

STUDIES ON WAKE VORTICES (II): Experimental Investigation of the Wake behind Cylinders and Plates at Low Reynolds Numbers

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STUDIES ON WAKE VORTICES (II)

Experimental Investigation of the Wake behind Cylinders and Plates at Low Reynolds Numbers

By Sadatoshi TANEDA

The wakes behind cylinders and plates were photographically investigated at Reynolds numbers from 0.1 to 2,000. Main results are as follows.

In the case of a circular cylinder, the rear twin-vortices begin to form at $R=5$ (R is Ud/ν), then become more and more enlarged as R increases, and finally become asymmetrical at about $R=45$. The laminar wake, on the other hand, begins to oscillate sinusoidally some distance downstream at $R=30$.

In the case of a flat plate parallel to the flow, the wake begins to oscillate sinusoidally some distance downstream at about $R=700$ (R is Ul/ν , where l is the breadth of the plate).

Lastly, some detailed knowledges about the structure and the nature of the Karman vortex street are obtained by a new method.

1. Purpose and Outline of the Experiment. As for the flow of a fluid at low Reynolds numbers, some experimental and theoretical investigations have been made up to the present, but not a few problems are left still unsolved. It is well known that the calculations are successfully made with the approximations of Stokes or Oseen for very small Reynolds numbers, but as Reynolds number is increased, Oseen's as well as Stokes's approximations become more and more remote from the exact solution of the Navier-Stokes equations. On the other hand, the experiments which serve as a verification or guidance of the theories seem to be in great lack. The present author planned to investigate minutely the wake behind circular cylinders and flat plates at Reynolds numbers from 0.1 to 2,000.

First, it was tried to determine the critical Reynolds number at which twin-vortices make their appearance in the rear of a circular cylinder. Concerning this problem there are several theoretical calculations. It is from this aspect that the author was interested in the subject. Though some photographs of the flow around cylinders at low Reynolds numbers have already been taken by several experimenters, they are not fit for determining the critical Reynolds number, because the stream-line in the immediate neighborhood of the rear stagnation point is not clear. The author has succeeded in getting the clear stream-lines by giving long duration of the exposure.

Secondly, the experiments were made concerning the oscillation of the wake behind a circular cylinder. It is extremely difficult to detect the oscillation which appear in the wake for the first time, as it is very faint. In the present experiment, however, the oscillation was sensibly detected utilizing the reflection of light from the aluminium dust suspended in the water. And it was found that the wake begins to oscillate at about $R=30$ and that the twin-vortices are independent, at least directly, of the oscillation of the wake.

Thirdly, the experiments were made using a model coated with condensed milk. This method enables us to see clearly the positions of separation point and the structure of twin-vortices. This series of experiments showed that the solutions of Navier-Stokes equations are in good agreement with the experimental data even in the case of $R=40$.

Fourthly, the wake behind flat plates parallel to the direction of motion was investigated. In the case of a flat plate the separation does not occur, in other words twin-vortices do not appear, for this reason it is very interesting to investigate the relationship between the wake of a circular cylinder and that of a plate.

Lastly, the experiments by a new method clarifying the structure and the nature of the Karman vortex street were carried out and some detailed knowledges concerned were obtained.

2. Apparatus and Experimental Method. The experiments were carried out in the glass water tank of 100 cm in length, 20 cm in breadth and 30 cm in depth. The carriage, to which the model was attached, was moved on the straight rails above the tank by means of a nut engaging with the lead screw which was turned by a 200 watt a.c. motor through a gear box. The velocity of the carriage was varied in 10 steps by exchanging the gears. According to circumstances, a d.c. motor could also be used. To obtain a uniform velocity, every part of the apparatus was carefully adjusted for a long period. Fig. 1 shows the general arrangement of the apparatus.

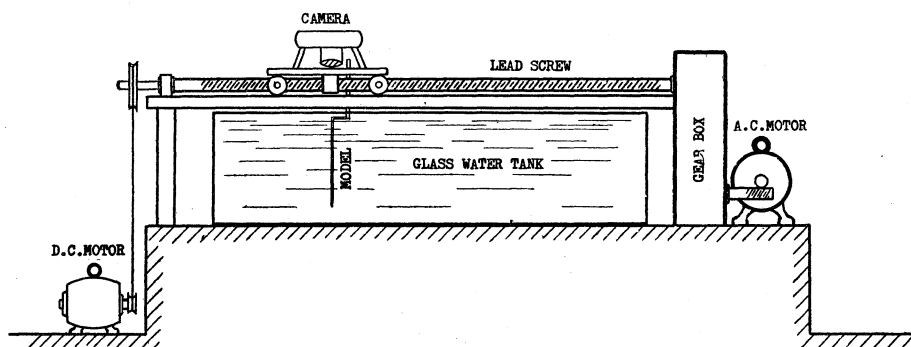


FIG. 1. Schematic diagram of the apparatus.

In order to visualize the motion of water below the free surface, aluminium dust was suspended uniformly in the water. And illuminating it by a sheet of intense light through a narrow slit, we could clearly observe the motion of water in the illuminated layer from the direction normal to the light. A 500 watt incandescent lamp with a line filament was used as the source of light.

The camera used was a Nikon F 1.4, and when necessary a Mirax (an enlarging equipment) was put on.

The sizes of the models are given in the Table 1.

Circular Cylinder		
d (mm)	L (mm)	d/D
6.00	300	1/33
3.93	250	1/51
2.95	250	1/68
1.99	200	1/100
1.54	200	1/130
1.02	180	1/196
Flat Plate		
l (mm)	b (mm)	L (mm)
37.0	0.24	280
20.1	0.24	250

d : Diameter of the cylinder
 L : Length of the model
 D : Width of the tank
 l : Breadth of the flat plate
 b : Thickness of the flat plate

TABLE 1.

3. The Reynolds Number at which Twin-Vortices begin to form in the Rear of a Cylinder. Concerning the critical Reynolds number at which twin-vortices begin to form in the rear of a circular cylinder, there has been no investigation except that by H. Nisi & A. Porter, and little is known of the accurate value of the critical Reynolds number. This seems to be chiefly due to the extreme difficulties of the experimental technique.

In the present experiments the method of moving a model through still water was adopted. For such a low Reynolds number, even a very small residual motion of water cannot be neglected, therefore great care was taken to allow the water to come to rest before beginning an experiment.

A careful photographic technique was also required to get a sufficiently clear and sharp picture. A Mirax enlarging equipment was put on and the duration of the exposure was made sufficiently long.

In Plate 1 are shown the examples of the photographs obtained.

It can be noticed from the photographs that the twin-vortices are seen at $R=7$,*) though they are very small, but they are not seen at $R=6$. (R is Ud/ν , where d is the diameter of the cylinder; U is the model speed and ν is kinematic viscosity.)

In the neighborhood of the critical Reynolds number, it is very difficult to detect the twin-vortices accurately for two reasons: (i) the dimension of twin-vortices is still very small, (ii) the position they appear is situated in the immediate neighborhood of the rear stagnation point where the velocity of the flow is extremely small.

The size of twin-vortices, however, can be accurately measured at Reynolds numbers exceeding 7.

*) The photograph for the case $R=7$ has been broken by accident and is not inserted here.

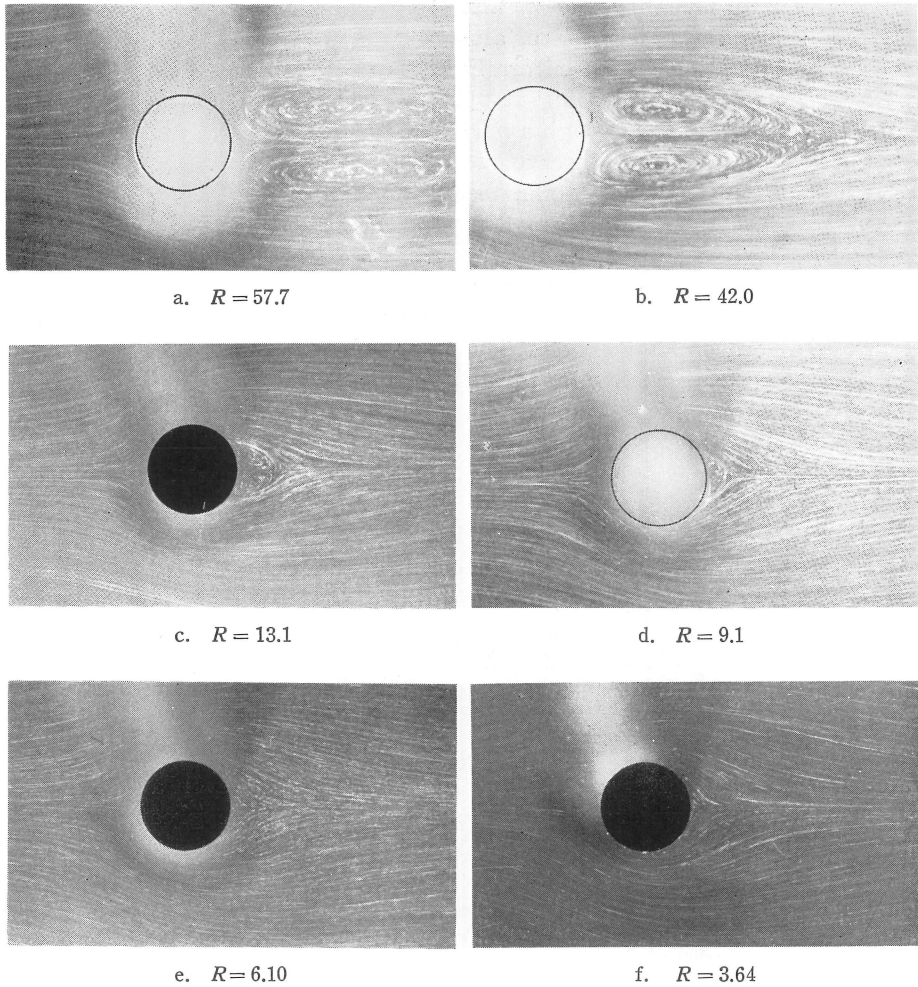


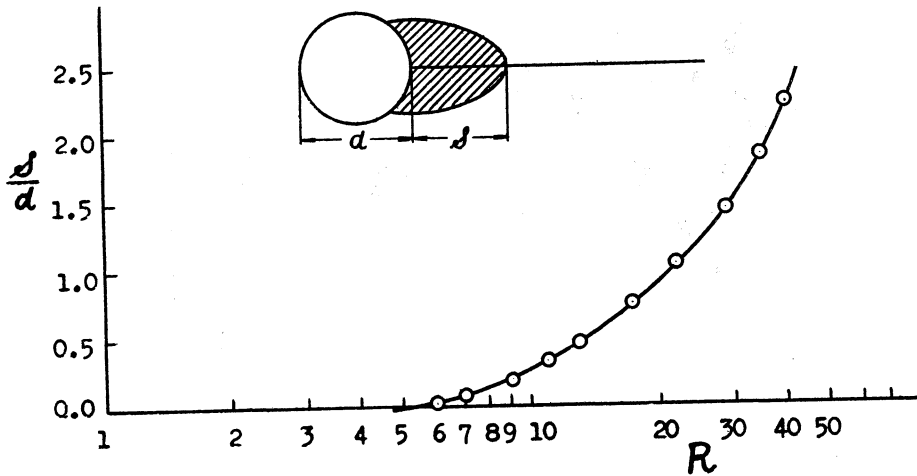
PLATE 1.

In Fig. 2 the size of twin-vortices is plotted against the Reynolds numbers, and from this curve the critical Reynolds number can be deduced.

It may be concluded that the critical Reynolds number at which the twin-vortices begin to form in the rear of a circular cylinder is 5.

Further, the experiments indicate that the wall effect of the tank can be neglected when $d/D < 1/50$ (where d is the diameter of the cylinder and D is the tank width).

An experiment using microscopic photography, for the determination of more accurate value of the critical Reynolds number, will be carried out in the near future.

FIG. 2. Plot of s/d against R .

4. Twin-Vortices. The twin-vortices appear in the rear of a cylinder at $R=5$, and as the Reynolds number is increased these two eddies grow and become more and more elongated in the flow direction. But they adhere stably on the cylinder until the Reynolds number of about 45 is reached.

Thus, the wake behind a circular cylinder consists of two parts, twin-vortices and a trail, at Reynolds numbers between 5 and 45. (See Fig. 3.)

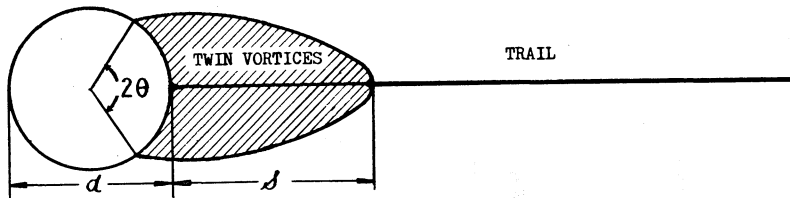
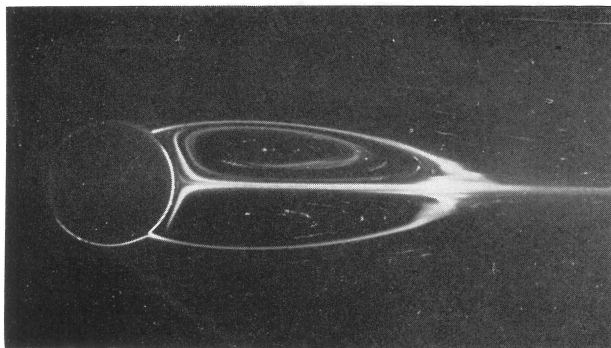
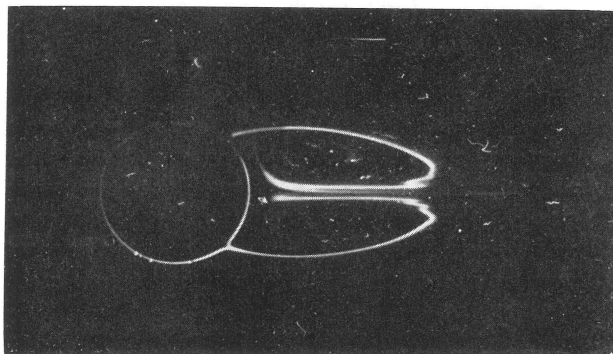
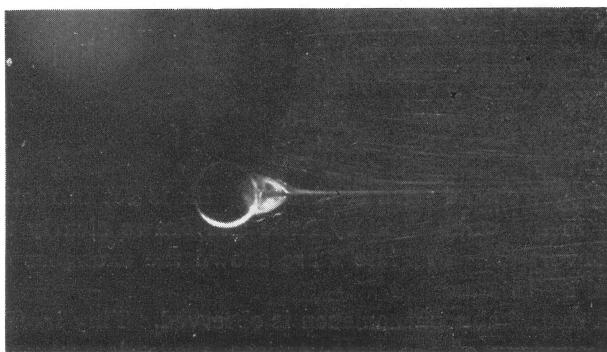


FIG. 3.

When a cylinder coated with condensed milk is set in motion in water, the milk melts in the wake little by little and draws clearly the boundary line of the twin-vortices. In Plate 2 are shown the examples of the photographs obtained by this method.

At $R > 35$ a remarkable phenomenon is observed. Tiny irregular gathers appear in the boundary of the twin-vortices, move downstream along the boundary line till they reach the rear end of the twin-vortices, tremble for a short time and die away. This small irregular motion, however, does not produce any effect on the flow behind the twin-vortices. The trail is sensibly laminar in the neighborhood of twin-vortices until the Reynolds number of about 45 is reached.

a. $R = 41.0$ b. $R = 28.4$ c. $R = 12.5$

The trail, on the other hand, begins to oscillate sinusoidally some distance downstream at $R=30$, and its amplitude becomes larger as the Reynolds number is increased. When the Reynolds number of 45 is reached, the oscillation of the trail begins to exert an influence upon the twin-vortices. At Reynolds numbers above 45, the gathers of the twin-vortices are elongated along the trail, shed alternately from each of the twin-vortices, and pass downstream, where they arrange themselves in a configuration corresponding to that of a Karman vortex street. But a large portion of the twin-vortices is still stably attached on the cylinder. The size of the gather which travel downstream increases with the Reynolds number, and for Reynolds numbers above 150 a large portion of the twin-vortices are shed.

In 1953 Kawaguchi carried out calculations about the flow around a circular cylinder for the case $R=40$, integrating numerically the exact Navier-Stokes equations. Comparison between the calculations and the experimental data is shown in Table 2. It may be concluded from Table 2 that the solutions of Navier-Stokes equations are in good agreement with the experimental data in the neighborhood of a circular cylinder even for $R=40$.

	Calculated	Measured
Position of separation (θ)	52.2°	53°
Size of twin-vortices (s/d)	1.9	2.1

TABLE 2. Comparison of theory and experiment.

5. Oscillation of the Wake behind a Circular Cylinder. The experiments about the oscillation of the wake behind a circular cylinder were carried out in the past by Thom (1933) and Kovasznay (1949), but comparatively little is known about the process of its development. The author planned to investigate this problem minutely.

As the oscillation which appear for the first time in the trail is very faint, its detection is extremely difficult. In the present study, however, the oscillation was sharply detected by means of the aluminium dust method. When the light comes into the water with suspended aluminium dust, we see innumerable tiny particles continually twinkling in the illuminated layer, and of course the brightness distributes homogeneously over the illuminated section so far as no motion of water exists. However, as soon as the wake begins to oscillate, there appears periodically striped pattern of brightness. As was clarified in the previous paper, this pattern relates to the direction of the local flow. Thus the wave length of the wake was determined by examining the photographs. The relation between the wave length λ and the frequency N is given by

$$\frac{d}{\lambda} = \frac{Nd}{U} \cdot \frac{U}{U-v},$$

where U is the velocity of the cylinder and v the velocity of the vortex filament. The wave length shows a tendency to increase downstream, so

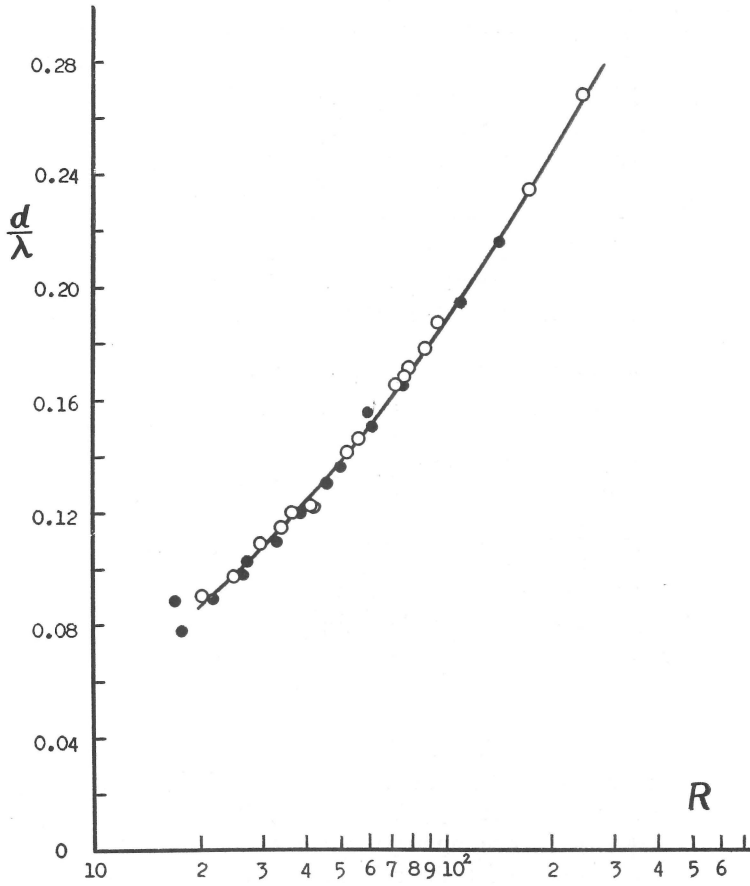
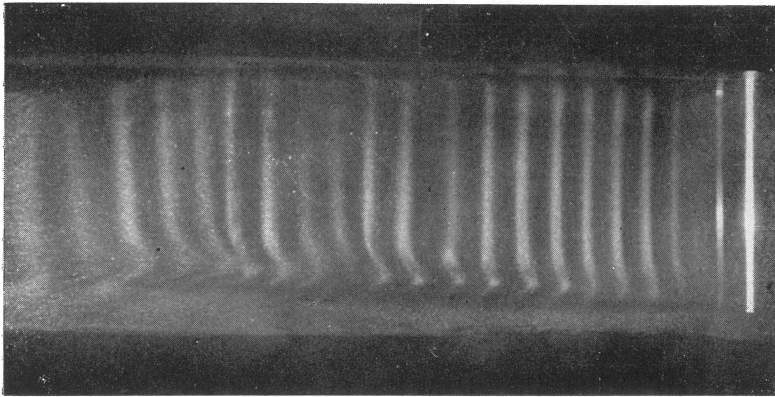


FIG. 4. Plot of d/λ against R .



$R = 42.0$

PLATE 3. Oscillating wake behind a circular cylinder.

all the measurements were made at 20 diameters downstream from the cylinder.

The results are illustrated graphically in Fig. 4, and an example of the photographs is shown in Plate 3. The present experiment indicates that the critical Reynolds number at which the wake begins to oscillate is 30.

Of course, if the cylinder is given a slight shock, the oscillation of the wake appears even at Reynolds numbers below 30. It should be noticed that the $d/\lambda - R$ curve is continuous at Reynolds numbers above 20, and does not show any sign of discontinuity at $R = 30$ and at $R = 45$.

As oscillation of the wake is affected strongly by the wall of the water tank, the present experiment was carried out at $d/D < 1/100$.

The process of the development of the wake behind a circular cylinder is summarized in Fig. 5.

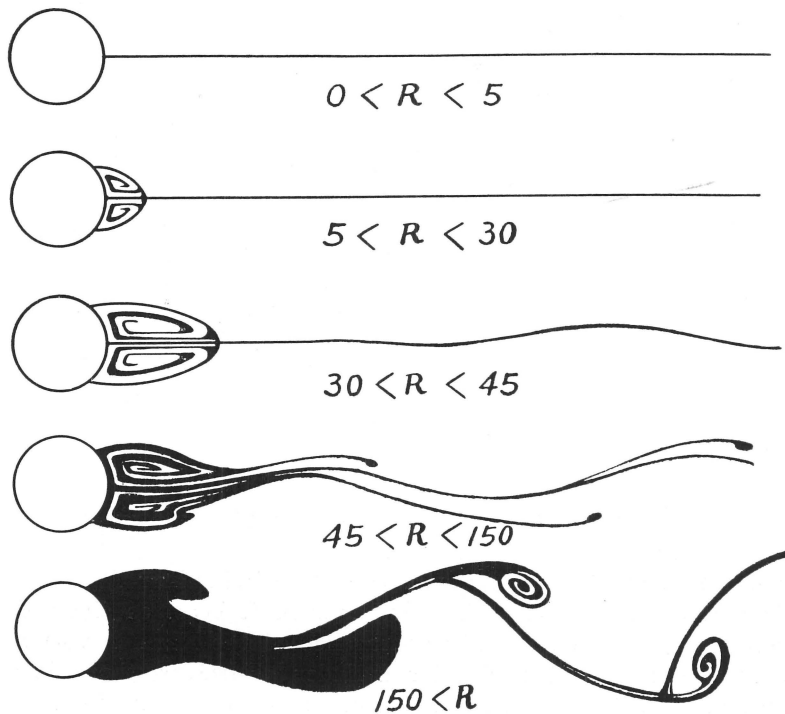


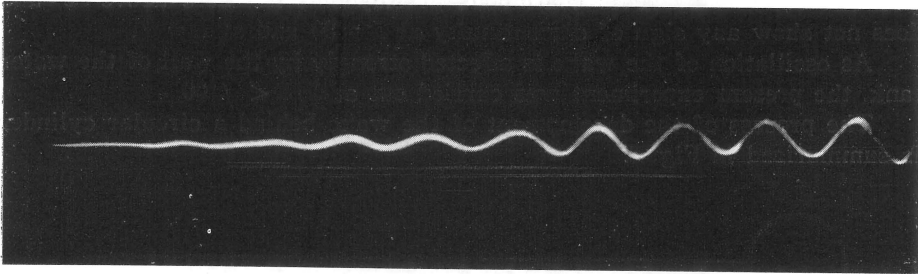
FIG. 5. Process of the development of the wake behind a circular cylinder.

6. Oscillation of the Wake behind a Flat Plate. In the case of the circular cylinder the phenomenon is complicated as the wake is constituted from two parts, the twin-vortices and the trail. On the other hand, in the case of the flat plate parallel to the direction of motion, the phenomenon is simple because it consists of only one part, i.e. the trail.

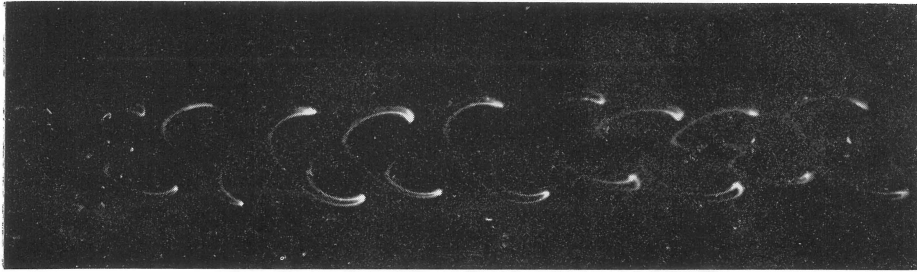
The plates used were made of steel and carefully tapered smoothly to sharp edges. The dimensions are shown in Table 1.

The results of the experiment indicated that the wake begins to oscillate at about $R=700$ (R is Ul/ν , where l is the breadth of the plate) some distance downstream from the plate.

Some examples of the photographs obtained are shown in Plate 4.



a. Immediately behind the flat plate.



b. About 50 cm behind the flat plate.

($R=1815$)

PLATE 4. Oscillation of the wake behind a flat plate.

As is seen from the photographs, the wake is laminar for a short distance behind the flat plate while further back it becomes sinusoidal and vortices appear in its loops. The appearance of the Karman vortex street is similar to the case of a circular cylinder. However, there is a little difference between the two cases of a plate and a cylinder; in the case of a flat plate the vortex filaments are produced from the general flow, while in the case of a circular cylinder they are shed from the twin-vortices. For this reason the shape of the wake behind the flat plate is a little different from that of the wake behind the circular cylinder.

7. Structure of the Karman Vortex Street. In order to observe the fine structure of the Karman vortex street, experiments were carried out by two methods at the same time; one is the aluminium dust method and the other the condensed milk method. The aluminium dust enables us to

see the velocity field, and the milk clearly shows the boundary between the flow which passed the right hand side of the cylinder and that passed the left hand side.

Thus, when two methods are applied at the same time, we can acquire the detailed knowledges about the structure and the nature of the Karman vortex street. The sketch of the Karman vortex street is shown in Fig. 6.***)

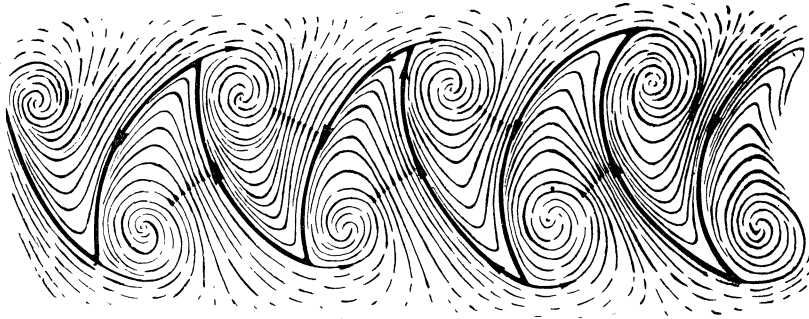


FIG. 6. Sketch of the Karman vortex street.

The Karman vortex street is deformed as time goes on and finally it is destroyed into pieces. But after an interval these scattered vortices are united and there appears again a new Karman vortex street, the dimension of which is much larger than that of the primary one. This does not, however, always take place. It was often observed that the primary Karman vortex street became directly turbulent flow.

In conclusion, the author wishes to express his thanks to Professor Hikoji Yamada of Research Institute for Applied Mechanics, for suggesting this investigation as well as for constant guidance in the course of this work.

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***) This is the picture observed downwards by eyes through the tank water, and seen from the view-point which rests relatively to the vortex street, the cylinder far left of the picture running to the left.

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