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THE EFFECT OF LOADING CONDITIONS ON THE NATURAL FREQUENCY OF HULL VIBRATION

By Toyoji KUMAI

Abstract An analysis into the factor β in Todd's formula for estimation of the two-node natural frequencies of the flexural hull vibrations is carried out by the calculations of the characteristic values of the flexural vibration of a beam with such distributions of mass as both cases of light and full load conditions of modern tankers and cargo ships in the vertical and horizontal transverse vibrations. The ratios of the factor β in full and light load conditions which have been obtained from the measurements in actual ships are compared with those obtained by theoretical ones.

The ratios of the factor β in full load condition to that in the light condition are obtained with various displacement ratios for estimation of the natural frequencies of the two-node flexural vibrations of tankers and cargo ships in full load condition by the use of Todd's formula in early stage of design. The vibration profiles in various cases mentioned above are also presented in the last paragraph of the present paper.

1. Introduction. In early stage of design of a ship, the estimation of the two-node natural frequency of the hull vibration in various loading conditions is one of the important works. The natural frequency of the hull vibration depends not only on the displacement including added virtual mass of water but also on the distribution of loading, for instance, if the sister ships have the same draught but the conditions of the loading are different between two ships, the natural frequencies differ from each other. It is clearly explained that the cause of the above difference is due to the characteristic value which corresponds to the factor β in Todd's formula. The factor β of the formula for estimation of the natural frequency of the hull vibration which developed by F. H. Todd[1] has good constancy for the type ships with similar loading, however, β varies between light and full load conditions even in a same ship.

The difference of the factor β in both conditions of light and full load in a same ship has already been pointed out by J. Calderwood,[2] however, no analysis has been presented. The approximate method for correction of the criticals considering the excess of loading from normal condition as concentrated loads has been carried out by Prof. M. Yoshiki[3] and the

1) F. H. Todd; "Some Measurements of Ship Vibration" Trans. N.E.C. Vol. 48. 1931-32.
2) J. Calderwood; discussion for Todd's paper 1). p. D 51.
3) M. Yoshiki; "Simple Estimation of Natural Frequency of Ship Vibration" Jour Soc. N.A. Japan Vol. 73. 1951.

application of this method to the cargo ships has been shown by T. Tomita.[4] Also, the correction for the distribution of load by the other method was presented by C. W. Prohaska.[5]

The present paper deals with an analysis into the factor β in Todd's formula considering as the characteristic values of the natural frequencies of the flexural vibrations of the beam of uniform rigidity with approximately similar distributions of loading as with the light and full load conditions of modern large tankers and cargo ships in both cases of two node vertical and horizontal vibrations.

As the correction factor of β in full load conditions, the ratios of β in full load condition to that in the light condition in the above cases are presented versus various displacement ratios. Also the vibration profiles are obtained in the above various cases.

2. Correction Factors of Coefficient β in Todd's Formula for Full Load Conditions.

The natural frequency of the beam with various distributions of load is generally presented as follows:

$$N=\frac{\lambda^2}{2\pi}\sqrt{\frac{gEI_0}{w_0 L^4}} \tag{1}$$

where,
- N natural frequency
- g acceleration due to gravity
- EI_0 flexural rigidity at the datum section of beam
- w_0 weight per unit length at the datum section of beam, such as midship section of vessel
- L length of beam
- λ characteristic value which depends on the variable section and the distribution of load along the length of beam

Now, we denote W the total load of the beam and also put

$$c=\frac{W}{w_0 L} \tag{2}$$

thence, (1) becomes

$$N=\frac{\sqrt{c}\,\lambda^2}{2\pi}\sqrt{\frac{gEI_0}{WL^3}} \tag{3}$$

In the above formula, $\sqrt{c}\,\lambda^2$, W, and I_0 are correspond to β, displacement Δ_1 and BD^3 respectively in Todd's formula which is shown as follows.

Todd's formula for estimation of natural frequency of the ship's hull is presented as the well known form;

$$N_V=\beta_V\sqrt{\frac{BD^3}{\Delta_{1V}L^3}} \tag{4}$$

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4) T. Tomita; "Estimation of the Natural Frequency of the Hull in Early Stage of Design" Jour. Soc. N.A. Japan. Vol. 86. 1953.

5) C. W. Prohaska; "Vibrations Verticales du Navire" Assoc. Tech. Martime et Aéro. Juin. 1947.

where, N_V two-node natural frequency of the vertical vibration
β_V Todd's factor for vertical vibration
L length between perpendiculars
B moulded breadth
D moulded depth
Δ_{1V} displacement of ship including added virtual mass of water, $\Delta_{1V}=\Delta(1.2+B/3d)$, where Δ and d are the displacement and the draught of a ship respectively

Todd's formula is also applicable to horizontal hull vibration by some modification. The modified formula has been presented by T. W. F. Brown.[6] Also, the reasonable estimation of the factor of added virtual mass of water in the horizontal vibration has been deduced by R. L. Townsin.[7] These formulae are presented respectively as follows;

$$N_H=\beta_H\sqrt{\frac{DB^3}{\Delta_{1H}L^3}} \tag{5}$$

$$\Delta_{1H}=\Delta\left(1+1.1\frac{d}{B}\right)$$

where, N_H two-node natural frequency of horizontal vibration
β_H Todd's factor for horizontal vibration

The factor β in the above two formulae (4) and (5) just corresponds to the characteristic value $\sqrt{c}\,\lambda^2$ in (3).
Thence, the β depends on the distributions of the load along the length of a ship. The product of $\sqrt{c}$ and λ^2 which correspond to the factor β is theoretically obtained provided that the mathematical expression of the distribution of the load along the length of a ship and the solutions of the fundamental equation of the vibration with such distribution are obtained.

In the present paper, we assume that the distribution of the rigidity of a ship is uniform over the length of a hull and the load distribution is symmetrical about midship section. The effects of the shear deflection and the rotatory energy on the natural frequency are not accounted in the present paper because these effects will be negligibly small provided that the ratio of the factor β in full and light load conditions is discussed.

The simplified form of the loading distributions in tankers and cargo ships in both typical conditions of light and full load are assumed to be of the form as shown in Fig. 1. The values of w_1 and w_2 in the figure show the load which includes added virtual mass of water per unit length. The ratios of w_1 to w_2 and the length ratio ξ_1 are determined from the displacement ratios of actual ships and from the distributions of the cargo and the virtual mass of water which are presented in design plan. Some examples of the assumptions of the values of ξ_1, w_1/w_2 and of the displace-

[6] T. W. F. Brown; "Vibration Problems from the Marine Engineering Point of View" Trans. N.E.C. Vol. 55. 1938-39.
[7] R. L. Townsin; discussion for A. J. Johnson's paper. Trans. N.E.C. Vol. 67. 1950-51.

ment ratios of large tankers and cargo ships in vertical and horizontal vibrations are shown in Table I.

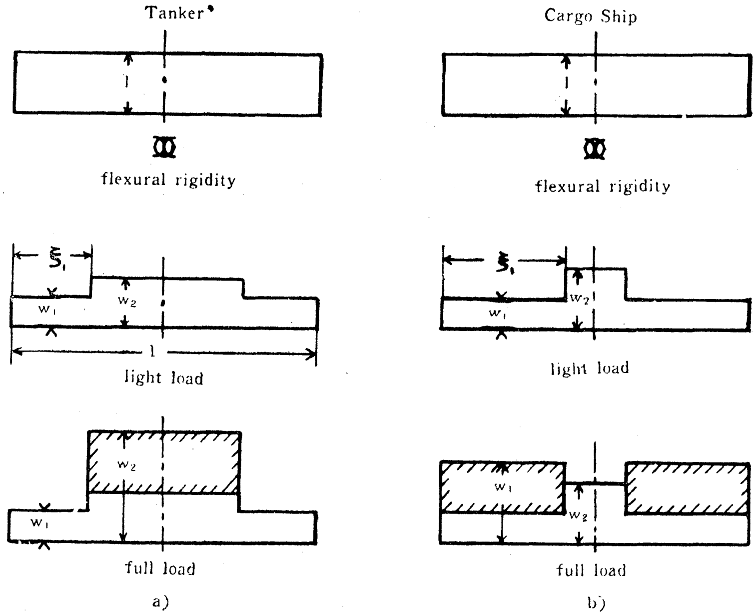


Fig. 1. Typical loading conditions in tankers and cargo ships.

Table I. Assumed values of ξ_1, w_1/w_2 and results of calculated values of β_f/β_l, in light and full load conditions of tankers and cargo ships for flexural vibrations.

Type of vessels	Vib.	ξ_1	Loading conditions	w_1/w_2	Δ_{1l}/Δ_{1f}	Virtual mass factors	Δ_l/Δ_f	$\sqrt{c}\,\lambda^2$	β_f/β_l
Tanker	Vert.	0.25	light	0.600	0.770	2.30	0.67	23.40	1.072
			full	0.407		2.00		25.08	
	Horiz.	0.25	light	0.980	0.601	1.30	0.67	22.37	1.110
			full	0.429		1.45		24.80	
Cargo Ship	Vert.	0.30	light	0.492	0.625	2.50	0.50	23.40	0.956
			full	1.110		2.00		22.38	
	Horiz.	0.40	light	0.578	0.435	1.30	0.50	22.50	1.045
			full	1.650		1.50		23.51	

$$\frac{\Delta_{1l}}{\Delta_{1f}}=\frac{w_{1l}\,\xi_1+w_{2l}\,(0.5-\xi_1)}{w_{1f}\,\xi_1+w_{2f}\,(0.5-\xi_1)}, \quad c=1-2\xi_1\left(1-\frac{w_1}{w_2}\right)$$

The value of c is obtained from the above assumption of the w_1, w_2 and ξ_1 as follows;

$$c = 1 - 2\xi_1(1 - w_1/w_2) \tag{6}$$

With regard to the characteristic values λ for given distribution of load, this is computed from the fundamental equation of the vibration of the beam with variable load. The determinant for computation of the characteristic value is shown in Appendix I. In the results, the values $\sqrt{c}\,\lambda^2$ are presented versus various w_1/w_2 with the parameter ξ_1 as shown in Fig. 2. As will be seen in the figure, the characteristic values $\sqrt{c}\,\lambda^2$ which correspond to the factor β in Todd's formula vary considerably with w_1/w_2 and ξ_1.

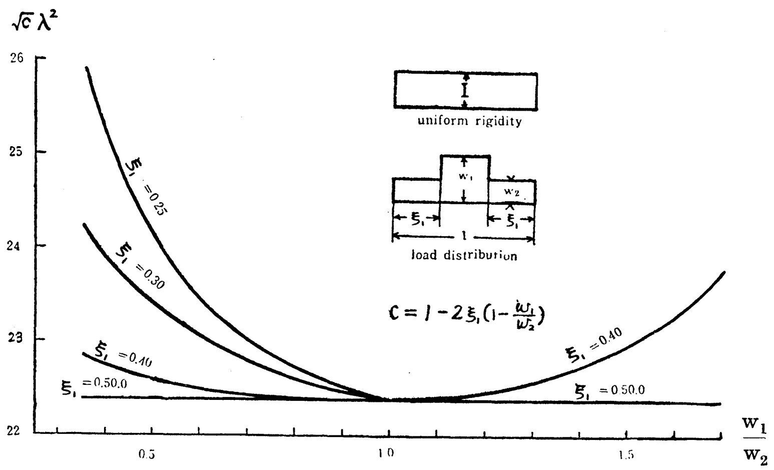


Fig. 2. Theoretical results of characteristic value $\sqrt{c}\,\lambda^2$ of flexural vibration of beam with symmetrical distribution of loads w_1 and w_2.

The $\sqrt{c}\,\lambda^2$ for given values of ξ_1 and w_1/w_2 which correspond to the two conditions of loading of tankers and cargo ships in vertical and horizontal vibrations are computed with respect to the corresponding displacement ratios as will be seen in Table I. The added virtual mass factors in the vertical and the horizontal vibrations are assumed respectively as the mean values in actual ships as will also be seen in Table I.

In the last stage, the ratios of β in full load condition to that in the light condition are obtained from those of $\sqrt{c}\,\lambda^2$. The relations between the ratios of β_f/β_l and the corresponding displacement ratios are plotted as shown in Fig. 3 a), b), c) and d).

The ratios of β which are obtained from the results of the measurements of the vibrations in actual ships by several foreign investigators and some of leading shipbuilders in Japan are shown in the above figures. (see also Table II, Appendix II.)

As will be seen in the figure, the discrepancies between measured and the theoretical results are considerable, however, the tendencies of the

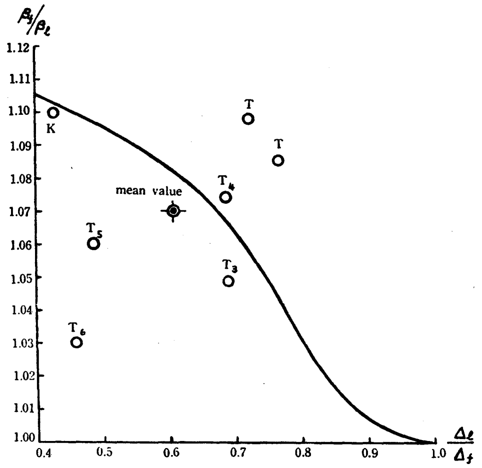


Fig. 3. a) β_f/β_l for vertical vibration of tankers.

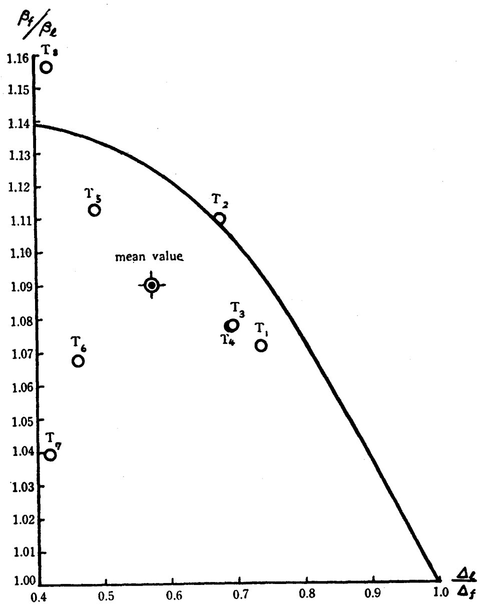


Fig. 3. b) β_f/β_l for horizontal vibration of tankers.

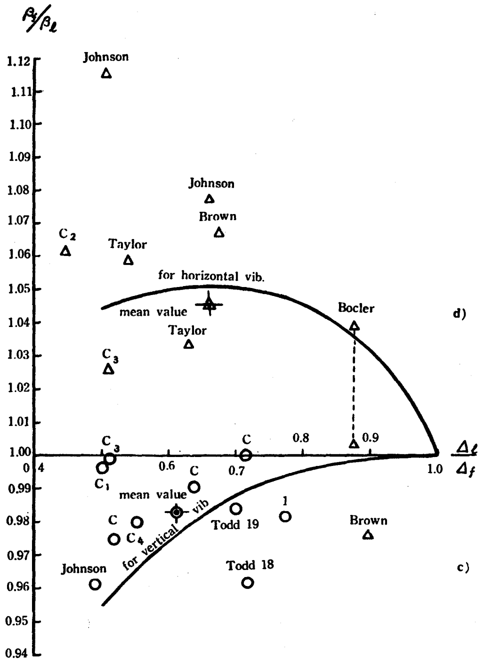


Fig. 3. c), d) β_f/β_l for flexural vibration of cargo ships.

increase and decrease of the ratio of β to the displacement ratio show fairly good agreement in both results.

It is interesting that the ratio of β in the full load to light load condition of tankers increase more than unity, however, in the same conditions of cargo ships, the ratio takes less than unity in the case of vertical vibration.

Curves showing in Fig. 3 a), b), c) and d) will be available for estimation of the natural frequencies in various cases in the early stage of hull design.

3. Comparisons of the Vibration Profiles. The vibration profiles in the above cases were calculated from respective characterestic values as shown in Fig. 4 a), b), c) and d). The amplitudes of the vibration at the ends of the hull of the tankers are larger than that of cargo ships in full load conditions as will be seen in the figure.

The same tendencies in the vibration profiles in light and full load conditions are shown as the results of two examples of the vibration measurements of a 32,000 ton d.w. tanker and a 10,000 ton d.w. cargo ship

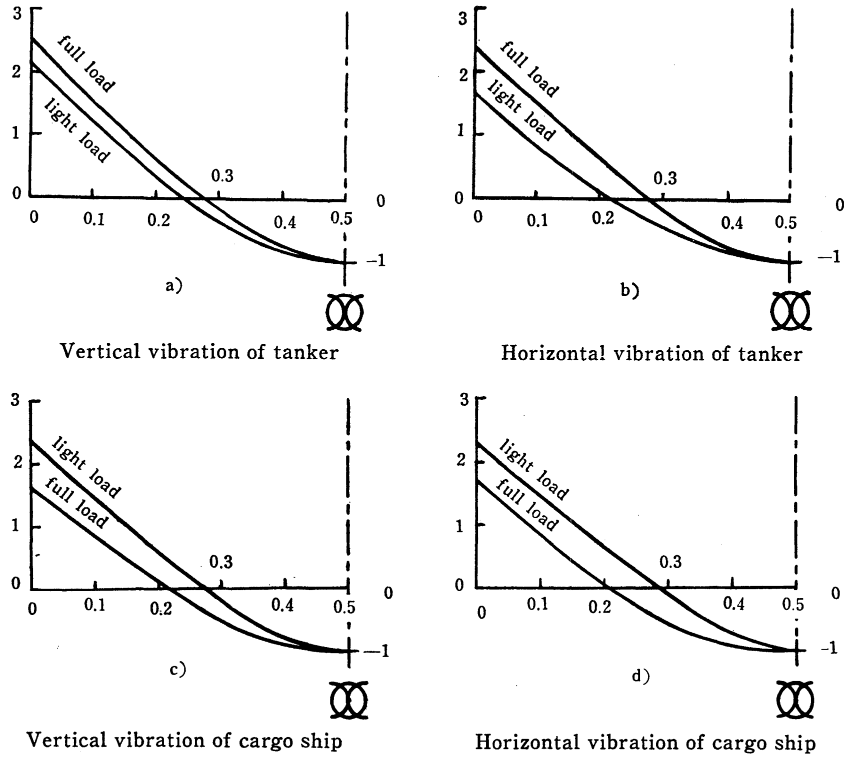
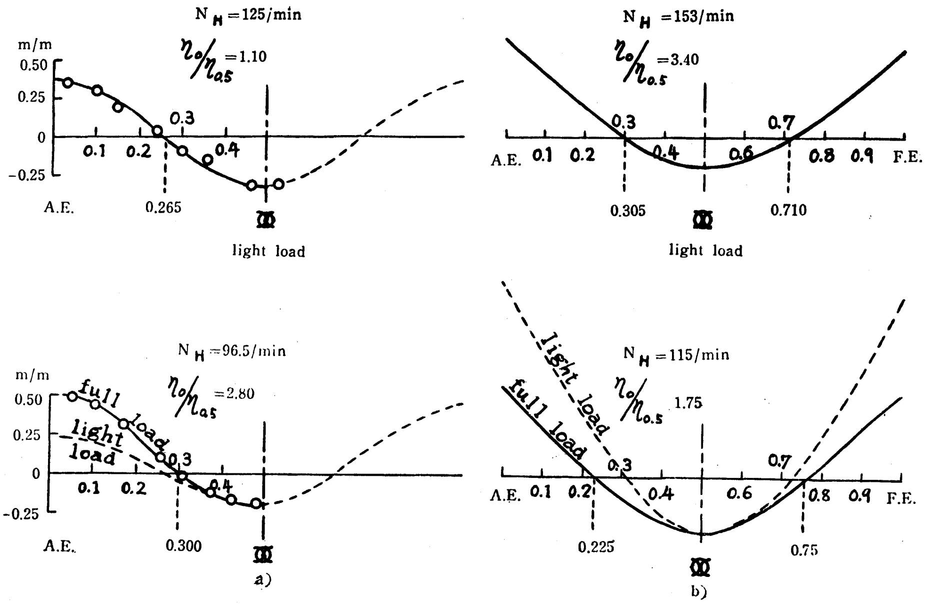


Fig. 4. Calculated profiles of two-node flexural vibrations of tanker and cargo ship in light and full load conditions.



32,000 ton d.w. tanker (measured by T. Okabe, K. Hirata and T. Kumai)

10,000 ton d.w. cargo ship (measured by A. J. Johnson)

Fig. 5. Examples of measured profiles of two-node horizontal vibration in light and full load conditions.

which have recently been taken place as shown in Fig. 5 a) and b respectively.[10), 11)]

4. Conclusions. The effect of loading conditions on the factor β in Todd's formula was investigated by the calculations of the characteristic values of the vibration of beam with various distributions of mass including added virtual mass of water with such distributions of load as tankers and cargo ships in the vertical and the horizontal vibrations.

The computed results were compared with those obtained by the vibration measurements. As the correction factor obtaining β in the full load condition from given factor in light condition, the ratios of β in the full load to that in light conditions are presented for various displacement ratios in the vertical and horizontal vibrations of tankers and cargo ships. The results will be available for estimation of natural frequencies of flexural hull vibrations in early stage of hull design.

In conclusion the author expresses his thanks to Professor Y. Watanabe, Director of the Institute, for his constant interest in the work, and also thanks to the shipbuilders for providing the data referred in this paper. This paper was read at the meetings of the Western Section of Ship Structure Committee of Japan Socity of Naval Architecture.

Appendix I.

Calculations of the Characteristic Values of Flexrual Vibration of Beam with Variable Mass.

We assume that the distribution of load over the length is symmetrical about the centre of the beam. The load is assumed to be uniformly distributed along the length of the beam with a discontinuity of loading between a half length of the beam. For the sake of simplicity, the flexural rigidity of the beam is assumed to be of constant along the length of the beam.

Take the origin of the length coordinate at the end of the beam, the fundamental equations of the beam are presented as the following two parts;

$$\left.\begin{aligned} EI\frac{\partial^4 y}{\partial x^4}+\frac{w_1}{g}\frac{\partial^2 y}{\partial t^2}&=0, \quad 0\leqq x\leqq x_1 \\ EI\frac{\partial^4 y}{\partial x^4}+\frac{w_2}{g}\frac{\partial^2 y}{\partial t^2}&=0, \quad x_1\leqq x\leqq x_c \end{aligned}\right\} \tag{1}$$

where, x length co-ordinate
y amplitude of vibration of beam
t time co-ordinate
w_1 uniformly distributed load between 0 to x_1
w_2 uniformly distributed load between x_1 to x_c $(=L/2)$

Now we put $y=u\cos pt$, where p is the natural frequency of the vibration of the beam, and also put $u/L=\eta$, $x/L=\xi$, where L is the total length of the beam, and substitute them to (1), we have

$$\left.\begin{aligned} \frac{d^4\eta}{d\xi^4}-(\alpha\lambda)^4\eta&=0, \quad 0\leqq\xi\leqq\xi_1 \\ \frac{d^4\eta}{d\xi^4}-\lambda^4\eta&=0, \quad \xi_1\leqq\xi\leqq 0.5 \end{aligned}\right\} \tag{2}$$

where
$$\lambda^4=\frac{p^2 w_2 L^4}{gEI}, \quad \alpha=\sqrt[4]{\frac{w_1}{w_2}}.$$

The normal mode is obtained from (2) provided that the two end conditions and conditions of the center of the free-free bar in the even number of nodes of the vibration are satisfied.

$$\left.\begin{aligned} \eta&=A(\cos\alpha\lambda\xi+\cosh\alpha\lambda\xi)+B(\sin\alpha\lambda\xi+\sinh\alpha\lambda\xi), \quad 0\leqq\xi\leqq\xi_1 \\ \eta&=F\cos(0.5-\xi)\lambda+K\cosh(0.5-\xi)\lambda, \quad \xi_1\leqq\xi\leqq 0.5 \end{aligned}\right\} \tag{3}$$

where, A, B, F and K are the integration constants.

These constants and the characteristic value λ are determined by satisfying four conditions at the section ξ_l of the beam and the results are shown as follows;

$$\begin{vmatrix} \cos\alpha\lambda\xi_l+\cosh\alpha\lambda\xi_l & \sin\alpha\lambda\xi_l+\sinh\alpha\lambda\xi_l & -\cos(0.5-\xi_l)\lambda & -\cosh(0.5-\xi_l)\lambda \\ \alpha\,(-\sin\alpha\lambda\xi_l+\sinh\alpha\lambda\xi_l) & \alpha\,(\cos\alpha\lambda\xi_l+\cosh\alpha\lambda\xi_l) & -\sin(0.5-\xi_l)\lambda & \sinh(0.5-\xi_l)\lambda \\ \alpha^2(-\cos\alpha\lambda\xi_l+\cosh\alpha\lambda\xi_l) & \alpha^2(-\sin\alpha\lambda\xi_l+\sinh\alpha\lambda\xi_l) & \cos(0.5-\xi_l)\lambda & -\cosh(0.5-\xi_l)\lambda \\ \alpha^3(\sin\alpha\lambda\xi_l+\sinh\alpha\lambda\xi_l) & \alpha^3(-\cos\alpha\lambda\xi_l+\cosh\alpha\lambda\xi_l) & \sin(0.5-\xi_l)\lambda & \sinh(0.5-\xi_l)\lambda \end{vmatrix}=0. \quad (4)$$

The computed results of the above determinant are shown in Fig. 2, and the vibration profiles are calculated by (3) as shown in Fig. 4 for given values of λ and the respective integration constants which are obtained from (4).

Appendix II.

Table II. Particulars of various ships and values of β and displacement ratios computed from the results of measurements of ship vibrations.

a) Vertical Vibration of Tankers

Ships (ref. no.)	$L\times B\times D$ m (ft)	d m	Δ ton	N_{2V}/min.	β	β_f/β_l	Δ_l/Δ_f
K (3)	92.96×15.24 × 7.24 (305′×50′×23.75′)	1.71 3.71	1,877 4,377	115 105.5	119,800 131,800	1.100	0.428
T (4)	106.68×14.25 × 8.61 (350′×46.75′×28.25′)	5.59 7.00	6,400 8,200	112 112	148,200 160,700	1.084	0.780
T_1	192.00×26.80 ×13.72	7.71 9.50	30,910 42,091	60 59	153,500 179,400	1.096	0.734
T_3	160.63×21.895×12.04	6.90 9.60	18,285 26,424	71 66.5	150,200 157,600	1.049	0.692
T_4	160.63×21.895×12.04	6.92 9.61	18,233 26,496	70 67	147,900 158,900	1.074	0.688
T_5	167.63×21.895×12.20	4.95 9.38	13,172 27,003	72 62	147,000 15,9600	1.060	0.487
T_6	167.00×22.00 ×12.00	4.59 9.24	12,530 27,200	76 63	153,600 158,300	1.030	0.460
					mean	1.070	0.610

b) Horizontal Vibration of Tankers

Ships (ref. no.)	$L \times B \times D$ m (ft)	d m	Δ ton	N_{2H}/min.	β	β_f/β_l	Δ_l/Δ_f
T_1	192.00×26.80 ×13.72	7.71 9.50	30,910 42,091	108 96.5	112,700 120,800	1.071	0.734
T_2	192.00×26.80 ×13.70	7.256 10.378	29 200 43,260	111 96.5	111,900 124,100	1.109	0.674
T_3	160.63×21.895×12.04	6.90 9.60	18,285 26,424	116.5 99.5	104,600 112,700	1.077	0.692
T_4	160.63×21.895×12.04	6.92 9.61	18,233 26,496	115 98	93,300 100,500	1.077	0.688
T_5	167.63×21.895×12.20	4.95 9.38	13,172 27,003	126 91	72,300 81,200	1.122	0.487
T_6	167.00×22.00 ×12.00	4.59 9.24	12,530 27,200	143 95	106,200 113,400	1.087	0.460
T_7	192.02×26.52 ×13.87	4.64 10.45	18,000 43,120	147 90	112,600 117,000	1.039	0.417
T_8	192.52×26.52 ×13 87	4.74 10.45	18,000 43,143	129 88	99,400 114,900	1.156	0.417
					mean	1.090	0.571

c) Vertical Vibration of Cargo Ships

Ships (ref. no.)	$L \times B \times D$ m (ft)	d m	Δ ton	N_{2V}/min.	β	β_f/β_l	Δ_l/Δ_f
Todd. 18 (7)	151.13×20.73×14.02 (495′-5′ 68 46′)	5.15 6.83	10,390 14,500	102 89	128,700 123,800	0.961	0.716
Todd. 19 (7)	114.30×16.61×11.58 (375′ 54.5′ 38′)	4.51 6.10	5,376 7,683	133 117.5	115,500 113.700	0.984	0.699
C (4)	129.54×16.92×12.19 (425′ 55.5′ 40′)	4.57 6.17 7.93	6,550 9,050 12,700	100 89 79	106,200 103,600 103,600	0.975 1.000	0.515 0.712
I	143.26×20.42×14.78 (470′ 67′ 48.5′6)	6.65 8.31	13,950 18,050	94 85.5	110,400 108,300	0.981	0.772
Johnson (10)	126.80×17.34×11.37 (416′ 56.9′ 37.3′)	4.33 8.45	6,745 13,790	100 78	116,900 112,400	0.961	0.489
C_1	145.00×19.4 ×12.50	4.22 5.24 7.68	7.245 9.23 14,500	108 102 88	136,200 137,200 135,800	0.996 0.990	0.499 0.636
C_3	134.00×18.50×11.40	4.12 7.45	6,710 13,150	113 93	142,400 142,300	0.999	0.510
C_4	140.00×19.00×12.50	4.35 7.90	7,470 13,510	105.3 88.4	127,500 125,000	0.980	0.552
					mean	0.983	0.610

d) Horizontal Vibration of Cargo Ships

Ships (ref. no.)	$L \times B \times D$ m (ft)	d m	Δ ton	N_{2H}/min.	β	β_f/β_l	Δ_l/Δ_f
Johnson (10)	126.80×17.34×11.37 (416′ 56.9′ 37.3′)	14.37 4.78 7.93	6,710 7,430 13,220	157 153 115	85,220 88,250 95,030	1.115 1.076	0.505 0.662
Brown (4)	143.26×20.42×14.78 (470′ 67′ 48.5′)	6.65 8.47 9.33	13,950 18,500 20,655	126 115.5 105	83,820 91,620 89,390	1.066 0.975	0.675 0.895
Taylor (4)	129 54×16.61×12.19 (425′ 54.5′ 40′)	4.72 5.41 7.93	6,850 8,000 12,700	147 137 106	86,930 89,070 92,030	1.058 1.033	0.540 0.629
Bocler (9)	151.03×20.73×15.16 (495.5′ 68′ 49.75′)	6 95 7.79	14,950 17,070	115~119 110	83,090 ~85,980 86,280	1.038 ~1.003	0.875
C_2	134.00×18.80×11.80	3.70 7.56	6,070 13,600	184 120	87,670 93,060	1.061	0.446
C_3	134 00×18.50×11.40	4.12 7.45	6.710 13,150	172 117	90,770 93,050	1.025	0.510
					mean	1.045	0.661

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