Coupled ToriSion-Horizontal Bending Vibrations of Ships

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NOTE

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By Toyoji KUMAI

1. Introduction. When a ship is excited by the horizontal or torsional vibratory generating force, the vibratory motion of the hull is generally presented as the coupled torsion-horizontal bending vibration, because the location of the centre of gravity of a ship along her length deviate from that of the shear centre or the torsion centre in the vertical plane include centre line of the hull [1]. Thence, when the critical frequencies of the horizontal vibration of the hull which are calculated with torsion-free or that of torsional one computed with bending-free are compared with the results of the measurements of corresponding natural frequencies of the hull vibration, there needs the correction of the natural frequencies for the above coupling interference. Strictly speaking, the coupled torsion-bending vibrations of the ship's hull should be considered as that of the curved bar, because of both of the profiles of the locations of the centre of gravity and of the torsion centre of the element of the length of a ship's hull along her length are of slightly curved ones. In the present paper, however, the studies on the coupled torsion-bending vibration of the ship-like beam of variable cross section with straight axes of the centre of gravity and of the torsion centre are carried out as a first approximation, and also since the coupled action of the ship's hull mentioned above mainly occur in the higher modes of the horizontal bending vibration, the shearing flexural vibration of the vibration of the hull that have been studied in the author's previous paper [2] is applied to the present calculations of the coupled vibration of the hull.

2. Some Examples of Coupled Vibration Presented in Vibration Measurements of Ships. If the resonant amplitude of the vertical vibration which measured at the side of the poop or upper deck of the hull are recorded as well as that of the horizontal one, it may be decided that this type of vibration is the torsional one. When the coupled vibration occur, the resonant amplitudes of the vibration appear as two peaks of frequences and whose frequencies are close to each other with the order of 10% of uncoupled peaks. Some of the results of the vibration measurements of these coupled vibration are shown in Fig. 1. a) and b).

The uncoupled natural frequencies of the shear flexural and of the torsional vibrations are calculated respectively as followings, [2], [3].

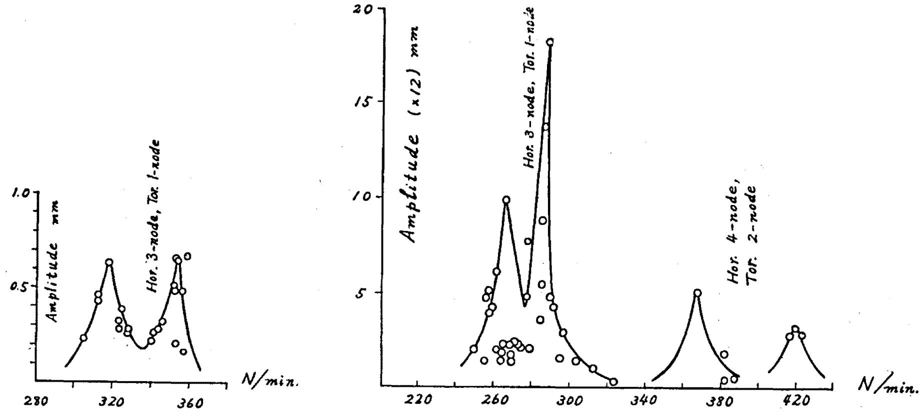
$$N_H = \frac{60}{2\pi}\sqrt{c}\,\lambda_1\sqrt{\frac{g k_0' G A_0}{\Delta L(1+\gamma_b)(1+\gamma_r)(1+\gamma_{wh})}} \tag{1}$$

$$N_T = \frac{60}{2\pi}\sqrt{c}\,\lambda_2\sqrt{\frac{g G J_{teo}}{\Delta r_p^2 L(1+\gamma_{wt})}} \tag{2}$$

[1] J. Lockwood Taylor: "Ship Vibration Periods" Trans. N.E.C.I., 1927–28. p. 153.

[2] T. Kumai: "Shearing Vibrations of Ships" European Shipbuilding Vol. 5, No. 2, 1956. Rep. Res. Inst. Appl. Mech. Kyushu Univ. Vol. V, No. 15, 1956.

[3] T. Kumai: "Estimation of Natural Frequencies of Torsional Vibration of Ships" Rep. Res. Inst. Appl. Mech. Kyushu Univ. Vol. IV, No. 13, 1955.



a) 10000 t.d.w. cargo ship b) 20000 t.d.w. tanker

Fig. 1. Two examples of measured results of peaks of coupled torsion-horizontal bending vibrations, the data are proposed by courtecies of Nagasaki Mitsubishi Works and Kobe Shin-Mitsubishi Works.

where,

N_H, N_T	natural frequencies of horizontal and torsional vibrations respectively per min.
$\sqrt{c}\lambda_1$, $\sqrt{c}\lambda_2$	characteristic values of horizontal and torsional vibrations respectively
$k'GA_0$, GJ_{teo}	sheare rigidity and torsional rigidity of hull amidship respectively
Δ, L	displacement and length of ship respectively
r_p'	radious of gyration of mass moment of inertia of hull amidship
$(1+\gamma_b)$, $(1+\gamma_r)$	correction factors for bending moment and rotational inertia of hull section
$(1+\gamma_{wh})$, $(1+\gamma_{wt})$	virtual mass and inertia factors for horizontal bending and torsional vibrations respectively

In the case of considerably coupled vibration, N_H equals to N_T, thence, the ratio of characteristic values in this case is shown as follows

$$\beta=\frac{\lambda_2}{\lambda_1}=\sqrt{\frac{k'A_0 r_p'^2(1+\gamma_{wt})}{J_{teo}(1+\gamma_b)(1+\gamma_r)(1+\gamma_{wh})}}. \tag{3}$$

The values of β depend not only on the ratio of shear flexural rigidity and torsional one, but on the number of nodes and the virtual mass and its inertia as will be seen in (3).

Also, the β varies in the range from about 0.3 to 0.5 in general cargo ships and large tankers. The theoretical uncoupled charateriatic values which are obtained from assumed ship-like beam of variable cross section versus β with the parameter of the number of nodes of the torsional and the flexural vibration of the beam are shown in Fig. 2.

When β is given by such a value as the characteristic value of uncoupled flexural criticals which coincides with torsional one, the effect of the action of interference upon the natural frequencies of uncoupled criticals is considerable. By the results of vibration measurements of S. S "Gopher Mariner" [4], as an example, the one-noded, two-noded

[4] R. T. McGoldrick and V. L. Russo: "Hull Vibration Investigation on S.S. Gopher Mariner" reprinted paper No. 8, Soc. N.A.M.E., 1955.

and three noded modes of the torsional vibrations coupled with each of three-noded, four-noded and five-noded modes of the horizontal flexural vibrations respectively, thence, two peaks are closely measured in each three criticals of the above nodal vibrations of the hull. When the peaks of the uncoupled natural frequencies cf torsional and flexural hull vibrations are considerably part from each other, the coupling action of nodal vibrations is out of question.

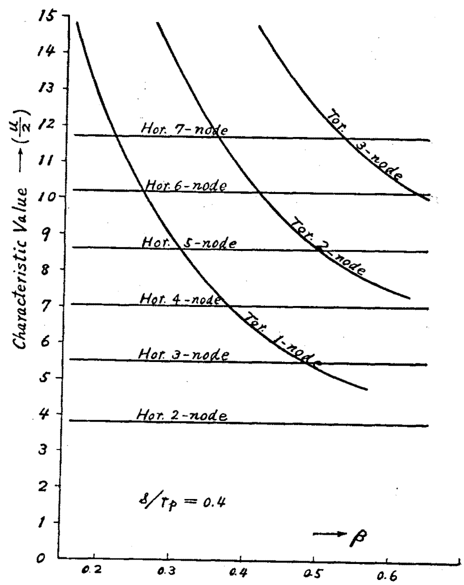
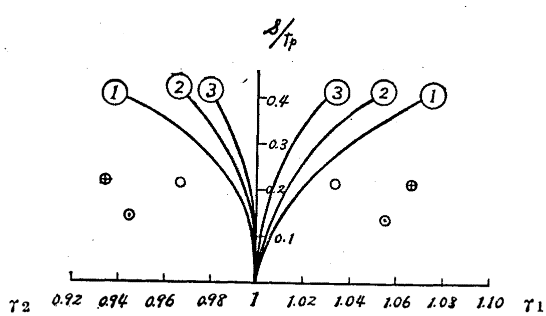


Fig. 2. Relation between charateristic values of torsional and horizontal shear bending vibrations of hull.

3. Calculations of Coupled Natural Frequencies. For the sake of simplicity, we assume that the ship form is symetrical about midship, and the distributions of the mass, the polar mass moment of inertia, the shear and the torsional rigidities are assumed to be of the shapes of linearly increasing to the midship section along her length in the present calculation. Some numerical results of the calculations which are presented in Appendix by the above assumptions of ship's form are shown in Fig. 3.

Figure shows the percentage separations of coupled natural frequencies for uncoupled one provided the uncoupled criticals coincide. As will be seen in the figure, the separation of coupled frequencies are small even the distance of both axes of the centre of grarvity and the torsion centre is considerable. Some results of the measurements of



Theoretical ① Hor. 5-node, Tor. 2-node, $\beta = 0.5$
② Hor. 7-node, Tor. 2-node, $\beta = 0.36$
③ Hor. 4-node, Tor. 1-node, $\beta = 0.375$

Measured ⊙ Hor. 3-node, Tor. 1-node, 10000 t.d.w. cargo ship
○ Hor. 3-node, Tor. 1-node, } 20000 t.d.w. tanker
⊕ Hor. 4-node, Tor. 2-node, }

Fig. 3. Comparisons of theoretical and measured results of percentage separations of coupled peaks.

coupled criticals which presented in Fig. 1 are plotted in the same figure for references. The discrepancies of the results of the calculations and the measurements will be caused by the rough assumptions for the ship's form in the calculations, the order of the percentage separation of the two peaks of coupled natural frequencies, however, may be estimated as of the order of 10 % of the mean value of the frequencies of two peaks.

APPENDIX.

Solutions of Coupled Torsion-Shear Flexural Vibration of Beam of Variable Cross Section.

We assume that the ship-like beam is symetrical amidship, and the distributions of mass, mass moment of inertia, shear flexural rigidity and of torsional rigidity of the beam are linearly increased toward amidship, also both of the axes of the centre of gravity and of the torsional centre are straight, but two axes do not coincide. Moreover, the flexural vibratory motion of the beam is assumed to be of the shearing vibration.

Symbols

M_t	twisting moment of beam
Q	shear force
w	mass of unit length of beam
g	acceleration of gravity
s	distance between both axes of centre of gravity and of torsion centre
r_p	radious of gyration of mass moment of inertia of section of beam
x, y	length coordinate and deflection of beam respectively
	$\xi = x/L,\ \eta = y/L,$
L	length of beam
ϕ	torsional angle of beam
p	circular frequency of coupled vibration of beam
$k'GA$	shear rigidity of beam
GJ_t	torsional rigidity of beam
ε	$=s/L$
μ	$= sL/r_p^2$
t	time coordinate

The basic equations for equilibrium of vertical force and of torsional moment of the element of the beam are presented respectively as following coupled equations,

$$\left.\begin{aligned} \frac{\partial Q}{\partial x}dx &= \frac{w}{g}\left(\frac{\partial^2 y}{\partial t^2} + s\frac{\partial^2 \phi}{\partial t^2}\right)dx \\ \frac{\partial M_t}{\partial x}dx &= \frac{w\, r_p^2}{g}\left(\frac{\partial_2 \phi}{\partial t^2} + \frac{s}{r_p^2}\frac{\partial^2 y}{\partial t^2}\right)dx \end{aligned}\right\}. \tag{1}$$

According to the assumptions of the beam of variable cross section mentioned above, the equations which present the mode of vibration are obtained from (1) as following forms;

$$\left.\begin{aligned} &\frac{1}{\xi}\frac{d}{d\xi}\xi\frac{d\eta}{d\xi}+\lambda_1^2(\eta+\varepsilon\psi)=0 \\ &\frac{1}{\xi}\frac{d}{d\xi}\xi\frac{d\psi}{d\xi}+\lambda_2^2(\psi+\mu\eta)=0, \end{aligned}\right\}. \tag{2}$$

where, $$\lambda_1^2=p^2\frac{w_0L^2}{gk_0'GA_0}, \quad \lambda_2^2=p^2\frac{w_0r_p^2L^2}{gGJ_{to}},$$

w_0, A_0···present the values of w, A···at the midship section.
Eliminate ψ from the above coupled equations, the following differential equation of the fourth order with respect to η is obtained,

$$\left(\frac{1}{\xi}\frac{d}{d\xi}\xi\frac{d}{d\xi}+u^2\right)\left(\frac{1}{\xi}\frac{d}{d\xi}\xi\frac{d\eta}{d\xi}+v^2\eta\right)=0, \tag{3}$$

where,

$$\left.\begin{matrix}u^2\\v^2\end{matrix}\right\}=\frac{\lambda_1^2}{2}\left\{(1+\beta^2)\pm\sqrt{(1-\beta^2)^2+4\mu\varepsilon\beta^2}\right\}, \tag{4}$$

$$\beta=\frac{\lambda_2}{\lambda_1}. \tag{5}$$

General solutions of (3) are obtained as follows;

$$\eta=AJ_0(u\xi)+BJ_0(v\xi)+CY_0(u\xi)+DY_0(v\xi), \tag{6}$$

similarly, $$\psi=\overline{A}J_0(u\xi)+\overline{B}J_0(v\xi)+\overline{C}Y_0(u\xi)+\overline{D}Y_0(v\xi), \tag{7}$$

where $A, B, \cdots, \overline{A}, \overline{B}\cdots$, integration constants

J_0, Y_0 Bessel functions of zero order of first kind and second kind respectively

The four constants and the charateristic values are obtained by given boundary conditions of the beam. If we take the origine at the end of the beam and take $\xi=\xi_2=1/2$, also, in accordance with the conditions of free-free bar, we obtain $C=D=0$. For an example, if we assume that the mode of the odd number nodes of the shear flexural vibration coupled with made of the even number of nodes of the torsional vibration in the above case, the determination equation of the coupled charateristic values is presented as follows;

$$\begin{vmatrix} J_0\left(\frac{u}{2}\right) & J_0\left(\frac{v}{2}\right) \\ u(\lambda_1^2-u^2)J_1\left(\frac{u}{2}\right), & v(\lambda_1^2-v^2)J_1\left(\frac{v}{2}\right) \end{vmatrix}=0. \tag{8}$$

As against the coupled charateristic values obtained above equation, the uncoupled criticals of which obtained by $J_0(u/2)=0$ and $J_0(v/2)=0$ with $\mu\varepsilon=0$, may be compared with above coupled charateristic values. The ratios of coupled criticals with uncoupled one, for some numerical examples, are shown in Fig. 3.

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