglected.  $\xi = 0$  and  $\xi = 1$  correspond to the left and right ends of stringers at any span, suffix 01 or 12 is omitted so far as there is no fear of causing confusion, and in the same way, the following coefficients  $\lambda_{uu}$  etc. are given by

$$\lambda_{uu} = EI_{u} \mu_{uu} s / l^{4}, \quad \lambda_{ul} = EI_{l} \mu_{ul} s / l^{4} 
\lambda_{lu} = EI_{u} \mu_{lu} s / l^{4}, \quad \lambda_{ll} = EI_{l} \mu_{ul} s / l^{4}$$
(3)

where, l is the span of stringer, and  $\mu_{ul}$  is the deflection at the crossing point of a given frame and the upper stringer when it is applied to unit concentrated load at the crossing point of the frame and the lower stringer, and  $\mu_{uu}$ ,  $\mu_{lu}$ ,  $\mu_{lu}$  are denominated similarly.

Parameters determined by relative flexual rigidities between side frames and stringers are defined as follows,

$$\alpha_{II} = \left\{ (\lambda_{uu} + \lambda_{ll} + \sqrt{(\lambda_{uu} - \lambda_{ll})^2 + 4\lambda_{ul}\lambda_{lu}}) / 8(\lambda_{uu}\lambda_{ll} - \lambda_{ul}\lambda_{lu}) \right\}^{1/4}$$

$$\alpha_{II} = \left\{ (\lambda_{uu} + \lambda_{ll} - \sqrt{(\lambda_{uu} - \lambda_{ll})^2 + 4\lambda_{ul}\lambda_{lu}}) / 8(\lambda_{uu}\lambda_{ll} - \lambda_{ul}\lambda_{lu}) \right\}^{1/4}$$

$$\left\{ (\lambda_{uu} + \lambda_{ll} - \sqrt{(\lambda_{uu} - \lambda_{ll})^2 + 4\lambda_{ul}\lambda_{lu}}) / 8(\lambda_{uu}\lambda_{ll} - \lambda_{ul}\lambda_{lu}) \right\}^{1/4}$$

In this case, we put a = b, and define following constants.

$$K = \left(rac{I_u}{I_{fr}}
ight)\!\!\left(rac{s}{l_{fr}}
ight)\!\!\left(rac{l_{fr}}{3\,l}
ight)^{\!4}, \hspace{0.5cm} r = I_l/I_u$$
 ,

then we have

$$\lambda_{uu} = (8/27)K, \quad \lambda_{ll} = (8/27)rK, \quad \lambda_{ul} = (11/54)rK,$$

$$\lambda_{lu} = (11/54)K,$$

$$\alpha_{I} = \left[ \{8(1+r) + \sqrt{64 - 7r + 64r^2}\}/10rK \right]^{1/4},$$

$$\alpha_{II} = \left[ \{8(1+r) - \sqrt{64 - 7r + 64r^2}\}/10rK \right]^{1/4}$$
(5)

Denoting

$$\begin{split} \beta_{\rm I} &= \frac{1 - 4\,\alpha_{\rm I}{}^4\,\lambda_{uu}}{4\,\alpha_{\rm I}{}^4\,\lambda_{ul}} = \frac{27 - 32\,\alpha_{\rm I}{}^4\,K}{22\,r\,\alpha_{\rm I}{}^4\,K} \\ \beta_{\rm II} &= \frac{1 - 4\,\alpha_{\rm II}{}^4\,\lambda_{uu}}{4\,\alpha_{\rm II}{}^4\,\lambda_{ul}} = \frac{27 - 32\,\alpha_{\rm II}{}^4\,K}{22\,r\,\alpha_{\rm II}{}^4\,K} \end{split}$$

we obtain the slope deflection coefficients derived from equations (2)

$$A_{uu} = rac{1}{eta_{\Pi} - eta_{\Pi}} \left\{ eta_{\Pi} A(lpha_{\Pi}) - eta_{\Pi} A(lpha_{\Pi}) 
ight\}, \quad A_{ul} = rac{-1}{eta_{\Pi} - eta_{I}} \left\{ A(lpha_{I}) - A(lpha_{\Pi}) 
ight\}.$$

$$A_{lu} = \frac{r \beta_{\rm I} \beta_{\rm II}}{\beta_{\rm II} - \beta_{\rm I}} \{ A(\alpha_{\rm I}) - A(\alpha_{\rm II}) \} , \qquad A_{ll} = \frac{-r}{\beta_{\rm II} - \beta_{\rm I}} \{ \beta_{\rm I} A(\alpha_{\rm I}) - \beta_{\rm II} A(\alpha_{\rm II}) \}$$

$$B_{uu} = \frac{1}{\beta_{\rm II} - \beta_{\rm I}} \{ \beta_{\rm II} B(\alpha_{\rm I}) - \beta_{\rm I} B(\alpha_{\rm II}) \} , \qquad B_{ul} = \frac{-1}{\beta_{\rm II} - \beta_{\rm I}} \{ B(\alpha_{\rm I}) - B(\alpha_{\rm II}) \}$$

$$B_{lu} = \frac{r \beta_{\rm I} \beta_{\rm II}}{\beta_{\rm II} - \beta_{\rm I}} \{ B(\beta_{\rm I}) - B(\beta_{\rm II}) \} , \qquad B_{ll} = \frac{-r}{\beta_{\rm II} - \beta_{\rm I}} \{ \beta_{\rm I} B(\alpha_{\rm I}) - \beta_{\rm II} B(\alpha_{\rm II}) \}$$

$$(6)$$

where  $A(\alpha_I)$ ,  $B(\alpha_I)$  etc. are the functions of which are given by (12) in the 1st report and by inserting the value of  $\alpha_{\rm I}$  or  $\alpha_{\rm II}$  obtained from (4) or (5).

Coefficients of Flexure for Web Frames. Deflections of stringers at crossing points of web frames may be

$$v_{u} = \mu_{web, uu} P_{u} + \mu_{web, ul} P_{l}$$

$$v_{l} = \mu_{web, ul} P_{u} + \mu_{web, ll} P_{l}$$
(7)

where  $\mu_{web, uu}$  etc. are coefficients of flexure of web frame. Solving (7) with respect to  $P_u$  and  $P_l$ , we obtain

$$P_{u} = \frac{E I_{u12}}{l_{12}^{3}} (\overline{\mu} v_{u} + \overline{\mu}' v_{l})^{*}$$

$$P_{l} = \frac{E I_{u12}}{l_{12}^{3}} (\overline{\mu}' v_{u} + \overline{\mu} v_{l})$$
(8)

where

$$\overline{\mu} = \frac{l_{12}^3}{E I_{u,12}} \cdot \frac{\mu_{web, uu}}{\mu_{web, uu}^2 - \mu_{web, ul}^2}, \quad \overline{\mu}' = \frac{-l_{12}^3}{E I_{u,12}} \cdot \frac{\mu_{web, ul}}{\mu_{web, uu}^2 - \mu_{web, ul}^2}$$
(9)

When a = b, we have

$$\overline{\mu} = \frac{2592}{5} \left( \frac{I_{web}}{I_{u12}} \right) \left( \frac{l_{12}}{l_{web}} \right)^{3}$$

$$\overline{\mu}' = -\frac{1782}{5} \left( \frac{I_{web}}{I_{u12}} \right) \left( \frac{l_{12}}{l_{web}} \right)^{3}$$
(10)

Fixing moments and shearing forces of stringers, when they are fixed at all ends of spans, are

$$\overline{M}_{u} = \mp \frac{\eta E I_{u}}{(\beta_{\Pi} - \beta_{I}) l^{2}} \{ (\beta_{\Pi} - 1)(D_{I} - C_{I}) - (\beta_{I} - 1)(D_{\Pi} - C_{\Pi}) \}, 
\overline{M}_{l} = \mp \frac{r \eta E I_{u}}{(\beta_{\Pi} - \beta_{I}) l^{2}} \{ \beta_{I} (\beta_{\Pi} - 1)(D_{I} - C_{I}) - \beta_{\Pi} (\beta_{I} - 1)(D_{\Pi} - C_{\Pi}) \}, 
\overline{Q}_{u} = \pm \frac{\eta E I_{u}}{(\beta_{\Pi} - \beta_{I}) l^{3}} \{ (\beta_{\Pi} - 1)(H_{I} - G_{I}) - (\beta_{I} - 1)(H_{\Pi} - G_{\Pi}) \}$$
(11)

The values of  $I_u$  and l in  $(7) \sim (10)$  may be used those of any span, we used accordingly the respective values of  $l_{u12}$  and  $l_{12}$  for the case of analysis shown in Fig. 2.

$$\overline{Q}_{l} = \pm rac{r\eta E I_{u}}{(eta_{ ext{II}} - eta_{ ext{I}}) l^{3}} \{eta_{ ext{I}}(eta_{ ext{II}} - 1)(H_{ ext{I}} - G_{ ext{I}}) - eta_{ ext{II}}(eta_{ ext{I}} - 1)(H_{ ext{II}} - G_{ ext{II}})\}$$

Elastic Equations of Stringers for Two Consecutive Spans  $(\alpha - 1, \alpha)$  and  $(\alpha, \alpha + 1)$ . The equilibrium equation of shearing forces for upper stringer on the nodal point  $(u, \alpha)$  is

$$\left(\frac{EI_{u}}{l^{2}}\right)_{a, \alpha-1} \left\{ (D_{u}, u)_{\alpha, \alpha-1} \theta_{u, \alpha-1} + (D_{u}, l)_{\alpha, \alpha-1} \theta_{l, \alpha-1} \right\} \\
+ \left\{ \left(\frac{EI_{u}}{l^{2}}\right)_{\alpha, \alpha-1} (C_{uu})_{\alpha, \alpha-1} - \left(\frac{EI_{u}}{l^{2}}\right)_{\alpha, \alpha+1} (C_{uu})_{\alpha, \alpha+1} \right\} \theta_{u, \alpha} \\
+ \left\{ \left(\frac{EI_{u}}{l^{2}}\right)_{\alpha, \alpha-1} (C_{ul})_{\alpha, \alpha-1} - \left(\frac{EI_{u}}{l_{a}}\right)_{\alpha, \alpha+1} (C_{ul})_{\alpha, \alpha+1} \right\} \theta_{l, \alpha} \\
- \left(\frac{EI_{u}}{l^{2}}\right)_{\alpha, \alpha+1} \left\{ (D_{uu})_{\alpha, \alpha+1} \theta_{u, \alpha+1} + (D_{ul})_{\alpha, \alpha+1} \theta_{l, \alpha+1} \right\} \\
+ \left(\frac{EI_{u}}{l^{3}}\right)_{\alpha, \alpha-1} \left\{ (H_{uu})_{\alpha, \alpha-1} v_{u, \alpha-1} + (H_{u, l})_{\alpha, \alpha-1} v_{l, \alpha-1} \right\} \\
- \left\{ \left(\frac{EI_{u}}{l^{3}}\right)_{\alpha, \alpha-1} (G_{uu})_{\alpha, \alpha-1} + \left(\frac{EI_{u}}{l^{3}}\right)_{\alpha, \alpha+1} (G_{ul})_{\alpha, \alpha+1} \right\} v_{l, \alpha} \\
- \left\{ \left(\frac{EI_{u}}{l^{3}}\right)_{\alpha, \alpha-1} (G_{ul})_{\alpha, \alpha-1} + \left(\frac{EI_{u}}{l^{3}}\right)_{\alpha, \alpha+1} (G_{ul})_{\alpha, \alpha+1} \right\} v_{l, \alpha} \\
+ \left(\frac{EI_{u}}{l^{3}}\right)_{\alpha, \alpha-1} \left\{ (H_{u, u})_{\alpha, \alpha-1} v_{u, \alpha-1} + (H_{u, l})_{\alpha, \alpha-1} v_{l, \alpha-1} \right\} \\
- \left\{ \left(\frac{EI_{u}}{l^{3}}\right)_{\alpha, \alpha+1} \overline{\mu}_{\alpha} v_{u, \alpha} + \left(\frac{EI_{u}}{l^{3}}\right)_{\alpha, \alpha+1} \overline{\mu}'_{\alpha} v_{l, \alpha} \right\} - \left\{ \overline{Q}_{u, \alpha, \alpha-1} + \overline{Q}_{u, \alpha, \alpha+1} \right\} = 0$$

Do. for lower stringer is omitted, as it is obtainable by interchanging the suffix u to l in the above equation.

The equilibrium equation of moments of forces for upper stringer on the nodal point  $\boldsymbol{\alpha}$  is

$$\begin{split} &\left(\frac{EI_{u}}{l^{2}}\right)_{\alpha, \alpha-1}\{(B_{u, u})_{\alpha, \alpha-1}\theta_{u, \alpha-1}+(B_{u, l})_{\alpha, \alpha-1}\theta_{l, \alpha-1}\}\\ &+\left\{\left(\frac{EI_{u}}{l^{2}}\right)_{\alpha, \alpha-1}(A_{u, u})_{\alpha, \alpha-1}+\left(\frac{EI_{u}}{l^{2}}\right)_{\alpha, \alpha+1}(A_{u, u})_{\alpha, \alpha+1}\right\}\theta_{u, \alpha}\\ &+\left\{\left(\frac{EI_{u}}{l^{2}}\right)_{\alpha, \alpha-1}(A_{u, l})_{\alpha, \alpha-1}+\left(\frac{EI_{u}}{l^{2}}\right)_{\alpha, \alpha+1}(A_{u, l})_{\alpha, \alpha+1}\right\}\theta_{l, \alpha}\\ &+\left(\frac{EI_{u}}{l^{2}}\right)_{\alpha, \alpha+1}\{(B_{u, u})_{\alpha, \alpha+1}\theta_{u, \alpha+1}+(B_{u, l})_{\alpha, \alpha+1}\theta_{l, \alpha+1}\}\end{split}$$

$$+ \left(\frac{EI_{u}}{l^{3}}\right)_{\alpha, \alpha-1} \left\{ (D_{u, u})_{\alpha, \alpha-1}v_{u, \alpha-1} + (D_{u, l})_{\alpha, \alpha-1}v_{l, \alpha-1} \right\}$$

$$- \left\{ \left(\frac{EI_{u}}{l^{3}}\right)_{\alpha, \alpha-1} (C_{u, u})_{\alpha, \alpha-1} - \left(\frac{EI_{u}}{l^{3}}\right)_{\alpha, \alpha+1} (C_{u, u})_{\alpha, \alpha+1} \right\} v_{u, \alpha}$$

$$- \left\{ \left(\frac{EI_{u}}{l^{3}}\right)_{\alpha, \alpha-1} (C_{u, l})_{\alpha, \alpha-1} - \left(\frac{EI_{u}}{l^{3}}\right)_{\alpha, \alpha+1} (C_{u, l})_{\alpha, \alpha+1} \right\} v_{l, \alpha}$$

$$- \left\{ \left(\frac{EI_{u}}{l^{3}}\right)_{\alpha, \alpha+1} (D_{u, u})_{\alpha, \alpha+1} v_{u, \alpha+1} + \left(\frac{EI_{u}}{l^{3}}\right)_{\alpha, \alpha+1} (D_{u, l})_{\alpha, \alpha+1} v_{l, \alpha+1} \right\}$$

$$+ \left\{ \overline{M}_{u, \alpha, \alpha-1} + \overline{M}_{u, \alpha, \alpha+1} \right\} = 0$$

Do. for lower stringer is omitted by the same reason as stated above.

**End Condition.** If the ends of stringers connecting to the bulkheads are assumed to be fixed, we may write according to the notation shown in Fig. 2.

$$\begin{cases} v_{u0} = v_{u3} = v_{l0} = v_{l3} = 0 \\ \theta_{u0} = \theta_{u3} = \theta_{l0} = \theta_{l3} = 0 \end{cases}$$

$$\begin{cases} v_{u1} = v_{u2} = v_{u}, & \theta_{u1} = -\theta_{u2} = \theta_{u} \\ v_{l1} = v_{l2} = v_{l}, & \theta_{l1} = -\theta_{l2} = \theta_{l} \end{cases}$$
(14)

Applying the elastic equations (12) and (13) etc. to the span (0, 1) and (1, 2), we obtain the equations referred to the deflections  $v_u$ ,  $v_l$  and rotations  $\theta_u$ ,  $\theta^l$  at the crossing points of upper and lower stringers respectively

$$\begin{cases} \psi_{1\,uu}\,\theta_{u} + \psi_{1\,ul}\,\theta_{l} - \psi_{2\,uu}\,\varphi_{u} - \psi_{2\,ul}\,\varphi_{l} = X_{1\,u} \\ \psi_{1\,lu}\,\,\theta_{u} + \psi_{1\,ll}\,\,\theta_{l} - \psi_{2\,lu}\,\,\varphi_{u} - \psi_{2\,ll}\,\varphi_{l} = X_{1\,l} \\ \psi_{2\,uu}\,\theta_{u} + \psi_{2\,ul}\,\theta_{l} - \psi_{3\,uu}\,\varphi_{u} - \psi_{3\,ul}\,\varphi_{l} = X_{2\,u} \\ \psi_{2\,l\,u}\,\,\theta_{u} + \psi_{2\,ll}\,\,\theta_{l} - \psi_{3\,lu}\,\,\varphi_{u} - \psi_{3\,ll}\,\varphi_{l} = X_{2\,l} \end{cases}$$

$$(15)$$

where

$$v_{u}/l_{12} = \varphi_{u}, \quad v_{l}/l_{12} = \varphi_{v}$$

$$I_{u01}/I_{u12} = \gamma, \quad l_{12}/l_{01} = \rho$$

$$\psi_{1uu} = \gamma \rho A_{uu01} + (A_{uu12} - B_{uu12}), \quad \psi_{1ul} = \gamma \rho A_{ul01} + A_{ul12} - B_{ul12}$$

$$\psi_{1lu} = \gamma \rho A_{lu01} + (A_{lu12} - B_{lu12}), \quad \psi_{1ll} = \gamma \rho A_{ll01} + A_{ll12} - B_{ll12}$$

$$\psi_{2uu} = \gamma \rho^{2} C_{uu01} - (C_{uu12} - D_{uu12}), \quad \psi_{2ul} = \gamma \rho^{2} C_{ul01} - (C_{ul12} - C_{ul12})$$

$$\psi_{2lu} = \gamma \rho^{2} C_{lu01} - (C_{lu12} - D_{lu12}), \quad \psi_{2ll} = \gamma \rho^{2} C_{ll01} - (C_{ll12} - D_{ll12})$$

$$\psi_{3un} = \gamma \rho^{3} G_{uu01} + (G_{uu12} - H_{uu12}) + \overline{\mu}$$

$$\psi_{3lu} = \gamma \rho^{3} G_{ul01} + (G_{lu12} - H_{lu12}) + \overline{\mu}'$$

$$\psi_{3ul} = \gamma \rho^{3} G_{ul01} + (G_{ul12} - H_{ul12}) + \overline{\mu}'$$

$$\psi_{3ul} = \gamma \rho^{3} G_{ul01} + (G_{ul12} - H_{ul12}) + \overline{\mu}'$$

$$\psi_{3ul} = \gamma \rho^{3} G_{ul01} + (G_{ul12} - H_{ul12}) + \overline{\mu}'$$

And

$$X_{1u} = -\left(\frac{l}{EI_{u}}\right)_{12} \cdot (\overline{M}_{u \, 10} + \overline{M}_{u \, 12})$$

$$X_{1l} = -\left(\frac{l}{EI_{u}}\right)_{12} \cdot (\overline{M}_{l \, 10} + \overline{M}_{l \, 12})$$

$$X_{2u} = \left(\frac{l^{2}}{EI_{u}}\right)_{12} \cdot (\overline{Q}_{u \, 10} + \overline{Q}_{u \, 12})$$

$$X_{2l} = \left(\frac{l^{2}}{EI_{u}}\right)_{12} \cdot (\overline{Q}_{l \, 10} + \overline{Q}_{l \, 12})$$

$$(17)$$

Solving equation (12) and substituting the obtained values of  $\theta_u$ ,  $\theta_l$ ,  $\varphi_u$  and  $\varphi_l$  into the following equations, we obtain the end moments and shearing forces at every ends of the stringers

$$\begin{split} M_{u \, 10} &= \frac{EI_{u \, 10}}{l_{10}} \{A_{uu \, 10} \theta_u + A_{ul \, 10} \theta_l - C_{uu \, 10} \varphi_u - C_{ul \, 10} \varphi_l\} + \overline{M}_{u \, 10} \\ M_{l \, 10} &= \frac{EI_{u \, 10}}{l_{10}} \{A_{lu \, 10} \theta_u + A_{ll \, 10} \theta_l - C_{lu \, 10} \varphi_u - C_{ll \, 10} \varphi_l\} + \overline{M}_{l \, 10} \\ Q_{u \, 10} &= \frac{EI_{u \, 10}}{l_{10}^2} \{-C_{uu \, 10} \theta_u - C_{ul \, 10} \theta_l + G_{uu \, 10} \varphi_u + G_{ul \, 10} \varphi_l\} + \overline{Q}_{u \, 10} \\ Q_{l \, 10} &= \frac{EI_{u \, 10}}{l_{10}^2} \{-C_{lu \, 10} \theta_u - C_{ll \, 10} \theta_l + G_{lu \, 10} \varphi_u + G_{ll \, 10} \varphi_l\} + \overline{Q}_{l \, 10} \\ M_{u \, 12} &= \frac{EI_{u \, 10}}{l_{10}^2} \{(A_{uu \, 12} - B_{uu \, 12}) \theta_u + (A_{ul \, 12} - B_{ul \, 12}) \theta_l \\ &\quad + (C_{uu \, 12} - D_{uu \, 12}) \varphi_u + (C_{ul \, 12} - D_{ul \, 12}) \varphi_l\} + \overline{M}_{u \, 12} \\ M_{l \, 12} &= \frac{EI_{u \, 12}}{l_{12}} \{(A_{lu \, 12} - B_{lu \, 12}) \theta_u + (A_{ll \, 12} - B_{ll \, 12}) \theta_l \\ &\quad + (C_{lu \, 12} - D_{lu \, 12}) \varphi_u + (C_{u \, 12} - D_{ul \, 12}) \varphi_l\} + \overline{M}_{l \, 12} \\ Q_{u \, 12} &= \frac{EI_{u \, 12}}{l_{12}^2} \{(C_{uu \, 12} - D_{uu \, 12}) \theta_u + (C_{u \, 12} - D_{u \, 12}) \theta_l \\ &\quad + (G_{uu \, 12} - H_{uu \, 12}) \varphi_u + (G_{u \, 12} - H_{u \, 12}) \varphi_l\} + \overline{Q}_{u \, 12} \\ Q_{l \, 12} &= \frac{EI_{u \, 12}}{l_{12}^2} \{(C_{lu \, 12} - D_{lu \, 12}) \theta_u + (C_{lu \, 12} - H_{u \, 12}) \varphi_l\} + \overline{Q}_{u \, 12} \\ + (G_{lu \, 12} - H_{u \, 12}) \varphi_u + (G_{lu \, 12} - H_{u \, 12}) \varphi_l\} + \overline{Q}_{l \, 12} \end{split}$$

Preliminary functions for deflection of stringers are defined as follows:

$$\begin{split} h_{1\,\mathrm{I}} &= \left[h_{1}(\xi,\alpha)\right]_{\substack{\xi=1/2\\\alpha=\alpha_{\mathrm{I}}}} = \frac{2}{\cos2\,\alpha_{\mathrm{I}} + \cos2\,\alpha_{\mathrm{I}} - 2} \left\{2\,\sin\alpha_{\mathrm{I}}\,\sinh\alpha_{\mathrm{I}}\,\sin\frac{\alpha_{\mathrm{I}}}{2}\,\sinh\frac{\alpha_{\mathrm{I}}}{2} \\ &+ (\sin\alpha_{\mathrm{I}}\,\cosh\alpha_{\mathrm{I}} + \cos\alpha_{\mathrm{I}}\,\sinh\alpha_{\mathrm{I}}) \left(\cos\frac{\alpha_{\mathrm{I}}}{2}\,\sinh\frac{\alpha_{\mathrm{I}}}{2} - \sin\frac{\alpha_{\mathrm{I}}}{2}\,\cosh\frac{\alpha_{\mathrm{I}}}{2}\right)\right\} \end{split}$$

$$\begin{split} h_{2\,\mathrm{I}} &= \left[h_2(\xi,\alpha)\right]_{\substack{\xi=1/2\\\alpha=\alpha_\mathrm{I}}} = \frac{-2}{\alpha_1(\cosh2\;\alpha_\mathrm{I} + \cos2\alpha_\mathrm{I} - 2)} \Big\{ \{\sin\alpha_\mathrm{I} \sinh\alpha_\mathrm{I} \Big(\sin\frac{\alpha_\mathrm{I}}{2} \cosh\frac{\alpha_\mathrm{I}}{2} \\ &-\cos\frac{\alpha_\mathrm{I}}{2} \sinh\frac{\alpha_\mathrm{I}}{2} \Big) + (\cos\alpha_\mathrm{I} \sinh\alpha_\mathrm{I} - \sin\alpha_\mathrm{I} \cosh\alpha_\mathrm{I}) \sin\frac{\alpha_\mathrm{I}}{2} \sinh\frac{\alpha_\mathrm{I}}{2} \Big\} \end{split}$$

And putting similarly,

$$egin{aligned} h_{1\, ext{II}} = [h_{1}(\xi,lpha)]_{\substack{\xi=1/2 \ lpha=lpha_{ ext{II}}}}, & h_{2\, ext{II}} = [h_{2}(\xi,lpha)]_{\substack{\xi=1/2 \ lpha=lpha_{ ext{II}}}} \end{aligned}$$

where the functions  $h_1(\xi, \alpha)$  and  $h_2(\xi, \alpha)$  were already introduced in the 1st report.

Deflection of upper stringer at the middle of (0, 1) span is

$$egin{aligned} egin{aligned} egin{aligned\\ egin{aligned} egi$$

Do. of lower stringer is

$$\begin{split} y_{l10} &= \frac{1}{\beta_{\text{II}} - \beta_{\text{I}}} \left[ \eta \{ -\beta_{\text{I}} \beta_{\text{II}} (h_{\text{II}} - h_{\text{III}}) + \beta_{\text{I}} h_{\text{II}} - \beta_{\text{II}} h_{\text{III}} \} \right. \\ &+ (v_{u_{\text{I}}} - \eta) \beta_{\text{I}} \beta_{\text{II}} (h_{\text{II}} - h_{\text{III}}) - (v_{\text{I}l} - \eta) (\beta_{\text{I}} h_{\text{II}} - \beta_{\text{II}} h_{\text{III}}) \\ &- \theta_{u_{\text{I}}} l_{01} \beta_{\text{I}} \beta_{\text{II}} (h_{21} - h_{2\text{II}}) + \theta_{l \cdot l} l_{01} (\beta_{\text{I}} h_{21} - \beta_{\text{II}} h_{2\text{II}}) \right] + \eta \end{split}$$

Deflection of upper stringer at the middle of (1, 2) span is

$$\begin{aligned} y_{u12} &= \frac{2}{\beta_{11} - \beta_{1}} \{ (v_{u1} - \eta)(\beta_{11}h_{11} - \beta_{1}h_{111}) + (v_{t1} - \eta)(h_{111} - h_{11}) \\ &+ \theta_{u1}l_{12}(\beta_{11}h_{21} - \beta_{1}h_{211}) - \theta_{t1}l_{12}(h_{21} - h_{211}) \} + \eta \end{aligned}$$

Do. of lower stringer is

$$\begin{aligned} & y_{l \mid 12} = \frac{2}{\beta_{\text{II}} - \beta_{\text{I}}} \{ (v_{u \mid 1} - \eta) \, \beta_{\text{I}} \beta_{\text{II}} (h_{\text{I} \mid 1} - h_{\text{I} \mid \text{II}}) + (v_{u \mid 1} - \eta) (\beta_{\text{II}} h_{\text{I} \mid \text{II}} - \beta_{\text{I}} h_{\text{I} \mid \text{I}}) \\ & + \theta_{u \mid 1} l_{12} \beta_{\text{I}} \beta_{\text{II}} (h_{2 \mid 1} - h_{2 \mid \text{II}}) - \theta_{l \mid 1} l_{12} (\beta_{\text{I}} h_{2 \mid 1} - \beta_{\text{II}} h_{2 \mid \text{II}}) \} + \eta \end{aligned}$$

Equations described below in this paragraph are common for each span of stringer, therefore we omit the suffix (0, 1) or (1, 2) in following formulae.

Reduction rates of deflection of side frames due to stringers are

$$\Delta u = \frac{\eta - y_u}{\eta} \times 100 \%$$

$$extit{ extit{d}}_l = rac{\eta - extit{y}_l}{\eta} imes 100 \, \%$$

Supporting forces of stringers to side frames in the case of a = b are

$$R_{u} = \frac{324 E I_{fr}}{5 l_{fr}^{3}} \{8 (\eta - y_{u}) - 5.5 (\eta - y_{l})\}$$

$$R_{l} = \frac{324 E I_{fr}}{5 l_{fr}^{3}} \{-5.5 (\eta - y_{u}) + 8 (\eta - y_{l})\}$$

Fixing moments of side frames, when a = b for upper end,

$$M_A = \frac{w \, s \, l_{fr}^2}{12} - \frac{2 \, l_{fr}}{27} (2 R_u + R_l)$$

for lower end,

$$M_B = \frac{w s l_{fr}^2}{12} - \frac{2 l_{fr}}{27} (R_u + 2 R_l)$$

for the middle of the span of side frames

$$M_C = \frac{w \, s \, l_{fr}^2}{24} - \frac{l_{fr}(R_u + R_l)}{18}$$

Reduction rates of them due to the stringers respectively,

$$\Omega_A = \frac{8(2R_u + R_l)}{9 w s l_{fr}} \times 100 \%$$

$$\Omega_B = \frac{8(R_u + 2R_l)}{9 \, w \, s \, l_{fr}} \times 100 \, \%$$

Reduction rate of the middle of the span of side frames

$$\Omega_C = \frac{4(R_u + R_l)}{3 w s l_{tr}} \times 100 \%$$

Table I

		1st Step Calculation		2nd Step Cal.	3rd Step Cal.
	Item	Upp. Stringer	Lower Stringer	$I = \frac{1}{2}(I_u + I_l)$	$I = \frac{1}{2} \left( \frac{1}{I_u} + \frac{1}{I_l} \right)$
General	$l_{10}$ cm $l_{12}$ cm $\rho = l_{12}/l_{01}$ $s$ cm $l_{fr}$ cm $l_{fr}$ cm $I_{fr}$ cm $I_{fr}$ cm $I_{(st)01}$ cm $I_{(st)12}$ cm $I_{(st)12$	-0, 1, -0,	$\begin{array}{c} 476\\ 544\\ 1.143\\ 68\\ 615\\ 0.5666\times10^4\\ 205\\ 14.281\times10^4\\ 14.834\times10^4\\ 2.365\\ 1.546\\ 2.680\\ 1.752\\ 7821\\ 0464\\ 7756\\ 048\\ \end{array}$	$ \begin{array}{c} 476 \\ 544 \\ 1.143 \\ 68 \\ 615 \\ 0.5666 \times 10^4 \\ 205 \\ 13.013 \times 10^4 \\ 13.493 \times 10^4 \\ \end{array} \\ \left. \begin{array}{c} \alpha_{10} = 1.546 \\ \end{array} \right\}  \boldsymbol{\alpha}_{12} = 1.752 \\ \end{array} $	$\begin{array}{c} 476\\ 544\\ 1.143\\ 68\\ 615\\ 0.566\times10^4\\ 205\\ 12.855\times10^4\\ 13.302\times10^4\\ 1.551\\ 1.758\\ \end{array}$

Table I (cont'd)

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,	Item	1st Step Calculation		2nd Step Cal.	3rd Step Cal.
	Item	Upp. stringer	Lower stringer	$I = \frac{1}{2} \left( I_u + I_l \right)$	$I = \frac{1}{2} \left( \frac{1}{I_u} + \frac{1}{I_l} \right)$
Stringers	$egin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 0.5584\times10^{-3}\\ 0.3334\times10^{-3}\\ 0.1814\\ +364.66\times10^{4}\\ -462.17\times10^{4}\\ -46.58\times10^{3}\\ -52.07\times10^{3}\\ +343.87\times10^{4}\\ -42.291\times10^{3}\\ -44.958\times10^{3}\\ -87.249\times10^{3}\\ \end{array}$	$\begin{array}{c} 0.5546\times10^{-3}\\ 0.3352\times10^{-3}\\ 0.1823\\ +377.80\times10^{4}\\ -487.60\times10^{4}\\ -47.87\times10^{3}\\ -54.26\times10^{3}\\ +353.34\times10^{4}\\ -353.36\times10^{4}\\ -43.044\times10^{3}\\ -46.731\times10^{3}\\ -89.775\times10^{3}\\ \end{array}$	$\begin{array}{c} 0.5520\times10^{-3} \\ 0.3336\times10^{-3} \\ 0.1815 \\ +371.26\times10^{4} \\ -473.23\times10^{4} \\ -47.22\times10^{3} \\ -52.97\times10^{3} \\ +347.96\times10^{4} \\ -347.69\times10^{4} \\ -42.665\times10^{3} \\ -45.701\times10^{3} \\ -88.356\times10^{3} \end{array}$	$\begin{array}{c} 0.5557\times10^{-3}\\ 0.3341\times10^{-3}\\ 0.1817\\ +370.88\times10^{4}\\ -473.51\times10^{4}\\ -47.27\times10^{4}\\ -53.01\times10^{4}\\ +348.36\times10^{4}\\ -348.36\times10^{4}\\ -42.72\times10^{3}\\ -45.72\times10^{3}\\ -88.44\times10^{3}\\ \end{array}$
Web <sup>.</sup> Fr	R kg	$+87.319\times10^{3}$	$+89.685 \times 10^{3}$	$+88.332 \times 10^{3}$	+88.44×10 <sup>3</sup>
Ord. Fr	$y_{01}$ cm $y_{12}$ cm $\eta$ cm $A_{10}$ % $A_{12}$ % $R_{01}$ kg $R_{12}$ kg	$\begin{array}{c} 0.1580 \\ 0.3996 \\ 1.6822 \\ 90.61 \\ 76.25 \\ 12.370 \times 10^{3} \\ 10.308 \times 10^{3} \end{array}$	$\begin{array}{c} 0.1437 \\ 0.3820 \\ 1.6822 \\ 91.46 \\ 77.29 \\ 13.010 \times 10^{3} \\ 11.095 \times 10^{3} \end{array}$	$\begin{array}{c} 0.1501 \\ 0.3875 \\ 1.6822 \\ 91.08 \\ 76.96 \\ 12.697 \times 10^{3} \\ 10.729 \times 10^{3} \end{array}$	$\begin{array}{c} 0.1487 \\ 0.3946 \\ 1.6822 \\ 91.16 \\ 76.54 \\ 12.708 \times 10^{3} \\ 10.670 \times 10^{3} \end{array}$
	Item	1st Step (	Calculation B end (lower)	2nd Step Cal. $I = \frac{1}{2}(I_u + I_l)$	$\begin{array}{ c c c }\hline & 3rd & Step & Cal. \\\hline & \frac{1}{I} = \frac{1}{2} \left( \frac{1}{I_n} + \frac{1}{I_l} \right) \\\hline \end{array}$
me	$egin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{c} 214.32 \times 10^{4} \\ 42.35 \times 10^{4} \\ 69.86 \times 10^{4} \\ 20.45 \end{array}$	214.32×10 <sup>4</sup> 39.43×10 <sup>4</sup> 66.27×10 <sup>4</sup> ×10 <sup>4</sup> ×10 <sup>4</sup>	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{ c c c c }\hline 1 & 2 & 1n & 1t \\ \hline & 214.32 \times 10^4 \\ & 40.65 \times 10^4 \\ & 68.49 \times 10^4 \\ & 20.62 \times 10^4 \\ & 34.25 \times 10^4 \\ \hline\end{array}$

	T4	1st Step Calculation		2nd Step Cal.	3rd Step Cal.
	Item	A end (upper)	B end (lower)	$I=\frac{1}{2}(I_u+I_l)$	$\frac{1}{I} = \frac{1}{2} \left( \frac{1}{I_n} + \frac{1}{I_l} \right)$
Ord, Frame	$egin{array}{lll} M & { m kg} { m cm} \ M_{01} & { m kg} { m cm} \ M_{12} & { m kg} { m cm} \ M_{c01} & { m kg} { m cm} \ M_{c12} & { m kg} { m cm} \ M_{c12} & { m kg} { m cm} \ \Omega_{01} & { m \%} \ \Omega_{c01} & { m \%} \ \Omega_{c12} & { m \%} \ \sigma_{01} & { m kg/cm^2} \ \sigma_{c12} & { m kg/cm^2} \ \end{array}$	34.03 80.24 67.40 80 68 1518 2504	$ \begin{vmatrix} 214.32 \times 10^4 \\ 39.43 \times 10^4 \\ 66.27 \times 10^4 \\ \times 10^4 \\ & 69.08 \\ .92 \\ .24 \\ & 1414 \\ 2376 \\ .733 \\ .220 \end{vmatrix} $	$\begin{array}{c} 214.32\times10^4\\ 40.80\times10^4\\ 67.69\times10^4\\ 20.40\times10^4\\ 33.85\times01^4\\ 80.96\\ 68.41\\ 80.96\\ 68.41\\ 1463\\ 2427\\ 732\\ 1214\\ \end{array}$	$\begin{array}{c} 214.32\times10^{4} \\ 40.65\times10^{4} \\ 68.49\times10^{4} \\ 20.62\times10^{4} \\ 34.25\times10^{4} \\ 81.03 \\ 68.04 \\ 81.03 \\ 68.04 \\ 1457 \\ 2455 \\ 729 \\ 1228 \end{array}$
String- er	σ <sub>(st) 10</sub> kg/cm <sup>2</sup>	1604	1291	1447	1445
Str	$\sigma_{(st)12}$ kg/cm <sup>2</sup>	1581	1273	1425	1428

Bending stresses of side frame are respectively

$$\sigma_A = M_A/Z_{fr}$$
 $\sigma_B = M_B/Z_{fr}$ 

where  $Z_{fr}$  is section modulus of the side frame.

Numerical results obtained from above equations are shown in Table I.

As it was expected, the results derived from the assumption  $(1/I) = (1/2)\{(1/I_u) + (1/I_l)\}$ , give the results of best approximation to that from accurate theory.

Analysis for the Structure with Actual Arrangement of Stringers and Web Frames.

	span	01	12	23	34
$1_{st}$	cm	272	476	544	366
s	cm	68	68	68	61
$I_{fr}$	cm <sup>4</sup>	$0.5725 \times 10^{4}$	0.5725×10 <sup>4</sup>	0.5614×104	$0.5498 \times 10^{4}$
77	cm	1.665	1.665	1.648	1.344
$I_{st}$	cm <sup>4</sup>	10.88×10 <sup>4</sup>	13.11×10 <sup>4</sup>	13.19×10 <sup>4</sup>	$11.81 \times 10^4$
$I_{web}$	cm <sup>4</sup>	1 $38.69 \times 10^4$ 2 $33.28 \times 10^4$ 3 $43.74 \times 10^4$		74×10 <sup>4</sup>	
α		0.926	1.548	1.758	1.243

Table II

In this paragraph we treat of the structure shown in Fig. 3 of which scantlings are given in Table II, but using the facts obtained in the preceding paragraph, we assume that the moments of inertia of upper and lower stringers are both equal to I, which is given by

$$rac{1}{I}=rac{1}{2}\Big(rac{1}{I_u}+rac{1}{I_l}\Big)$$

for respective span of stringers. Other assumptions referred to scantlings are already stated in the preceding paragraph.

Applying the elastic equations of stringers (16) given in the 1st report, p. 73 to the crossing points 1, 2 and 3, we obtain

$$-(\lambda_{1}\rho_{1}A_{01} + A_{12})\theta_{1} - B_{12}\theta_{2} + (\lambda_{1}\rho_{1}^{2}C_{10} - C_{12})\varphi_{1} + \rho_{2}D_{12}\varphi_{2} = X_{1}$$

$$-\lambda_{2}\rho_{2}B_{12}\theta_{1} - (\lambda_{2}\rho_{2}A_{12} + A_{23})\theta_{2} - B_{23}\theta_{3}$$

$$-\lambda_{2}\rho_{2}D_{12}\varphi_{1} + (\lambda_{2}\rho_{2}^{2}C_{12} - C_{23})\varphi_{2} + \rho_{3}D_{23}\varphi_{3} = X_{2}$$

$$-\lambda_{3}\rho_{3}B_{23}\theta_{2} - (\lambda_{3}\rho_{3}A_{23} + A_{34})\theta_{3} - \lambda_{3}\rho_{3}D_{23}\varphi_{3} + (\lambda_{3}\rho_{3}^{2}C_{23} - C_{34})\varphi_{3} = X_{3}$$

$$(\lambda_{1}\rho_{1}^{2}C_{10} - C_{12})\theta_{1} - D_{12}\theta_{2} - (\lambda_{1}\rho_{1}^{3}G_{10} + G_{12} + \overline{\mu}_{1})\varphi_{1} + \rho_{2}H_{12}\varphi_{2} = X_{4}$$

$$\lambda_{2}\rho_{2}^{2}D_{12}\theta_{1} + (\lambda_{2}\rho_{2}^{2}C_{12} - C_{23})\theta_{2} - D_{23}\theta_{3}$$

$$+\lambda_{2}\rho_{2}^{2}H_{12}\varphi_{1} - (\lambda_{2}\rho_{2}^{3}G_{12} + G_{23} + \overline{\mu}_{2})\varphi_{2} + \rho_{3}H_{23}\varphi_{3} = X_{5}$$

$$\lambda_{3}\rho_{3}^{2}D_{23}\theta_{2} + (\lambda_{3}\rho_{3}^{2}C_{23} - C_{31})\theta_{3} + \lambda_{3}\rho_{3}^{2}H_{23}\varphi_{2}$$

$$-(\lambda_{3}\rho_{3}^{3}G_{23} + G_{34} + \overline{\mu}_{3})\varphi_{3} = X_{6}$$

$$(18)$$

where

$$egin{align*} I_{10}/I_{12} &= \lambda_1 \,, \quad I_{12}/I_{23} &= \lambda_2 \,, \quad I_{23}/I_{34} &= \lambda_3 \ l_{12}/l_{01} &= 
ho_1 \,, \quad l_{23}/l_{12} &= 
ho_2 \,, \quad l_{34}/l_{23} &= 
ho_3 \ X_1 &= \left(rac{l_{st}}{EI}
ight)_{12} (\overline{M}_{10} + \overline{M}_{12}) \,, \quad X_4 &= \left(rac{l_{st}^2}{EI}
ight)_{12} (\overline{Q}_{10} + \overline{Q}_{12}) \ X_2 &= \left(rac{l_{st}}{EI}
ight)_{23} (\overline{M}_{21} + \overline{M}_{23}) \,, \quad X_5 &= \left(rac{l_{st}^2}{EI}
ight)_{23} (\overline{Q}_{21} + \overline{Q}_{23}) \ X_3 &= \left(rac{l_{st}}{EI}
ight)_{34} (\overline{M}_{32} + \overline{M}_{34}) \,, \quad X_6 &= \left(rac{l_{st}^2}{EI}
ight)_{34} (\overline{Q}_{32} + \overline{Q}_{34}) \ \overline{\mu}_1 &= rac{l_{12}^3}{EI_{12}} \left(rac{162\,EI_{web\,1}}{l_{web}^3}
ight) \,, \qquad \overline{\mu}_2 &= rac{l_{23}^3}{EI_{23}} \left(rac{162\,EI_{web\,2}}{l_{web}^3}
ight) \ \overline{\mu}_3 &= rac{l_{34}^3}{EI_{34}} \left(rac{162\,EI_{web\,3}}{l_{web}^3}
ight) \ \varphi_1 &= v_1/l_{12} \,, \quad arphi_2 = v_2/l_{23} \,, \quad arphi_3 = v_3/l_{34} \ . \end{gathered}$$

 $I_{10}$ ,  $I_{12}$  etc. are the moments of inertia of upper and lower stringers of respective span.

 $\theta$  (radian)
  $0.8063 \times 10^{-3}$   $0.2234 \times 10^{-3}$   $-0.8969 \times 10^{-3}$ 
 $\varphi$   $0.2410 \times 10^{-3}$   $0.3641 \times 10^{-3}$   $0.3227 \times 10^{-3}$  

 v cm
 0.1147 0.1981 0.1218

Table III

Solving the simultaneous equations (18), we obtain rotation  $\theta$ 's and deflections v's of stringers at each connecting point of web frames, as shown in Table III, and the deflections at the middle of the spans are obtained from (17) and (18) in the 1st report p. 73. Numerical results obtained from above equations are shown in Table IV along with the results derived by conventional method already stated in the preceding paragraph.

#### Couclusions and Acknowledgements

From Table IV, we obtain following conclusions:

- (1) Effectiveness of panting stringers and web frames arranged to support the side frames of panting structure designed as "Web and Stringer system" is practically sufficient in this case.
- (2) Analysis of them by the proposed conventional method give fairly good approximation compared to the results derived from accurate theory.

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Table IV

	Item	4th Step Cal. $I_u \neq I_l$	Approx. method
	$eta_1$ radian $eta_2$ radian $eta_3$ radian	$0.8063 \times 10^{-3} \ 0.2234 \times 10^{-3} \ -0.8969 \times 10^{-3}$	- - -
	$egin{array}{c} arphi_1 \ arphi_2 \ arphi_3 \end{array}$	$0.2410\times10^{-3}\ 0.3641\times10^{-3}\ 0.3227\times10^{-3}$	- - -
	$egin{array}{ll} oldsymbol{v_1} & \operatorname{cm} \ oldsymbol{v_2} & \operatorname{cm} \ oldsymbol{v_3} & \operatorname{cm} \end{array}$	0.1147 0.1981 0.1218	0.1355 0.2147 0.1458
rs	$egin{array}{ll} M_{10} = -M_{12} &  ext{kg cm} \ M_{21} = -M_{23} &  ext{kg cm} \ M_{32} = -M_{34} &  ext{kg cm} \end{array}$	$180.13 \times 10^{4}$ $414.12 \times 10^{4}$ $296.72 \times 10^{4}$	—. —. —.
Stringers	$egin{array}{ll} Q_{10} & \mathrm{kg} \\ Q_{12} & \mathrm{kg} \\ Q_{21} & \mathrm{kg} \\ Q_{23} & \mathrm{kg} \\ Q_{34} & \mathrm{kg} \\ Q_{34} & \mathrm{kg} \end{array}$	$\begin{array}{l} -26.15\!\times\!10^3 \\ -38.76\!\times\!10^3 \\ -48.03\!\times\!10^3 \\ -48.37\!\times\!10^3 \\ -44.65\!\times\!10^3 \\ -33.25\!\times\!10^3 \end{array}$	— — — —
	$\mathbb{R}_1 = Q_{10} + Q_{12} \text{ kg}$ $\mathbb{R}_2 = Q_{32} + Q_{34} \text{ kg}$ $\mathbb{R}_3 = Q_{21} + Q_{23} \text{ kg}$	$\begin{array}{c} -64.90 \times 10^{3} \\ -96.40 \times 10^{3} \\ -97.90 \times 10^{3} \end{array}$	=
	$\sigma_{10}$ kg/cm <sup>2</sup> $\sigma_{12}$ kg/cm <sup>2</sup> $\sigma_{21}$ kg/cm <sup>2</sup> $\sigma_{23}$ kg/cm <sup>2</sup> $\sigma_{32}$ kg/cm <sup>2</sup> $\sigma_{34}$ kg/cm <sup>2</sup>	801 750 1724 1685 1207 1274	_ _ _ _ _
Web. Frame	$egin{array}{ccc} R_1 &  ext{kg} \ R_2 &  ext{kg} \ R_3 &  ext{kg} \end{array}$	$ 64.91 \times 10^{3}  96.39 \times 10^{3}  77.90 \times 10^{3} $	$76.67 \times 10^{3}$ $104.55 \times 10^{3}$ $93.28 \times 10^{3}$

Table IV (cont'd)

Item	4th Step Cal. $I_u  eq I_l$	Approx. method
y <sub>01</sub> cm y <sub>12</sub> cm y <sub>23</sub> cm y <sub>34</sub> cm	0.04250 0.3009 0.3721 0.05203	0.0805 0.2746 0.3491 0.1115
$egin{array}{ccccc} A_{01} & \% & & & & & & & & & & & & & & & & & $	97.44 81.93 77.41 96.12	95.2 83.5 78.8 91.7
$egin{array}{ll} R_{01} &  ext{kg} \ R_{12} &  ext{kg} \ R_{23} &  ext{kg} \ R_{34} &  ext{kg} \end{array}$	$13.59 \times 10^{3}$ $11.42 \times 10^{3}$ $10.48 \times 10^{3}$ $10.39 \times 10^{3}$	$\begin{array}{c} 13.57 \times 10^{3} \\ 11.91 \times 10^{3} \\ 10.91 \times 10^{3} \\ 10.14 \times 10^{3} \end{array}$
$egin{array}{lll} M_{01} & { m kg} & { m cm} \ M_{fre01} & { m kg} & { m cm} \ M_{12} & { m kg} & { m cm} \ M_{fre12} & { m kg} & { m cm} \ M_{23} & { m kg} & { m cm} \ M_{34} & { m kg} & { m cm} \ M_{fr34} & { $	$\begin{array}{c} 214.32 \times 10^{4} \\ 28.65 \times 10^{4} \\ 214.32 \times 10^{4} \\ 58.22 \times 10^{4} \\ 214.32 \times 10^{4} \\ 71.15 \times 10^{4} \\ 192.26 \times 10^{4} \\ 50.28 \times 10^{4} \end{array}$	$\begin{array}{c} 214.32 \times 10^{4} \\ 89.89 \times 10^{4} \\ 214.32 \times 10^{4} \\ 51.61 \times 10^{4} \\ 214.32 \times 10^{4} \\ 65.26 \times 10^{4} \\ 192.26 \times 10^{4} \\ 53.75 \times 10^{4} \end{array}$
$egin{array}{cccccccccccccccccccccccccccccccccccc$	86.63 72.84 66.80 73.84	86.52 75.92 69.55 72.04
$\begin{array}{ccc} \sigma_{01} & \text{kg/cm}^2 \\ \sigma_{12} & \text{kg/cm}^2 \\ \sigma_{23} & \text{kg/cm}^2 \\ \sigma_{34} & \text{kg/cm}^2 \end{array}$	1024 2080 2560 1824	1032 1844 2348 1949

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