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<https://doi.org/10.5109/7160909>

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出版情報 : Evergreen. 10 (4), pp.2317-2324, 2023-12. 九州大学グリーンテクノロジー研究教育センター

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# Reliability Estimation of a Degradable System using Intuitionistic Fuzzy Weibull Lifetime Distribution

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(Received August 19, 2023; Revised November 8, 2023; accepted November 16, 2023).

**Abstract:** The basis of reliability engineering is reliability theory. Reliability analysis is critical for developing different alternatives for system improvement during the design, configuration, and tuning stages, which are essential to the efficient operation of a complicated system. This paper deals with the estimation of the reliability of a degradable system with imperfect coverage and uncertain information about system components. The Weibull intuitionistic fuzzy set (WIFS) concept was used to deal with the uncertainties in the data. The knowledge of a trapezoidal intuitionistic fuzzy number (TriFN) is presented, as well as its arithmetic operations. Trapezoidal intuitionistic fuzzy numbers (TriFN) are applied to signify the failure rate of system.

Keywords: Reliability, IFS, Failure Rate, TriFN

## 1. Introduction

The ability of a system or part of the system to fulfil its intended purpose under stated limits for an extended period of time is referred to as reliability. The objective of achieving high reliability has been important throughout human history, as it is necessary for the effective and safe operation of tools and equipment. The development of statistical methods for assessing the reliability of technical systems began after World War I, when engineers were tasked with assessing the operational safety of airplanes. However, it was not until World War II that the use of statistical methods to evaluate reliability became widespread. During this time, engineers focused on the problem of low reliability in products<sup>1)</sup>, even when they were made of components with high reliability. One important development during this time was the product probability law<sup>2)</sup> of series components, proposed by German mathematician Robert Lusser. This law provided insight into the problem of low reliability in V-1 missiles, which were made up of highly reliable components but still had low overall reliability. Over the next decade, the theory of reliability became an important part of the development of space programs, nuclear power plants, and other industrial processes. Initially, reliability was primarily focused on minimizing economic losses due to system failures. However, over time, the concepts of quality and safety were integrated into the study of reliability, with the

goal of minimizing not only financial losses but also the loss of property and human life.

Today, the study of reliability is an important part of many fields, including engineering<sup>3)</sup> manufacturing, and quality control. By quantifying the parameters for assessing reliability<sup>4)</sup>, engineers and researchers can design more reliable systems and components, and better understand the causes of system failures<sup>5)</sup> when they occur. This development is presently evident in our daily life in the form of high reliability and long operational life of home appliances like Televisions, Refrigerators, Washing machines, LED bulbs, tube lights etc. Similar developments in the areas of reliability and quality<sup>6)</sup> are observable in all forms of transportation and tele-communication services. A reliable industrial manufacturing process not only makes profit for itself but also plays an important role to up-lift Gross Domestic Product (GDP) of a nation. The reliability models are broadly classified into the following categories<sup>6)</sup>.

Hardware reliability models are used to estimate the reliability of a system that is composed of various sub-systems or components<sup>7)</sup>. Failure of any of these components may cause the whole system to fail.

The physical approach of modelling hardware reliability entails examining the system's architecture to control the dependability of each component.

The strength and load of each component are assumed to be described by their probability distribution function<sup>8)</sup>

$F(t)$  of time to failure in the actuarial approach to hardware reliability modelling. In this method, the probability distribution function<sup>9)</sup> is used to estimate each component's reliability parameters.

These models are used in different fields to measure the reliability of various systems and components and can be combined and adapted to fit specific applications.

Overall, hardware reliability models are useful for estimating the reliability of complex systems<sup>10)</sup> and identifying potential points of failure. They are widely used in engineering and manufacturing industries to design. The goal of reliability analysis is to design and maintain systems that are safe, efficient, and cost-effective while minimizing the risk of failure and downtime.

As all of us know, traditional probabilistic reliability evaluation<sup>11)</sup> has been proven to be insufficient in managing failure statistics and modelling uncertainty<sup>12)</sup>. The available information in traditional studies<sup>13)</sup> is the idea of as crisp values. However, due to incorrect assessment (either by humans or machines), the ambiguity of the related notions, or a convinced level of unawareness<sup>14)</sup> regarding the actual values, some data are related with an inherent imprecision.

However, in practice, the exact values of the parameters<sup>15)</sup> are frequently problematic to regulate due to data uncertainty. In these circumstances, it is assumed that the parameters of component life are not clear.

In classical research, the existing data are studied as sharp values. Though, in real-world situations, some data is linked with fundamental inaccuracy<sup>16)</sup> because of inaccuracy in the measurement process (human or machine errors). As a result, the fuzzy set theory<sup>17)</sup> offers the fuzzy number (FN), which is more effective than taking into account just one value, as a suitable tool for modelling uncertain data.<sup>18)</sup> first projected the idea of fuzzy sets assuming that an object whose membership degree is 1 which is less than its non-membership degree. However, in real-world circumstances<sup>19)</sup>, there might be uncertainty regarding the extent to which an object belongs to set A. It is possible to use IFS to model this uncertainty in the membership degree<sup>20)</sup> suggested the concept of IFS using intuitionistic index, which occurs due to information uncertainty.<sup>21)</sup> An IFS can be used to represent vague or imprecise concepts, where the boundary between the set and its complement is not well-defined. In other words, IFS can be viewed as one of the fuzzy sets theory's generalisations<sup>22)</sup>.

The fuzzy set theory normally offers a suitable tool for modelling imprecise data known as fuzzy number (FN), which is much more effective than analyzing only a specific value or classification<sup>23)</sup>. used the fuzzy sets method to measure the system's Reliability.<sup>24)</sup>states that the reliability engineering industry covers a varied range of activities the most important of which is reliability modelling. Efforts have been made by<sup>25)</sup> for a long time in the design and improvement of huge-scale structures.

For the duration of that point, academics put in a lot of effort to develop an organized theory of reliability that depends totally on the concept of probability theory.

Various researchers<sup>26)</sup> work on a coherent system for reliability analysis<sup>27)</sup> depends on the ambiguous state presumption<sup>28)</sup> conveyed reliability assessment in fuzzy environments, fuzzy Weibull lifetime distributions, and fuzzy lifetime distributions<sup>29)</sup>. <sup>30)</sup>gave an example of how to analyse the dependability of fuzzy systems using fuzzy number arithmetic operations.<sup>31)</sup> employed intervals of confidence to examine the reliability of fuzzy systems. For reliability analysis,<sup>32)</sup> proposed a fuzzy set-based method.<sup>33)</sup> estimated the dynamic dependability of a failing system involving the notion of probist validity as a fuzzy triangular number.<sup>34)</sup> The membership functions of a degradable system with inadequate coverage and fuzzy parameters<sup>35)</sup> were constructed using the concept of non-linear<sup>36)</sup> parametric programmes.<sup>37)</sup> uses triangular intuitionistic fuzzy numbers<sup>38)</sup> to investigate reliability functions. Different operations on IFS<sup>39)</sup> were defined by<sup>40)</sup> mapping operations between fuzzy sets and intuitionistic fuzzy sets.<sup>41)</sup> define that IFS and TrIFN are extensions of classical fuzzy sets that allow for a more accurate depiction of uncertain and ambiguous information. A TrIFN is a type of IFS that uses a trapezoidal membership function to give each possible value within its domain a degree of membership. The complement of the membership degree determines the non-membership degree<sup>42)</sup> and the width of the trapezoidal function determines the degree of indeterminacy. The TrIFN is particularly useful for representing information that is imprecise or ambiguous, but where there is some level of confidence in the values being represented. The degree of indeterminacy in the form of TrIFN is determined by<sup>43)</sup> allows for the representation of the uncertainty associated with the values being represented, and this information can be used in decision-making and other applications where ambiguity and uncertainty are present.

Overall, the use of IFS and TrIFN allows for a more flexible and nuanced representation of information than classical fuzzy sets, and can be particularly useful in decision-making applications where there is a need to capture uncertainty and ambiguity.

The Weibull distribution is commonly used in reliability analysis to model the failure rate of a system over time. It is a flexible distribution that can represent various failure patterns and hence is suitable for a wide range of applications. The Weibull distribution can model the failure behavior of components or systems, while intuitionistic fuzzy sets can capture the degrees of membership, non-membership, and hesitation associated with reliability assessments. Thus the intuitionistic fuzzy Weibull distribution along with TrIFNs is being used to model the uncertain and imprecise nature of the system's reliability characteristics in this paper. This approach

allows for a more comprehensive and accurate representation of system reliability.

The paper is systematized as: The fuzzy hazard function and fuzzy Weibull distribution are introduced in Section 2 along with some basic concepts. In Section 3, the fuzzy reliability function is presented. Section 4 discuss the modelling of a degradable system followed by the fuzzy reliability function. The final section discusses the study's conclusion and includes a presentation of numerical results.

## 2. (a) FUZZY WEIBULL DISTRIBUTION

In reliability analysis, the most common distribution function is weibull distribution function. The definition of the probability distribution function is

$$f(x) = \frac{\beta}{\theta} \left(\frac{x}{\theta}\right)^{\beta-1} \exp\left(-\left(\frac{x}{\theta}\right)^\beta\right), x > 0, \theta > 0, \beta > 0$$

Where  $\theta$  is scale parameter,  $\beta$  is shape parameter.

This distribution is capable of modelling a wide range of lifetime data by varying the value of the shape parameter. Weibull distribution becomes an exponential distribution if  $\beta = 1$ ;  $\beta = 2$ , it becomes a Rayleigh distribution.

The determination of exact values of lifetime parameters becomes extremely challenging due to the vagueness and imprecision of data. To simplify this, intuitionistic fuzzy numbers  $\tilde{\theta}$  are used instead of lifetimes of Weibull distributions  $\theta$ .

The fuzzy probability of event  $x \in [c, d], c \geq 0, P(c \leq x \leq d)$  and its  $\alpha$ -cut set in this situation is given by

$$\tilde{P}(c \leq x \leq d)[\alpha] = \int_c^d \frac{\beta}{\theta} \left(\frac{x}{\theta}\right)^{\beta-1} \exp\left(-\left(\frac{x}{\theta}\right)^\beta\right) dx \mid \theta \in \tilde{\theta}[\alpha] \\ = [P^L[\alpha], P^U[\alpha]]$$

Where  $\tilde{\theta}[\alpha]$  is the  $\alpha$ -cut of trapezoidal institutional fuzzy number and

$$P^L[\alpha] = \min \left\{ \int_c^d \frac{\beta}{\theta} \left(\frac{x}{\theta}\right)^{\beta-1} \exp\left(-\left(\frac{x}{\theta}\right)^\beta\right) dx \mid \theta \in \tilde{\theta}[\alpha] \right\} \forall \alpha$$

$$P^U[\alpha] = \max \left\{ \int_c^d \frac{\beta}{\theta} \left(\frac{x}{\theta}\right)^{\beta-1} \exp\left(-\left(\frac{x}{\theta}\right)^\beta\right) dx \mid \theta \in \tilde{\theta}[\alpha] \right\} \forall \alpha$$

## (b) FUZZY HAZARD FUNCTION

The fuzzy hazard function  $\tilde{h}(t)$ , given that an item has not failed by time  $t$ , Here the fuzzy conditional probability defined by  $\tilde{h}(t)$  lies in the interval  $t$  to  $(t + d \ t)$ . The failure rate is another name for the hazard function. The mathematical formulation of the fuzzy

hazard function is defined as

$$\tilde{h}(t)[\alpha] = \lim_{\Delta t \rightarrow 0} \frac{\tilde{P}(t < x < t + \Delta t \mid X > t)}{\Delta t} \\ = \left\{ \frac{f(t)}{S(t)} \mid \theta \in \tilde{\theta}[\alpha] \right\}$$

If the life time follows fuzzy Weibull distribution, then the corresponding fuzzy hazard function can be expressed as:

$$\tilde{h}(t)[\alpha] = \left\{ \frac{\beta}{\theta} \left(\frac{t}{\theta}\right)^{\beta-1} \mid \theta \in \tilde{\theta}[\alpha] \right\} \\ = \left[ \frac{\beta t^{\beta-1}}{(a_1 + (a_2 - a_1)\alpha)^\beta}, \frac{\beta t^{\beta-1}}{(a_4 - (a_4 - a_3)\alpha)^\beta} \right] \\ \left[ \frac{\beta t^{\beta-1}}{(a_2 - (1 - \alpha)(a_2 - a'_1))^\beta}, \frac{\beta t^{\beta-1}}{(a_3 + (1 - \alpha)(a'_4 - a_3))^\beta} \right]$$

It should be noticed that in terms of  $\alpha$  and  $t$  ( $0 < \alpha < 1$  and  $t > 0$ ),  $\tilde{h}(t)$  is a two-dimensional function. The hazard rate curve is like a band in this manner for every  $\alpha$ -cut.

## 3. FUZZY RELIABILITY FUNCTION

Fuzzy survival function or fuzzy reliability  $\tilde{S}(t)$  is the hazy chance that a unit exists or survives the passage of time  $t$ . Let  $x$  be a random variable that represents the lifespan of a system component. It also includes a density function as well as a cumulative distribution function  $f(x, \theta)$ . The following definition describes the fuzzy reliability function at time  $t$ .

$$\tilde{S}(t) = \tilde{P}(x > t) = 1 - F_X(t), t > 0$$

The likelihood of failure or the chance of an item failing in the time span  $[0, t]$  is provided by the unreliability function  $\tilde{Q}(t)$  such as

$$\tilde{Q}(t) = \tilde{P}(x \leq t) = F_X(t), t > 0$$

Assume that a component's lifespan parameter has an IFWLD and that the lifetime parameter  $\tilde{\theta}$  represents a TrIFN as  $\tilde{\theta} = \{a_1, a_2, a_3, a_4; a'_1, a'_2, a'_3, a'_4\}$ .

Then a membership and non-membership function is defined by  $\xi_{\tilde{\theta}}(x)$  and  $\eta_{\tilde{\theta}}(x)$  can be designated as:

$$\xi_{\tilde{\theta}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \frac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4 \\ 0, & \text{otherwise} \end{cases}$$

and

$$\eta_{\bar{\theta}}(x) = \begin{cases} \frac{a_4 - x}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 0, & a_2 \leq x \leq a_3 \\ \frac{x - a_3}{a_4 - a_3}, & a_3 \leq x \leq a_4 \\ 1, & \text{otherwise} \end{cases}$$

Where  $\{a_1, a_2, a_3, a_4; a'_1, a_2, a_3, a'_4\}$  are wholly real numbers. The  $\alpha$ -cut set of a trapezoidal lifetime fuzzy parameter  $\bar{\theta}$  is given by

$$\tilde{\theta}[\alpha] = [\{a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha\}; \{a_2 - (1 - \alpha)(a_2 - a'_1), a_3 + (1 - \alpha)(a'_4 - a_3)\}]$$

Consequently, a function's fuzzy reliability component defined as

$$\begin{aligned} \tilde{S}(t)[\alpha] &= \int_t^\infty \frac{\beta}{\theta} \left(\frac{x}{\theta}\right)^{\beta-1} \exp\left(-\left(\frac{x}{\theta}\right)^\beta\right) dx \mid \theta \in \tilde{\theta}[\alpha] \\ &= \exp\left(-\left(\frac{x}{\theta}\right)^\beta\right) \mid \theta \in \tilde{\theta}[\alpha] \end{aligned}$$

The intuitionistic fuzzy reliability function's - cutoff value is given by

$$\tilde{S}(t)[\alpha] =$$

$$\left[ \exp\left(-\left(\frac{t}{a_1 + (a_2 - a_1)\alpha}\right)^\beta\right), \exp\left(-\left(\frac{t}{a_4 - (a_4 - a_3)\alpha}\right)^\beta\right), \right. \\ \left. \exp\left(-\left(\frac{t}{a_2 - (1 - \alpha)(a_2 - a'_1)}\right)^\beta\right), \right. \\ \left. \exp\left(-\left(\frac{t}{a_3 + (1 - \alpha)(a'_4 - a_3)}\right)^\beta\right) \right]$$

In terms of and  $(0 < \alpha < 1$  and  $t > 0)$   $\tilde{S}(t)$  is a two-dimensional function which is important to note. Given at time  $t_0$ , fuzzy reliability is a TrIFN. If the membership and non-membership function is defined as  $\xi_{\bar{\theta}}(x)$  and  $\eta_{\bar{\theta}}(x)$  of a fuzzy number, respectively, then

$$\xi_{t_0}(x) = \begin{cases} \left(\frac{x - e^{-\left(\frac{t_0}{a_1}\right)^\beta}}{e^{-\left(\frac{t_0}{a_2}\right)^\beta} - e^{-\left(\frac{t_0}{a_1}\right)^\beta}}\right), & \text{if } e^{-\left(\frac{t_0}{a_1}\right)^\beta} \leq x \leq e^{-\left(\frac{t_0}{a_2}\right)^\beta} \\ 1 & \text{if } e^{-\left(\frac{t_0}{a_2}\right)^\beta} \leq x \leq e^{-\left(\frac{t_0}{a_3}\right)^\beta} \\ \left(\frac{e^{-\left(\frac{t_0}{a_4}\right)^\beta} - x}{e^{-\left(\frac{t_0}{a_4}\right)^\beta} - e^{-\left(\frac{t_0}{a_3}\right)^\beta}}\right) & \text{if } e^{-\left(\frac{t_0}{a_3}\right)^\beta} \leq x \leq e^{-\left(\frac{t_0}{a_4}\right)^\beta} \end{cases}$$

and

$$\eta_{t_0}(x) = \begin{cases} \left(\frac{e^{-\left(\frac{t_0}{a_2}\right)^\beta} - x}{e^{-\left(\frac{t_0}{a_2}\right)^\beta} - e^{-\left(\frac{t_0}{a_1}\right)^\beta}}\right), & \text{if } e^{-\left(\frac{t_0}{a_1}\right)^\beta} \leq x \leq e^{-\left(\frac{t_0}{a_2}\right)^\beta} \\ 0 & \text{if } e^{-\left(\frac{t_0}{a_2}\right)^\beta} \leq x \leq e^{-\left(\frac{t_0}{a_3}\right)^\beta} \\ \left(\frac{x - e^{-\left(\frac{t_0}{a_3}\right)^\beta}}{e^{-\left(\frac{t_0}{a_4}\right)^\beta} - e^{-\left(\frac{t_0}{a_3}\right)^\beta}}\right) & \text{if } e^{-\left(\frac{t_0}{a_3}\right)^\beta} \leq x \leq e^{-\left(\frac{t_0}{a_4}\right)^\beta} \end{cases}$$

#### 4. SYSTEM MODELLING

Markov models are commonly used in modelling and reliability analysis, where events like a module failure or repair could happen at any time. With three identical and independent units, a redundant degradable system can have its reliability examined using this model. In this system, each unit can be in one of three states: functional, damaged, or failed.

The transition rates of the Markov model between states are determined by the failure and repair rates of the individual units, as well as the coverage factor. The Markov model is a better modelling strategy when the coverage factor is taken into account because failures of covered and uncovered components are thought of as mutually exclusive events.

The coverage factor denotes the percentage of time that the system is operational (i.e., protected from environmental factors that could cause deterioration or failure). The transition rates in the Markov model would be selected based upon whether the structure is covered or uncovered to model the coverage factor.

Once the Markov model is constructed and the transition rates are determined, various reliability metrics can be calculated, such as the steady-state availability, the expected number of failures, and the expected downtime. These metrics can provide insights into the performance and maintenance requirements of the redundant degradable system.

Based on the following assumptions, it appears that the reliability and system availability made up of numerous identical units are assessed using the Markov model,

- Each unit can be performed in one of two states: operative or unsuccessful. The lifetimes for each unit follow an exponential distribution with parameter  $\mathcal{L}$ .
- When a unit fails, the failed module is immediately detected by a reconfiguration process and removed from the system.
- The success of a reconfiguration operation means that all other fault-free components will continue working. The

coverage factor for this operation is  $C$ .

At time  $t$ , the system may be one of the following states:

$$\delta(t) = \begin{cases} 3, & \text{All three units are working properly} \\ 2, & \text{One of the operating unit is failed} \\ 1, & \text{Two of the operating units are failed} \\ 0, & \text{All three units are failed} \end{cases}$$

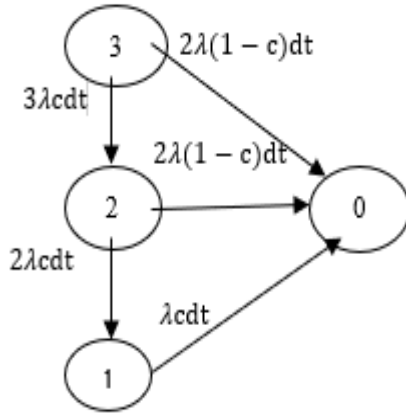


Fig. 2. State transition diagram of a 3-unit degradable system

#### 4.(a) Reliability evaluation under IFS Calculation

For the system depicted in Fig. 2, let  $\tilde{S}(t)$  stand for the fuzzy reliability of each component, and let  $\lambda$  represent the fuzzy failure rate as a TriFN

$$\lambda[\alpha] = [110, 120, 130, 140, 100, 120, 130, 150]$$

The intuitionistic fuzzy reliability function's  $\alpha$ -cut is specified as by

$$\begin{aligned} \tilde{S}(t)[\alpha] &= \left[ \exp\left(-\left(\frac{t}{a_1 + (a_2 - a_1)\alpha}\right)^\beta\right), \right. \\ &\exp\left(-\left(\frac{t}{a_4 - (a_4 - a_3)\alpha}\right)^\beta\right) \\ &\exp\left(-\left(\frac{t}{a_2 - (1 - \alpha)(a_2 - a'_1)}\right)^\beta\right), \\ &\exp\left(-\left(\frac{t}{a_3 + (1 - \alpha)(a'_4 - a_3)}\right)^\beta\right) \\ \tilde{S}(t)[\alpha] &= \\ &\left[ \exp\left(-\left(\frac{t}{110 + 10\alpha}\right)^\beta\right), \exp\left(-\left(\frac{t}{140 - 10\alpha}\right)^\beta\right); \right. \\ &\exp\left(-\left(\frac{t}{120 - 20(1 - \alpha)}\right)^\beta\right), \\ &\exp\left(-\left(\frac{t}{130 + 20(1 - \alpha)}\right)^\beta\right) \end{aligned}$$

The equations for fuzzy reliability under IFS are given in Appendix – A

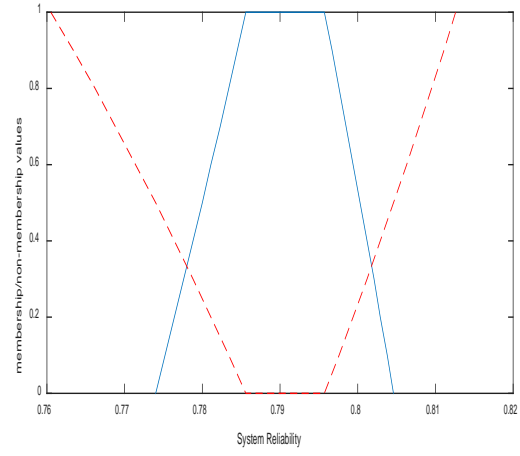


Fig. 3(a)  $\alpha$ -cut of fuzzy reliability at  $t = 50$ ,  $\beta = 0.5$  and  $c = 0.9$

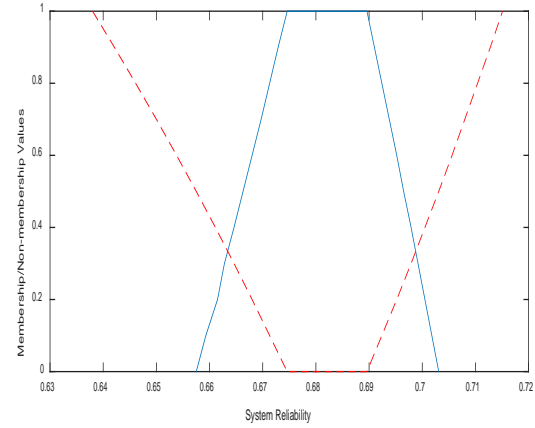


Fig. 3(b):  $\alpha$ -cut of fuzzy reliability at  $t = 100$ ,  $\beta = 0.5$  and  $c = 0.9$

Based on these expressions, the membership values of reliability parameters along with left and right spreads are analyzed for the system. The major advantage of using intuitionistic fuzzy sets over the fuzzy sets is that it separates the acceptance and rejection evidence, which can lead to a more precise and flexible representation of uncertainty in the context of system reliability analysis.

#### 5. Conclusion

A crucial tool for evaluating the dependability of systems that deteriorate over time is degradation analysis. The uncertainty that results from the use of inaccurate or incomplete data can be managed using intuitionistic fuzzy sets (IFS). The degradable system being analyzed in the paper is likely one in which the reliability of the components degrades over time, which can be caused by factors such as wear and tear, corrosion, or other forms of damage. The intuitionistic fuzzy Weibull distribution is a probabilistic model that is used to describe the distribution of failure times for components in a system.

We analyze the fuzzy reliability of a degradable system with inadequate coverage using the intuitionistic fuzzy Weibull lifetime rate. Inadequate coverage refers to a situation where the system does not have sufficient redundancy or backup components to ensure continued operation in the event of component failures. This can lead to reduced reliability and increased risk of system failure. The paper appears to be using a TriFN (which stands for "triangular intuitionistic fuzzy number") to represent the lifetime rate of the system's components. We also introduced arithmetic operations of TriFN. The FMTTF and fuzzy hazard function, with their  $\alpha$ -cut set have all been successfully explored using IFWLD. To demonstrate the method of analysis, numerical examples are used, and the results are plotted. The use of IFS and the IFWLD can provide a more accurate and robust estimate of the reliability of degradable systems, by taking into account the inherent uncertainty associated with such systems.

### Abbreviations

In this manuscript, the subsequent acronyms are utilized:

|       |  |
|-------|--|
| IFS   | Intuitionistic fuzzy set                           |
| TriFN | Trapezoidal Intuitionistic Fuzzy Numbers           |
| FN    | Fuzzy Number                                       |
| MTTF  | Mean Time to Failure                               |
| FMTTF | Fuzzy Mean time to failure                         |
| IFWLD | Intuitionistic fuzzy Weibull lifetime distribution |

### Acknowledgements

We would like to express our sincere gratitude to our institution and all those who have contributed to the completion of this journal article.

### Availability of data and material

The authors certify that no data is associated with this research.

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## Appendix – A

$$\tilde{R}_L(t)[\alpha] = 3c^2 e^{-\left(\frac{t}{110+10\alpha}\right)^\beta} + 3c(1-2c)e^{-2\left(\frac{t}{110+10\alpha}\right)^\beta} + (1-3c+3c^2)e^{-3\left(\frac{t}{110+10\alpha}\right)^\beta}$$

$$\tilde{R}_L(t)[\alpha] = 3c^2 e^{-\left(\frac{t}{140-10\alpha}\right)^\beta} + 3c(1-2c)e^{-2\left(\frac{t}{140-10\alpha}\right)^\beta} + (1-3c+3c^2)e^{-3\left(\frac{t}{140-10\alpha}\right)^\beta}$$

$$\tilde{I\dot{R}}_L(t)[\alpha] = 3c^2 e^{-\left(\frac{t}{120-20(1-\alpha)}\right)^\beta} + 3c(1-2c)e^{-2\left(\frac{t}{120-20(1-\alpha)}\right)^\beta}$$

$$+ (1-3c+3c^2)e^{-3\left(\frac{t}{120-20(1-\alpha)}\right)^\beta}$$

$$\tilde{I\dot{R}}_U(t)[\alpha] = 3c^2 e^{-\left(\frac{t}{130+20(1-\alpha)}\right)^\beta} + 3c(1-2c)e^{-2\left(\frac{t}{130+20(1-\alpha)}\right)^\beta}$$

$$+ (1-3c+3c^2)e^{-3\left(\frac{t}{130+20(1-\alpha)}\right)^\beta}$$