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SHEARING VIBRATION OF SHIPS

By Toyoji KUMAI*

Abstract; Some investigations into the flexural vibrations of the ships' hulls are carried out. The author pointed out that such a higher mode of the hull vibration as which resonant to the propeller blade force excited through the hydrodynamic action is considered as the shearing vibration. The vibration profiles of the higher mode of the hull vibrations under some assumptions of the distributions of the shear rigidity and of the mass are calculated based on the shearing vibration. Also, the natural frequencies of the fundamental mode to the eight-noded mode of the vertical vibration of the cargo vessel, for an example, are calculated based on the shearing vibration with taking the corrections of the bending moment and the rotatory inertia of the hull section into account. For the sake of realizing the requirement of simple method of the estimation of the natural frequencies of the hull vibration including higher modes in the stage of the initial design, the empirical formula founded on the shearing vibration is presented. The empirical factors of the formula were determined based on the data which have been sent from the leading shipbuilders in Japan to the author.

1. Introduction: The investigations into the propeller blade forces which excite the hull vibrations are of great importance in several vibration generating systems of a ship. This investigation has already been carried out by F. M. Lewis.[1)2)3)] One of the methods of the reduction of the vibration excited by the propeller blade is to increase the number of the propeller blades, for examples, the three-bladed propeller had been replaced by the four-bladed one in the "Mauretania." the "Lusitania" and the "Normandie."[4)] In the present time, the same method of the vibration prevention as mentioned above is taken into consideration, namely, the four-bladed propellers are frequently replaced by the five-bladed ones in the modern large tankers and the other high speed cargo boats in practice.

The mode of the vibration whose criticals resonant to the propeller blade frequency is higher than of our expectation. For instance, the five-noded mode was measured in the large passenger boat with the three bladed twin

1) F. M. Lewis. "Propeller Vibration" Trans. Soc. N.A.M.E. 1935.
2) F. M. Lewis. "Propeller Vibration" Trans. Soc. N.A.M.E. 1936.
3) F. M. Lewis and A. J. Tachminji. "Propeller Forces Exciting Hull Vibration" Reprinted paper No. 8 of Soc. N.A.M.E. 1954.
4) F. Coqueret and P. Romano. "Some Particulars Concerning The Design of The *Normandie* and the Elinination of Vibration." Trans. Soc. N.A.M.E. 1936.

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screw by F. M. Lewis,[1] and the seven-noded mode of the vibration in the 10,000 ton d.w. cargo vessels with four-bladed twin screw, and up to the nine-noded mode in 32,000 ton d.w. tanker with four-bladed single screw were both observed by T. Okabe and his coworkers.[5)6)] The calculations of such higher modes of the hull vibrations as mentioned above are hardly able to obtain by the use of the classical method based on the bending theory, because the shear flexibility and the rotational inertia of the cross section of the hull have considerably influence upon the natural frequencies of such a higher mode of the flexural hull vibration as resonant to the propeller blade frequency.

In the present paper, the vibration profiles and the natural frequencies of the two-noded to the eight-noded modes are obtained as those of the shearing vibration taking the corrections for the bending moment and the rotational inertia into account.

So far as the author is aware, no investigation into the shear rigidities of the hull sections under the considerations of the deck openings and of the sectional form in way of the entrance and the run has been carried out, thence the present paper deals with those effects on the shear rigidity.

The empirical formula for estimation of the natural frequencies including higher modes based on the shearing vibration is presented in the last paragraph of the present paper.

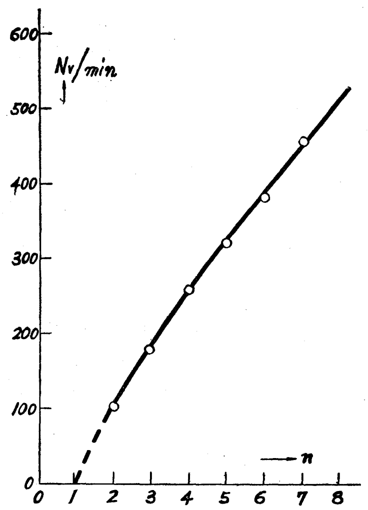


Fig. 1. Measured natural frequencies up to the seven-noded mode of vertical vibration of 10,000 ton d.w. cargo ship in ballast condition.

2. Shearing Vibration of Ship's Hull. The natural frequencies of the flexural vibration of the elastic beam vary as the square of its corresponding characteristic values by the classical theory of the bending vibration without the considerations of the other effects. The results of the vibration measurements of the ship's hull, however, are greatly different from that calculated by the classical theory, namely, the natural frequencies of the hull vibration are not presented by the square of the characteristic values, but are almost propertional to its values like as the torsional vibration of the elastic beam.[5)6)] Fig. 1 shows an example of the relation between the

[5)] T. Okabe, S. Fujita and Y. Matsuyama. Report of Western Soc. of N.A. Japan No. 6. 1953.

[6)] T. Okabe, K. Hirata and T. Kumai. Rep. of Res. Inst. Applied Mech. Vol. IV. No. 13. 1955.

natural frequencies of the vertical vibration and its number of nodes which have been measured in 10,0)0 ton d.w. cargo boat.[5)]

It is pointed out that the cause of the above discrepancy between the classical theory and the observation is due to the effects of the shear deflection and the rotatory inertia of the hull section. Since the discrepancy is very much considerable in the higher node modes of the hull vibration, the motion is approximately considered as the shearing vibration in such a higher mode of the vibration of ship's hull which resonant to the propeller blade excitation.

An example of the shearing vibration has already been investigated into the vibration of the low buildings by Dr. K. Suyehiro.[7)] The ships' hulls, however, have variable stiffness and irregular mass distributions along her length, then the solutions are different from the case of the beam of uniform section like as low buildings. If we assume that the hull form is symmetrical amidship, and the distributions of the shear rigidity and of the mass along her length are represented by the powers of m and n-th order of the length coordinate respectively, the solutions of the equation of the shearing vibration are presented by the Bessel functions as follows, (see Appendix.)

$$\eta = A\xi^{\mu} J_{-\frac{\mu}{\mu+\nu}}\left(\frac{\lambda_1}{\mu+\nu}\xi^{\mu+\nu}\right) + B\xi^{\mu} J_{\frac{\mu}{\mu+\nu}}\left(\frac{\lambda_1}{\mu+\nu}\xi^{\mu+\nu}\right), \tag{1}$$

for the beam of uniform section, or in way of the parallel body of the hull, η becomes as the following well known form,

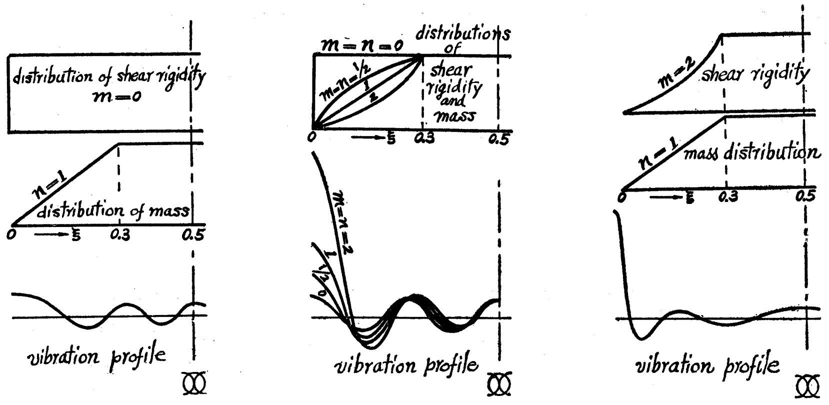
$$\eta = A\sin\lambda\xi + B\cos\lambda\xi. \tag{2}$$

Since the vibration profiles of the higher mode of the hull vibrations are approximately presented by the solutions of the shearing vibration, (see paragraph 3) the effects of the non-uniformities of the shear rigidity and of the mass upon the higher mode of the hull vibrations are qualitatively investigated by the computed results of the modes of the shearing vibration with the parameters m and n.

The effects of some combinations of the distributions of the shear rigidity and the mass upon the vibration profiles of the hull vibrations are shown in Fig. 2. (a), (b) and (c). As will be seen in the figures, the amplitude of the hull vibration in the vicinity of the stern or the bow of the ship is larger than that of midship section except uniform box shaped one in the case of the higher mode of the flexural vibration. It will be pointed out that one of the reasons of the undue vibration which occurs at the stern or the bow of the ship is caused by the considerable reduction of the shear rigidity along the length of the run or of the entrance.

The shearing vibration of the beam in the higher mode of the flexural vibration is also deduced from the fundamental equation of the vibration of the elastic beam based on the bending theory with the consideration of the

7) K. Suyehiro. "Scientific and Technical Papers" p. 278. 1934.



(a) $m < n$ (b) $m = n$ (c) $m > n$

FIG. 2. Vibration profiles of the eight-noded mode with various distributions of shear rigidities and of masses along the length of hull.

effects of the shear deflection and of the rotatory inertia which is deduced by S. Timoshenko.[8] (see Appendix.)

In the case of the calculations of the natural frequencies of the hull vibration based on the shearing vibration mentioned above, the corrections for the effects of the energy of the bending moment and of the rotatory inertia of the hull section must be carried out as will be seen in the following paragraph.

3. Corrections of Bending Moment and Rotatory Inertia for the Natural Frequeucies of the Shearing Vibration of the Hull. In the range of the natural frequencies of the lower modes of the hull vibration, the strain energy of the bending moment is more considerable than that of the higher modes, on the other hand, the rotational inertia effect upon the criticals of the higher modes is more remarkable than of which in the lower modes. The corrections for those effects, therefore, should be taken through all the natural frequencies into account.

Now, the total strain energy of the beam is presented by the sum of the strain energy of the bending moment and the shear strain energy, which is shown as the following form,

$$V = \frac{1}{2}\int_0^L EI\left(\frac{d^2y_b}{dx^2}\right)^2 dx + \frac{1}{2}\int_0^L k'GA\left(\frac{dy_s}{dx}\right)^2 dx\,, \qquad (3)$$

where, EI bending rigidity of the hull
$k'GA$ shear rigidity of the hull
y_b deflection due to bending moment
y_s deflection due to shear force

[8] S. Timoshenko. "Vibration Problems in Engineering." p. 338.

x length coordinate
L overall length of the hull

On the other hand, the maximum kinetic energy of the vibration is presented as follows,

$$T=\frac{1}{2}\frac{(2\pi N)^2}{g}\left\{\int_0^L wy^2dx+\int_0^L I_r\left(\frac{dy}{dx}\right)^2dx\right\}, \tag{4}$$

where
g acceleration of gravity
N natural frequency of the hull vibration
w mass per unit length of the hull
$y(=y_b+y_s)$ total deflection of the hull
I_r mass moment of inertia of the section about its neutral axis

Equating (3) and (4), the natural frepuency of the hull vibration based on the shearing vibration with the correction factors becomes as the following form,

$$N=\frac{60}{2\pi}\sqrt{\frac{g\int_0^L k'GA\left(\frac{dy_s}{dx}\right)^2dx}{\int_0^L wy_s^2dx\,(1+\gamma_b)(1+\gamma_r)}}, \tag{5}$$

where,
$$(1+\gamma_b)=\frac{\int_0^L wy^2dx\Big/\int_0^L wy_s^2dx}{1+\left\{\int_0^L EI\left(\frac{d^2y_b}{dx}\right)^2dx\Big/\int_0^L k'GA\left(\frac{dy_s}{dx}\right)^2dx\right\}}, \tag{6}$$

$$(1+\gamma_r)=1+\frac{\int_0^L I_r\left(\frac{dy}{dx}\right)^2dx}{\int_0^L wy^2dx}. \tag{7}$$

For the computations of the above factors 6) and 7), the functional form of the shear deflection y_s should be required. Fortunately, since the exact solutions of the shearing vibration are obtained as presented in the previous paragraph and in Appendix, the results of the above computations are correctly obtained. On the other hand, the deflection due to the bending moment denoted by y_b in the above factors is calculated by the use of the relations presented as follows,

$$M=\int Q\,dx=\int k'GA\frac{dy_s}{dx}dx,$$

and also,
$$M=-EI\frac{d^2y_b}{dx^2}dx,$$

then,
$$\frac{dy_b}{dx}=-\int\frac{1}{EI}\int k'GA\frac{dy_s}{dx}dx^2+C_1,$$

where, M bending moment
Q shear force
C_1 integration constant

Finally, the correction factors presented by (6) and (7) are both computed under the assumption of the value of k'. The numerical calculations of the correction factors and the natural frequencies are shown in the paragraph 5.

4. Shear Rigidity of a Ship's Hull. The shear rigidity of the section of the thin hollow beam like as the ships' hulls is presented by the form,

$$k'A = \frac{I^2}{\int_A \frac{m^2}{(\sum t)^2} dA}, \tag{8}$$

where, I moment of inertia of the section of the beam about its neutral axis
A sectional area
m moment of area above considered line about the neutral axis of the section
t thickness of the shell at the cross point of the line considered

The examples of the numerical computation of the shear rigidity of thin hollow rectangular section have already been carried out by J. L. Taylor[9] and Prof. Y. Watanabe.[10] Also, rough estimation of the shear rigidity is presented by W. Hovgaard[11] as the well known form,

$$k'A = A_w, \tag{9}$$

where, A_w area of the vertical web plate for vertical shear force

The effects of the under two or three decks upon the rigidity in way of the parallel body of the hull have been carried out by the above two investigators. However, so far as the author is aware, no investigation of the influences of the deck openings and of the forms of the hull section in way of the entrance or the run has been carried out except the calculations of k' for the solid beam of ovaloid section presented by R. D. Mindlin and H. Deresiewicz.[12] In the first place, the effects of the ratio of the thickness of the flange and the web, and also of the ratio of the breadth and the depth of the section of the thin hollow rectangular tube upon the

[9] J. L. Taylor. "The Theory of Longitudinal Bending of Ships." Trans. N.E.C.I. 1924–25.
[10] Y. Watanabe. "Effect of Shear Flexibility on the Flexural Vibtation of Ships." Rep. of Kyushu Soc. of N.A. Japan. 1934.
[11] W. Hovgaard. "Design of Warship."
[12] R. D. Mindlin and H. Deresiewicz. "Timoshenko's shear Coefficient for Flexural Vibrations of Beams." The reprinted paper of Proceedings of the 2nd U.S. National Congress of Applied Mechanics. 1954.

shear rigidity were computed in the present paper. The computed results are shown in Fig. 3. The values of k' obtained by the approximation (9) is presented in the same figure by $\bar{k}'$. As will be seen in the figure, k' is comparatively low in both cases of the vartical and horizontal shear provided the values of B/D take 1.5 to 2 and with the small value of t_f/t_w like as the ordinary hull form with deck openings. In the second place, the shear rigidities of the typical sections in way of the run, the parallel body and the entrance of the ship's hull with deck openings are calculated and the results are shown in Fig. 4 (a), (b), (c) respectively.

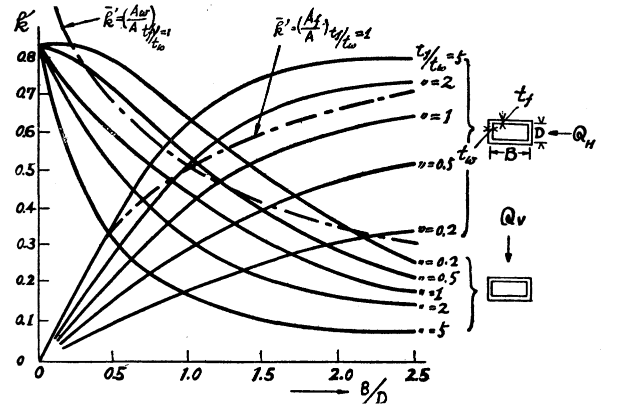
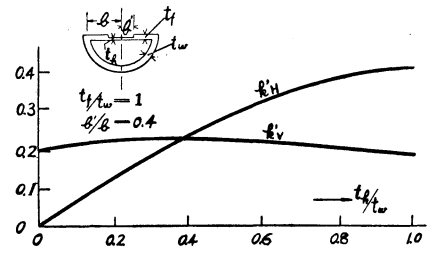
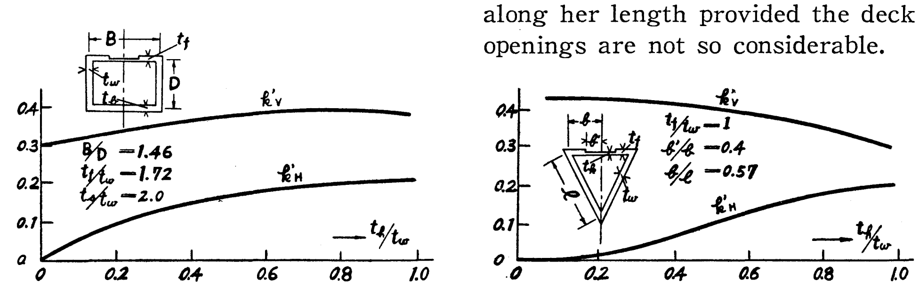


FIG. 3. Values of k' of the thin hollow rectangular tube.

The equivalent thickness of the deck plate in way of the deck openings is calculated considering the suggestion of J. L. Taylor.[13] There is no considerable variation of the k' except the section with large deck openings and sharp V-shaped section in horizontal shear and the section of the semi-circular hollow tube in vertical shear. In general case, the distribution of the shear rigidity may be considered almost proportional to the sectional area of the hull along her length provided the deck openings are not so considerable.



(a)



(b) (c)

FIG. 4. Values of k' of various section of thin hollow beams with deck openings.

5. Calculations of the Natural Frequencies of the Hull Vibration.

The equation of the calculation of the natural frequencies including higher modes of the hull vibrations obtained from the results of the foregoing investigation is presented as follows,

$$N=\frac{60}{2\pi}\sqrt{c}\,\lambda\sqrt{\frac{gk_0'GA_0}{\Delta L(1+\gamma_b)(1+\gamma_r)(1+\gamma_w)}}, \tag{10}$$

where N natural frequencies of n-noded mode per min

$\sqrt{c}\,\lambda$ characteristic values of n-noded mode depend on the ship's form and distributions of the shear rigidity and of the mass (see Appendix.)

$k_0'GA_0$ shear rigidity of the midship section

$(1+\gamma_b)$, $(1+\gamma_r)$, $(1+\gamma_w)$ correction factors of bending moment, rotational inertia and the virtual mass,[14) 15) 16) 17) 18)] respectively in n-noded mode

Δ, L displacement and over all length of a ship respectively

The natural frequencies up to the eight-noded mode of the vertical vibration of a hull were computed, for an example, by the use of the data of 10,000 ton d.w. modern cargo vessel, whose paticulars are shown in Table 1.[5)] The numerical calculations were carried out under the assumption of the four kinds of the distributions of the shear rigidities and of the masses, namely, $m\ (=n)=0$, 1/2, 1, 2. The vibration profiles of the fundamental node produced by the shear force and by the bending moment which denoted by η_s and η_b respectively, are presented with above four kinds of the hull forms in Fig. 5, and the ratio of η_b and η_s for the above values of m and n are shown in the Table. 2.

The correction factors of the bending moment and the rotatory inertia

TABLE 1. Particulars of 10,000 ton d.w. cargo ship whose natural fraquencies were calculated and meassered.

L, B, D	$140^M \times 19^M \times 10.5^M$
d	4.3^M
Δ	7,470 ton
A_0	1.632 M^2
k'_0	0.3
ξ_1	0.3
$m(=n)$	0, 1/2, 1, 2

TABLE 2. Ratios of η_b and η_s in cases of varions forms and nodes.

	$(\eta_b/\eta_s)_{k'=0.3,\ \xi_1=0.3}$			
$m(=n)$	2-node	4-node	6-node	8-node
0	5.03	0.568	0.254	0.142
1/2	3.80	0.305	0.130	0.070
1	3.43	0.250	0.105	0.040
2	2.58	0.190	0.050	0.015

13) J. L. Taylor. "Ship Vibration Periods." Trans. N.E.C.I. 1927–28.

14) T. Terada. "On the Vibration of a Bar Floating on a Liguid Surface." Proc. Tokyo Math-Phys. Soc. III, pp. 103–109, 1906.

15) H. W. Nicholls. "Vibration of Ships." Trans. I.N.A. 1924.

16) F. N. Lewis. "The Inertia of the Water Surrounding a Vibrating Ship." Trans. Soc. N.A.M.E. 1929.

17) C. W. Prohaska. "Vibrations Verticales du Navier." Associa. Tech. M. et A. 1947.

and of the virtual mass for the natural frequencies were computed as shown in Fig. 6. The shear rigidity coefficient is assumed to be equals to 0.3 in the present calculations.

In the results, the natural frequencies of the vertical vibration of the typical cargo boat mentioned above were computed as shown in Fig. 7 with the parameters of various distributions of the rigidities and the masses. The results of the measurements of the natural frequencies of the above ship are plotted in the same figure.

The relation between natural frequency and the number of node is found in fairly good agreement provided $m\,(=n)=2$ as will be seen in Fig. 7.

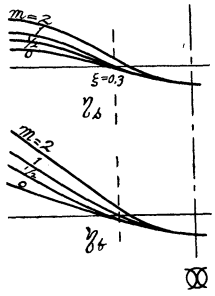


FIG. 5. Two-noded vibration profiles of shear flexural and of bending flexural vibration with various distributions of rigidities and masses.

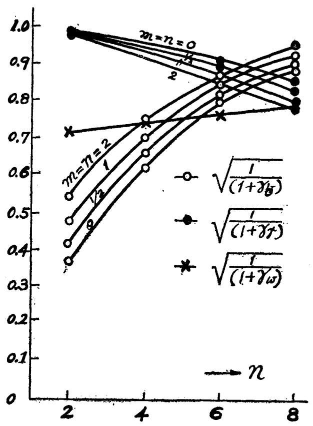


FIG. 6. Computed values of correction factors for bending flexibility, rotatory inertia and virtual mass of water.

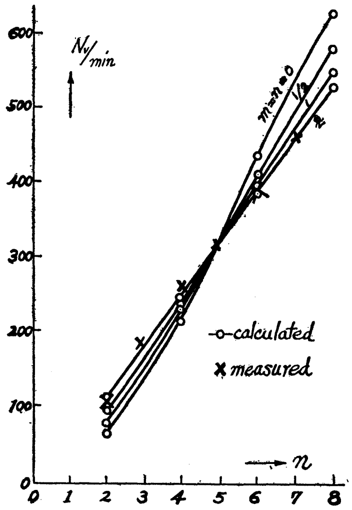


FIG. 7. Comparison of the natural frequencies obtained by the theory and the results of measurements.

6. Empirical Formula Estimating the Natural Frequencies of the Hull Vibtations. The foregoing method of the calculation of the natural frequency of the hull vibration is applicable under the assumptions of the distributions of the rigidity and the mass along the length of a ship's hull. In the actual ships, however, these distributions are not so simple like as those of the above assumption, moreover, the calculations are much laborious in the design room. Thence, the empirical formula of the estimation of the natural frequencies of the hull vibration based on the shearing vibration for the practical use in the stage of the initial design of the ships is presented in this paragraph.

The formula computing the natural frequencies based on the result of the foregoing investigation is presented as the following form,

$$N_n = a(n-1)^{\mu}\sqrt{\frac{A_w \times 10^5}{\Delta_1 L}}, \tag{11}$$

where
N_n n-noded natural frequency of the flexural vibration of the hull per minute
n number of nodes of flexural vibration
A_w sectional area of the web plate amidship in M^2
Δ_1 displacement with virtual mass of water, for instance, $\Delta_1 = \Delta\left(1{,}2 + \frac{B}{3d}\right)$, B, d, breadth and draft respectively in vertical vibration, Δ is displacement in ton[18]
L length between perpendiculars of ship in M
μ, a empirical factors obtained from the results of vibration measurements

The empirical factors μ and a are computed by the following equations respectively,

$$\mu = \frac{\log N_n/N_2}{\log(n-1)}, \quad (n = 3, 4, \cdots),$$

$$a = \frac{N_n}{(n-1)^{\mu}\sqrt{\frac{A_w 10^8}{\Delta_1 L}}}, \quad (n = 2, 3, \cdots),$$

where N_2 and N_n are measured natural frequencies of the hull vibrations. The mean values of μ and of a which obtained from the results of the vibration test of the thirteen cargo vessels of 10,000 ton d.w. class shelter decker type in the trial trip condition, in the case of the vertical vibration, for an example, are accounted as follows,*

$$\mu = 0.842, \quad a = 26.85.$$

The results of calculations of the natural frequencies of

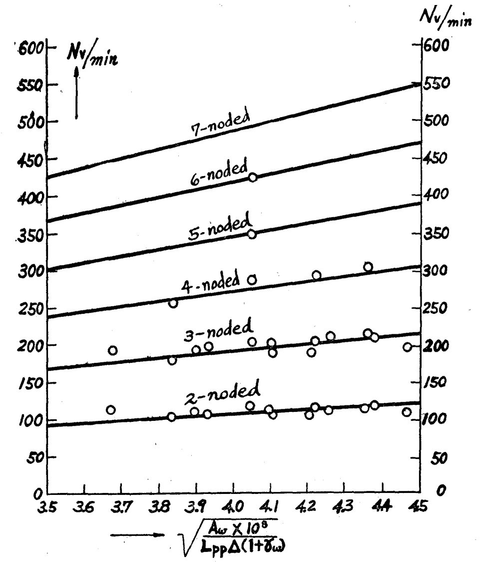


FIG. 8. Natural frequencies of the vertical vibration of 10,000 ton d.w. class shelter decker type cargo ships obtained by the use of the empirical formula and comparisons with measured ones.

* μ and a of three islander type and tankers are accounted of slightly different from those of shelter deker type.

the vertical vibration up to seven noded mode by the use of (11) with the above values of μ and a are shown in the Fig. 8. The measured results are plotted in the same figure.

Fig. 8 will be available for the estimation of the natural frequencies including higher modes of the vertical hull vibrations in the design stage.

The same method as above is also applicable to the horizontal transverse vibration of ships' hulls and to the other types of the vessels.

Replacing A by BD^3/L^2 and always put $n=2$ in the present formula, the same formula as which has been presented by F. H. Todd[18] is obtained. Also, Baier's rough estimation formula[19] of the frequencies of the higher node mode of the hull vibration is obtained provided $\mu=1$ and N_2 is known. In general case, however, μ takes the value less than unity, thence the Baier's formula is applicable only in the special type of vessels, for instance, in the case of the vertical vibration of tankers of large type.

7. Conclusions. The theoretical investiation into the higher mode profiles and its natural frequencies of the hull vibration was carried out taking the shearing vibration into consideration. The vibration profiles of the beam of variable cross section like as ship's hull are obtained and the effects of the distributions of the shear rigidity and of the mass along her length upon the amplitude of the vibration were clearly shown in the high mode of the hull vibration. By the process of the corrections of the bending moment and the rotational inertia of the section, the natural frequencies up to the eight-noded modes of the vibration of the typical hull formed beams are obtained. The calculations of the natural frequencies of the 10,000 ton d.w. cargo vessel were carried out as an example, the results of the calculations are clearly explained the relation between the natural frequencies and its corresponding nodes of the measured ones. The convenient formula based on the foregoing theory of the shearing vibration was presented and whose empirical factors were determined by the data of the measured natural frequencies of the vertical vibrations of the thirteen modern shelter decker cargo vessels built in Japan in trial trip condition. The empirical formula will be usefull for estimation of the criticals up to the blade frequencies of the vertical vibrations including higher modes of this type of cargo ships in the stage of initial hull design.

The investigations described in this paper were discussed in the meetings of the Western Section of Shipstructure Committee of Japan in 1955. Valuable guidance and help is afforded by Prof. Y. Watanabe in the preperation of this paper. Also, the data of vibration measurements of cargo ships recentely have been built were given by the courtesies of the leading shipbuilders in Japan.

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18) F. H. Todd. "Some Measurements of Ship Vibration." Trans. N.E.C. 1933.

19) L. A. Baier and J. Ormondroyd. "Vibration of the Stern of Single Screw Vessel." Trans. S.N.A.M.E. Vol. 60. 1952.

Appendix.

Shearing Vibration of the Beam with Variable Cross Section

1. Solutions of the equation of the shearing vibration.

The equilibriun of the element of the beam which is subjected by the vibratory shear force Q and the inertia force is presented as the following form,

$$\frac{\partial Q}{\partial x}dx = \frac{w}{g}\frac{\partial^2 y}{\partial t^2}dx\,, \tag{12}$$

where
- x length coordinate
- y deflection perpendicular to x direction
- w mass of element per unit length
- g acceleration of gravity
- t time coordinate

Now, the shear force is represented by the slope of the element, namely,

$$Q = k'GA\frac{\partial y}{\partial x}\,, \tag{13}$$

where, k' is the coefficient of the shear rigidity which is defined by S. Timoshenko.[8] A is the sectional area of the element of the beam and G is the shear modulus of the material of the beam.

The beam is now under the action of the simple harmonic motion, or $y = y\cos pt$, provided $x/L = \xi$ and $y/L = \eta$, (13) becomes

$$\frac{d}{d\xi}k'GA\frac{d\eta}{d\xi} + p^2\frac{w}{g}\eta = 0\,. \tag{14}$$

For the sake of simplicity, the section of the beam is assumed to be of the symmetrical form about its midpoint and has the variable cross section between the length $0 \leqq \xi \leqq \xi_1$, also the origine is taken at the end of the beam. The shear rigidity and the mass are assumed to be of the forms,

$$\left.\begin{aligned} k'GA &= k_0'GA_0(\xi/\xi_1)^m \\ w &= w_0(\xi/\xi_1)^n \end{aligned}\right\} 0 \leqq \xi \leqq \xi_1\,, \tag{15}$$

where, m and n are both positive number, $k_0'GA_0$ and the w_0 are shear rigidity and the mass respectively at the section of the parallel body $\xi_1 \leqq \xi \leqq \xi_2$, the coordinate at the midpoint of the beam is denoted by ξ_2. Substitute the relation (15) in (14), the following Bessel equation is obtained,

$$\frac{d^2\eta}{d\xi^2} + \frac{m}{\xi}\frac{d\eta}{d\xi} + \lambda_1^2\xi^{n-m}\eta = 0\,. \tag{16}$$

The solutions of (16) is obtained as following form provided $m - n < 2$

$$\eta = A\xi^\mu J_{-\frac{\mu}{\mu+\nu}}\left(\frac{\lambda_1}{\mu+\nu}\xi^{\mu+\nu}\right) + B\xi^\mu J_{\frac{\mu}{\mu+\nu}}\left(\frac{\lambda_1}{\mu+\nu}\xi^{\mu+\nu}\right) \tag{17}$$

and also provided $m-n=2$

$$\eta = A\xi^{-\frac{m-1}{2}}\sin(\beta\log\xi) + B\xi^{-\frac{m-1}{2}}\cos(\beta\log\xi)\,, \tag{18}$$

where, $\lambda_1^2 = \xi_1^{m-n}\lambda^2$

$$\lambda^2 = \frac{p^2 w_0 L}{k_0' G A_0} \tag{17$'$}$$

$$\mu = \frac{1-m}{2}\,, \quad \nu = \frac{1+n}{2}$$

$J_{\frac{\mu}{\mu+\nu}}$, J-Bessel function of $\frac{\mu}{\mu+\nu}$-th order

$$\beta = \sqrt{\lambda_1^2 - \frac{(m-1)^2}{4}}\,.$$

The solutions in way of the parallel body are presented as the well known form,

$$\eta = A\sin\lambda\xi + B\cos\lambda\xi\,. \tag{19}$$

The characteristic value λ and the integration constants A and B are determined by the boundary conditions at the sections $\xi=0$, $\xi=\xi_1$ and $\xi=\xi_2$. The determination equation of λ of (17) is presented as the following form

$$\frac{J_{-\frac{\mu}{\mu+\nu}}\left(\frac{\lambda}{\mu+\nu}\xi_1\right)}{J_{-\frac{\mu}{\mu+\nu}+1}\left(\frac{\lambda}{\mu+\nu}\xi_1\right)} + \begin{cases} \cot\lambda(\xi_2-\xi_1) \\ -\tan\lambda(\xi_2-\xi_1) \end{cases} = 0\,, \tag{20}$$

in the above equation, upper side of the second term takes the λ in the case of even number nodes, and the lower side the odd number nodes.

The foregoing calculation is applicable for the beam of variable cross section with parallel body like as ship's hull form. In the ship's hull, however, the mass of the corresponding beam is replaced by the displacement of a ship. Thus, the natural frequencies of the ship's hull is presented by the following form, which is deduced from (17)$'$.

$$N' = \frac{60}{2\pi}\sqrt{c}\,\lambda\sqrt{\frac{g k_0' G A_0}{\Delta L}}\,, \tag{21}$$

where, N' natural frequencies of the hull vibration per min. without corrections

Δ displacement of ship

$$c = \frac{\int_0^{\xi_1} w\,d\xi + w_0(\xi_2-\xi_1)}{w_0\xi_2}\,.$$

The actual natural frequencies of the hull vibration are obtained from N' with the corrections of bending moment and rotatory inertia of the section and of the virtual mass of water surrounding hull surface which described in the paragraph 3 of this paper.

2. Equation of the shearing vibration deduced from the bending theory under the consideration of the shear deflection and the rotatory inertia.

For the sake of simplicity, the vibration of the uniform beam is investigated in the following discussion. The fundamental equation of the vibration of the beam taking the effects of the shear flexibility and of the rotatory inertia into account is presented as follows,[8]

$$EI\frac{\partial^4 y}{\partial x^4}-\left(\frac{\rho I}{g}+\frac{EI\rho}{gk'G}\right)\frac{\partial^4 y}{\partial x^2\partial t^2}+\frac{\rho I}{g}\frac{\rho}{gk'G}\frac{\partial^4 y}{\partial t^4}+\frac{\rho A}{g}\frac{\partial^2 y}{\partial t^2}=0. \qquad (22)$$

The equation is easily deduced to ordinary form provided $y = y\cos pt$, namely

$$EI\frac{d^4 y}{dx^4}+p^2\left(\frac{\rho I}{g}+\frac{EI\rho}{gk'G}\right)\frac{d^2 y}{dx^2}+\left(\frac{\rho I}{g}\frac{\rho p^4}{gk'G}-\frac{\rho A p^2}{g}\right)y=0, \qquad (23)$$

as will be seen in the above equation, the coefficient of y consists of two terms, the first term of this coefficient is smaller than the second term and is ignored for the second term provided the critical p takes small value, for instance, in the case of fundamental mode of the hull vibration. The second place, the first term of the coefficient of y becomes large when the value of p increases, consequently when both terms take the equal values, or $p = p_1$, the last term in (23) vernishes. The third place, p becomes higher than p_1, the coefficient of y takes positive sign. The solutions of the equation (23) in the foregoing three cases caused by the values of p take different forms as follows,[20]

$$p < p_1;\quad y = A\sin qx + B\cos qx + C\sinh rx + D\cosh rx, \qquad (24)$$

$$p = p_1;\quad y = A\sin sx + B\cos sx + Cx + D, \qquad (25)$$

$$p > p_1;\quad y = A\sin q_1 x + B\cos q_1 x + C\sin r_1 x + D\cos r_1 x. \qquad (26)$$

The frequency of the hull vibration excited by the blade flequency is approximately estimated by the second place in the above three cases, which has alreaby been presented in the author's previous paper.[21] The fundamental equation whose solutions present by (25) is of the shearing vibration provided the coefficient of y vernishes and replacing y by y_s, moreover ignoring the rotational effect in the equation (23). Thence, the fundamental equation of the shearing vibration is also obtained from the equation based on the bending moment taking the effects of the shear flexibility and of the rotatory inertia into account.

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