THE TRANSITIONAL RATE OF STRAINING CORRESPONDING TO A RELAXATION TIME

HIGUCHI, Masakazu
Research Institute for Applied Mechanics, Kyushu University : Professor

https://doi.org/10.5109/7159859

出版情報：Reports of Research Institute for Applied Mechanics. 4 (15), pp.93-98, 1956-01. 九州大学応用力学研究所
バージョン：
権利関係：

Reports of Research Institute for Applied Mechanics
Vol. IV, No. 15 January, 1956

THE TRANSITIONAL RATE OF STRAINING CORRESPONDING TO A RELAXATION TIME

By Masakazu HIGUCHI*

The relationship,

$$Transitional\ strain\ rate = \frac{Elastic\ strain\ limit}{Relaxation\ time}$$

is presented (cf. Eq. (4) in the text) to establish the concept "correspondence of a transitional strain rate to a relaxation time" which thus far has been used vaguely, and it is shown that the predicted temperature dependence of the rate agrees with the tendency having been recognized by several investigators.

The order of magnitude of the plastic strain limit and that of the elastic strain limit of the employed model are calculated with the data of some textile materials and estimated to be 10^{-1} and 10^{-2} respectively.

Refering in the introduction to the limiting breaking velocity and the critical impact velocity, the writer gives his view that neither has direct relation to the transition phenomenon here considered.

1. Introduction. The energy to break the specimen of a plastic material by an impulsive tensile force is in general dependent on the rate of elongation and attains a maximum value at a certain rate, beyond which the energy is rather reduced; some viscoelastic materials break almost without recognizable elongation at a certain rate of straining in which case the fracture surfaces have an obviously different pattern from those of the specimens breaking after considerable elongation. Such transitional rates of straining were formulated from the standpoint of fracture dualism in the previous papers.[1)]

Further development of the dualistic understanding will be introduced in the present paper; the definite concept of the correspondence of a transitional rate of straining to a relaxation time is formed by the analytical expression, from which the temperature dependency of the transitional rate is deduced, and the validity of the interpretation is examined.

There are two other *critical* (in the meaning that it does) velocities having something similar to the transitional velocity; one of them is the *limiting breaking velocity* recently suggested by W. K. Stone *et al.*,[2)] and the other is the *critical impact velocity* due to Th. von Karman and was investigated by P. E. Duwez *et al.*[3)] with metals.

Both of them, however, differ from the above mentioned transitional

* Member of the Institute, Professor of the Kyushu University.

rate of straining in that the former in the first place is related to the problem whether a specimen breaks or not when a certain amount of momentum is given its head mass attached to the specimen. Naturally it depends upon the masses attached to the specimen at each end as well as on the initial tensile velocity, and the investigators suggest that the specimen will not break when the velocity v satisfies the relation

$$\log v \leq -\frac{1}{2}\log\frac{\left(n+\frac{1}{2}\right)\left(m+\frac{1}{2}\right)}{m+n+1}+\log v_b\,, \tag{1}$$

where m and n are the ratios of the masses attached to the specimen at the head end and at the tail end respectively to the mass of the specimen.

The limiting breaking velocity is the v_b in this formula and evaluated by

$$\frac{1}{2}\rho v_b^2=\int_0^{\varepsilon_r}\sigma\, d\varepsilon\,,$$

ρ being the density, σ the stress, ε the strain and ε_r the rupture strain of the specimen. As a matter of course, the velocity of the head mass will decrease as the specimen elongates. Nevertheless, the velocity can be regarded actually to keep its initial value when m is large enough. Such a condition is satisfied in the usual impact testing, in which case at the same time n is infinitely large because of clamping the specimen at the tail end.

Thus,

$$v\to 0$$

is obtained from (1). This is nothing strange and tells only that the specimen will necessarily break, if one, clamping the specimen at one end, goes on pulling the other end, and has nothing to do with the existence of a transition in the mode of breaking.

Next, the critical impact velocity v_c is defined as

$$v_c=\int_0^{\varepsilon_r}\sqrt{E/\rho}\, d\varepsilon \tag{2}$$

E being $d\sigma/d\varepsilon$, the modulus of deformation, elastic or plastic, and other notations the same as the already used.

This velocity contrasts with our transitional velocity in temperature dependency. In order to compare the critical impact velocities for the differing temperatures θ_1 and θ_2, we shall consider the ratio v_{c_1}/v_{c_2}, where the suffices 1 and 2 refer to the values for the temperatures θ_1 and θ_2 respectively (and the same applies to the following other notations).

Provided that the material is regarded as not predominantly entropy-elastic for the temperature of the experiment,

$$E_1<E_2\,,\quad \text{for}\quad \theta_1>\theta_2\,,$$

when the strains are of equal amount. Since the change of ρ can be neglected relatively to the change of E, we obtain by supposing $\varepsilon_{r1}>\varepsilon_{r2}$ as is usually the case,

$$\frac{v_{c1}}{v_{c2}} \fallingdotseq \frac{\int_0^{\varepsilon_{r1}} \sqrt{E_1}\, d\varepsilon_1}{\int_0^{\varepsilon_{r2}} \sqrt{E_2}\, d\varepsilon_2} = \frac{\int_0^{\varepsilon_{r2}} \sqrt{E_1}\, d\varepsilon_1 + \int_{\varepsilon_{r2}}^{\varepsilon_{r1}} \sqrt{E_1}\, d\varepsilon_1}{\int_0^{\varepsilon_{r2}} \sqrt{E_2}\, d\varepsilon_2}$$

$$< \frac{\int_0^{\varepsilon_{r2}} \sqrt{E_1}\, d\varepsilon_1 + \int_{\varepsilon_{r2}}^{\varepsilon_{r1}} \sqrt{E_1}\, d\varepsilon_1}{\int_0^{\varepsilon_{r2}} \sqrt{E_1}\, d\varepsilon_1} = 1 + \frac{\int_{\varepsilon_{r2}}^{\varepsilon_{r1}} \sqrt{E_1}\, d\varepsilon_1}{\int_0^{\varepsilon_{r2}} \sqrt{E_1}\, d\varepsilon_1}.$$

The modulus of deformation for the strain ε_{r2} at the temperature θ_1, say E_{r1}, is in general larger than the moduli for the larger strains than ε_{r2}, and is smaller than those for the smaller strains than ε_{r2}. Consequently,

$$\int_{\varepsilon_{r2}}^{\varepsilon_{r1}} \sqrt{E_1}\, d\varepsilon_1 < (\varepsilon_{r1} - \varepsilon_{r2}) \sqrt{E_{r1}},$$

$$\int_0^{\varepsilon_{r2}} \sqrt{E_1}\, d\varepsilon_1 > \varepsilon_{r2} \sqrt{E_{r1}},$$

and hence

$$\frac{v_{c1}}{v_{c2}} < 1 + \frac{(\varepsilon_{r1} - \varepsilon_{r2}) \sqrt{E_{r1}}}{\varepsilon_{r2} \sqrt{E_{r1}}} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}.$$

According to the experiment of Gotô *et al.* with a viscoelastic compound of rosin-olive oil system,[4] the velocity ratio

$$\frac{v_{c1}}{v_{c2}} \fallingdotseq 100$$

is obtained for two brittle points $\theta_1 = 30^\circ$ C and $\theta_2 = 15^\circ$ C. This leads to the unlikely relation,

$$\frac{\varepsilon_{r1}}{\varepsilon_{r2}} > 100$$

while both the strains are of brittle fracture. Thus the critical impact velocity is not likely to have direct relation to the above experimental facts.

It is a matter of course that a number of more or less independent factors may be called into play and their effects are not of the same amount for different testing conditions. The strain wave effect suggested by von Karman is to become influential in the higher range of strain rate, for example $10^5 \sim 10^6$ percent per second for viscoelastic such as textile materials, as stated by W. K. Stone *et al.*

The energy-to-fracture transition observed by Negishi *et at.* for some sorts of textile materials[5] was, however, in the range of the order of $10 \sim 100$ cm/sec, or $10^3 \sim 10^4$ percent per second for their specimens 1 cm long, which was far below that given by (2).

On account of the slow speed of elongation, the strain wave must propagate along the specimen and be reflected at each end in succession a good many times before breaking. Since the wave velocity is of the order of 10^3 m/sec, the wave is supposed to go and return about fifty times along the specimen of the length of 1 cm during the time of the order of 10^{-3}

seconds which is expected to pass before the specimen breaks. Even when we must take less value for E (or $d\sigma/d\varepsilon$) in the range of large strain, going back and forth must have been repeated ten times or so before breaking, because the wave velocity decreases only as $\sqrt{E}$ does. On account of the viscoelasticity of the materials, we can consider the strain wave to be damped meanwhile and can regard the strain to be leveled and to become uniform along the specimen.

It may be an evidence for the uniformity of the strain when breaking, that the relationship between the energy to fracture the specimen l cm long and the velocity to extend it, is much the same, when divided by l, as the relationship for the specimen 1 cm long.

2. Transitional Rate of Straining Corresponding to a Relaxation Time. The above-mentioned transition may be therefore understood from the standpoint of the fracture dualism and the rate of straining at transition may have the following general expression:

$$\beta_{tr} = (E k_e - \sigma_0)^n / \xi , \tag{3}$$

where β_{tr} = transitional rate of straining,
E = spring constant of Maxwell model,
k_e = strain limit of the spring,
σ_0 = yield strength of the dashpot of the model,
n, ξ = constants to describe the flow of the dashpot.

When the flow of the dashpot is regarded to be simply Newtonean, i.e., $\sigma_0 = 0$, $n = 1$, the equation (3) is to become

$$\beta_{tr} = \frac{k_e}{\tau} , \tag{4}$$

τ being η/E, and η used in place of ξ for the Newtonean flow.

Some materials may have more than one values of k_e/τ, according to which as many transitions are to appear in the energy to fracture. The several points of transition observed for nylon filament, viscose rayon and other materials can be considered to correspond to the distribution of k_e/τ of the materials.

Thus, the word "to correspond to a relaxation time" is given a definite concept in the meaning of the equation (4), though the word thus far has not been connected to so manifest a concept as in vibration phenomena and is being used somewhat carelessly in considering the impact behavior of materials.

$\beta_{tr}/2\pi k_e$ is, if any, an equivalent frequency of an impact load analogous to oscillating load; because the correspondence $2\pi f = 1/\tau$ (f being the frequency of the load) has long been used in this case.

3. Temperature Dependency. Actually the expression for the transitional rate is examined by the observations of its dependency on temperature. As is well known, η is related to temperature approximately as

$$\eta = A e^{B/T} ,$$

A and B being constants and T temperature in °K. Accordingly the expression (4) becomes

$$\ln \beta_{tr} = \ln \frac{E k_e}{A} - \frac{B}{T_{tr}}, \tag{5}$$

where the related transitional temperature to β_{tr} is denoted with T_{tr}. Representing the length of the specimen by l, we can derive the relationship

$$\ln v_{tr} = \ln \frac{E k_e l}{A} - \frac{B}{T_{tr}}, \tag{5'}$$

from (5) provided that the experiment is done in such a range of impact velocity that the strain wave effect can be neglected.* The above relationship is identical with the results of the experimental works.[4)6)7)]

The Young's moduli of high polymeric materials, indeed, decrease remarkably as the temperature rises. However, the E which we are using in (5') as a spring constant is different from them, because all part of the plastic strain is attributed in our model to the elongation of the dashpot. Hence, Ek_e may be nearly independent of the temperature, while k_p, the plastic strain limit, increases evidently with the temperature rise.

This increase of k_p reveals another energy peak with the rise of the temperature as shown in the experiment with some textile materials, because the energy to fracture per unit volume corresponding to one value of k_e/τ's is Ek_pk_e as is deduced from the previous study.

4. Evaluation of k_p and k_e. The order of magnitude of k_p as well as k_e can be calculated by using the value of the transitional rate given in the paper of Negishi *et al.* The transitional velocity is in the range of $30 \sim 90$ cm/sec, or $\beta_{tr} = 30 \sim 90\ \mathrm{sec}^{-1}$ for their specimens 1 cm long, while the viscosity coefficient η for textile materials is in the range of $(2 \sim 6) \times 10^7$ poise. Thus, from (4)

$$Ek_e = \eta \beta_{tr} = (10 \sim 50) \times 10^8\ \mathrm{dyne/cm^2}. \tag{6}$$

This is the breaking strength in the range above the transitional rate, which may be of a reasonable magnitude, though of the same order as that of the static strength; because it is known that the dynamic strength is not so much increased even by increasing the extending velocity by several orders.

Next, the energy to break becomes

$$W_R = Ek_pk_e = (10 \sim 50)\, k_p \times 10^8\ \mathrm{erg/cm^3}$$

at the transitional rate of straining. Since the values of W_R are in the range of $(5 \sim 12) \times 10^8\ \mathrm{erg/cm^3}$, k_p will be of the order of 10^{-1}.

---

* Such a direct evaluation of the velocity from the rate of straining is not permissible when the strain wave effect should be considered. If one takes into account the effect by the dualistic understanding for a Maxwell body,

$$v_{tr} = c k_e$$

is the transitional velocity, where c is the *elastic* strain wave velocity of propagation. The temperature dependency of this velocity will be comparatively slight and rather of opposite sign as inferred from the expression.

On the other hand, k_e is considered to be of the order of 10^{-2} according to (6) in which 10^{11} dyne/cm^2 is used for the order of E, the order of the greatest value measured by the method of vibration.

As shown in the previous paper, the strain of the specimen at break is to be nearly equal to k_p in the static test and, increasing with the straining rate, to become $k_p + k_e$ at the transitional rate, though it may be considered not to change actually because of the smallness of k_e compared with k_p. At the transitional rate, however, the strain at break is to decrease suddenly and to tend to k_e with the increase of the rate.

The orders of the magnitudes of k_p and of k_e estimated above agree with the fact that the elongation is a few scores percent in the static test with textile materials and the breaking strength at the highest tensile rate is of the order of one hundredth of the Young's modulus.

Working the material by drawing is supposed to cause the decrease of k_p, as the result of which the elongation at break of the material of large draw ratio might not suffer much change over the whole range of straining rate. This may be considered to be the reason why there prevail both the opinions, contradicting each other, that the elongation at break increases with the straining rate, and that the increasing is scarcely recognized.

Acknowledgement. It is a pleasure to record here a debt of gratitude to Professor T. Isibasi and the members of the Institute for their interest and helpful discussions relative to this study.

References

1) M. Higuchi, Rep. Res. Inst. Appl. Mech., Kyushu University, Vol. III, No. 12, 1955.
M. Higuchi, Proc. 4th Japan Nat. Congr. for Appl. Mech., pp. 203-206, 1954.
2) W. K. Stone *et al.*, J. NBS, 54, 5, 1955.
F. L. McCrackin *et al.*, Ditto, 54, 5, 1955.
J. C. Smith *et al.*, Ditto, 55, 1, 1955.
3) Th. von Karman *et al.*, JAP, 21, 10, 1950.
P. E. Duwez *et al.*, Proc. ASTM, 47, pp. 502-522, 1947.
D. S. Clark *et al.*, Trans. ASM, 42, pp. 45-74, 1950.
4) R. Gotô *et al.*, Kikai no Kenkyu, 7, 11, 1955.
5) M. Negishi *et al.*, Sen-i Gakkaishi, 7, 5, 1951.
M .Negishi *et al.*, Ditto, 7, 6, 1951.
M. Negishi *et al.*, Ditto, 7, 7, 1951.
6) H. Itô, Kohbunshi, 3, 242, 1954.
7) D. R. Morey, Ind. Eng. Chem., 37, 3, 1954.

(Received January 7, 1956)

ERRATA

On the Dualism in the Fracture Theory

By Masakazu Higuchi

(Reports of Research Institute for Applied Mechanics, Vol. III, No. 12, 1955)

Page 164, line 3, for $(1-\sigma_0)^n$, read $\left(1-\frac{\sigma_0}{Ek_e}\right)^n$.

" " " 29, for $k_p/k_e \leq 1$, read $k_p/k_e \geq 1$.

" 167, Fig. 9, for 1.0 and 2.0 on the axis of ordinates, read 70.31 and 140.62.