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Envisaging Modularity Detecting Communities in Networks: Gephi Visuals

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Abstract: Understanding the intricate connections within networks relies on exploring their connection patterns and extracting valuable statistics about their overall framework. Among the various aspects of complex networks, community structure plays a crucial role in contemporary times. Detecting and analyzing these communities is paramount in applications such as information sharing, dissemination, recommendation systems, and classification. Modularity optimization presents a formidable approach to discerning both the community structure within a complex network and the internal structure of its nodes. This paper aims to contribute to the identification and visualization of communities within networks, highlighting their distinctive attributes that set them apart from the typical network structure. By employing graph theoretical analysis, our study utilizes the Gephi software to detect and represent communities through their visuals with the aid of modularity and also provides key statistical insights into the network. It explores the application of Gephi, a graph visualization software, to visually represent these communities, providing insights that extend beyond traditional network analysis. Additionally, we present a comparison between modularity and the clustering coefficient, shedding further light on the network's characteristics. By using Gephi visualization capabilities, we present a novel approach to gain deeper insights into network structures, thus contributing to a more profound understanding of complex networks and their community dynamics.

Keywords: Graphs, Community detection, Clustering, Modularity

1. Introduction

Internet, social and biological relationships etc. are interesting and complicated system[1,2,3]. These systems are made up of nodes and links forming networks[4,5,6]. Small-world phenomenon[7], power-law degree distribution, and the presence of community structure[8,9,10] where nodes cluster normally into closely linked modules, often called communities, with only the barest of ties between them, have all been discovered through research in a variety of academic domains. A key structural component of networks, which can simulate a variety of complicated systems, is community structure[11]. The network design could be enhanced for better system performance with a better understanding of community structures[9,10,12,13]. Identifying this ,one can unveil the framework and its connections with the operational components of the network like; a community in web as a group of pages with the similar topics ,community as per interactions among web services, a community in the network of protein-protein interactions as a collection of proteins that supports a single activity , a community in a social network as a collection of users with similar interests or professions, or a community structure of sensor data related to information dependence among sensors[14,15,16,17].

Despite the enormous work done by a vast multidisciplinary group of scientists, uncovering community structure is a primary as well as difficult subject in the field of network system research that is to be adequately answered even today[18]. This area has become quite noticeable and popular. The most widely used technique is modularity optimization.[19] established it as an important evaluation function for gauging community structure in networks. To achieve a sizeable community structure in a network, a high modularity value is desired[20,21]. It determines the quality of module division in a network. Good divisions with high modularity values have substantial internal links between nodes inside modules but few links between modules. Its adaptability led to a wide range of applications. Modularity is most commonly used as a foundation for optimization method for recognizing network community structure.

In this study, we visualize the network's community structure in various forms using Gephi software to assess the significant details of its vital statistics encrypted in the community detection procedure. The research work done here is aimed to do deep mining and visualize the data of the network. The work here is organized as follows: Section 1 introduces the overall problem, Section 2 summarizes the community structure, Section 3 describes the community detection, Section 4 describes clustering and clustering coefficient, Section 5 describes modularity, Section 6 compares modularity and clustering coefficient, Section 7 visualizes community detection by experiment on real data and in Section 8 concluding remarks are given.

2. Community Structure

A variety of traits have been discovered to be prevalent in many networks. Many networks have the small-world trait, heavy-tailed degree distributions, and clustering such as networks in computing, information, biology, and society. Another frequent attribute in networks is community structure. The nodes of a network constitute community structure if they can be readily united into nodes group and each set is closely packed immanently.

This structure may be non-overlapping or overlapping as per network construction. The network typically separates into clusters of nodes with intense internal links and a smaller number of links between groupings. The concept asserts that if two nodes belong to the same community, apparently, they are connected, and if they don't, they are less likely to be connected. Using networks, a number of technical significant mysteries can be depicted and experimentally analyzed.

Social connections, biological patterns, internet, metabolic networks, interconnections in food chains, brain networks, and pathogenic networks and many more so are naturalistic cases that can be mathematically described and topologically investigated to exhibit some surprising structural aspects[22]. The majority of these networks have a community structure that is critical to understanding the network's behaviour[23,24]. A tightly connected social society, for example, will have a faster rate of information or rumour transmission than a loosely connected community.

The community structure is represented by a network (Figure1), which is made up of nodes and links, where community represents subset of nodes that are closely associated to one another in same community and sparsely associated to nodes in different communities[25].

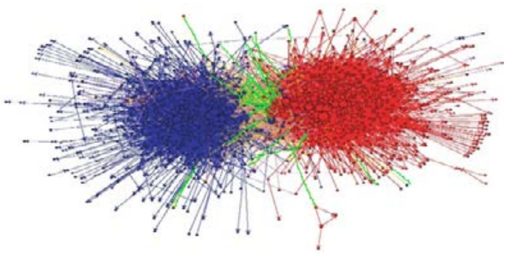

Fig.1: Community Structure in Networks

As a result, graph theory is useful for identifying communities in networks, because communities may differ from the typical network in terms of node degree, clustering coefficient, betweenness, and centrality[26].

3. Community Detection

Community detection is used to unveil the structural properties and dynamic behaviour of networks[27,28]. Interacting and sharing knowledge has never been easier because of the internet's democratization. This digital age has ushered in a whole new manner of consuming, learning, interacting, and forming relationships[29], as well as a massive amount of data that we can analyze to gain a better understanding.

We can utilize the information obtained from grid of connections between people or objects like Instagram followers, Facebook users, co-purchasing of products on Amazon etc[30]. The discipline of community detection tries to find strongly related groupings of people or items inside these networks[31], which are referred to as communities[32].

Community detection can assist a brand to comprehend distinct groups of people's opinions about its products, can target specific categories of people or find experts. It can also aid trading websites to form a suggestion system based on buying.

Figure 2 shows community detection in a network where different colours show individual communities.

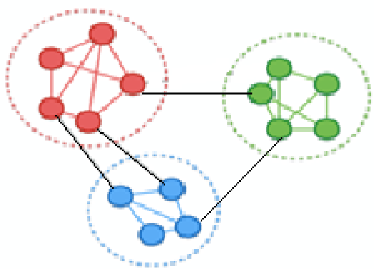

Fig.2: Community Detection in Graphs

4. Clustering

Clustering is the process of arranging items into sets known as clusters, with each cluster including pieces that are similar in some way[33,34,35]. Depending on the areas of interest and the goals that the clustering intends to attain, there are numerous ways to define similarity criteria. The clustering coefficient C is one of the most basic measuring degrees of a network internal structure. The clustering coefficient estimates the probability that two nodes with a common neighbour are joined and is related to local cohesion in a network.

Clustering coefficient is a measure used in graph theory. It evaluates how intimately nodes in a graph incline to cluster with each other. Research done in this field exposes that in most networks, especially in social media platforms, nodes build tightly knit groupings relatively with good number of connections; this likelihood is higher than the average probability of a connection forming arbitrarily between two nodes[36].

There are two variations of this metric: global and local. The global version is intended to show the comprehensive clustering in the network, whereas the local version shows the embeddedness of individual nodes.

For a given vertex η_i and its d_i adjacent vertices in an undirected network, there are $\mathcal{E}_{max} = d_i(d_i - 1)/2$ potential edges between the adjacent vertices. The clustering coefficient C_i of vertex η_i is then calculated as the ratio of the number of edges $\mathcal{E}_i$ among adjacent vertices to the maximum number $\mathcal{E}_{max}$ correlations[31]. The clustering coefficient is also affected by the edge density in the network. To distinguish this local clustering coefficient with global clustering coefficient within the network, an average value of C has to be computed and to be examined that whether this value is indeed important or not.

According to the definition, C can be represented using the number of triads (three vertices connected to each other) in a network. As a result, it is based on node triplets. Three nodes are connected by undirected links to form a triplet, either two open triplet or three closed triplets. As a result, each node in a triangular graph has three closed triplets, one for each node. Thus, global clustering coefficient is the ratio of closed triplets (or 3 triangles) to the total number of triplets including open and closed both. Figure 3 represents clustering of data.

Fig.3: Representation of 3 clusters from 2D data set

5. Modularity

One measure of framework of graphs is modularity [37]. It was created to valuate the effectiveness of a network's segmentation into different communities. A decent network partitioning into modules should have as many within-module edges as possible and bare minimum links between-module. Although, if we merely strive to reduce the number of between-module edges (or, conversely, increase the number of within-module edges), the best partition is one module with no between-module edges. Modularity refers to the grouping of nodes and edges into distinct packets. When the density of intra-modular connections exceeds that of inter-modular connections, a threshold is set. The fundamental concept behind the various threshold-creating algorithms is the same. The fundamental principle of modularity draws attention: At what point do intra-modular connections outnumber inter-modular connections in density? To get this answer, better methods for assessing the quality of decomposition are required.

In order to quantify the intuitive feeling of community organization, counting edges is not a useful technique. A good and justified network partition is one in which less margins across communities exist than expected.

Modularity can be used to quantify the assumption that a statistically surprising positioning of edges corresponds with network community structure [38]. Thus, the difference of number of edges lying into modules and the predicted number in an analogous network with edges inserted at random, up to a multiplicative constant, is the modularity.

Assume that our network has η nodes. Let $s_i = 1$ if node i is in group 1 and $s_j = 1$ if it is in group 2 for a specific network partition into two groups. Let A_{ij} be the number of edges between node i and j, which is typically 0 or 1, though bigger values are conceivable in networks with multiple edges. The quantities A_{ij} constitute adjacency matrix A. At the same time, if edges are inserted at random, the predicted number of edges between vertices i and j is $\frac{d_i d_j}{2m}$, where $d_i \, and \, d_j$ denote degrees of nodes and $m = ½\sum_i d_i$ represents the total number of edges in the network. The summation of $A_{ij} - \left(\frac{d_i d_j}{2m}\right)$ across all pairs of vertices i, j that belong to the identical group, gives the modularity Q[22].

The expression $1/2\,(s_i s_j + 1)$ value is 1 if i and j belong to identical group and 0 otherwise, so mathematical modularity can be given as

$$Q = \frac{1}{4m}\sum_{ij}\left(A_{ij} - \frac{d_i d_j}{2m}\right)(s_i s_j + 1) = \frac{1}{4m}\sum_{ij}\left(A_{ij} - \frac{d_i d_j}{2m}\right) s_i s_j \qquad (1)$$

where the second term in equation is derived from the observation that $2m = \sum_i d_i = \sum_{ij} A_{ij}$ [7]. The 1/4m leading factor is just conventional. It is included since it is compatible with A_{ij}[19].

We shall get $Q = 0$ if the number of within-modules is no better than random. A strong community structure is indicated by values nearing $Q = 1$, which is the highest value. If there is a community structure present, the values for social networks analyzed by Newman range are roughly from 0.3 to 0.7 [39].

Positive number indicates the presence of community structure, whereas negative values indicate the absence of community organization. As a result, checking for network partitions can be used to explicitly search for community structure. So, we can specifically locate community structure by identifying network splits with positive, and preferably large, modularity values.

Looking for modules with high modularity, according to the evidence, is a very effective technique to find communities.[40,41] maximized modularity over possible partitions of computer-generated test networks by employing simulated annealing. It was observed that this strategy of community detection surpassed all other approaches that they were familiar of in direct comparisons using standard measurements, in most cases by a significant margin[41)]. Based on these findings, we believe that maximization of modularity is currently the best strategy for detecting communities. In this article visualization of network has been done by Gephi software, which further partitions the graph into different communities based on modularity. It is also to note that modularity has a resolution limit, so small communities are not detectable.

6. Modularity vs Clustering coefficient

For graph clustering, modularity is a new quality metric. In graph theory, clustering coefficient is a metric of how intimately nodes in a network cluster together and modularity measures edge density within communities as compare to among communities. Both concepts are interesting to measure clustering. One looks at density of triangles compared to induced density of 2-paths and in other edge densities in given clusters are compared to edge densities between clusters.

Modularity quantifies the intensity of a network in terms of modules (also called clusters, groups, or communities[42,43)]. High-modularity networks possess huge number of connections between nodes inside modules, but limited number of connections between nodes in distinct modules whereas clustering coefficient is a measure of the degree to which nodes in a graph gravitate to cluster together and specially in social networks, nodes prefer to form integrated groupings with a big volume of links. A graph with no triangles has clustering coefficient 0 but may have high modularity.

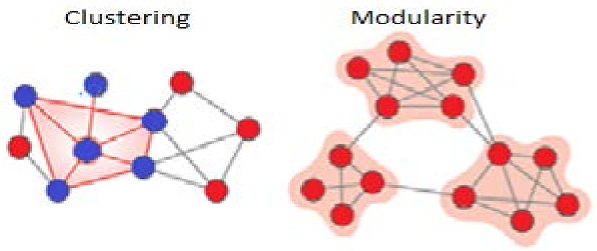

Fig. 4: Clustering coefficient vs Modularity in Network

Figure 4 clearly differentiates between clustering coefficient and modularity in a graph. It quantifies the number of neighbours of a specific node that are interconnected and quantifies the local cliquishness, on the other hand, modularity means that within clusters, there is intense interconnectedness, while connections between clusters are sparse[3)].

7. Community detection by experiment on real data

Everything in the world is connected. These associations are plain and convoluted at the same time. Identifying these links and understanding their complexity is not simple. To find these connections, complete data of a data set is modelled as a network to emphasise, uncover, or reveal the linkages or associations between distinct components or nodes. Alternatively, network analysis provides a 360-degree graphical representation of all the connections between nodes. Several factors of network analysis, such as degree, betweenness, closeness, and network centrality, are key characteristics of a network and can help you understand it better. Understanding networks and their metrics is critical since these networks serve as the foundation for quick information distribution[44)].Community detection,or community under standing, informs about the clusters and partitions within the community in network so it plays a pivotal role to understand patterns of collaborations and visualizing networks. It aids with the comprehension of new ways to present and handle data, as well as the conversion of that data into useful and unknown information.

Zachary's karate club network is used in this study to comprehend network community detection. Wayne W. Zachary described the dynamics of university karate club, Zachary's Karate Club, in his paper "An Information Flow Model for Conflict and Fission in Small Groups". This file record is well acknowledged for displaying community structure, which results when nodes in a network can be clustered together into strongly connected clusters. Michelle Girvan and Mark Newman utilized Zachary's Karate Club to demonstrate community structure in their paper "Community structure in social and biological networks" in 2002[8)]. We used Gephi to design a network using the Zachary's Karate Club dataset for visualization. Figure 5 portrays Zachary’s karate club visualization. Table 1 presents its some statistics as output whereas Figure 6 depicts the community detection in this club network. 4 communities are perfectly shown by four different colour and to give best understanding and visualize the network nodes have also been shown increasing in size. Bigger the size, bigger its degree which actually shows the number of acquaintances of particular node.

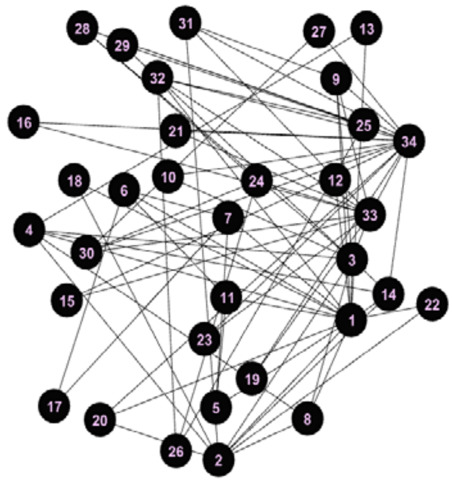

Fig. 5: Zachary’s Karate Club Network

Table 1. Node degree, clustering coefficient in Zachary Club Network

Node	Di	Ci	Node	Di	Ci
1	16	0.150000006	18	2	1
2	9	0.333333343	19	2	1
3	10	0.244444445	20	3	0.333333343
4	6	0.666666687	21	2	1
5	3	0.666666687	22	2	1
6	4	0.5	23	2	1
7	4	0.5	24	5	0.400000006
8	4	1	25	3	0.333333343
9	5	0.5	26	3	0.333333343
10	2	0	27	2	1
11	3	0.666666687	28	4	0.166666672
12	1	0	29	3	0.333333343
13	2	1	30	4	0.666666687
14	5	0.600000024	31	4	0.5
15	2	1	32	6	0.200000003
16	2	1	33	12	0.196969703
17	2	1	34	17	0.110294119

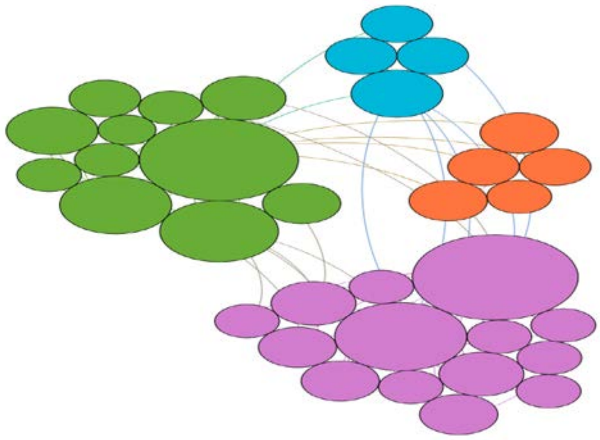

Fig. 6: Modularity based community detection in Zachary's Karate club network

Another example has been taken from Marvel Entertainment. It is been around for more than 70 years, churning out book after book, building its different characters and franchises, and even branching out into films, television series, and video games. Marvel's universe has grown so large over the years that keeping track of everything via traditional ways has become nearly impossible for this massive network.[45]created the Marvel Comics character collaboration graph and analyzed the features of this universe. The issue is that the network is far too big to be visualized well. As a result, taking a closer look at the Marvel universe could supply us with some intriguing information. Large clusters of superheroes who debuted in the same franchise can be further investigated using a network visualization of this dataset. Marvel Universe Social network has 19090 nodes and over 5 lakhs edges. Visualization of this huge network is done with the help of Gephi. Marvel Universe dataset network is visualized by community detection and different community structures. Following figures give glimpse of this huge network.

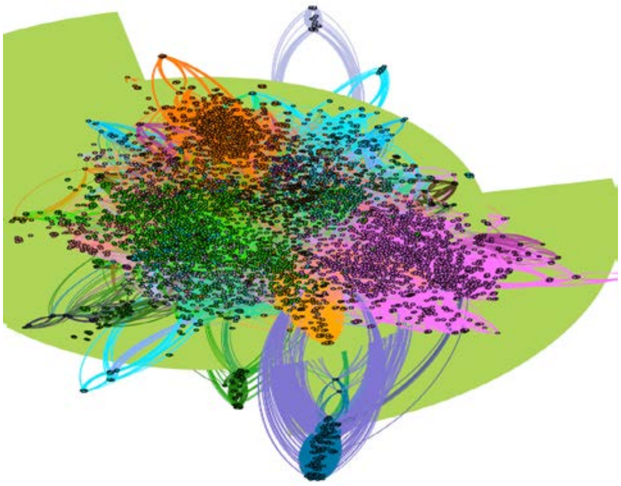

Fig. 7: Marvel Universe Network

Table 2. Vital statistics output for Marvel Universe network based on Figure 7

S.No.	Statistics	Output
1	Number of Nodes	19090
2	Number Of Edges	5,74,467
3	Average Degree of Nodes	13.824
4	Modularity	0.518
5	Number of Communities	19134
6	Number of Triangles	3804793

Figure 7 represents Marvel universe with its different communities and Table 2 informs about various statistics of this huge network where modularity is .518. Gephi has beautiful aspect of showcasing network in different forms and from different angle, so another layout of Marvel universe has been shown in circle pack layout. Various colors in Figure 8 show different communities of Marvel universe. This community detection has been done on the basis of modularity. The best-attained outcomes in terms of modularity Q and the community partitions are shown in Table 3. To visualize it properly nodes of degree less than 10 have been omitted and again statistics have been stored as shown in Table 3.

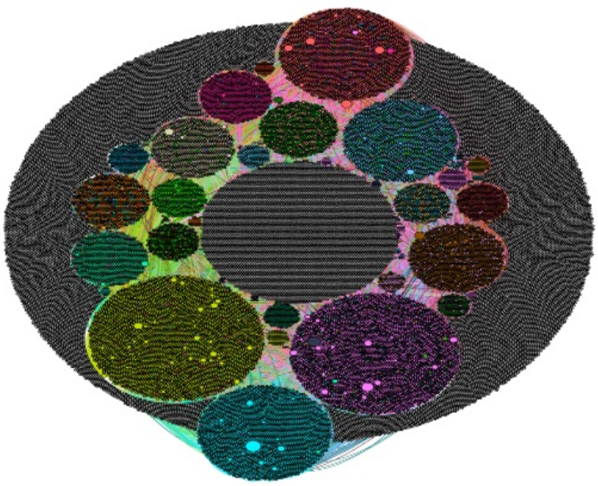

Fig. 8: Circle pack Layout of Marvel Universe Network

Table 3. Vital statistics output for Marvel Universe network based on Figure 8

S.No.	Statistics	Output
1	Number of Nodes	19090
2	Number of Edges	5,74,467
3	Average Degree of Nodes	5.034
4	Modularity	0.675
5	Number of Communities	19144
6	Number of Triangles	0
7	Number of paths (length 2)	12555954
8	Global Clustering Coefficient	0

The Gephi visualization indicates that the characters appear together and the strength of the connection between characters of the Marvel universe. By setting range for degrees of nodes in this huge network modularity has been calculated. The stronger/super clusters are visible and can be easily compared with the weaker ones on colour basis. On comparing Figure 7 and Figure 8 we notice significant difference between average degree of nodes, global clustering coefficient and modularity with the aid of Table 2 & 3.

8. Conclusion

The information can be displayed in an informative manner using an interactive graphical representation. An important challenge in Big Data is the graph-based presentation of networks and the subsequent analysis of those networks, which includes detecting relationships and clustering them. This study attempted to establish a framework for analyzing network data using community detection. To visualize the network, graph theory metrics are applied to network analysis. Nodes forming communities in networks based on communication among them have been partitioned.

With the growing growth of real-world network data, new computational research has become increasingly significant, and community detection has become a critical component of network analysis. Apart from the degree of each vertex, the work here reveals mutual interactions among connected vertices. Because it determines the shape and community partitions, the tuning parameter modularity is important in network analysis. It is a normalized dealing between edges covered by clusters and squared cluster degree sums. This paper visualizes the network on the basis of this popular clustering index modularity. Different values of modularity and clustering coefficient in each graph of Figure 7 and 8 also clarify the difference between the two. Figure 8 graph with no triangles have a clustering coefficient of 0 but have a modularity as 0.675 which justifies the statement of being modularity and clustering as two different measures in a network. All visuals are prepared with Gephi software. These nodes and connections can be displayed as a 360-degree graph using Gephi visualization.

Acknowledgements

We would like to express our sincere gratitude to our institution and all those who have contributed to the completion of this journal article.

Availability of data and material

The authors certify that the data supporting the findings of this study are available online and their links are:

1. http://networkdata.ics.uci.edu/data/karate/karate.paj (Karate club data)

2.https://www.kaggle.com/csanhueza/the-marvel-universe-social-network?select=hero-network.csv(data used for Marvel Universe)

Code availability: Gephi usage

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