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ON THE DUALISM IN THE FRACTURE THEORY

By Masakazu HIGUCHI

The strength properties are analytically investigated at the variable rates of straining on the reasonable but somewhat hypothetical bases both of the inherently discontinuous structure on a colloidal scale of the solid material and of the duality of the micromechanism of fracture that broadly there are classified two modes of local fracture, either of which is caused by attaining to the limit of the tensile local elongation of the material. The resulting characteristics of the energy to fracture and the ultimate strain are shown together with the general expression of the transition rate of straining. The principle is extended and applied to the rather simple cases of loading of the specimens; and there are presented the linear relationship between the breaking strength and the rate of straining when plotted on the log-log scale, the existence of the positive as well as the negative velocity sensitivity of the energy to fracture turning to each other at the transition rate, and the several possible types of the energy characteristics dependent on the parameter, k_p/k_e. The experimental results of some research workers are thus explained satisfactorily in terms of the dualism in the fracture theory.

1. Introduction. Fracturing may be called a series of discontinuous phenomena having a number of phases, each of which is supposed to require considerable investigations. Thus the studies of a certain phase of the fracturing of materials are not satisfactory for those who are keenly interested in its other phases. However, since fracturing is too complicated to investigate comprehensivly, the present study is intended only to formulate the dualistic strength property of the material, especially of plastics, without considering such other phases as the nucleation and propagation of cracks and the statistical distribution of the strength values.

Though it sounds paradoxical, we can probably say that the development of the molecular theory has been so in advance of the formulation of the fracture problem to be done from the mechanical point of view, that we have come to see too much the submicroscopic structures. While the properties not affected by the far minor singular portion of the material, such as modulus of elasticity, viscosity and electric resistance, are being analysed successfully in the light of the structure theory of the matter, that is not the case as concerns the breaking strength. Cracks are probably nucleated at the very sites of those minor constituents of the least strength and breaking is invited by the ensuing unstable process of the successive growth of them.[1] What we are to consider, accordingly, is not an arbitrary small portion of such an extending homogeneous domain as the ocean of

the molecules of the major constituents, but that of the microscopic scale as bounded by the line connecting the sites of some minor constituents, including flaws, of the weaker strength.[2]

Plastics are to be regarded with good reason to have such inner boundaries which consist, for the most part, of some minor products yielded accidentally in the process of polymerization reaction[3][4] and divide more or less discretely the bulk of the material component into a great number of small regions of irregular shape. It is, if any, indeed in those boundaries that R. Houwink's flaws[5] similar to those supposed by A. A. Griffith[6] in glass will be found and work as harmful stress raisers as well as behave as considerable component parts on the rheological phase of the material. In the interior of the minute granular regions, of course, such flaws may be found, too. However, it appears that we have on the present case no direct bearing on those flaws scattered separately and confined firmly in their several compact clusters of the polymer molecules polymerized in the relatively early stage of reaction.

Consequently, I think that the estimate of the bonding forces in some polymers, which were from the chemicophysical view-point worked by J. de Boer[7] before and then by K. Ohmori[8] in the more reasonable and thorough way, is to be applied in the first place to the bonding forces acting in the boundary regions. Nevertheless, their strength values may be with some dynamical modification utilized to the final process of the brittle fracture in which the crack must necessarily propagate in drastic measures because of the plentiful strain energy released on fracturing and is possibly not given ample time to select its path.

2. Micromechanism of fracture and its model. The inner boundaries supposed above must be essential to the nucleation of minute cracks and to the ensuing connection of the cracks with each other; and it is supposed to happen that *tensile local* displacements are caused in the first place somewhere between the minute grains whenever the material is subjected to any external force, be it tensile or compressive, or otherwise, because of the higher rigidity of the grains. When the local displacements attain to a certain limit of magnitude, cracks will be found to arise there. There may be found the primary and the secondary bonds likewise on the boundaries. However, the secondary ones are *not* expected necessarily to break prior to the other, because the primary ones have in general surpassing moduli of elasticity and in such portions do the stresses concentrate as a general rule; and we have not yet any accurate knowledge of the moduli of the minor components.

In either case, the stiff links in the polymer material are capable of local rearrangements by way of turning to the direction of the local tensile stress and it takes naturally some time for the local displacements as well as for the deformations of the grain boundaries to take place. Consequently,

cracks will arise not only at diverse sites but also in different manners depending on whether the time to rearrange is sufficient or not.

This process can be described in another expression: Let us imagine a topography on which the initial state and the fractured state are located together with the intermediate states in the process of fracturing. We have then a mountain range of potential having the mountain passes between the initial and the fractured states, the lowest of which is not always the only way wended up as may be supposed. Depending on the loading conditions such as the rate of straining, the other somewhat high but shortest pass might be preferred. Probably the fracturing takes place in the lowest but time-taking (in a certain sense, *long*) course at the low rate of straining, the work done during which will be almost dissipated every moment. Impact load, on the contrary, will cause the fracturing to occur in the shortest, though higher, course. Most part of the energy given then will be stored temporarily in the material as the elastic strain energy and, being released on fracturing, become the force for nucleating the new cracks or for their growth and fracturing, as well as the energy of the vibration which will be dispersed away in course of time.

Though we are thinking of the prescribed height when considering the mountain range, it can be also regarded, on the contrary, to change with the external force; then the time dependent change of the slopes of the paths controls the course of fracture, which is compared to the path of a ball rolling down and being accelerated in the direction of the steapest descent of that time. The elastic strain energy, aiding the external force, to force the cracks to propagate may be likened to the kinetic energy acquired by the ball.

3. A simple formulation. In order to formulate most simply such a process the diversity will be represented by a couple of courses, in other words, a couple of kinds of the rheological local responses and their respective displacement limits. And for the purpose of explanation we shall resort to the simplest model, i.e. the Maxwell model in the theory of rheology, which should be however modified in order to be adopted in our fracture theory; let us assign the respective displacement limits to the two elements of the model and consider that the spring of the model snaps off as well as the plunger of the dashpot slips out of the mating cylinder at each inherent elongation limit, and a local fracture will be caused when *either* of the two members breaks. The circumstances are visualized with the modified Maxwell model such as shown in Fig. 1. The spring of the model has a connection between its component parts and is uncoupled when pulled with the force just surpassing the connecting force and equal to Ek_e where E is the spring constant and k_e the elongation limit; while the dashpot has the definite length of the cylinder, k_p.

Thus, with the notation;

t, time

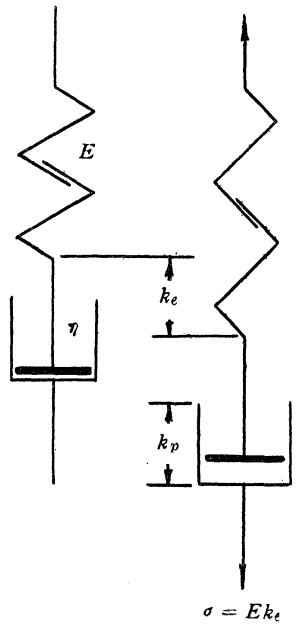

FIG. 1. The simplest model to explain the duality of breaking, constructed with a dashpot and a spring in series.
Left, no load state.
Right, just before the breaking of the elastic element under the higher rate of straining.

ε, total elongation at t
σ, tensile force at t
E, spring constant of the model
η, flow constant of the model
and $\tau = \eta/E$,

the behavior of the model preceding breaking is represented by

$$\frac{d\varepsilon}{dt} = \frac{\sigma}{\eta} + \frac{1}{E}\frac{d\sigma}{dt}, \qquad \text{(I)}$$

from which, considering the case $\frac{d\varepsilon}{dt}$ = constant $(= \beta$, say),

$$\frac{d\sigma}{\beta\eta - \sigma} = \frac{dt}{\tau}$$

and, putting $\sigma = 0$ when $t = 0$,

$$\frac{\sigma}{\beta\eta} = 1 - e^{-t/\tau} \qquad \text{(II)}$$

are obtained.

The elongation of the dashpot (say, plastic elongation, ε_p) and that of the spring (say, elastic elongation, ε_e) are derived respectively from the first and the second terms of the right-hand side of (I) using (II), i.e.

$$\left.\begin{aligned} \varepsilon_p &= \int_0^t \frac{\sigma}{\eta}\,dt = \beta\int_0^t (1 - e^{-t/\tau})\,dt = \beta\tau\left\{\frac{t}{\tau} - (1 - e^{-t/\tau})\right\}, \\ \varepsilon_e &= \int_0^\sigma \frac{1}{E}\,d\sigma = \frac{\sigma}{E} = \beta\tau(1 - e^{-t/\tau}). \end{aligned}\right\} \qquad \text{(III)}$$

We can see in Fig. 2 as well how ε_p and ε_e increase with time at the constant rate of straining, as we can recognize the characteristic difference between the breakings at the lower rate and at the higher rate of straining.

Another schematic explanation is done in Fig. 3, which is obtained by eliminating t from ε_e and ε_p in (III) and drawing the loci for three values of β in the plane of the coordinates of ε_e and ε_p. On reaching either of the elongation limit lines, k_e or k_p, the model breaks. At the higher rate of straining, β_1, the breaking is caused by crossing over the k_e-limit, while at the lower rate by attaining to the k_p-limit. Both elements happen to break at the same time at the intermediate rate of the definite value, which is called transitional.

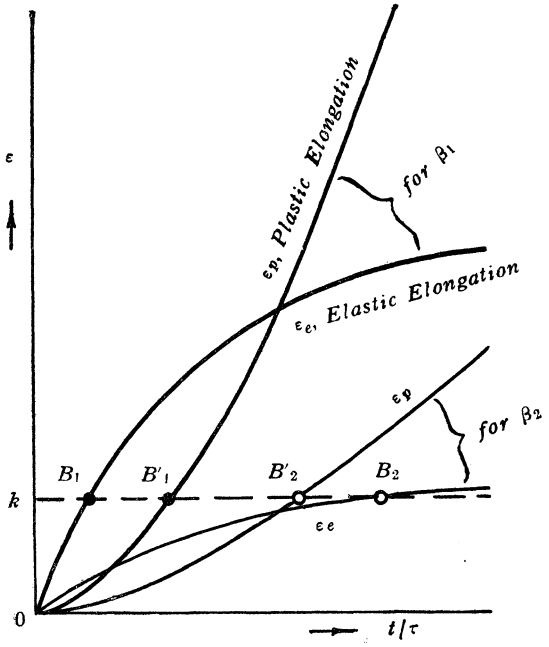

FIG. 2. Schematic explanation of the dependency of the breaking mode on the rate of straining.

When the elongation limits are supposed k_e, $k_p = k$ for this explanation, the breaking of the elastic element occurs at B_1 for β_1 in advance of the other, while for $\beta_2 (< \beta_1)$ the plastic element breaks first at B_2'.

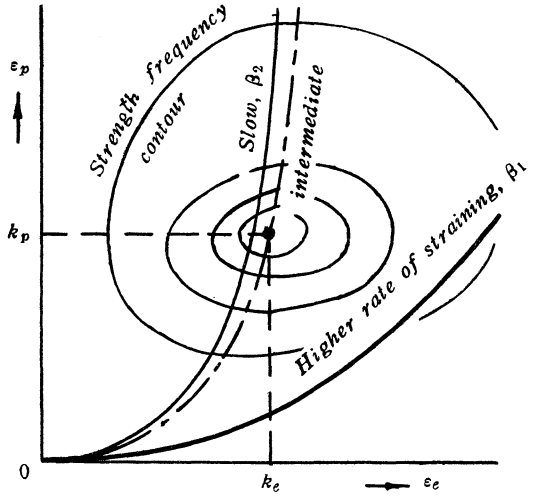

FIG. 3. The loci of the elongations of the elements in the plane $(\varepsilon_e, \varepsilon_p)$ at the constant rates of straining. The elastic strain ε_e at the higher rate is larger than that at the lower rate and the plastic strain ε_p conversely.

Considering the strength of the model in any case, we are supposing a certain definite value representable with a coordinate point in the plane $(\varepsilon_e, \varepsilon_p)$. In order to investigate breaking strength in full, not of the model but of the material, the above considerations are as a matter of course not sufficient and are lacking, of all things, in the crack propagation and in the development to the macromechanism of fracture out of the microscopic local strength theory. Such closer investigation will require to take into consideration the distribution of the ***strength values***, briefly called, in the above described coordinate plane. Our next step thus will be to consider as the model the array consisting of a number of such Maxwell units in parallel as the adopted above; the distribution of the strength values of the units then forms a curved surface rising above the ε_e, ε_p-plane, which is represented schematically by the contour lines in Fig. 3. The fact that the condition of loading itself changes the strength of the material is represented here by the distinction of the loci followed in the ε_e, ε_p-plane. Further consideration on these phases of fracture however will be done at another occasion.

4. A numerical example. We had better use a Voigt unit in place of the dashpot of the Maxwell model, i.e. employ the three-element model, for describing the behavior at the lower rate of straining. However, I have no intention here to cover the whole range of the rheological behaviors of the material with our model, especially that of the behavior at the extremely low rate of straining at the higher temperature in which case some material or other might behave chemorheologically.[3] [9]

A slightly modified model of the Maxwell type will suffice now and the essential features of the duality of breaking strength are manifested by applying the result of the above consideration to such a simple case as the uniform tensile loading, where the criteria of breaking are represented as a pair of the strain limits which must be, reasonably speaking, determined respectively by summing up and averaging, along the direction of loading, the local displacements when the specimen starts to break over the length and breadth of the specimen.

In this development, you must not confuse this conclusion with the St. Venant's maximum principal strain theory.[10] As for the combined stress state, we should go back to the original micromechanism and derive the consistent criteria. Therfore the following formulas as it stands can not be used in the cases of compression or other load, though the rheologists are usual to use an identical formula.

Now, making the alterations of some notations and adding some new ones as;

ε, total strain at t
σ, stress at t
σ_0, Bingham yield stress
ξ, flow constant of the model
n, exponential power of the flow

the behavior of the model preceding breaking is expressed as

$$\left.\begin{aligned} \frac{d\varepsilon}{dt} &= \frac{1}{E}\frac{d\sigma}{dt}, && \text{for } \sigma \leqq \sigma_0, \\ &= \frac{1}{\xi}(\sigma-\sigma_0)^n + \frac{1}{E}\frac{d\sigma}{dt}, && \text{for } \sigma > \sigma_0, \end{aligned}\right\} \quad \text{(IV)}$$

which equations become

$$\left.\begin{aligned} \frac{E}{\xi}dt &= \frac{1}{\beta\xi}d\sigma, && \text{for } \sigma \leqq \sigma_0, \\ &= \frac{d\sigma}{\beta\xi-(\sigma-\sigma_0)^n}, && \text{for } \sigma > \sigma_0, \end{aligned}\right\} \quad \text{(V)}$$

when the rate of straining β is constant. Thereform, using the suffix B to denote the values at break, we obtain the relation of the breaking strength σ_B and the time t_B to break.

$$t_B = \int_0^{\sigma_0}\frac{d\sigma}{E\beta} + \frac{\xi}{E}\int_{\sigma_0}^{\sigma_B}\frac{d\sigma}{\beta\xi-(\sigma-\sigma_0)^n} = \frac{\sigma_0}{E\beta} + \frac{\xi}{E}\int_{\sigma_0}^{\sigma_B}\frac{d\sigma}{\beta\xi-(\sigma-\sigma_0)^n}. \quad \text{(V')}$$

The ultimate strain ε_B and the energy to fracture W_B are

$$\varepsilon_B = \beta t_B, \quad \text{(VI)}$$

and

$$W_B = \frac{\sigma_0^2}{2E} + \frac{\beta\xi}{E}\int_{\sigma_0}^{\sigma_B}\frac{\sigma\, d\sigma}{\beta\xi-(\sigma-\sigma_0)^n}, \quad \text{(VII)}$$

respectively.

In the subtransitional range in which the dashpot, attaining to its strain limit k_p, first breaks, the breaking strength is obtained from the first term of the right-hand side of (IV) as

$$k_p = \int_{t_0}^{t_B}\frac{1}{\xi}(\sigma-\sigma_0)^n\, dt = \frac{1}{E}\int_{\sigma_0}^{\sigma_B}\frac{(\sigma-\sigma_0)^n}{\beta\xi-(\sigma-\sigma_0)^n}d\sigma,$$

t_0 being the time when the stress σ attains to the Bingham yield stress, σ_0. Rewriting the above integral, we obtain the following expression for the breaking strength as the function of the rate of straining, β,

$$\frac{E\,k_p}{\beta\xi} + \frac{\sigma_B-\sigma_0}{\beta\xi} - \int_{\sigma_0}^{\sigma_B}\frac{d\sigma}{\beta\xi-(\sigma-\sigma_0)^n} = 0, \quad \text{(VIII)}$$

and for the ultimate strain

$$\varepsilon_B = \frac{\sigma_B}{E} + k_p, \quad \text{(VI')}$$

and the energy to fracture is given by (VII) in which the σ_B is determined by (VIII).

In the supertransitional range where the spring, first attaining to its strain limit k_e, forestalls the other member,

$$\sigma_B = E k_e , \tag{IX}$$

is the breaking strength. Then t_B from (V′), ε_B from (VI) and W_B from (VII) are in succession derived as the functions of β respectively.

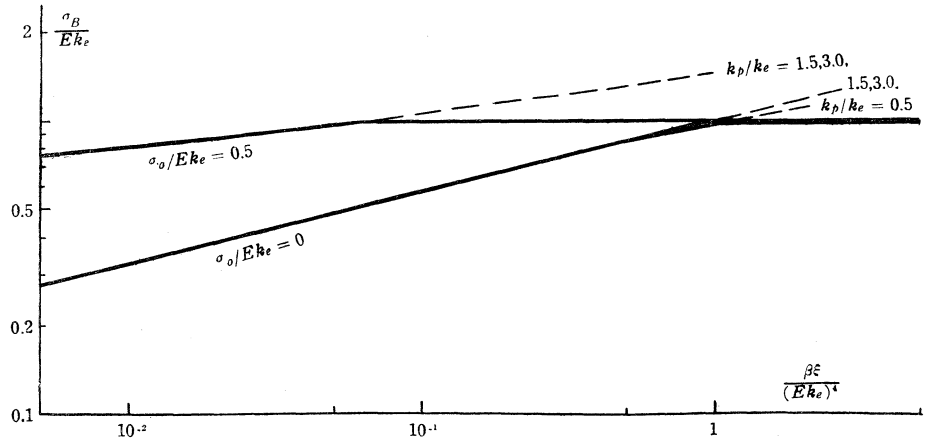

FIG. 4. Breaking strength, σ_B, *vs.* rate of strain, β, for $n = 4$ The finite value of the Bingham yield stress, σ_0, effects the decrease of the transitional rate of straining.

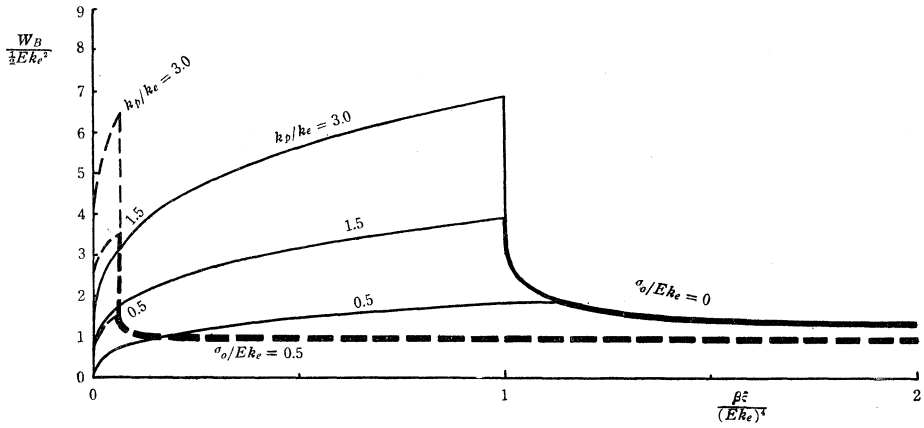

FIG. 5. Energy to fracture, W_B, *vs.* rate of straining, β. The supertransitional part of the curves does not depend on k_p/k_e. At the extremely higher rate W_B tends to $1/2\,E\,k_e^2$ for all values of σ_0/Ek_e.

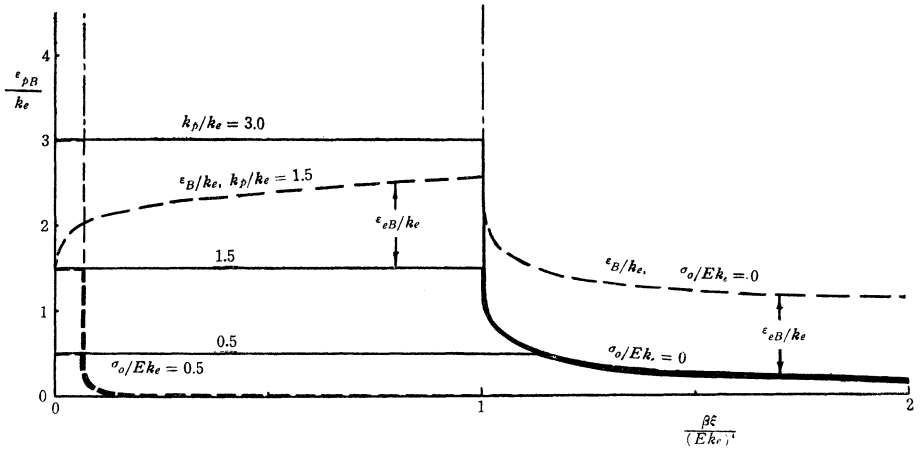

FIG. 6. Ultimate plastic strain, ε_{pB}, *vs.* rate of straining, β. The magnitudes of the ultimate plastic strains in the subtransitional range of straining are constant for the respective parameters, k_p/k_e, while the total strain including the elastic one is for instance as shown by the broken line for $k_p/k_e = 1.5$.

For example, supposing $n = 4$ and putting

$$a = (\beta\,\xi)^{1/4}$$

we obtain for the subtransitional range

$$\varepsilon_{pB} = k_p\,, \qquad \varepsilon_B = k_p + \frac{\sigma_B}{E}\,, \tag{VIa}$$

$$\frac{W_B}{\frac{1}{2}E\,k_e{}^2} = \frac{1}{2}\left(\frac{a}{E\,k_e}\right)^2 \ln\frac{a^2+(\sigma_B-\sigma_0)^2}{a^2-(\sigma_B-\sigma_0)^2} + \frac{2\,\sigma_0}{E\,k_e}\cdot\frac{\varepsilon_B}{k_e} - \left(\frac{\sigma_0}{E\,k_e}\right)^2, \tag{VIIa}$$

and the breaking strength, σ_B, is obtained from the equation

$$\frac{a}{2\,E\,k_e}\left\{\frac{1}{2}\ln\frac{a+(\sigma_B-\sigma_0)}{a-(\sigma_B-\sigma_0)} + \arctan\frac{\sigma_B-\sigma_0}{a}\right\} - \frac{\sigma_B-\sigma_0}{E\,k_e} = \frac{k_p}{k_e}\,, \tag{VIIIa}$$

which is derived from (VIII) and will be solved numerically.

The numerical calculations are worked out with the non-dimensional variables and a parameter k_p/k_e for $\sigma_0/E\,k_e = 0$ and 0.5 and reproduced in Figs. 4, 5 and 6. The relation given by (VIIIa) of the breaking strength and the strain rate, being plotted on the log-log scale, is almost linear independently of the parameter, k_p/k_e, and has a slope of n, except in the vicinity of the transition point, as shown in Fig. 4, and in the range of the lowest rate.

The transition rate of straining can be regarded as expressible by $(E k_e)^n/\xi$ unless k_p/k_e is so small, while the finite value of the Bingham yield stress, σ_0, effects its decrease by a factor $(1-\sigma_0)^n$. The energy to fracture (VIIa) and the strain at break (VIa) are shown for $k_p/k_e = 3.0$, 1.5 and 0.5 in Figs. 5 and 6 respectively.

In the subtransitional range of the strain rate, the energy to fracture increases gradually with the rate, while, at the transition point, the sudden decrease of the energy takes place and is succeeded by the asymptotic decrease to $1/2\,E k_e^2$ independently of the parameter, k_p/k_e. For such small values of k_p/k_e as 0.5, however, the sudden decrease is not remarkable and the transition point is shifted slightly to the higher side of the rate; though such is not the case for $\sigma_0/E k_e = 0.5$. In Fig. 6, ε_B/k_e is given by a broken line only for $k_p/k_e = 1.5$ and $\sigma_0/E k_e = 0$; all other full lines are for ε_{pB}/k_e, plastic strain remaining after break, for the several values of k_p/k_e and σ_0/Ek_e. They tend to zero alike at the extremely high rate of straining.

5. The experiments of some research workers. Such linear relations as shown in Fig. 4 between the breaking strength and the rate of straining in its subtransitional range was pointed out by R. N. Haward in his experimental work.[11] His results are reproduced in Fig. 7 by taking as $\sigma_{B\max}$, or $E k_e$, the maximum short time fracture stresses suggested there and estimating the respective transitional rates of straining, or $(E k_e)^n/\xi$. We obtain $n = 6.5$ and 4.9 for his plasticized cellulose acetate and methyl methacrylate respectively from the slopes of the lines in the subtransitional range. We shall be able to investigate strictly the relationship by using those values of n's in (VIII) and calculating numerically, the closed forms being underivable on account of the non-integral values of n's, only to find the results agreeing almost exactly with the original lines tried in so far as $k_p/k_e \leqq 1$.

Haward sought in his book[11] an explanation of the fact that the methacrylate as compared with the cellulose derivative showed much the greater extensibility and energy absorption when deformed slowly, but nevertheless had a lower impact strength; and that the cellulose derivative gave a similar limiting fracture strength at short times. His explanation is probably pertinent and expressible concisely in our terms as: Both materials had the nearly equal maximum strength values, $E k_e$. But the transition rate of straining, $(E k_e)^n/\xi$, was found lower with the acrylate than with the other. Consequently, it occurs as conceived from Fig. 5 that while the cellulose derivative is possibly almost in the transition, the acrylate is entirely in the supertransitional range at the same rate. Further, we can probably add that the acrylate falls behind the cellulose acetate in the value of $1/2\,E k_e^2$ in spite of their similar maximum strength, $E k_e$; accordingly, the k_e of the acrylate is smaller than that of the other. Nevertheless, or rather, as a result of it, the k_p/k_e of the acrylate is undoubtedly larger than the other, because the former is said having shown much the greater

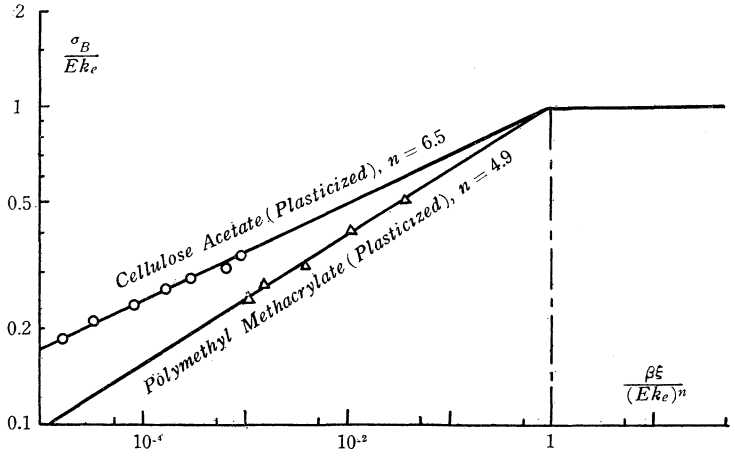

FIG. 7. Reproduction of Haward's experimental work, the parameter, k_p/k_e, being supposed > 1.

The maximum short time fracture stresses suggested there are taken as $\sigma_{B\max}$, i.e.

$E k_e = 218.8\ \mathrm{kg/cm^2}$ for Cellulose Acetate,

$\quad\quad = 251.2\ \mathrm{kg/cm^2}$ for Methyl Methacrylate;

then the transitional rates of strain are $574\ \mathrm{min^{-1}}$ and $39.8\ \mathrm{min^{-1}}$ respectively, $(Ek_e)^n/\xi$ being our analytical expression for them.

extensibility and energy absorption when deformed slowly. As a consequence, we may employ two ways differing from each other for the purpose of increasing the shock resisting capacity; one to increase the magnitude of $1/2\,E k_e^2$, the other to raise the critical rate of straining, $(E k_e)^n/\xi$, without decreasing the value of k_p/k_e.

Further considerations on the interesting functions of plasticizers and temperature in connection with these characteristics will be made at other occasions.

The negative velocity sensitivity is undoubtedly characteristic of the supertransitional rate of straining and the positive one may be of the subtransitional. On account of the larger value of $(E k_e)^n/\xi$ of a material, the positive velocity sensitivity may happen even at such higher rates of impact that some other plastic materials are already in the negative sensitivity range.

It agrees also with the observation that there is in general a sudden decrease of impact strength just above the rate of transition. However, the transition of gradual nature can result from the smaller value of k_p/k_e, for which the sudden change of the impact strength is not supposed to occur. The weight factor and frictional effects to which Haward more or less ascribed the positive velocity sensitivity will be such that is seen also in the supertransitional range when tested by changing the height of the hammer of the impact testing machine.

In other workers' experiments, we can also recognize the similar characteristics, linearity in the relationship between the fracture strength and the strain rate in the log-log plotting, and the limiting fracture strength at the higher rate of straining than a certain definite rate, though their results are almost plotted on the semi-log scale according to the usual practice. Thus, when reproduced on the log-log scale, the ambiguous curves in the figures in the paper of W. George[12] taken from LaTorre's work for the nylon filament can be replaced by the straight lines such as in Fig. 8.

How can we however explain the obvious difference between the real rheological behavior of the nylon filament (Fig. 9) and that which will be described according to the simple model (IV)? As mentioned in the preceding paragraph, the equation representing the behavior of a material for some small time before break will suffice for us to consider on the dualistic behavior of the breaking. It is proper for the materials of complicated type of rheological behavior that the equation should be solved with respect to the later part of the strain before break under other conditions than $\sigma = 0$ and $\varepsilon = 0$ when $t = 0$; it may be said an instance of such cases to take the Bingham yield stress, σ_0, into account. In our analytical treatment, the nylon

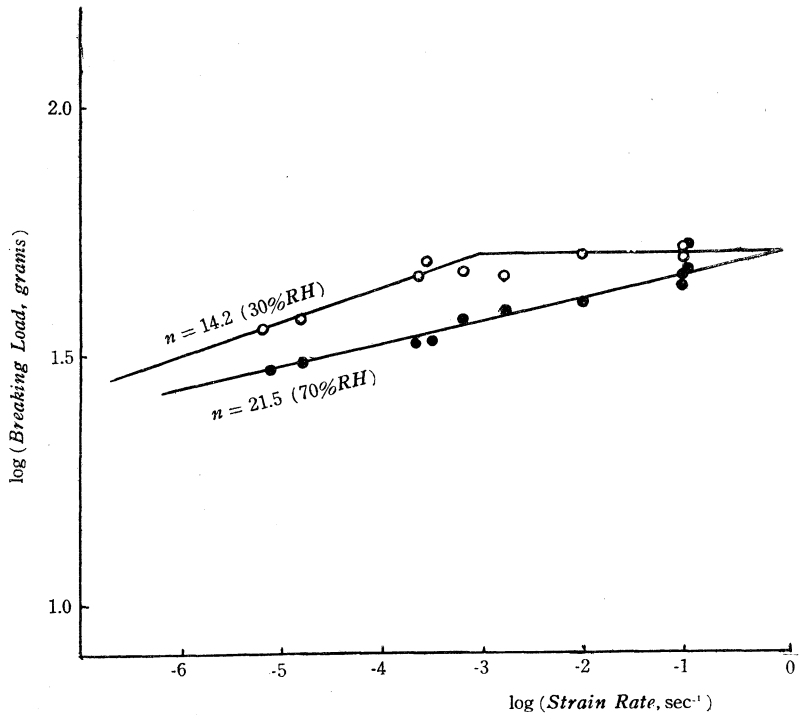

FIG. 8. The linear dependency of the strength of the nylon monofilament Type 300 (after LaTorre)[12] on the strain rate replotted on the log-log scale.

filament can be regarded to be elongated not along ABC but AA'BC (Fig. 9) and A' will be taken as the initial state, which is the imaginary state of the material of being crystallized without any tensile force.

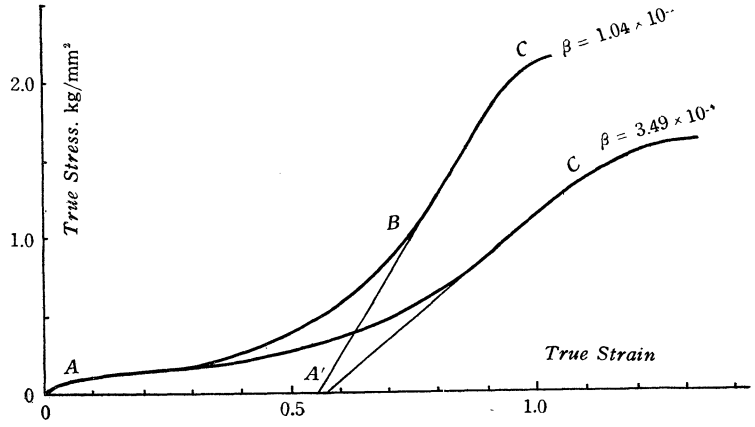

FIG. 9. Typical true stress-strain curves for Type 300 Nylon at 65% R.H., 23° C. (after LaTorre).[12]

You will find another illustration of the similar property of the breaking strength in the experiments carried out at the constant rates of loading by A. G. H. Dietz *et al.*,[13] in which case the circumstances are similar, too. The breaking strength, for example, of phenolic paper laminate, phenolic cloth laminate and cellulose acetate butyrate reach obviously the top values and do not increase at the higher rate of loading, though the linear relation appears to have been supposed approximately by the authors between log (load rate) and the bending strength itself. It may be possibly due to the comparatively slight effect of the rate on the strength (the greater value of n) that the relationship is almost linear in the semi-log plotting at the lower rate; or otherwise possibly due to not a small value of the Bingham yield stress. Referring to our expression of the transitional rate, $(E k_e)^n/\xi$, it is also reasonable that the transitional rate for cellulose acetate butyrate is shifted to the lower side as its type changes from hard to soft, and it is because Ek_e, top value of the maximum bending stress, decreases with the change of the type as seen in Fig. 22(a) annexed to the authors' paper and the decrease may be enough to make up for the possible decreasing of the value of ξ with the change of the type.

Similarly to the fracture energy change, the ultimate elongation measured after break may be classified into several characteristic types of change with the rate of straining according to the variation of the parameter, k_p/k_e. In any case, the higher the rate of straining in the supertransitional range is, the more the ultimate elongation decreases.

It is doubtful, on the contrary, that the residual strain after break is constant throughout the subtransitional range as predicted from the results of our treatment, because precisely we should take the delayed elasticity into account. It may happen that the strain ε_{pB} shown in Fig. 6 is including the component of the delayed elasticity, in which case the ultimate strain is no longer constant but decreases with the rate of straining. We can say nothing about the circumstances at the extremely low rate which are excluded from the present treatment.

6. Concluding remarks. Although there are no data enough to evaluate k_p, k_e and ξ yet, the several tendencies of the breaking strength of the time sensitive materials can be explained reasonalby with the above analysis. If we consider the distributions of the inherent limits of strain, the smoother curves are to be obtained, in particular for the energy to fracture and the ultimate strain. Anyhow, that the slopes of the curves of the ultimate strain and of the energy to fracture are steeper on the right-hand side, as seen in the annexed figures, is characteristic of most materials tested. It is much similar for the constant stress rate, the analysis for which will be done more easily.

However, there is contained an important hypothetical standpoint in the present analysis that microfracture will be caused by the *limiting tensile elongation* not only under the tensile, but also arbitrary, external forces; the validity of which should be verified by testing the materials under the states of combined stresses.

In connection with this, it is interesting that the three types of the transition of the tensile impact energy observed by D. S. Clark and D. S. Wood[14] with some specimens of steel are explained by the variation of our parameter, k_p/k_e. It is also plausible from our view-point that the critical impact velocity was almost independent of the types other than that having no evident transition, for which type the critical velocity, as seen in the figures annexed to their paper, is slightly shifted to the higher side. As suggested by Haward, it is sure that there may be found the similar fracture mechanism in the structure of metal.

Recently I have read one of the papers written by H. Itô[15] from the somewhat similar standpoint on the strength property, in which he tried an interesting qualitative explanation of the impact strength property of the textile fibers.

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