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Praveen Kumar Agrawal
Department of Mathematics, GL Bajaj Group of Institutions

Jain, Anamika
Department of Mathematics & Statistics, Manipal University Jaipur

Thakur, Sonali
Department of Mathematics & Statistics, Manipal University Jaipur

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Optimization of Multicomponent Machine System with Reneging

Praveen Kumar Agrawal[1], Anamika Jain[2*], Sonali Thakur[2]

[1]Department of Mathematics, GL Bajaj Group of Institutions, Mathura-281406 India

[2]Department of Mathematics & Statistics, Manipal University Jaipur, Jaipur-303007

*Author to whom correspondence should be addressed:
E-mail: anamikajain_02@rediffmail.com

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Abstract- The system reliability of a multi-spares repairable system with reneging under "N-Policy" is considered. There is a provision of warm and cold reserve units and single repairman. Spares are designed to have exponentially distributed life durations and restoration times. A set of differential-difference equations has been constructed in terms of state dependent letdown and restoration rates. The time dependent analysis has been provided to obtain the states probabilities and to determine 'system reliability' and 'mean time to failure'. Laplace transform of the equations and the matrix technique is used to obtain the probabilities. The cost optimization is also presented. Optimal system parameters have been obtained using a heuristic approach. Various performance indices will be evaluated to explore the outcome of different strictures on the system reliability and 'mean time to failure'.

Keywords: - N-Policy, Multi-component system, Transient Analysis, Reliability, MTTF, Reneging.

1. Introduction

Due to various applications of machining system in many areas, including manufacturing system, production system, multi-programmed computer system[15], time sharing-computer systems, communication system, and transportation system etc., reliability indices of such systems are very useful features[16]. For increasing the reliability of a machining system, the spare part support to the structure can be facilitated. Hot, cold, and warm are the three categories of reserves[14]. The letdown rate of hot reserve unit is equal to the letdown rate of operative unit, the letdown rate of cold reserve unit is zero and the letdown rate of warm reserve unit is non-zero or a lesser amount of than that of operative unit. Numerous academics have investigated the subject of repairable problem with standbys in different frameworks[13]. Authors worked on asynchronous policy machine repair problem under the guidance of multi-server and optimized machine repair model under the observation of two server with different service rates by using SOR technique[18]. In the relevant study server may breakdowns according to the Poisson process[17]. This sensitivity analysis is also illustrated to endorse the results obtained. MTTF, availability and reliability of machines where the server may take multiple vacations during the operation considered by the authors[24]. The multi-components queueing system and optimized cost function in various research paper and displayed results through numerical illustration in tabular form are carried out by the authors[3-6].

Due to complication in the time dependent analysis most of the work endeavored on machines maintenance with standbys is restricted to time independent solution. The concept of 'N-Policy' for general service queues, in which a server switches on only on the accumulation of N units in the system, is a cost-effective strategy because the server's time can be used for other tasks studied and optimal reliability, MTTF for a machining system where system may repair with the help of available spares and repair facility[19].

Analyzed a model subjected to multiple broken-down servers, repairable problem with M operative and S warm reserves units under N-policy strategy and removable repairman[20]. Studied line up model via working vacation[21]. The removable repairman turns on only when there are N units available for repair derived MTTSF and Reliability using Laplace Transform technique for a controllable reliable machining system[10]. They also obtained some important results for the model and validated their results through numerical illustrations. Discussion on transient single server Markovian model under the consideration of secondary task when there was no customer present for service[26]. Embedded Markov Chain technique has been used for the formulation of the mathematical equations. Sensitivity analysis has also taken place to the verification of the analytical results. Investigated N policy' waiting line model by using supplementary variable technique where the server stops the service on becoming the system empty and start service after the accumulation of N customers[9]. Investigated the topic of 'admission control' where customers arrive and join the system depend on customer availability[12]. In the rising scenario, a method for dimensioning the system is established[8]. Focused on real life problem by using discharge scenario of electric machines[7]. Considered unreliable system for minimizing the total cost by using simulation-based optimization approach[1]. Focused on green energy[15]. Analyzed a machining system by using matrix method[22]. Enhanced power transmission model by using numerical techniques[23]. Observed attributes and gave a model on Hexapod Robot[25].

In this investigation we demonstrate reliability and MTTF of repairable machines containing working units, cold reserve, and warm reserve spares in the process of 'N-policy' strategy. Reneging is also a concept that is included. A job waiting in the queue for service may get impatient and renege from the system after some time without getting service. Our model is more practical in a real-time machining system because to the reneging. A differentiation is also prepared for the MTTF by taking dissimilar reserves sets. The cost function was created with the aim optimal cost's function and spares.

2. Assumptions and Notation

The section provides the subsequent notations for the mathematical construction of the model dealing with repairable system and the provision of mixed reserve units:

- W_a Quantity of warm reserve units
- Co Quantity of cold reserve units.
- Op Quantity of operative units
- T Overall quantity of components i.e. T = Op+Wa+Co.
- $\lambda(\alpha)$ Letdown rate of an operative (warm reserve) unit
- λ_d Degraded letdown rate when all reserves are being depleted.
- μ_i Repair rate of failed unit (i=0, 1, 2).
- α_i Reneging parameter (i=0,1,2).
- $\overline{P}_{0,\tau}(t)$ 'Prob. of repairman's turn off state with τ ($0 \le \tau \le N-1$) failed unit at time t'.
- $\overline{P}_{1,\tau}(t)$ 'Prob. of repairman's turn on state with τ ($1 \le \tau \le T$) failed units at time t'.
- $f_{0,\tau}(s)$ 'Laplace transform function' of $\overline{P}_{0,\tau}(t)$.
- $f_{1,\tau}(s)$ 'Laplace transform function' of $\overline{P}_{1,\tau}(t)$.

3. The Model Descriptions

M operative units are compulsory in the system for effective operation. If a functional part fails, is swapped with warm reserve part. If all warm reserve units are depleted, a cold reserve unit is substituted. The fruitless part is directly sent to repairman; if repairman is unavailable, the fruitless unit is placed on the waiting list. If there are less than O_p operative units, the futile unit is returned to the operative group; otherwise, it is moved to the reserve group. The repairman repairs the futile units in order of their letdown i.e. according to FCFS. It is believed that operating and spare units will fail independently of one another. Operative unit and warm reserve unit lifetimes are exponentially distributed with rate λ and α, respectively. When all spare parts are utilized, the operational unit degrades and fails at a rate of λ_d ($>\lambda$).

Depending on the presence of reserve units, the repair time has an exponential distribution with parameter μ_0. If warm reserve unit is available to replace the failed unit, the letdown rate is assumed to be zero. When all warm units are used then failed units swapped by cold reserve unit if unfilled and the repairman repair the failed unit with faster rate $\mu_{1(>\mu_0)}$. If all cold reserve units are used, the repair is completed at a quicker pace of μ_2 ($>\mu_1$). When waiting in line, failing units may also renege exponentially with parameter α_0, α_1 and α_2 respectively, depending upon whether warm reserves, cold reserves and no reserves are available to replace the units upon letdown. When a reserve unit is utilized in the system for procedure, its letdown and operative characteristics is similar as that of the operative unit. It is assumed that while a letdown unit is refurbished it becomes as good as new one. The transition from reserve to operating, and from repair to reserve, takes very little time. We consider machining system where the repairman turns according to N-policy and turns off as soon as repairman restores all failed units.

The state dependent letdown and restoration rates are given by

$$\lambda(\tau)=\begin{cases} O_p\lambda+(W_a-\tau)\alpha, & 0\le\tau\le W_a \\ O_p\lambda, & W_a<\tau<W_a+C_o \\ (O_p+W_a+C_o-\tau)\lambda_d, & W_a+C_o\le\tau<T \end{cases} \tag{1}$$

$$\mu(\tau)=\begin{cases} \mu_0+(\tau-1)\alpha_0, & 1\le\tau\le W_a \\ \mu_1+(\tau-1)\alpha_1, & W_a<\tau\le W_a+C_o \\ \mu_2+(\tau-1)\alpha_2, & W_a+C_o<\tau\le T \end{cases} \tag{2}$$

4. Transient State Equations

The time dependent state equations constructing the mathematical structure are as follows:

$$\frac{d\bar{P}_{0,0}(t)}{dt} = -\lambda(0)\bar{P}_{0,0}(t) + \mu(0)\bar{P}_{1,1}(t) \quad (3)$$

$$\frac{d\bar{P}_{0,\tau}(t)}{dt} = -\lambda(\tau)\bar{P}_{0,\tau}(t) + \lambda(\tau-1)\bar{P}_{0,\tau-1}(t) \quad (4)$$

$$1 \le \tau \le N$$

$$\frac{d\bar{P}_{1,1}(t)}{dt} = -[\lambda(1) + \mu(1)]\bar{P}_{1,1}(t) + \mu(2)\bar{P}_{1,2}(t) \quad (5)$$

$$\frac{d\bar{P}_{1,\tau}(t)}{dt} = -[\lambda(\tau) + \mu(\tau)]\bar{P}_{1,\tau}(t) + \lambda(\tau-1)\bar{P}_{1,\tau-1}(t)$$
$$+\mu(\tau+1)\bar{P}_{1,\tau+1}(t) \qquad 2 \le \tau \le \quad (6)$$

$$\frac{d\bar{P}_{1,N}(t)}{dt} = -[\lambda(N) + \mu(N)]\bar{P}_{1,N}(t) + \lambda(N-1)\bar{P}_{1,N-1}(t)$$
$$+\mu(N+1)\bar{P}_{1,N+1}(t) + \lambda(N-1)\bar{P}_{0,N-1}(t) \quad (7)$$

$$\frac{d\bar{P}_{1,\tau}(t)}{dt} - [\lambda(\tau) + \mu(\tau)]\bar{P}_{1,N}(t) + \lambda(N-1)\bar{P}_{1,N-1}(t)$$
$$+\mu(N+1)\bar{P}_{1,N+1}(t) + \lambda(N-1)\bar{P}_{0,N-1}(t) \quad (8)$$

$$\frac{d\bar{P}_{1,\tau}(t)}{dt} = -[\lambda(\tau)+\mu(\tau)]\bar{P}_{1,\tau}(t) + \lambda(\tau-1)\bar{P}_{1,\tau-1}(t)$$
$$+\mu(\tau+1)]\bar{P}_{1,\tau+1}(t), \qquad N \le \tau \le T-1 \quad (9)$$

$$\frac{d\bar{P}_{1,T}(t)}{dt} = \lambda(T-1)\bar{P}_{1,T-1}(t) - \mu(T)\bar{P}_{1,T}(t) \quad (9a)$$

Denote the Laplace transform of probabilities by

$$\Theta_{0,\tau}(s) = \int_0^\infty e^{-st}\,\bar{P}_{0,\tau}(t)dt \;\; \tau = 0, 1, \ldots. N-1 \quad (10a)$$

and

$$\Theta_{1,\tau}(s) = \int_0^\infty e^{-st}\,\bar{P}_{1,\tau}(t)dt \;\; \tau = 0, 1, \ldots. N-1 \quad (10b)$$

Since initially all units are good, the initial conditions are given by

$$\bar{P}_{0,0}(0) = 1; \bar{P}_{0,\tau}(0) = 0; (\tau = 1, 2, \ldots..N-1)$$
$$\bar{P}_{1,\tau}(0) = 0; (\tau = 1, 2, \ldots..T) \quad (11)$$

We get the following equations by taking the Laplace transform of eq. 3 to eq. 9a and substituting the values of $\lambda(\tau)$ and $\mu(\tau)$.

$$(s + O_p\lambda + W_a\alpha)\Theta_{0,0}(s) - \mu_0\Theta_{1,1}(s) = \bar{P}_{0,0}(0) \quad (12)$$

$$[s + O_p\lambda + (W_a-\tau)\alpha]\Theta_{0,\tau}(s) - [O_p\lambda + (W_a-\tau+1)\alpha]$$
$$\Theta_{0,\tau-1}(s) = \bar{P}_{0,\tau}(0), \qquad 1 \le \tau \le N \quad (13)$$

$$[s + O_p\lambda + (W_a-\tau)\alpha + \mu_0]\Theta_{1,1}(s) - [\mu_0 + \alpha_0]\Theta_{1,2}(s)$$
$$= \bar{P}_{1,1}(0), \quad (14)$$

$$[s + O_p\lambda + (W_a-\tau)\alpha + \mu_0 + (\tau-1)\alpha_0]\Theta_{1,\tau}(s) - [O_p\lambda$$
$$+ (W_a-\tau+1)\alpha]\Theta_{1,\tau+1}(s) - (\mu_0 + \tau\alpha_0)\Theta_{1,\tau+1}(s) = \bar{P}_{1,\tau}(0) \quad (15)$$

$$[s + O_p\lambda + (W_a-N)\alpha + \mu_0 + (N-1)\alpha_0]\Theta_{1,N}(s) - [O_p\lambda$$
$$+ (W_a-N+1)\alpha]\Theta_{0,N-1}(s) - [O_p\lambda + (W_a-N+1)\alpha] \quad (16)$$
$$\Theta_{1,N-1}(s) - (\mu_0 + N\alpha_0)\Theta_{1,N+1}(s) = \bar{P}_{1,N}(0)$$

$$[s + O_p\lambda + (W_a-N)\alpha + \mu_0 + (\tau-1)\alpha_0]\Theta_{1,\tau}(s) - [O_p\lambda$$
$$+ (W_a-\tau+1)\alpha]\Theta_{0,\tau-1}(s) - (\mu_0 + \tau\alpha_0)\Theta_{1,\tau+1}(s) = \quad (17)$$
$$\bar{P}_{1,\tau}(0) \qquad N \le \tau \le W_a$$

$$[s + O_p\lambda + \mu_1 + (\tau-1)\alpha_1]\Theta_{1,\tau}(s) - O_p\lambda\Theta_{1,\tau+1}(s)$$
$$-(\mu_1 + \tau\alpha_1)\Theta_{1,\tau+1}(s) = \bar{P}_{1,\tau}(0) \quad W_a \le \tau \le W_a + C_0 \quad (18)$$

$$[s + (O_p + W_a + C_0 - \tau)\lambda_d + \mu_2 + (\tau-1)\alpha_2]\Theta_{1,\tau}(s) -$$
$$[O_p + W_a + C_0 - \tau + 1)\lambda_d]\Theta_{1,\tau-1}(s) - (\mu_2 + \tau\alpha_2) \quad (19)$$
$$\Theta_{1,\tau+1}(s) = \bar{P}_{1,\tau}(0) \quad W_a + C_0 \le \tau \le T-1$$

$$[s + \mu_2 + (T-1)\alpha_2]\Theta_{1,T}(s) - [O_p + W_a + C_0 - T + 1)\lambda_d]$$
$$\Theta_{1,T-1}(s) = \bar{P}_{1,\tau}(0) \quad (20)$$

5. The Analysis

We can be write eq. 12 to eq. 20 in the matrix form as

$$G(s)\,F(s) = \bar{\mathbf{P}}(0) \quad (21)$$

where

$$F(s) = [\,\Theta_{0,0}(s), \Theta_{0,1}(s), \ldots., \Theta_{0,N-1}(s), \Theta_{1,1}(s),.$$
$$\Theta_{1,N-1}(s), \Theta_{1,N}(s), \Theta_{1,N+1}(s), \ldots., \Theta_{1,T}(s)], \quad (22)$$

$$F(0) = [\,0,0(0), 0,1(0), \ldots., 0,N-1(0), 1,1(0), \ldots.,$$
$$1,N-1(0), 1,N(0), 1,N+1(0), \ldots., 1,T(0)], \quad (23)$$

Here **G**(s) is an (N+T) $\times$(N+T) matrix.

Using Cramer's rule, eq. 23 can be solved and we get an obvious expression for $\Theta_{1,T}(s)$ as

$$\Theta_{1,T}(s) = \frac{|G_{N+T}(s)|}{|G(s)|} \quad (24)$$

where $|G(s)|$ and $|G_{N+T}(s)|$ respectively are the determinants of matrix **G**(s), and matrix found by swapping the (N+T) th column of **G**(s) by the initial vector

$$\bar{\mathbf{P}}(0) = [1, 0, \ldots, 0]^T$$

It is obvious that the root of $|G(s)| = 0$ is s = 0. Substituting $s = -\beta$, we obtain

$$G(-\beta) = A - \beta I \quad (25)$$

where **A** = **G** (0) is an (N+T) $\times$ (N+T) matrix and **I** is the identity matrix. Using eq. 25 in eq. 19, we obtain

$$G(-\beta) F(s) = A - \beta I F(s) = F(0) \qquad (26)$$

Let $d_1, d_2, d_3, \ldots, d_i$ $i \neq 0$ be the i real distinct eigen values of the matrix [**A** $-\beta$ **I**]. By setting its determinant to zero, one may determine the eigenvalue d_k $1 \le k \le i$. Let $((d_{i+1}, \bar{d}_{i+1}), (d_{i+2}, \bar{d}_{i+2}) \ldots\ldots (d_{i+i}, \bar{d}_{i+j})$ be the j pairs of conjugate complex eigen values.

Note there are one zero and i+2j=N+T-1 non-zero eigen values. Also

$$|G(s)| = s\left[\prod_{k=1}^{i}(s+d_k)\right]\left[\prod_{k=1}^{j}\left\{s^2 + \left(d_{i+k} + \bar{d}_{i+k}\right)s + d_{i+k}\bar{d}_{i+k}\right\}\right] \qquad (27)$$

We find an obvious formula of $|G_{N+T}(s)|$ by building a sequence of tri-diagonal matrices and applying their properties.

$$|G_{N+T}(s)| = \left[\prod_{k=0}^{T-1} d_k\right]\Delta_{N-1}(s) \qquad (28)$$

where $\Delta_{N-1}(s)$ is a matrix given as follows:

$$\begin{bmatrix} s+\lambda_1+\mu_0 & -\Phi_1 & 0 & \cdots & \cdots & 0 & 0 & 0 \\ -\lambda_1 & s+\lambda_1+\Phi_1 & -\Phi_2 & \cdots & \cdots & 0 & 0 & 0 \\ 0 & -\lambda_2 & s+\lambda_2+\Phi_2 & \cdots & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \cdots & -\Phi_{N-1} & 0 & 0 \\ 0 & 0 & 0 & \cdots & \cdots & s+\lambda_N+\Phi_{N-1} & -\Phi_N & 0 \\ 0 & 0 & 0 & \cdots & \cdots & -\lambda_N & s+\lambda_{N+1}+\Phi_N & -\Phi_{N+1} \end{bmatrix}$$

(29)

Where $\Phi_i = \mu_0 + i\alpha_0$

The recursive approach is employed to obtain $\Delta_{N-1}(s)$. Consuming eq. 27 and eq. 28 in eq. 24, we acquire

$$f_{1,T}(s) = \frac{\left[\prod_{k=0}^{T-1} d_k\right]\Delta_{N-1}(s)}{s\left[\prod_{k=1}^{i}(s+d_k)\left\{\prod_{k=1}^{j}\left(s^2 + (d_{i+k} + \bar{d}_{i+k})s + d_{i+k}\bar{d}_{i+k}\right)\right\}\right]}$$

$$= \frac{h_0}{s} + \frac{h_1}{s+d_1} + \ldots + \frac{h_i}{s+d_i} + \frac{e_1 s + g_1}{s^2 + (d_{i+1} + \bar{d}_{i+1})s + d_{i+1}\bar{d}_{i+1}} + \ldots + \frac{e_j s + g_j}{s^2 + (d_{i+j} + \bar{d}_{i+j})s + d_{i+j}\bar{d}_{i+j}}$$

(30)

Here h_l (l =0,1, 2... i) is given by

$$h_0 = \frac{\left(\prod_{k=1}^{T-1} d_k\right)\Delta_{N-1}(0)}{\left(\prod_{k=1}^{i} d_k\right)\left(\prod_{k=1}^{j} d_{i+k}\bar{d}_{i+k}\right)} \qquad (31)$$

and

$$h_l = \frac{\left[\prod_{k=1}^{T-1} d_k\right]\Delta_{N-1}(-d_l)}{(-d_l)\left[\prod_{\substack{k=1\\k\neq l}}^{i}(d_k - d_l)\right]\left[\prod_{\substack{k=1\\k\neq l}}^{j}\left\{(-d_l)^2 + (d_{i+k} + \bar{d}_{i+k})(-d_l) + d_{i+k}\bar{d}_{i+k}\right\}\right]}$$

(32)

Also from eq. 34, we obtain

$$e_l(-d_{i+l}) + g_l = \frac{\left[\prod_{k=1}^{T-1} d_k\right]\Delta_{k-1}(-d_{i+l})}{(-d_{i+l})\left[\prod_{k=1}^{i}(d_k - d_{i+l})\right]\left[\prod_{\substack{k=1\\k\neq l}}^{j}\left\{(-d_{i+l})^2 + (d_{i+k} + \bar{d}_{i+k})(-d_{i+l}) + d_{i+k}\bar{d}_{i+k}\right\}\right]}, (l = 1,2,\ldots, j)$$

(33)

Here α_i and β_i denote real and imaginary faction of complex eigen value of d_{i+l} then the inverse Laplace transform of eq. 30 is

$$\bar{P}_{1,T}(t) = h_0 + \sum_{l=1}^{i} a_l e^{-d_l t} + \sum_{l=1}^{j}\left[d_l e^{-\alpha_l t}\cos(\beta_l t) + \frac{g_l - f_l\alpha_l}{v_l} e^{-\alpha_l t}\sin(\beta_l t)\right]$$

(34)

Here h_0, h_l, d_l, g_l, α_l, and β_l are all real numbers.

Since in long run the system will be in failed state, we have $\lim_{t\to\infty} \bar{P}_{1,T}(t) = 1$

6. Some Performance Measures

The reliability function is obtained using

$$R_1(t) = 1 - \bar{P}_{1,T}(t) \qquad (35)$$

where $\bar{P}_{1,T}(t)$ denotes probability of failed units on or before time t. $P_{1,T}$ (t) is determined by eq. 34.

Let $R_1^*(s)$ symbolizes the 'Laplace Transform' of $R_1(t)$, so that

$$R_1^*(s) = \int_0^\infty e^{-st} R_1(t)\,dt$$

Since $\int_0^\infty R_1(t)\,dt = \lim_{s\to 0} R_1^*(s)$ (36)

(MTTF) is obtained by

$$\mathrm{MTTF} = \int_0^\infty R_1(t)dt = \sum_{k=1}^{j} \frac{h_k}{d_k} + \sum_{k=1}^{j} \frac{g_k}{d_{i+k}\bar{d}_{i+k}} \tag{37}$$

Here h_k and g_k can be obtained using eq. 32 and eq. 33, one-to-one.

7. Numerical Illustration

This segment provides the impact of variation of various parameters on performance indices. For this purpose, we develop the computation procedure in MATLAB to evaluate the performance of the model. Numerical outcomes are shortened in tables 1-3.

In table 1, we fix M=7, α=0.13, α_0=0.4, $\alpha_1=\alpha_0$, $\alpha_2=1.5\alpha_0$, μ_0=1.2, μ_1=1.3, μ_2=1.4, and compute the value of MTSF by changing the letdown rate (λ) of working parts and the quantity of reserve units S and Y. It is noticed that the MTSF changes with respected to λ as per the expected in real time system.

Table 1. MTTS by varying λ, S and Y

λ	MTTSF				
	S=2,Y=2	S=3,Y=2	S=2,Y=3	S=4,Y=3	S=3,Y=4
0.5	2470.27	388.77	380.37	51.69	49.59
0.6	779.80	135.74	134.14	22.07	21.52
0.7	319.96	61.73	61.30	12.14	11.94
0.8	157.59	33.61	33.46	7.84	7.74
0.9	88.60	20.77	20.71	5.60	5.54
1	54.99	14.08	14.04	4.29	4.25
1.1	36.82	10.21	10.19	3.45	3.42
1.2	26.16	7.80	7.79	2.87	2.85
1.3	19.48	6.20	6.19	2.45	2.43

In table 2, we fix λ=1, α=0.13, α_0=0.4, $\alpha_1=\alpha_0$, $\alpha_2=1.5\alpha_0$, μ_0=1.2, μ_1=1.3, μ_2=1.4, and compute the value of MTTSF by varying the quantity of functioning units (M) and the quantity of reserve units S and Y. It is noted that the MTSF declines with the inclines in M, S and Y.

Table 2. MTTSF by varying M, S and Y

M	MTTSF				
	S=2,Y=2	S=3,Y=2	S=2,Y=3	S=4,Y=3	S=3,Y=4
8	13.93	6.49	6.47	3.00	2.98
10	3.88	2.78	2.77	1.86	1.84
12	2.10	1.73	1.73	1.34	1.33
14	1.44	1.26	1.26	1.05	1.05
16	1.10	1.00	0.99	0.87	0.86
18	0.89	0.82	0.82	0.74	0.73
20	0.75	0.70	0.70	0.64	0.64
22	0.65	0.62	0.61	0.57	0.57
24	0.57	0.55	0.55	0.51	0.51

In table 3, we fix M=7, λ =1, α=0.13, $\alpha_1=\alpha_0$, $\alpha_2=1.5\alpha_0$, μ_0=1.2, μ_1=1.3, μ_2=1.4, and evaluate the value of MTSF by changing the reneging constraint (α_0) and quantity of reserve units S and Y. It is seen that the MTSF increases with the increase in α_0.

The effect of variation in different parameters can be easily visualized from the graph shown in figures 1-3.

Table 3. MTTSF by varying α_0, S and Y

α_0	MTTSF				
	S=2,Y=2	S=3,Y=2	S=2,Y=3	S=4,Y=3	S=3,Y=4
0.1	8.48	4.04	4.03	2.30	2.28
0.2	15.50	5.85	5.83	2.75	2.73
0.3	29.21	8.91	8.89	3.38	3.35
0.4	54.99	14.08	14.04	4.29	4.25
0.5	101.72	22.64	22.57	5.61	5.55
0.6	183.38	36.58	36.42	7.53	7.44
0.7	321.37	58.84	58.46	10.33	10.17
0.8	547.51	93.65	92.82	14.39	14.10
0.9	907.87	147.04	145.32	20.24	19.73

By fixing parameters M=7, Y=2, S=3, α=0.3, α_0=0.4, $\alpha_1=1.2\alpha_0$, $\alpha_2=1.5\alpha_0$, μ_0=1.2, μ_1=1.3, μ_2=1.4, the influence of letdown rate λ of working parts on system availability is depicted and noted that the system availability declines with the inclines in λ and t in figure 1. Again fixing λ=0.5, Y=2, S=3, α=0.3, α_0=0.4, $\alpha_1=1.2\alpha_0$, $\alpha_2=1.5\alpha_0$, μ_0=1.2, μ_1=1.3, μ_2=1.4, we explore, the consequence of the quantity of operative units M on system availability in figure 2. It is seen that the system availability declines with the incline in M. Finally, by fixing M=7, λ=0.5, Y=2, S=3, α=0.3, $\alpha_1=1.2\alpha_0$, $\alpha_2=1.5\alpha_0$, μ_0=1.2, μ_1=1.3, μ_2=1.4, we examine the influence of reneging parameter α_0 on system availability in figure 3 and noted that the system availability decline with the incline in α_0 and t.

Overall, we conclude that the MTTSF declines with the incline in λ, M, S, Y and inclines with the incline in α_0. Further-more, availability declines as t, λ, M and α_0 incline.

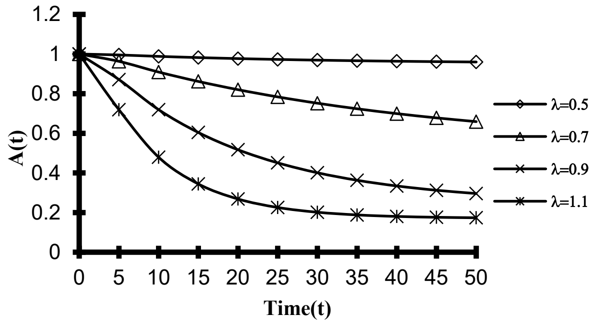

Fig. 1: Effect of failure rate λ of operating Units on system availability

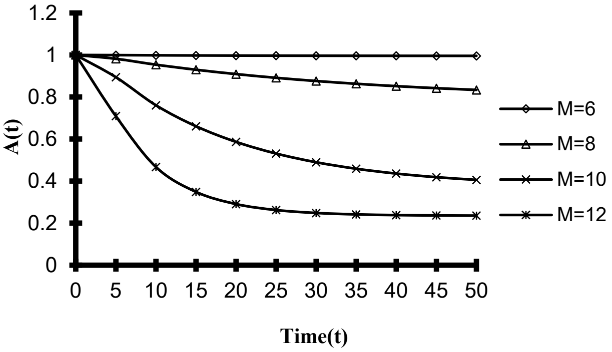

Fig. 2: Effect of number of operating units M on the system availability

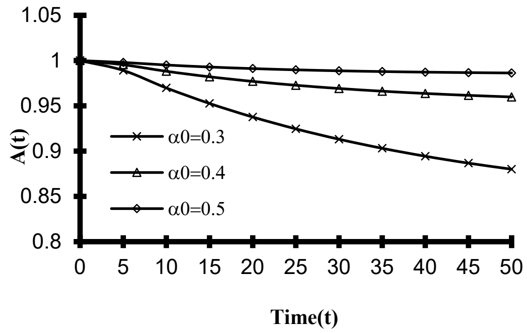

Fig. 3: Effect of reneging parameter α_0 on system Availability

8. Conclusion

N-Policy repairable system with mixed components is studied by employing matrix method. The transient solution is obtained for the probability of the system states, which is further used to get the MTTF. The providing of mixed spares upkeep in the system may be helpful in upgrading the 'system reliability' as commonly observed in machining environment of electronics, computer communication and production systems.

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