

Development of Regression Equations for Stochastic Inventory System

Srivastav, Achin

Department of Mechanical Engineering, Swami Keshvanand Institute of Technology, Management and Gramothan

Kumar, Deepak

Department of Mechanical Engineering, Swami Keshvanand Institute of Technology, Management and Gramothan

Singh, Prem

Department of Mechanical Engineering, Swami Keshvanand Institute of Technology, Management and Gramothan

Srivastav, Nidhi

Department of Computer Science and Engineering, Swami Keshvanand Institute of Technology, Management, and Gramothan

<https://doi.org/10.5109/7160924>

出版情報 : Evergreen. 10 (4), pp.2654-2660, 2023-12. Transdisciplinary Research and Education Center for Green Technologies, Kyushu University

バージョン :

権利関係 : Creative Commons Attribution 4.0 International

Development of Regression Equations for Stochastic Inventory System

Achin Srivastav^{1*}, Deepak Kumar¹, Prem Singh¹, Nidhi Srivastav²

¹Department of Mechanical Engineering, Swami Keshvanand Institute of Technology,
Management and Gramothan, Jaipur, India

²Department of Computer Science and Engineering, Swami Keshvanand Institute of Technology,
Management and Gramothan, Jaipur, India

*Author to whom correspondence should be addressed:

E-mail: achin.srivastav@skit.ac.in

(Received March 20, 2023; Revised December 19, 2023; accepted December 19, 2023).

Abstract: This paper addresses the intractability in stochastic inventory systems due to frequent crossover of shipments. The present scenario due to volatile markets, presence of manifold ways to place orders globally, vacillating requirements of customers, climate disruptions, surge in demand of military equipments and medical devices lead to numerous orders are getting placed at small interval of times, due to which shipments are arrived in different order in which they were booked. The fast-moving items are highly susceptible to order crossover. In this paper, based on the ratio of ordering cost to inventory carrying charge, fast moving items are categorized as fast movers not ordered enough and fast movers frequently ordered. The datasets are used for developing regression equations for optimal total inventory cost, order quantity and safety stock factor from the results of factorial experiments. The first group of regression equations are handy for decision systems lead to essentials items and second one is applicable in decision systems for discretionary products. The regression equations will help the practitioners to compute the right optimal quantity, estimate the correct inventory cost and compute safety stock factor, that will help their industry sustainability in the global world and provide a competitive edge over their competitors.

Keywords: Order Crossover; Fast-moving; Regression; Inventory; Sustainability

1. Introduction

The phenomenon when shipments arrive in disparate series as they were placed is termed as order crossover (Riezebos, 2006). In a supply chain, unprecedented factors such as traffic congestions, internet glitches, strikes, lockdowns, natural calamities, power shutdowns, assemblage of group of people, disruptions face by companies are few reasons when the shipments are not reached to the customer at the expected time and the shipments to be arrived in a particular sequence reaches in different order. Moreover, due to the factors discussed earlier, increase in lead times due to congestions will also increase carbon emissions, more fuel consumption and inconvenience^{1,2)}. The delay of consignments leads to increase the instances of shortages and to overcome for future the retailers' stock more than the actual requirement which raises the holding cost enormously. Industries which can acknowledge presence of crossover of shipments and process shipments according to their arrival rather than the sequence in which are placed, can save a lot of cost associated to holding and shortages. Scenarios

of crossover of shipments is observed more, when shipment orders are done between small amount of time. The fast-moving merchandises are usually experience crossover of shipments more in comparison to slow moving commodities. A lot of research has been carried out in the past on inventory and crossing of orders. In the upcoming section, significant contributions on order crossover made are discussed.

2. Literature Review

The studies on order crossover can be categorized as researchers who acknowledged existence of order crossover but ignored while computing total cost. The contributions made under this study are Galliher et al.³⁾, Hadley & Whitin⁴⁾. Generally, inventory literature of the above-mentioned school of thought made an assumption that consignments reach in similar fashion in which they were booked or considering lead time as deterministic. The assumption made is not practical in reality. Aliunir et al.⁵⁾ used Simulation and optimization for inventory management of spare parts. Response surface

methodology (RSM) used by Choudhary et al.⁶⁾ for optimizing military supplies. Some recent well-known studies which studied and developed inventory expert systems using Multi objective Particle Swarm Optimization (MOPSO) are Tsou⁷⁾, Srivastav & Agrawal^{8,9)}. Srivastav and Agrawal¹⁰⁾ formulated slow-moving system using multi objective cuckoo search (MOCS) and a two objective fast moving inventory system¹¹⁾.

Zalkind¹²⁾ explained crossover in orders with the distribution of outstanding orders. Hayya et al.¹³⁾ made use of classical approach along with concept of demand interchangeability to obtain an expression of crossover probability. Studies made by Robinson et al.¹⁴⁾, Bradley and Robinson¹⁵⁾ and Robinson & Bradley¹⁶⁾ on crossover of orders in periodically assessment of inventory systems. Riezebos¹⁷⁾ categorized order crossover and introduced different conditions of crossovers of orders. Hayya et al.¹⁸⁾ studied crossover in orders in perpetual assessment of inventory systems using effective lead time approach.

Studies on order crossover that considered case of shortages as complete backorder are mentioned here. Srinivisan et al.¹⁹⁾ made use of dynamic programming for developing order crossover inventory policies. Other prominent works on backorder inventory models that investigate crossover of orders are Hayya & Harrison²⁰⁾, Bischak et al.²¹⁾ and Wensing & Kuhn²²⁾. Some recent studies that used advance optimization techniques such as evolutionary computation MOCS to incorporate order crossover are Srivastav & Agrawal²³⁾.

Literature on partial backordering which is a more realistic case usually occurs in grocery shops, retail outlets, where some customers wait till the next replenishment arrives and others choose to buying from some other store is presented below. In the past, Montgomery et al.²⁴⁾, Kim & Park²⁵⁾, Padmanabhan & Vrat²⁶⁾, Kumar et al.²⁷⁾ etc., developed mixture inventory models ignoring considering order crossover. Few studies that included crossover of orders in mixture inventory systems are Srivastav and Agrawal^{23,28)}.

The literature suggests most of the work on crossovers of shipments, developed inventory models and conducted their study. The availability of few studies on inventory systems developed for mixture inventory systems suggest difficulty in incorporating order crossover in their work and due to stochastic nature of lead time correct estimation of inventory cost becomes intractable. It would be very difficult to understand the complexity of order crossover and to compute the inventory cost using above mentioned inventory models, by the practitioners. Therefore, in the present study, using exchange curve, regression equations are developed for the easy use of practitioners.

Rest of the paper is organized as follows. The upcoming section 3 shows the notations and assumptions of the models considered in the paper. In the section 4 cost equations expressions for ignoring order crossover situation and order crossover situation is discussed.

Development of Regression equations using Exchange curve is discussed in section 5. Section 6 concludes the paper.

3. Assumptions

- i. Regression equations are developed from the exchange curve, which is plotted for fast moving item, for multi-suppliers and single retailer / manufacturer supply chain.
- ii. The replenishment lead time is assumed as independently and identically (iid) exponentially distributed.
- iii. Ordering quantity is assumed to be fixed that is placed by the retailer.
- iv. The cost expressions are formulated with the assumption that distribution of demand during lead time following Normal distribution.

4. Cost Expression for Determining Inventory Cost

Total approximate cost equation for part of backorders and remaining lost sales is formulated by modifying the cost equation of Montgomery et al.²⁴⁾ and Srivastav & Agrawal²⁸⁾ for considering order crossover as below.

$$C = \frac{AD}{Q} + h \left(\frac{Q}{2} + z_0(aD + bQ) \right) + \left[h(1 - q) + \frac{D}{Q} [\pi + \pi_0(1 - q)] (aD + bQ) G(z_0) \right] \quad (1)$$

Considering order crossover, standard deviation of effective lead time (Hayya et al.¹⁸⁾) is computed as.

$$\sigma_{ELT} = D(a + bT) \quad (2)$$

Therefore, standard deviation for lead time is computed as.

$$\sigma_X = D\sigma_{LT} \approx D\sigma_{ELT} = D(a + bT) = aD + bQ \quad (3)$$

5. Development of Regression Equations

In this section, regression equations for optimal parameters are developed, so that practitioners can directly use them. Here two sets of regression equation are formulated for fast moving item. The fast-moving item can be classified on the basis of number of replenishments in two types namely fast movers not ordered enough (low number of replenishments per year) and fast movers frequently ordered (high number of replenishments). The factors that have taken into account to determine characteristic of fast-moving items are total average cycle stock (TACS) and total number of replenishments per unit time (N). The equations given by Silver et al.²⁹⁾ are used to find TACS and N as mentioned below (eqs. 4 to eq. 6).

The TACS is determined using below equation.

$$TACS = \sqrt{\frac{A}{r}} * \frac{1}{\sqrt{2}} * \sqrt{Dv} \tag{4}$$

The N is determined using below equation.

$$N = \sqrt{\frac{r}{A}} * \frac{1}{\sqrt{2}} * \sqrt{Dv} \tag{5}$$

The ratio of TACS to N is equal to ratio of fixed cost per replenishment to inventory carrying charge.

$$\frac{TACS}{N} = \frac{A}{r} \tag{6}$$

Considering a fast-moving item with $D= 700$, $v= \$5$ and $A/r [1, 1500]$, the exchange curve is drawn between TACS and N.

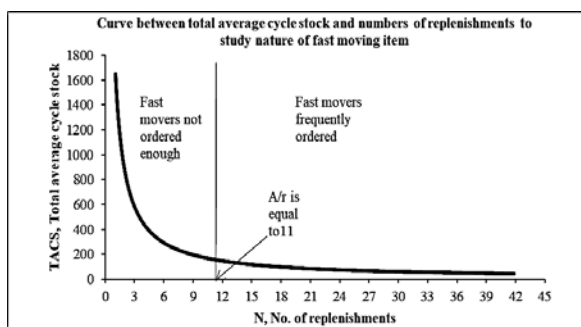


Fig. 1: Exchange curve for fast moving inventory item

The Figure 1 shows the exchange curve between total average cycle stock and number of replenishments for fast moving inventory item. Figure illustrates the two regions, one of fast movers not ordered enough ($A/r = 11$ or more) and other for fast movers frequently ordered (A/r less than 11). Based on these two regions, regression equations are developed, by selecting the range of A/r in between the desired region.

For conducting factorial experiments, the levels of different factors are to be selected. Here, seven different factors are considered. The different values of inventory carrying charge (r) for both types of fast moving item is considered as 0.24, 0.32 and 0.40. The cost of item (v) is \$5, so holding cost for both types of fast moving item is \$1.2, \$1.6 and \$2. The replenishment cost (A) is determined on basis of selection of A/r ratio. The shortage cost π is considered as backorder cost and π_0 , lost sales penalty is considered as 1.2 times π .

The shortage cost in two situation is determined by the following relation.

$$TBS = \frac{\left(\frac{q}{D}\right)}{P(z > z_0)} \tag{7}$$

From the eq. (4) the stockout risk, $P(z > z_0)$ for mixture model and eq.(6) of time between stockout is computed, and obtain the following expression for stockout cost for mixture inventory model.

$$\pi + \pi_0(1 - q) = \frac{hQ}{D} \left[\frac{1}{P(z > z_0)} - (1 - q) \right] \tag{8}$$

The shortage cost components, backorder cost and lost sales penalty over backorder are estimated from this formula by keeping Q equals to economic order quantity, D and h values as discussed above. The stockout risk is substituted as 0.05. Johnson & Montgomery³⁰ recommended to keep stock out risk as 0.05, as most companies prefer to operate at this stockout risk. The backorder cost and lost sales penalty are computed from equation is shown in Table 2 and Table 5 for fast moving not ordered enough and fast moving frequently ordered item respectively.

The regression coefficients obtain to estimate standard deviation of effective lead time considering order crossover from given standard deviation of lead time with exponential mean 2.5, 5 and 7.5 is shown in Table 1. These regression coefficients will be used in estimation of standard deviation of effective lead time for both types of fast moving items.

Table 1: Estimation of standard deviation of ELT ($0 < T \leq 2$)

σ_{LT}	σ_{ELT}	Coefficient a	Coefficient b	Adj. R ²
2.5	$0.4645 + 0.5481T$	0.4645	0.5481	94.97%
5	$0.5854 + 0.8846T$	0.5854	0.8846	94.53%
7.5	$0.8547 + 0.9322T$	0.8547	0.9322	95.70%

The Table 2 shows 7 factors and 3 levels for fast moving not ordered enough item. There are total 3^7 (2187) experiments are conducted to obtain regression equation considering order crossover for mixture inventory fast moving not ordered enough items.

The Table 3 shows the regression equations for the optimal parameters cost, order quantity and safety factor for fast moving not ordered enough item. The regression equation is obtained from 2034 data sets. The data sets, which gave high residual error, are removed. The adjusted R-square value 99% shows very good fit for $\ln(C^*)$ equation. The adjusted R square value for $\ln(Q^*)$ is 78.1%, which is tolerable and for safety stock factor z_0^* is 83.1% value of adjusted R square is good. All the p values are zero in the three regression equations, which shows each factor is significant. This set of regression equation is applicable in mass production systems where frequency of replenishment is few and average cycle stock is quite large.

The Table 4 shows the validation of regression equations given in Table 3 for fast moving item not ordered enough. The average cost, order quantity and safety stock factor for 2034 combinations of data sets are compared for both normal approximation and regression equations and error is determined. The error range for all optimal parameters is within permissible range, hence it validates the regression equation.

Table 2: Factors and levels of fast moving item not ordered enough

Factor	Level 1	Level 2	Level 3
<i>D</i>	500	750	1000
<i>A</i>	10	12	14
<i>h</i>	1.2	1.6	2
<i>q</i>	0	0.5	1
π	2.25	3.5	4.75
π_0	2.70	4.20	5.70
σ_{LT}	2.5	5.0	7.5

Table 3: Regression equations for optimal cost, order quantity and safety stock factor of fast moving item not ordered enough

Regression equations for mixture inventory model for fast moving item not ordered enough	Adj. R ²	p-Values
$\ln(C^*) = 5.04 + 0.00133 D + 0.00333 A + 0.508 h + 0.0363 \pi + 0.0137 \pi_0 - 0.155 q + 0.109 \sigma_{LT}$	99.0%	All p values are zero
$\ln(Q^*) = 3.82 + 0.00109 D + 0.0180 A + 0.0593 h - 0.0793 \pi - 0.0243 \pi_0 + 0.394 q + 0.0273 \sigma_{LT}$	78.1%	All p values are zero
$z_0^* = 1.76 + 0.000137 D - 0.00822 A - 0.315 h + 0.136 \pi + 0.0480 \pi_0 - 0.607 q - 0.0110 \sigma_{LT}$	83.1%	All p values are zero

The Table 5 shows the different levels selected and computed for seven factors for fast movers frequently ordered.

The Table 6 shows the regression equations for the optimal parameters cost, order quantity and safety factor for fast moving frequently ordered item. The adjusted R-square value 94.3% shows very good fit for $\ln(C^*)$ equation. The adjusted R square value 80.6% for $\ln(Q^*)$ and 84.9% for safety stock factor z_0^* are good. All the p values are zero in the $\ln(C^*)$ regression equation except value of $A = 0.202$, which suggests that factor A is moderately significant. The p value for factor A is little more than 0.1, as ordering cost component in frequent replenishments as JIT plays little dominant role as

compare to other factors. All the p values are zero in the $\ln(Q^*)$ regression equation is zero except $A = 0.002$, which shows all factors including A are significant. All the p values are zero in the z_0^* regression equation, except D and A . The p value of D is slightly more than 0.1 and p value of A is less than 0.1, therefore, all factors including D and A are significant. This set of linear regression equations are applicable in JIT systems, where frequent ordering is done in small order sizes.

Table 4: Validation of regression equations for fast moving item not ordered enough

Method	Average C*	Average Q*	Average z_0^*
Normal Approximation	2035.31	138.86	1.6212
Regression equation (Factorial Experiments)	2023.83	137.05	1.6195
Error in percentage	0.57%	1.32%	0.10%

Table 5: Factors and levels of fast moving item frequently ordered

Factor	Level 1	Level 2	Level 3
<i>D</i>	500	750	1000
<i>A</i>	1	2	3
<i>h</i>	1.2	1.6	2
<i>q</i>	0	0.5	1
π	1.20	1.70	2.20
π_0	1.44	2.04	2.64
σ_{LT}	2.5	5.0	7.5

The Table 7 shows the validation of regression equations given in Table 6 for fast moving item frequently ordered. The average cost, order quantity and safety stock factor for 2187 combinations of data sets are compared for both normal approximation and regression equations and error is determined. The error range for cost and safety stock factor is within permissible range, and for order quantity regression equation is little higher.

Table 6: Regression equations for optimal cost, order quantity and safety stock factor of fast moving item frequently ordered

Regression equations for mixture inventory model for fast moving item frequently ordered	Adj. R ²	p values
$ln(C^*) = 4.93 + 0.00138 D + 0.00329 A + 0.409 h + 0.175 \pi + 0.0346 \pi_0 - 0.387 q + 0.0992 \sigma_{LT}$	94.3%	All p values are zero, except $A = 0.202$
$ln(Q^*) = 3.08 + 0.00132 D + 0.0246 A + 0.408 h - 0.265 \pi - 0.0855 \pi_0 + 1.23 q + 0.0896 \sigma_{LT}$	80.6%	All p values are zero, except $A = 0.002$
$z_0^* = 1.89 + 0.000035 D - 0.0127 A - 0.582 h + 0.379 \pi + 0.126\pi_0 - 1.28 q - 0.0452 \sigma_{LT}$	84.9%	All p values are zero, except $D = 0.180$ and $A = 0.051$

Table 7: Validation of regression equations for fast moving item frequently ordered

Method	Average C*	Average Q*	Average z ₀ *
Normal Approximation	1617.51	238.49	0.9962
Regression equation (Factorial Experiments)	1604.06	221.58	0.9950
Error in percentage	0.84%	7.63%	0.12%

This section has discussed on factor analysis of datasets and development of optimal regression equations of fast-moving items for essential and discretionary products.

6. Concluding Comments

There are several industries of real world, which may experience the order crossover due to stochastic lead times of suppliers. Those industries can be benefitted by

recognising the phenomenon of order crossover when demands are substitutable. Two separate sets of regression equations are developed for fast moving items as fast movers that are not ordered enough and fast movers frequently ordered for the inventory managers, practitioners, industry to compute inventory cost, amount of quantity to order and safety stock to be maintained. The first group of regression equations are applicable for decision systems of discretionary products like skin care, grooming products in comparison to frequently ordered, that are fast movers but not ordered enough. The second set of regression equations is applicable in decision systems for fast moving frequently order items such as essential products like grocery, dairy items, consumables in industry such as oils, coolants, water etc. The results suggest that average operating inventory cost is higher for fast movers not ordered enough in comparison to fast movers that are frequently ordered.

Acknowledgements

Authors would like to thank the Swami Keshvanand Institute of Technology, Management and Gramothan Jaipur for providing the support and encouragement during the research.

Nomenclature

- C = Expression for cost of crossover of shipments
- A = Replenishment cost for an order
- D = Annual requirement (Demand)
- h = holding cost per unit per unit time
- q = part of shortages backordered
- 1-q = part of lost sales
- Π = backorder cost per unit shortage
- π₀ = extra lost sales cost over backorder cost per unit lost
- Q = order quantity with order crossover situation
- σ_x = standard deviation of demand during lead time
- σ_{LT} = standard deviation of lead time
- σ_{ELT} = standard deviation of effective lead time
- z₀ = safety stock factor, with z~ N(0,1)
- G(z₀) = expected shortage pe replenishment cycle on the standard normal curve

- a, b = coefficients to be determined for regression
 s = reorder point
 r = inventory carrying charge
 v = unit cost of item

References

- 1) Muhammad Idham Sabtu, Hawa Hishamuddin, N. Saibani, and Mohd Nizam Ab Rahman, "A review of environmental assessment and carbon management for integrated supply chain models," *Evergreen*, 8(3) 628–641 (2021). doi:10.5109/4491655.
- 2) Nurul Syafiqah Zulkefely, H. Hishamuddin, Fatin Amrina A. Rashid, N. Razali, N. Saibani, and Mohd Nizam Ab Rahman, "The effect of transportation disruptions on cold chain sustainability," *Evergreen*, 8(2) 262–270 (2021). doi:10.5109/4480702.
- 3) H.P. Galliher, P.M. Morse, and M. Simond, "Dynamics of two classes of continuous-review inventory systems," *Operations Research*, 7 (3) 362–384 (1959). doi:10.1287/opre.7.3.362.
- 4) G.F. Hadley, and T.M. Whitin, "Analysis of Inventory Systems," 1963rd ed., Prentice Hall, New Jersey, n.d.
- 5) F. Aliunir, Teuku Yuri M. Zagloel, and R. Ardi, "Discrete-event simulation and optimization of spare parts inventory and preventive maintenance integration model considering cooling down and machine dismantling time factor," *Evergreen*, 7(1) 79–85 (2020). doi:10.5109/2740949.
- 6) S. Choudhary, A. Sharma, K. Srivastava, H. Purohit, and M. Vats, "Read range optimization of low frequency rfid system in hostile environmental conditions by using rsm approach," *Evergreen*, 7(3) 396–403 (2020). doi:10.5109/4068619.
- 7) C. Tsou, "Multi-objective inventory planning using mopso and topsis," *Expert Systems with Applications*, 35 (1–2) 136–142 (2008). doi:10.1016/j.eswa.2007.06.009.
- 8) A. Srivastav, and S. Agrawal, "Multi-objective optimization of hybrid backorder inventory model," *Expert Systems with Applications*, 51 76–84 (2016). doi:10.1016/j.eswa.2015.12.032.
- 9) A. Srivastav, and S. Agrawal, "Multi-objective optimization of a mixture inventory system using a mopso–topsis hybrid approach," *Transactions of the Institute of Measurement and Control*, 39 (4) 555–566 (2017). doi:10.1177/0142331215611211.
- 10) A. Srivastav, and S. Agrawal, "Multi-objective optimization of slow moving inventory system using cuckoo search," *AUTOSOFT*, 24 (2) 343–350 (2018). doi:10.1080/10798587.2017.1293891.
- 11) A. Srivastav, and S. Agrawal, "Multi Objective Cuckoo Search Optimization for Fast Moving Inventory Items," in: E.-S.M. El-Alfy, S.M. Thampai, H. Takagi, S. Piramuthu, T. Hanne (Eds.), *Advances in Intelligent Informatics*, Springer International Publishing, Cham, 2015: pp. 503–510. doi:10.1007/978-3-319-11218-3_45.
- 12) D. Zalkind, "Order-level inventory systems with independent stochastic leadtimes," *Management Science*, 24 (13) 1384–1392 (1978). doi:10.1287/mnsc.24.13.1384.
- 13) J.C. Hayya, S.H. Xu, R.V. Ramasesh, and X.X. He, "Order crossover in inventory systems," *Communications in Statistics. Stochastic Models*, 11 (2) 279–309 (1995). doi:10.1080/15326349508807346.
- 14) L.W. Robinson, J.R. Bradley, and L.J. Thomas, "Consequences of order crossover under order-up-to inventory policies," *M&SOM*, 3 (3) 175–188 (2001). doi:10.1287/msom.3.3.175.9887.
- 15) J.R. Bradley, and L.W. Robinson, "Improved base-stock approximations for independent stochastic lead times with order crossover," *M&SOM*, 7 (4) 319–329 (2005). doi:10.1287/msom.1050.0085.
- 16) L.W. Robinson, and J.R. Bradley, "Note —further improvements on base-stock approximations for independent stochastic lead times with order crossover," *M&SOM*, 10 (2) 325–327 (2008). doi:10.1287/msom.1070.0160.
- 17) J. Riezebos, "Inventory order crossovers," *International Journal of Production Economics*, 104 (2) 666–675 (2006). doi: 10.1016/j.ijpe.2004.11.011.
- 18) J.C. Hayya, U. Bagchi, J.G. Kim, and D. Sun, "On static stochastic order crossover," *International Journal of Production Economics*, 114 (1) 404–413 (2008). doi: 10.1016/j.ijpe.2008.03.007.
- 19) M. Srinivasan, R. Novack, and D. Thomas, "Optimal and approximate policies for inventory systems with order crossover," *Journal of Business Logistics*, 32 (2) 180–193 (2011). doi:10.1111/j.2158-1592.2011.01015.x.
- 20) J. Hayya, and T.P. Harrison, "A mirror-image lead time inventory model," *International Journal of Production Research*, 48 (15) 4483–4499 (2010). doi:10.1080/00207540802616843.
- 21) D.P. Bischak, D.J. Robb, E.A. Silver, and J.D. Blackburn, "Analysis and management of periodic review, order-up-to level inventory systems with order crossover," *Prod Oper Manag*, 23 (5) 762–772 (2014). doi:10.1111/poms.12072.
- 22) T. Wensing, and H. Kuhn, "Analysis of production and inventory systems when orders may cross over," *Ann Oper Res*, 231 (1) 265–281 (2015). doi:10.1007/s10479-014-1546-1.
- 23) A. Srivastav, and S. Agrawal, "Multi-objective optimization of mixture inventory system experiencing order crossover," *Ann Oper Res*, 290 (1–2) 943–960 (2020). doi:10.1007/s10479-017-2744-4.
- 24) D.C. Montgomery, M.S. Bazaraa, and A.K. Keswani,

- “Inventory models with a mixture of backorders and lost sales,” *Naval Research Logistics*, 20 (2) 255–263 (1973). doi:10.1002/nav.3800200205.
- 25) D.H. Kim, and K.S. Park, “(Q,r) inventory model with a mixture of lost sales and time-weighted backorders,” *The Journal of the Operational Research Society*, 36 (3) 231 (1985). doi:10.2307/2582254.
- 26) G. Padmanabhan, and P. Vrat, “Inventory model with a mixture of back orders and lost sales,” *International Journal of Systems Science*, 21 (8) 1721–1726 (1990). doi:10.1080/00207729008910488.
- 27) V. Kumar, Sudesh Kumar Garg, S. Gupta, and Sanjay Kumar Sharma, “Two level storage inventory model with ramp type demand under inflationary environment with partial backordering,” *Evergreen*, 9(2) 367–377 (2022). doi:10.5109/4794161.
- 28) A. Srivastav, and S. Agrawal, “On single item time weighted mixture inventory models with independent stochastic lead times,” *IJSOM*, 22 (1) 101 (2015). doi:10.1504/IJSOM.2015.070885.
- 29) E.A. Silver, D.F. Pyke, and Thomas D.J., “Inventory and Production Management in Supply Chains,” 2016th ed., CRC Press, n.d.
- 30) J. Lynwood A., and D.C. Montgomery, “Operations Research in Production Planning, Scheduling, and Inventory Control,” 1974th ed., Wiley, n.d.