Experimental Study on the Discharge Coefficient of Type A Piano Key Weir for Water Resource Sustainable Development

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Abstract: The piano key weir (PKW) is a spillway with inlet and outlet keys that are similar to piano keys. Depending on the water head and the width of the outlet keys, the flow into the inlet keys and through the outlet keys is directed by partition walls, resulting in a flow characteristic that differs from that of a traditional spillway. This paper presents some flow characteristics of the type A rectangular PKW and research on physical models in a laboratory flume, 22 m in length, 0.8 m in width, and 1 m in height. We found the equation that tells us how much water flows through the PKW in the full regime and the incomplete regime of the outlet key when Ho/Wo = 0.5 is the boundary value. The equation was built for flow states and convenient calculations, with an accuracy of less than 8.03% compared to experimental data. Beside, statistical indicators between measured value and predicted values are good (RMSE = 0.022; MAE = 0.017; MAPE = 2.0%), especially in correlation coefficient is strong (R^2 = 0.98). Using the data of other authors, the calculation efficiency of the proposed formulas is also very suitable, giving a very strong correlation coefficient (R^2 > 0.9) and other statistical indicators close to the ideal point (zero). This shows that the proposed equations are very suitable to evaluate the flow capacity through the PKW and effective in water management and sustainable development at river basin.

Keywords: Piano key weir; PKW; characteristic flow; physical experimental model; type A

1. Introduction

In 2006, the Goulours dam was the first piano key weir (PKW) built in France[1]. This weir has been studied for 20 years. The structure of the weir includes an inlet key and outlet key. The keys can be interspersed, the bottom of the key has a slope, and the basic structure helps shrink the spill foot to 2/3 of the peak length. The flow of the PKW has not only increased 4 ÷ 5 times compared to the traditional spill but also easily builds in tight terrain conditions, along with the ability to increase the discharge, it will be effective in water resource management, environmental protection, stabilizing water supply for downstream (reducing energy costs for using pump)[2], meet the needs of agricultural irrigation, urban water supply and help the ecosystem of the flow become more stable, etc[4),5),6)]. Because of its low cost, large flow, safety and storage, and the flood control efficiency of many existing dams, it is the best solution[7),8)]. The energy dissipation issues downstream of the dam need to pay more attention when the flow volume increases, especially paying attention to the energy dissipation, stabilize the riverbank, and protection against scour[10].
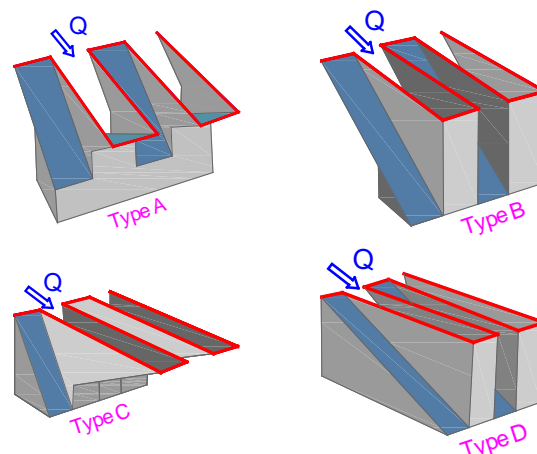

Fig.1: 3D sketch of types PKW[1),18),36)]

PKW has four basic types: type A, type B, type C, and type D, with 20 geometric parameters[8),9)] (Fig.3, 4 and 5). The establishment of the relationship between these factors is analyzed through Buckingham's Pi theory[11], from which empirical equations and databases need to be collected on physical models.

Fig.2: Van Phong dam (Type A-PKW) in Vietnam[11]

The type-A PKW has been widely used in the world (e.g. L'Etroit dam PKW - France and Tzaneen dam - South African[12], etc.). In Vietnam, this type of project has been built and operates stably, as Van Phong dam (Binh Dinh province, 2015)[11] and Da Dang dam (Lam Dong province, 2019)[14], etc. Besides, some projects are in the construction phase, as a Phu Phong Weir Project (Binh Dinh province)[15] or dam downstream on the Tra Khuc River (Quang Ngai province)[16], etc.
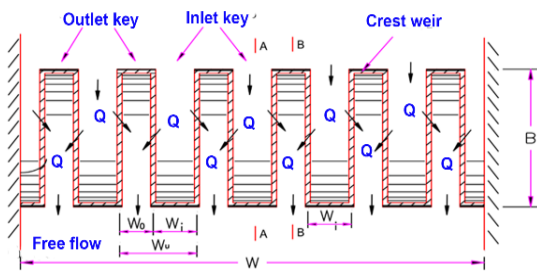
Fig.3: Design suggestion for A-PKW of plan view [19)20)39)]
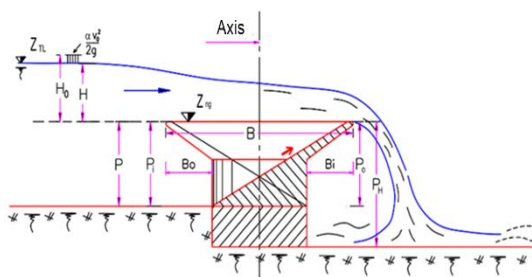
Fig.4: The side view of the studied type A-PKW[39]
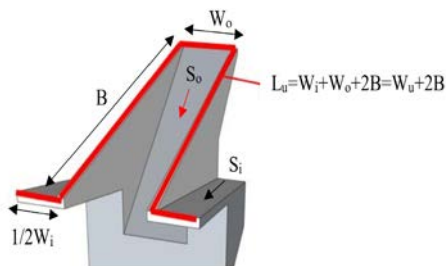
Fig.5: 3d sketches of the type A-PKW [19), 20)]

In all studies on the geometric parameters influencing the PKW hydraulic efficiency, the flow features over the PKW are studied by changing pairs of geometric quantities, like the height-to-unit width ratio P/Wu [21), 22)], the key width ratio Wi/Wo[23),24)], the crest length magnification ratio L/W, and the overhangs positions ratio Bo/Bi[1),25)].

The PKW Geometric Parameters, A.Ouamane et al.[24] showed that the discharge coefficient C_d increases if the height-to-unit width ratio (N=L/W) is large . A.Noui & A.Ouamane[25] showed that the height-to-unit width ratio P/Wu should influence flow (Q), but this is only obvious when H is small . M.Leite Ribeiro et al.[27] showed that the crest length magnification ratio N ranges from 3 to 7; all types of the PKW are similar in flow, and the flow is rising about 50% if the H/P ratio is about 0.2 . O.Machiels et al.[22] showed that the PKW geometric parameters have great affection for flow with N = 5 and the height of the inlet key and outlet key . Kam R. Eslinger and Brian M. Crookston[28] studied the energy dissipation efficiency after the PKW of Type A and established the H_1/H_o relationship with the influencing factors, showing that energy dissipation is a nonlinear function.

Singh, D. and Kumar, M[9] studied the application of PKW, showing its effectiveness for a spillway structure. Amr A. Bekheet et al.[29] investigated the effect of PKW shape on flow efficiency using a variety of dam shapes (rectangle, trapezoid, and triangle), demonstrating that the Wi/Wo ratio is the most influential factor on flow. Deepak Singh and Munendra Kumar[30] used the GEP model to predict the energy dissipation of the flow through type A of the PKW, and the results showed good efficiency, with an average error of less than 5.4%. Rezaei, V. et al.[20] conducted empirical research on the discharge coefficient (C_d) and optimization simulation through the FLOW-3D model, showing the factors affecting the discharge of the PKW, but they were not clarified using an equation.

Review more recent studies on the PKW, such as the study by S. Li et al.[3] and Vafa Rezaei et al.[20] studied the hydraulic characteristics on the flow through the Type A-PKW, the study compared the experimental physical model with the simulation model (e.g Flow-3D model), the study showed a good fit with the physical model and mathematical model. Mojtaba Kheilapour et al.[31] experimentally studied the flow coefficient through the triangular PKW, the study analyzed the removal capacity and the removal efficiency of the experiment. Research by Bhukya, R.K. et al.[32] generalized about the types of Piano spillway, the study analyzed some empirical work on the discharge coefficient (C_d).

Each study has its own aspects, it describes the characteristics of the flow through the Piano spillway with different types of shapes. However, studies have shown the need for a formula for calculating the flow through the Piano spillway. A new research equation can specifically generalize about the ability to flow through the Piano spillway in general and the Piano grade A spillway in particular, this is an important issue and in which empirical research palys leading role.

Most studies examined the influence of the geometry of the dam on the discharge capacity in terms of the variation in the upstream head (H_o or H) or the ratio of the head and the height of the weir (H/P). However, some studies have not examined the effects of the widths of the outlet key (W_o) on the flow shape characteristics and the transition of the state of the discharge on the PKW's key.

Therefore, this paper focuses on analyzing the impact of the width of the outlet key (Wo), height of the weir (P) and crest length (Lu) on the characteristic of the PKW. On the one hand, establishing the new experimental equations to determine the discharge coefficient (C_d) for the dam design and rehabilitation in water resource management and sustainable development of this river basin.

2. Experimental setup

2.1 Physical Model Parameters of the PKW

Systematic physical model tests were conducted in a straight rectangular channel at the Key Laboratory of River and Coastal Engineering (KLORCE) of the Vietnam Academy for Water Resources (VAWR). By studying type A of the PKW, we conducted model tests, with nine different the PKW geometries. The basic parameter variation included H/P = 0.2÷2.1; H/W_o = 0.3÷2.1.

Table 1. Parameters of the PKW case studies in model dimensions.

Factors	Authors			
	Present study	**O.Machiels et al. [16]**	**A.Noui et al. [19]**	**N.T Hai et al [29]**
N = L/W	5	5	5.9	4.3÷8.2
P/W_u	0.5÷1.1	0.33÷2.0	0.9	0.3÷2.4
W_i/W_o	1.25	1.5	0.96÷1.53	1.2
P_i/P_o	1	1	1	1
B_o/B_i	1	1	1	1
B/P	1.87÷4.52	1.00÷6.00	2.73	1.50÷4.50
W_u (m)	0.25	0.3	0.17÷0.25	0.17÷0.30
Type of PKW	A	A	A	A

2.2 Structures of the Physical Model

The model was set up at the KLORCE of VAWR and was built from organic glass. Its dimensions were as follows: 100 cm high, 50 cm wide, 1.5 cm thick, and 22 m long (Fig.6 and 7).

When the Reynolds number (Re) reaches Re_{max} = 34500 > Re_{gh} = 5000 (limiting Reynolds number), the working flow condition in the model automatic zone is satisfied.

The smallest upstream height value, H_{min} = 0.03 m, ensures that the flow on the model is not affected by surface tension.
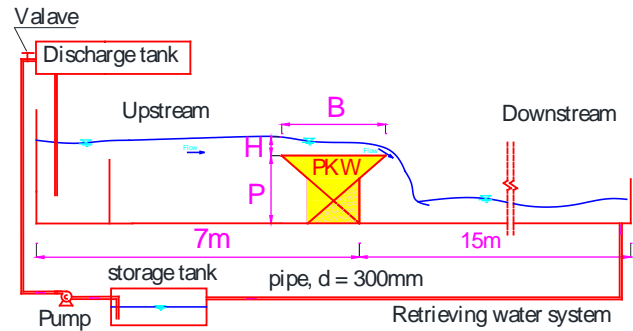

Fig.6: Plan of the experimental set-up

Fig.7: Experimental flume layout of type A-PKW

If the flow is over the PKW, reading the data based on the scale on the wall of the flume, and using a steel ruler, levelling staff and automatic level to measure the data.

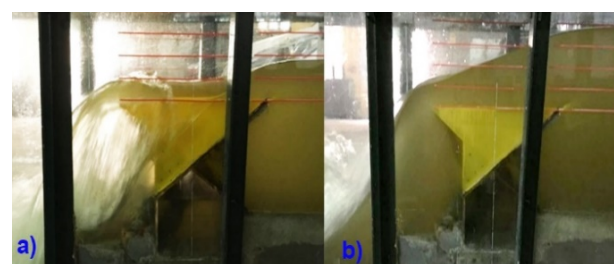**Fig.8:** Flow over the outlet and inlet keys of the type A-PKW with H_0/W_o = 0.5.

Fig.9: Model view with different energy head over crest values (a. *Medium approach flow head with Ho/Wo=0.8 and b. Big approach flow head with Ho/Wo=1.9*)

Based on collecting the data from the physical model, the flow characteristics through the PKW are shown in Figures 10 and 11:
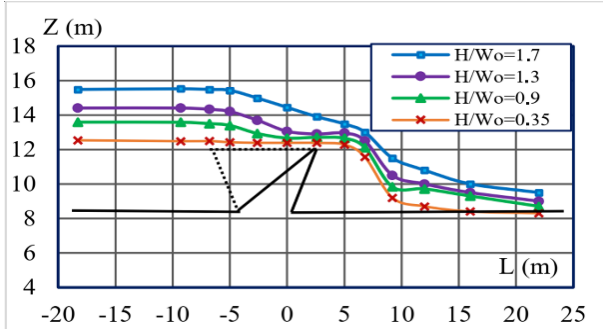
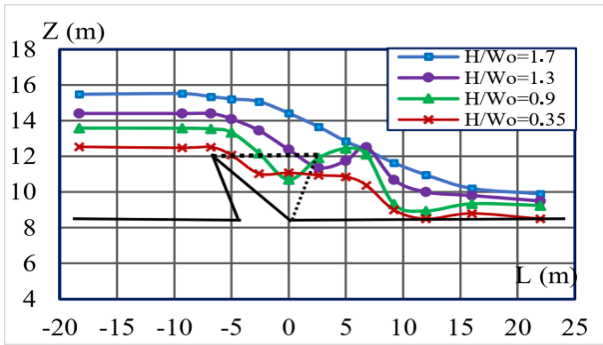**Fig.10:** The water surface profile for inlet keys of the PKW.

Fig.11: The water surface profile for outlet keys of the PKW.

The experimental data of the PKW's discharge are represented by the graph between the coefficient discharge (Cd) and the ratio of the total upstream head and the outlet keys' width (H_o/W_o), as shown in Figure 9:
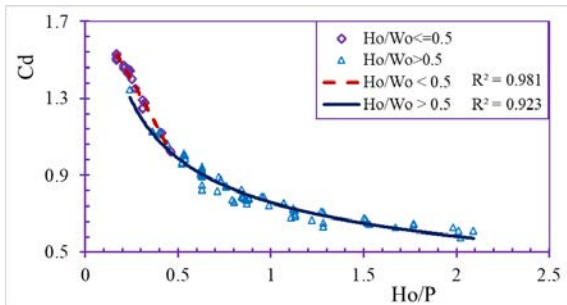

Fig.12: The experimental variation of C_d and H_o/P depends on H_o/W_o

Figure 12 depicts experimental results (91 observed values) that show the following:

+ When $H_0/W_o \leq .5$, the trend of the relationship between C_d and H_0/W_u has a steep slope and the discharge coefficient (Cd) decreases rapidly when the total upstream head increases. As can be seen, C_d decreased from 2.0 to nearly 1.0 when H_o/W_o changed from 0.1 to 0.5, and correlation coefficients is strong ($R^2 = 0.98$).

+ When $H_0/W_o > 0.5$, the trend of the relationship between (C_d) and H_o/P does not change much. It was found that C_d decreased slowly as the upstream head increased and Cd decreased from 1.4 to about 0.6 when H_o/P increased from 0.3 to 2.5 and correlation coefficients is 0.92.
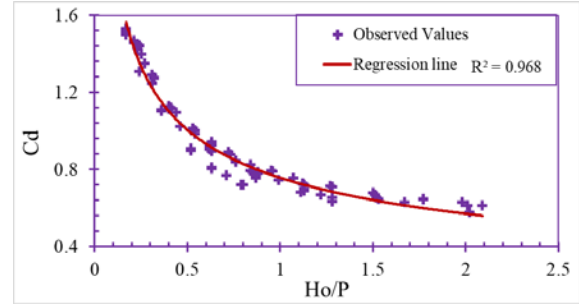

Fig.13: The experimental relationship between C_d and H_o/P

+ As observed in Figure13, when using all data sets (not grouped by Ho/Wo), there is also a strong correlation[38] ($R^2 \approx 0.97$), R^2 values are average between the 2 groups according to Ho/Wo values. Studying the coefficient C_d with the Ho/Wo non-grouping method has had many studies and achieved high computational efficiency[27][37]. So this research will focus on data groups divided according to Ho/Wo values.

3. Experimental study on the discharge coefficient

3.1 Establishing an empirical equation

The PKW is a free surface weir and the discharge (*Q)* over the PKW depends on the total upstream head (*H_o),* given as follows:

$$Q = C_d . W \sqrt{2g} H_o^{3/2} \qquad (1)$$

where C_{do} is a discharge coefficient that is unaffected by the downstream channel bottom and is only affected by crest shape and length.

A discharge coefficient C_d is a function of variables like H_0, P, Wu, and Lu, with the flowing state being full or not full over outlet keys. The study analyzes for one unit of the PKW, so the L/W ratio is a constant, so this study does not consider the L/W ratio. Besides, the influence of the downstream is not considered, because the study only considers the free flow through the spillway.

Using Buckingham's Pi theory[11][33][39] for equation (1), we have the following:

$$C_d = f (H_0/P,\ H_0/W_u,\ H_0/L_u) \qquad (2)$$

A nonlinear function was used to analyze the correlation between C_d and variables. The discharge coefficient is calculated using this method as follows:

+ If the state of flow is not full over the outlet key ($H_0/W_o \leq 0.5$),

$$C_d = f\left(\frac{H_o}{P};\frac{H_o}{L_u}\right) \qquad (3)$$

+ If the state of flow is full over outlet key ($H_0/W_o > 0.5$),

$$C_d = f\left(\frac{H_o}{P}; \frac{H_o}{W_u}\right) \qquad (4)$$

The empirical discharge coefficient (C_d) will be determined based on the observed data from the physical models. Accordingly, this equation format is mentioned in numerous studies, including those by Government standard of Vietnam[19] (TCVN 12262-2018), Rezaei, V.[20], Bhukya, R.K. et al. [32] and Xinlei Guo et al.[37].

To determine empirical coefficients, the author uses experimental datasets on physical models and tests through datasets of other authors.

Table 2. Experimental data for studying and testing.

Factors	Authors			
	Present study	O. Machiels et al.[16]	A. Noui et al.[19]	N. T. Hai et al.[29]
q (m³/s/m)	0.03÷0.32	0.04÷0.41	0.03÷0.17	0.03÷0.31
H_0/P	0.17÷2.09	0.06÷2.29	0.15÷0.95	0.13÷2.15
H_0/W_o	0.31÷2.08	0.26÷2.45	0.26÷2.05	0.23÷3.16
H_o/L_u	0.03 ÷ 0.18	0.02÷0.20	0.04÷0.14	0.03÷0.22

Using research data, combined with equations (3), (4) to determine the empirical coefficients, the experimental equations for determining the flow of PKW are shown as follows:

Table 3. Empirical coefficient equations for type A-PKW if $H_0/W_o \le 0.5$.

Symbol	Equation	R^2
C_{d1}	$C_d = 1.856 - 1.729\dfrac{H_0}{P} - 0.92\dfrac{H_0}{L_u}$	0.98
C_{d2}	$C_d = 0.565\dfrac{P^{0.345} \cdot L_u^{0.12}}{H_o^{0.465}}$	0.95

Table 4. Empirical coefficient equations for type A-PKW if $H_0/W_o > 0.5$.

Symbol	Equation	R^2
C_{d3}	$C_d = 0.878\left(\dfrac{H_0}{P}\right)^{-0.275} - 0.198\left(\dfrac{H_0}{W_u}\right)$	0.96
C_{d4}	$C_d = 0.694\dfrac{P^{0.294} \cdot W_u^{0.148}}{H_o^{0.442}}$	0.98

Analysis of the statistical indicators (MAE, RMSE, R^2, MAPE and ε) in evaluating between the observed values and the calculated values, has shown that the statistical indicators are very good, the indicators are shown in Table 5.

Table 5. Statistical indicators obtained from all available measurements.

Eq.	Statistical indicators			
	MAE	RMSE	R^2	MAPE (%)
C_{d1}	0.014	0.019	0.982	1.01
C_{d2}	0.026	0.031	0.951	1.97
C_{d3}	0.023	0.030	0.962	8.40
C_{d4}	0.018	0.023	0.978	2.27

The empirical coefficients found have a good coefficient of inspection and evaluation, as correlation coefficients $R^2 = 0.951 \div 0.982$, other statistical indicators are close to the ideal point (zero). This shows that the experimental equation is very suitable for hydraulic processes through the PKW.

Thus, a closer analysis of the statistical characteristics between the calculated values and measured values (Table 5) shows that in the case of ($H_o/W_o \le 0.5$), the equation (C_{d1}) is more efficient than the equation (C_{d2}). For the case $H_o/W_o > 0.5$, the equation (C_{d3}) has a statistical indicator that is not as good as the equation (C_{d4}), and the percentage error of the equation (C_{d3}) is quite large (The maximum percentage error of the equations C_{d3} and C_{d4} is 12.7% and 8.03%, respectively). Therefore, the equations (C_{d1} and C_{d4}) are the proposed equations for research.

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3.2 Evaluation of the Appropriateness of the Experimental Equation

The evaluation of new equations (C_{d1}) and (C_{d4}) is performed on more than 250 experimental data items from four authors (present study, N.T. Hai[35], A. Noui et al.,[26] and O. Machiels et al.[22]). Comparing the calculation results by an empirical equation with the experimental data of the studies in Table 1 shows that the relationship between the discharge coefficient and the height-to-unit width ratio ($C_d \sim H_o/P$) of the equation fits the distribution trend. In the experimental value zone of the studies, the C_d coefficient is large with small H_o/P and C_d decreases as H_o/P increases.
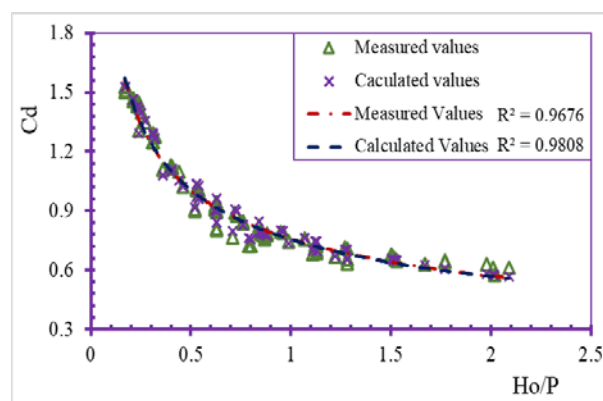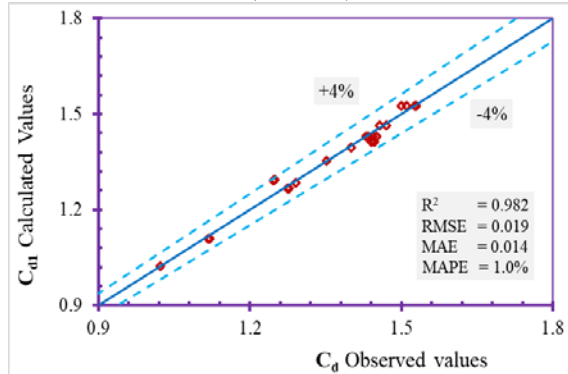

Fig.14: Comparison of results calculated by the empirical equation and the experimental data.

Considering the Figure 14, it is shown that the relationship between C_d and Ho/P ratio between experimental and calculated values according to equations (C_{d1}) and (C_{d4}) is consistent with the research results and the law of variable values, this is shown by a strong correlation coefficient[38] ($R^2 > 0.9$).


Fig.15: Comparison between the observed values and the the calculated values (Ho/Wo ≤ 0.5).

The Figure 15 also indicates that the results obtained by the Eq.C_{d1} show ±4% difference with the corresponding observed values and the maximum error 3.6%.
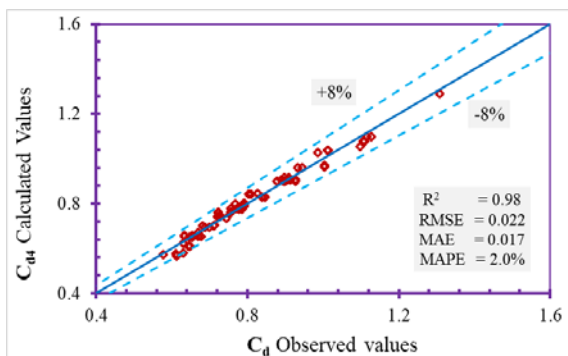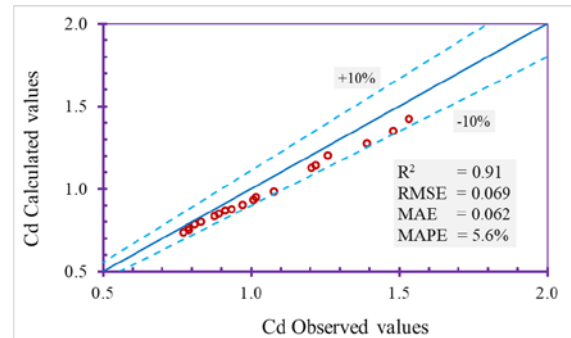

Fig.16: Comparison between the observed values and the the calculated values (Ho/Wo > 0.5).

Figure 16 shows the comparison between the computed and measured specific discharge, the computed values are close to the agreement line and indicate ±8% difference with the measured values, the maximum error is 8.03% (Eq.C_{d4} with Ho/Wo > 0.5).
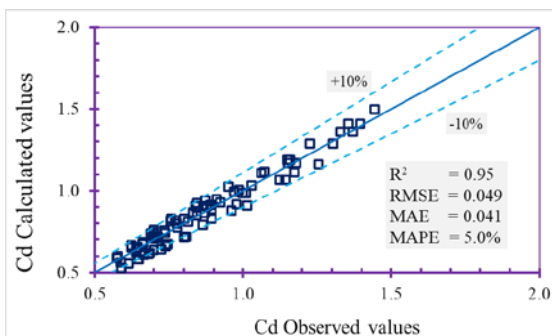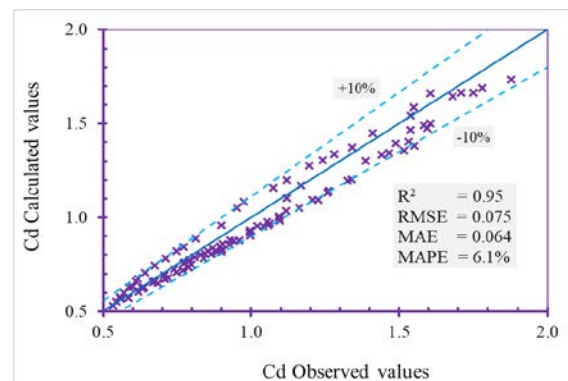

Fig.17: Relationship between calculated and measured discharge coefficient by Nguyen Thanh Hai et al.[35]

The Figure 17 also indicates that the results obtained by the equation (C_{d1} and C_{d4}) show ±10% difference with the corresponding observed values, only 4 out of 78 data (5% of the databases) fall beyond the error band ±10%. The indicators in the statistical evaluation of the measured data and calculated data are close to the ideal point (zero), the correlation coefficient is also very strong ($R^2 = 0.95$).

Fig.18: Relationship between calculated and measured discharge coefficient by A. Noui & A. Ouamane[25]

Figure 18 shows the comparison between the observed values study and those calculated by equations (C_{d1}) and (C_{d4}), The results indicate that the calculated values have ±10% difference with the observed values, so the calculated values is small (the maximum percentage error is 8.6%). However, the statistical indicators are also very good (RMSE = 0.069, MAPE = 5.6%) and the correlation coefficient is very strong ($R^2 = 0.91$)[38].

Fig.19: Relationship between calculated and measured discharge coefficient by O. Machiels et al.[22]

As shown in Figure 19, the calculated data are close to the agreement line, only 8 out of 99 data (8% of databases) fall beyond the error band ±10%, other statistical indicators have shown the effectiveness of the proposed equation (C_{d1} and C_{d4}).

When compared to other authors' studies, as in Figure 13-17, the new equation produces (Eq. C_{d1} and C_{d4}) smaller errors (The maximum error is about 10.9%, accounting for 5% to 8% of the research data). The statistical indicators are very close to zero (ideal point), besides the correlation coefficients are very strong ($R^2 > 0.9$).
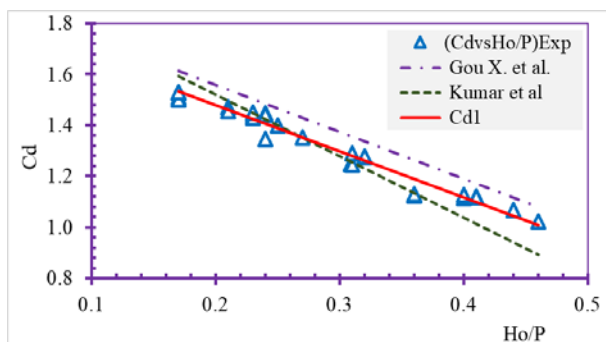
Fig.20: Calculating C_d by equations if Ho/Wo < 0.5

Considering the Figure 20, if Ho/Wo < 0.5 the equations of Kumar M. et al.[40] and Gou X. et al.[37] has the calculated values consistent with the measured values, in which the equation of Kumar M. et al.[40] tends to be equivalent to the equation (C_{d1}). Equation of Gou X. et al.[37] has large biased calculated values, but there are 27 out of 28 data with an error of less than 8.5% (only 1 out of 28 data has an error of 12.3%).
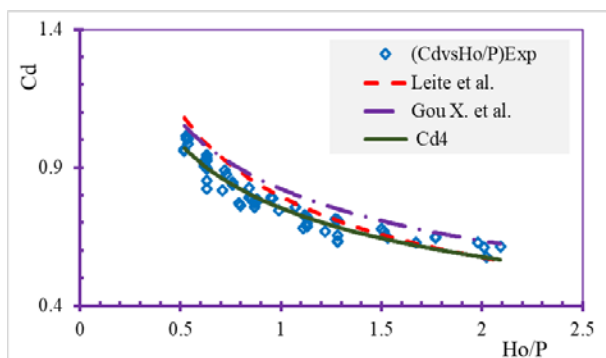

Fig.21: Calculating C_d by equations if Ho/Wo ≥ 0.5

Figure 21 shows the comparison between the equations for calculating the discharge coefficient (C_d), the equations of Leite et al.[27] and Gou X. et al.[37] tend to be similar to the equation (C_{d4}), the experience points are consistent with the calculated values, in which the maximum error from the equations of Leite et al.[27] and Gou X. et al.[37] is 16.8% and 13.5%, respectively, while the maximum error of the equation (C_{d4}) is 8.03%.

So, the results of this research, the equations (C_{d1}) and (C_{d4}), are effective and guarantee calculation results for type A-PKW; they can be used to optimize the parameters in type A-PKW design.

4. Conclusion

The current study examines the effect of the outlet keys width (Wo) on the discharge of the PKW (Q or C_d).

When compared to conventional dams, the PKW has a significant effect on flow ability. However, when selecting the PKW structure, three basic ratios must be considered: P/W_u, W_i/W_o, and $N = L/W$.

The study has developed an equation for calculating the discharge coefficient (C_d) through the PKW and comparing the calculation results from equations (C_{d1}) and (C_{d4}) with other alternatives to show that the error of the new equation is 3.6% (Ho/Wo ≤ 0.5) and 8.03% (Ho/Wo > 0.5), respectively. Therefore, the equation setting suitable for each H_0/W_o status was proposed.

Testing the proposed equations with the data of other authors and analyzing experimental data according to the existing equations of other authors, all show that the proposed equations have good computational efficiency and have high applicability in practice.

The equations built in this article allow for calculating, analyzing, and selecting parameters for the PKW quickly and appropriately. This helps in the design of new projects, renovating the old dams to improve Drain off the water, with stable flow through the PKW, it helps to ensure the ability to meet the water demand for agriculture and urban, etc. It is especially important to ensure the stable water sources for the ecosystem in the downstream of the river.

Nomenclature

H	Upstream head (m)
H_0	Total upstream head (m)
Q	Discharge (m³/s)
q	Specific discharge (m³/s.m)
L	Total crest length (m)
P	Height of the weir (m)
W	Width of PKW (m)
W_u	PKW-unit width (m)
	$W_u = W_i + W_o$
W_i	Inlet keys' widths (m)
W_o	Outlet keys' widths (m)
N	Length coefficient
	N = L/W
C_d	Discharge coefficient
B	Key length (m)
Bo	Upstream overhangs length (m)
Bi	Downstream overhangs length (m)
Ts	Wall thickness (m)
L_u	Developed crest length (m)
R^2	R Square
MAE	Mean absolute error
RMSE	Root Mean Square Error
MAPE	Mean Absolute Percentage Error (%)
ε	Percentage error (%)

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