

ESTIMATION OF NATURAL FREQUENCIES OF TORSIONAL VIBRATION OF SHIPS

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ESTIMATION OF NATURAL FREQUENCIES OF TORSIONAL VIBRATION OF SHIPS

By Toyoji KUMAI

Abstract.—The characteristic values of the torsional vibration of the beam of variable cross section and with variable mass distributions like as ship's form are computed from the solutions of the fundamental equation, and the estimation formula of the criticals of the torsional vibration of a ship's hull by the use of the computed one is presented. The effects of the deck openings upon the torsional rigidity and of the virtual inertia of the entrained water mass in the case of torsional vibration on the natural frequencies are taken into account. The results of calculations are compared with measured ones in two types of cargo vessels.

1. Introduction. The numerous methods of calculation of the natural frequencies of the torsional vibration of ships' hulls have already been carried out by many researchers, for instance, L. Gümbel,¹⁾ H. W. Nicholls,²⁾ J. L. Taylor³⁾ and Horn.⁴⁾ So far as the author is aware, the results of measurements of the torsional criticals in actual ships have been presented by Horn⁴⁾ and T. Okabe and others.⁵⁾ In regard to the comparisons of the calculations with the measurements of natural frequencies of the torsional vibrations of the hulls, the considerable discrepancies are always be seen between them. For agreement of both results, the empirical factor is used, for instance, this is 0.775 in the calculation of Horn's method.

The present paper deals with a method of estimation of the natural frequencies of the torsional vibration of ship's hull by the use of the characteristic values which obtained from the theoretical results of the torsional vibration of the beam with variable rigidity and variable mass distributions along length, and the convenient formula estimating the natural frequencies of the torsional vibration of the hulls without the use of empirical factor is presented. The formula includes the effective torsional rigidity which has been investigated by G. Vedeler⁶⁾ and the reduction of rigidity due to deck openings which has been suggested by J. L. Taylor.⁷⁾ Also, the effect

¹⁾ L. Gümbel, Trans. I.N.A. 1912.

²⁾ H. W. Nicholls, Trans. I.N.A. 1924.

³⁾ J. L. Taylor, Trans. N.E.C.I. 1927-1928.

⁴⁾ Horn, W.R.H. 1925.

⁵⁾ T. Okabe, S. Fujita and Y. Matsuyama, Trans. of Western Soc. of N.A., Japan, No. 6. 1953.

⁶⁾ G. Vedeler, Trans. I.N.A. 1924.

⁷⁾ *loc. cit.* 3).

of added virtual inertia of entrained water mass on the natural frequency of the torsional vibration which has already been investigated by Y. Watanabe⁸⁾ in the case of rolling of ships in the perfect fluid is taken into account. The natural frequencies and the mode of the torsional vibrations of two types of cargo ships measured by Horn and T. Okabe and his co-workers are compared respectively with those results obtained by the present calculations.

2. Calculations of Natural Frequency of Torsional Vibration of Ships.

The natural frequency of the beam with variable cross section is generally expressed by the following formula, (see Appendix.)

$$p = \lambda \sqrt{\frac{g G J_{t0}}{\rho A_0 r_0^2 L^2}}, \quad \dots\dots\dots (1)$$

where

p	natural circular frequency,
λ	characteristic value which depend on the distributions of the mass and of the torsional rigidity,
g	gravity acceleration,
G	shear modulus of material of the beam,
J_{t0}	torsional rigidity of datum section,
$\rho A_0 r_0^2 (= I_0)$	polar mass moment of inertia of datum section, A_0 , r_0 , ρ denote the sectional area, radius of gyration of the section and density of the material respectively,
L	length of the beam.

In the case of the torsional vibration of ship's hull which is afloat on water, mass of the beam in (1) is replaced by the displacement of the ship, namely,

$$\rho A_0 L = \Delta/c, \quad \dots\dots\dots (2)$$

where Δ , c and L denote displacement of a ship, prismatic coefficient and over all length of the ship respectively.

Now, put N_t the natural frequency per min. of the torsional vibration and η the additional virtual inertia factor of entrained water mass with the assumption of the same distribution as the mass moment of inertia of ship, the natural frequency is calculated by the following formula,

$$N_t = \frac{60}{2\pi} \sqrt{c} \lambda \sqrt{\frac{g G J_{t0e}}{\Delta r_0^2 L (1 + \eta)}}. \quad \dots\dots\dots (3)$$

The values of $\sqrt{c} \lambda$ in the above formula are exactly computed under reasonable assumption of the distribution functions of the rigidity and the mass moment of inertia which respond to the ship's hull. It will be expected

⁸⁾ Y. Watanabe, Trans. of Soc. of N. A. Japan 1933.

that the value of effective torsional rigidity J_{toe} is obtained by Bredt's formula and corrected by the use of G. Vedeler's experimental results for thin hollow rectangular section, more over the the effect of the deck openings on the rigidity of a hull should be taken into account. And also the value of η which has already been obtained by the theoretical investigations in the case of rolling of ships by Y. Watanabe is applicable for the present vibration problem as the first approximation. The natural frequency of torsional vibration of ship's hull is thus estimated by (3), and there is no need of the empirical factor in the above formula provided the adequate values of each terms are considered.

3. Distributions of Torsional Rigidity and Polar Mass Moment of Inertia and Determination of Characteristic Value. The distributions of the torsional rigidity and the polar mass moment of inertia of a ship along the length are assumed to be of the form,

$$\left. \begin{aligned} J_t &= J_{to} (\xi/\xi_1)^m, & w &= w_0 (\xi/\xi_1)^q \text{ for ballast condition,} & 0 \leq \xi \leq \xi_1 \\ & & w &= w_0 (\xi/\xi_1)^{2q} \text{ for full loaded cond.,} & \\ J_t &= J_{to}, & w &= w_0, & \xi_1 \leq \xi \leq 1/2, \end{aligned} \right\} \quad (4)$$

where, J_t and w are assumed to be distributed symmetrically about midship, and $\xi = x/L$, where x is the length coordinate with the origin at the end of the hull. The length of the entrance or the run is ξ_1 and $1/2 - \xi_1$ is half length of the parallel body of a hull, the values of m , q and ξ_1 are positive numbers determined by comparing with the curve of J_t , w , and the mean value of the length of the entrance and the run of those computed from the plans of actual ship respectively.

The value of c in (3) is easily obtained by the following equation for given value of q and ξ_1 ,

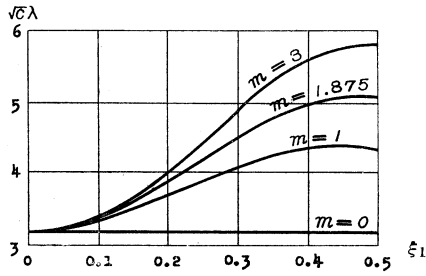
$$c = \frac{\int_0^{\xi_1} w d\xi + w_0 \int_{\xi_1}^{1/2} d\xi}{w_0 \int_0^{1/2} d\xi} = \frac{1 + (1 - 2\xi_1) q}{1 + q} \quad \dots\dots\dots (5)$$

The characteristic value λ of the hull with variable cross section above mentioned is determined by the equation deduced from the boundary conditions at the points of $\xi = 0$, $\xi = \xi_1$ and $\xi = 1/2$, this is shown as follows, (see Appendix),

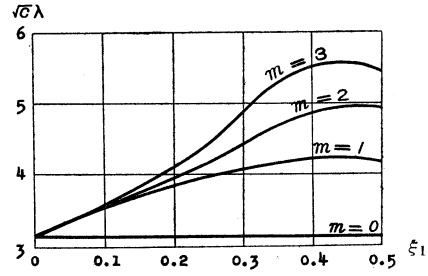
$$\frac{J_{-\frac{\mu}{\mu+\nu}} \left(\frac{\lambda}{\mu+\nu} \xi_1 \right)}{J_{-\frac{\mu}{\mu+\nu}+1} \left(\frac{\lambda}{\mu+\nu} \xi_1 \right)} + \frac{\sin \lambda \xi_1 + B' \cos \lambda \xi_1}{\cos \lambda \xi_1 - B' \sin \lambda \xi_1} = 0, \quad \dots\dots\dots (6)$$

where $\mu = \frac{1-m}{2}, \quad \nu = \frac{1+n}{2},$

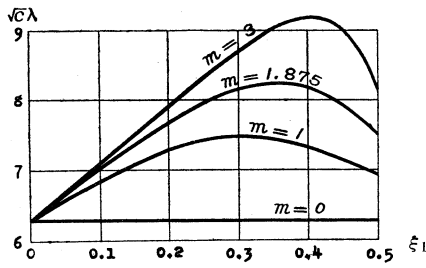
$$\begin{aligned} n &= 3q, & \text{for ballast condition,} \\ n &= 2q, & \text{for full loaded condition.} \end{aligned}$$



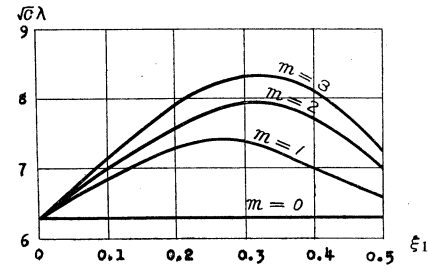
(a) 1-node mode, full loaded cond.



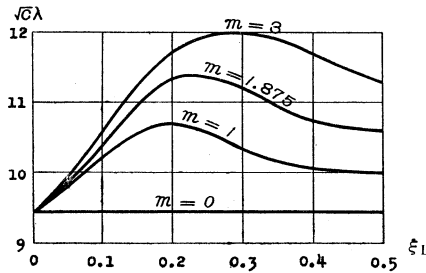
(b) 1-node mode, ballast cond.



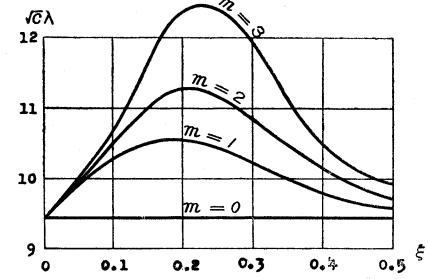
(c) 2-node mode, full loaded cond.



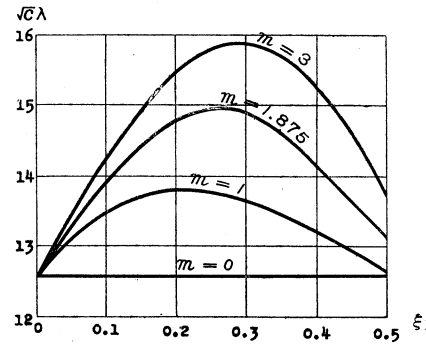
(d) 2-node mode, ballast cond.



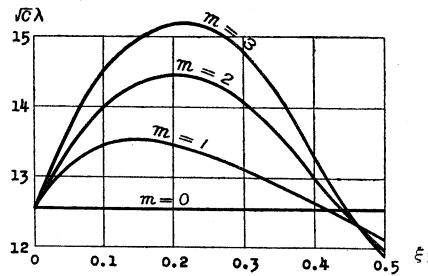
(e) 3-node mode, full loaded cond.



(f) 3-node mode, ballast cond.



(g) 4-node mode, full loaded cond.



(h) 4-node mode, ballast cond.

FIG. 1. Curves of $\sqrt{c} \lambda$ for given values of m and ξ_1 up to 4-noded mode for two loading conditions.

$J_{\mu/(\mu+\nu)}\left(\frac{\lambda}{\mu+\nu}\xi_1\right)$ is the Bessel function of the first kind of $\left(\frac{\mu}{\mu+\nu}\right)$ -th order. The values of c and λ are easily computed with given values of m , q , and ξ_1 from (5) and (6) respectively.

Thus, the terms $\sqrt{c}\lambda$ in (3) in the cases of the loaded condition and of the ballast condition were computed about three values of m , for six values of ξ_1 in the four cases from fundamental mode to four noded mode of the torsional vibrations in two loading conditions as shown in Fig. 1. As will be seen in the figures, it is clearly shown that the values of $\sqrt{c}\lambda$ are different from those of the uniform section beam.

4. Effective Torsional Rigidity. Since the deck girders and hatch coamings contribute to the local strength of the deck in the vicinity of the hatch ways and other deck openings, and since the longitudinal shear stress produced by the twisting moment is resisted by the deck plate between two adjacent openings, the rigidity of the hull having those openings is thus assumed to be that of the closed tube. Accordingly, the formula for obtaining the rigidity of the closed thin hollow tube is applicable to the ship's hull. The effective rigidity, however, should be taken for the deck openings and the sharp corner at the gunwale of the weather deck into consideration for the ordinary hull structure. Since the lower decks scarcely contribute to the torsional rigidity, increment of the rigidity of the hull due to second deck or lower decks are ignored. In the present paper, as the final result, the outermost continuous platings are taken into account as closed hollow tube with the consideration of the reduced thickness at the deck openings which presented by J. L. Taylor. With regard to the effective rigidity of thin hollow rectangular section like as the section of parallel part of ship's hull, so far as the author is aware, no investigation is carried out except the model experiments of the box-shaped tube which has been investigated by G. Vedeler. The empirical formula for the calculation of the effective torsional rigidity of the thin hollow rectangular tube obtained by G. Vedeler is presented as the following form

$$J_{toe} = C_e \frac{4F^2}{\sum \frac{l_n}{t_n}}, \quad \dots\dots\dots (7)$$

$$C_e = \frac{2 - D/B}{2},$$

where F the area of the figure bounded by the middle line of the thickness of the plate,
 l_n length of the plate along the middle line of the thickness t_n ,
 t_n thickness of the n -th strake in section.
 C_e empirical factor obtained by G. Vedeler, if C_e takes unity, (7) shows Bredt's formula,

The sum of the length l_n of the section are divided into two parts of different thickness of the closed tube, for an example, one is the breadth

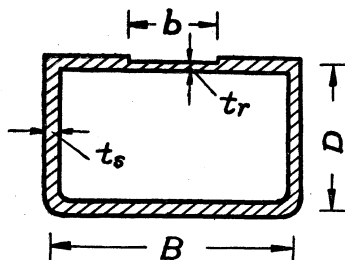


FIG. 2. Typical section of hull with deck opening as closed section tube.

of hatchway with equivalent thickness of deck plate and the other is remainder with mean thickness of shell and deck plate (see Fig. 2). The remainder part may be subdivided into three parts, namely, deck plate, side shell and bottom plate, provided more accurate result is required.

For the sake of simplicity, the closed section is assumed to be separated by two parts as mentioned above, the denominator of (7) becomes

$$\sum \frac{l_n}{t_n} = \frac{2(B+D)}{t_s} \left\{ 1 + \frac{b}{2B(1+D/B)} \left(\frac{t_s}{t_r} - 1 \right) \right\} \dots \dots \dots (8)$$

The terms b/B and t_s/t_r govern the effects of breadth and length of deck openings respectively upon the torsional rigidity. The value of J_{toe} decreases provided the ratios b/B or t_s/t_r increases. The reduced thickness t_r is easily computed from the thickness of the deck plates between adjacent deck openings multiplied by the ratio of the sum of length of deck openings and that of a ship.

The ratio of J_{toe} and J_{to} , for an example, is computed as about 0.655 for 10,000 ton d. w. cargo vessel.

5. Radius of Gyration of Polar Mass Moment of Inertia of Datum Section of Hull.

It is easily be supposed that the radius of gyration of the polar mass moment of inertia of the datum section of the hull takes different value for the loading conditions. The results of approximate computations of these values

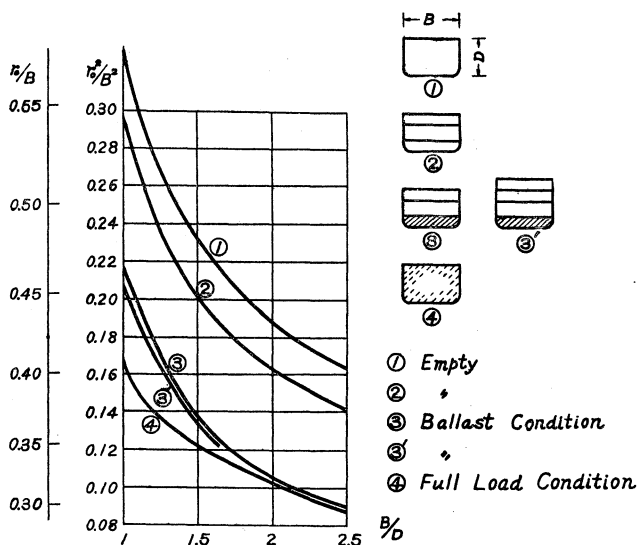


FIG. 3. Radius of gyration of polar mass moment of inertia as fraction of breadth of ship versus the ratio of breadth and depth in various loading conditions in 10,000 ton D. W. cargo vessel.

in various conditions of the type of 10,000 ton d.w. cargo vessel are shown by the ratio r_0/B versus B/D in Fig. 3. As will be seen in the figure, the values of r_0/B in the case of the empty condition are considerably different from those of ballast or full loaded conditions. In the case of full loaded condition, the radius of gyration is nearly equal to that of the solid rectangular section, and that has already been presented as simple form by Horn, namely

$$r_0 = 0.306 \sqrt{B^2 + D^2}, \dots\dots\dots (9)$$

where, B and D denote the breadth and the depth of a ship respectively.

6. Virtual Inertia of Entrained Water Mass in the Torsional Vibration of Ship.

The effects of the added virtual mass of entrained water upon the criticals of vertical vibration of ships has already been carried out by F. M. Lewis,⁹⁾ J. L. Taylor,¹⁰⁾ H. W. Nicholls¹¹⁾ and C. W. Prohaska.¹²⁾ However, so far as the author is aware, no paper on the effect upon torsional criticals is published except the theoretical investigation on the apparent inertia of the water in the case of rolling of ships in the perfect fluid which carried out by Prof. Y. Watanabe. He suggested in his lecture of vibration of ship that the results obtained in this investigation are also applicable to the problem on the torsional vibration of ship. If the distribution of the added inertia of the water mass along the length of a ship is assumed to be the same form as that of the polar moment of inertia of mass of ship, η presented in (3) may approximately takes the same value as that calculated at the midship section as two dimensional value which obtained by Prof. Y. Watanabe. This value shows 0.325 in the full loaded conditions, provided the mass moment of inertia takes $\frac{4B^2}{6}$, where Δ and B are displacement and breadth of a ship respectively. In the other loading conditions, the particular consideration should be taken into account. Since the effect of the draught of the ship on the value of η will probably be considered small, $\eta = 0.325$ is adopted in all cases of loading as the first approximation in the present paper. The results of some model tests on the above problem will be presented by the author in near future.

7. Comparisons of Natural Frequencies of Cargo Ships obtained by the above Calculations and Measured Ones. The principal dimensions and the other items of two cargo vessels "Wasgenwald" and "A" ship measured by Horn¹³⁾ and by T. Okabe and his coworkers¹⁴⁾ respectively are shown in the following table. The calculations of the items above mentioned and their results are also presented in the same table. As will be seen in

⁹⁾ F. M. Lewis, Tran. S. N. A. M. E. 1929.

¹⁰⁾ J. L. Taylor, Phil. Mag. 1930.

¹¹⁾ *loc. cit.* 2).

¹²⁾ C. W. Prohaska, A. T. M. A. 1947.

¹³⁾ *loc. cit.* 4).

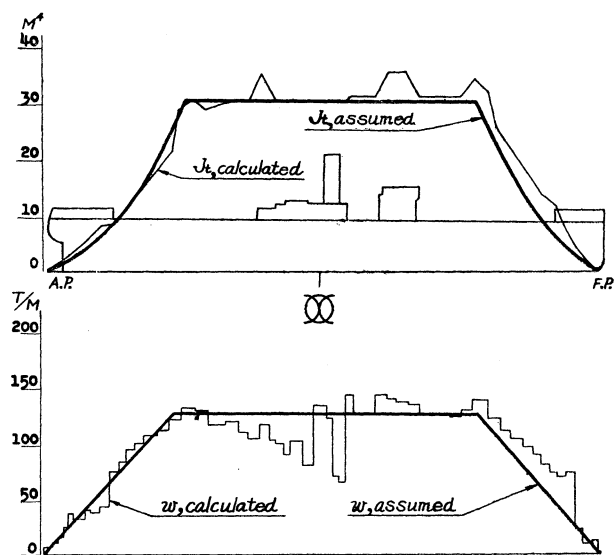
¹⁴⁾ *loc. cit.* 5).

the table, the results of calculations and measured ones are found in good agreement in two cargo vessels. The assumed curves of the form of the distributions of J_t and of w were each compared with those computed from actual ships' plans of "Wagenwald" and "A" ship as shown in Fig. 4 a) and b) respectively. Except the comparison of w curve in the ballast condition of "A" ship, the assumed curves of J_t and w are found fairly in good agreement with the calculated ones as will be seen in the figures.

TABLE.

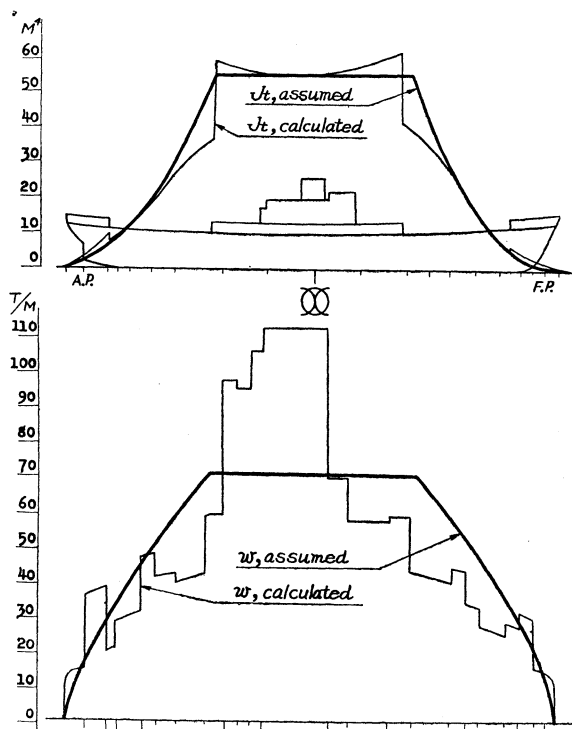
Principal dimensions and particulars for the calculations
of torsional criticals of "Wagenwald" and "A" ship.

Items	"Wagenwald"	"A" ship
L_{OA}	126.7 M	141.0 M
L_{PP}	121.4 M	132.0 M
B	16.45 M	18.0 M
D	9.237 M	10.0 M
D_{Br}	(shelter) 11.637 M	12.45 M
Speed	11 Kt	—
Engine	3 exp. steam	7 MS. Diesel
H. P.	2,850	5,000
R. P. M.	70 ~ 72	120 ~ 125
Number of propeller blade	4	4
Hull wt.	3,040 ton	—
Eng. wt.	880 ton	—
Cargo	8,830 ton	1,367 ton
Δ	12,750 ton	6,109 ton
d	8.003 M	3.860 M
Loading condition	full loaded	ballast
ξ_1	0.23	0.30
m	1.5	2.0
node of vib.	1-noded	2-noded
$\sqrt{c} \lambda$	3.95	7.49
r	5.745 M	6.710 M
η	0.325	0.325
$g = 9.8 \text{ M/sec}^2$		$G = 2/5 E = 0.8 \times 10^7 \text{ ton/M}^2$
J_{to}	31.4 M ⁴	40.4 M ⁴
J_{toe}	20.28 M ⁴	26.5 M ⁴
N_t , calculated	180/min.	510/min.
N_t , measured	186/min.	500/min.



a) "Wasgenwald"

(Calculated curves are taken from Horn's paper.)



b) "A" ship

FIG. 4. Comparisons of assumed curves of J_t and w with computed ones.

The results of the measurements and the calculations of the modes of the torsional vibration of the above two ships are compared in Fig. 5 a) and b) respectively.

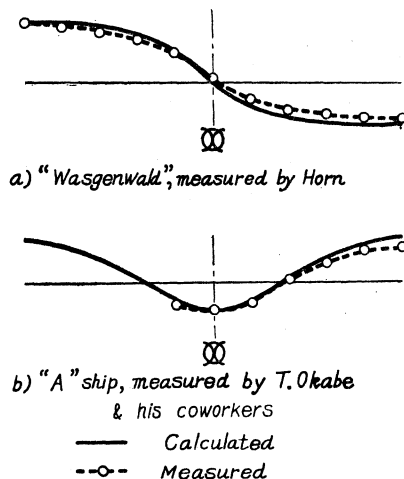


FIG. 5. Comparisons of mode of torsional vibration measured with calculated ones.

8. Conclusions. The estimation formula of the natural frequency of torsional vibration of the ship's hull for practical use in the design stage was proposed. The formula involves the effective torsional rigidity of hull and the additional inertia of entrained water mass in the case of torsional vibration. The characteristic values up to four node mode were computed for various ship's forms. For convenience' sake, these are shown by the curves. No empirical factor is needed in the present formula. The numerical examples are compared with measured natural frequencies and the modes of torsional vibrations about two types of cargo ships are presented, the agreements of both results are good in two ships for practical use.

This paper has been presented and discussed on the meeting of Western Section of Ship Structure Committee of Japan, at Kyushu University, Feb. 1955. The author desires to acknowledge the helpful advice and criticism received from Prof. Y. Watanabe.



Appendix

Solutions of Torsional Vibration of Beam of Variable Cross Section with Variable Mass

For the sake of simplicity, the beam is assumed to be of the symmetrical form about midpoint of the beam considered. The length coordinate of the beam denoted by x is measured from the end point. Denote the twisting angle of the torsional vibration by Ψ_1 , the equilibrium of the element of the beam of variable cross section with parallel part is presented by two equations respectively as follows,

$$G \frac{\partial}{\partial x} J_t \frac{\partial \Psi_1}{\partial x} = \rho I \frac{\partial^2 \Psi_1}{\partial t^2} \dots \dots \text{variable part,} \dots \dots (1)$$

$$G J_{t_0} \frac{\partial^2 \Psi_2}{\partial x^2} = \rho I_0 \frac{\partial^2 \Psi_2}{\partial t^2} \dots \dots \text{parallel part,} \dots \dots (2)$$

where G shear modulus of the beam,
 J_t variable torsional rigidity of the beam,
 ρ density of the beam,
 I polar moment of inertia of the section of the beam.

Now, put $\Psi_1 = \phi_1 \cos pt$, and $\xi = x/L$, where p and L denote circular frequency and the length of the beam, respectively, then, substitute from these values into (1) and (2), the above equations are shown by the following forms respectively,

$$\frac{d^2 \phi_1}{d\xi^2} + \frac{1}{J_t} \frac{dJ_t}{d\xi} \frac{d\phi_1}{d\xi} + \frac{p^2 \rho I L^2}{G J_t} \phi_1 = 0, \dots \dots \dots (3)$$

$$\frac{d^2 \phi_2}{d\xi^2} + \frac{\rho I_0 p^2 L^2}{G J_{t_0}} \phi_2 = 0, \dots \dots \dots (4)$$

$$p = \lambda \sqrt{\frac{G J_{t_0}}{\rho I_0 L^2}}, \dots \dots \dots (5)$$

provided $\lambda^2 = \frac{p^2 \rho I_0 L^2}{G J_{t_0}}$ in the above equation (4).

Now, if the distributions of J_t and I are assumed to be of the power curves, $J_t = J_{t_0} (\xi/\xi_1)^m$ and $I = I_0 (\xi/\xi_1)^n$ respectively, the equation (3) is cast into as the following form

$$\frac{d^2 \phi_1}{d\xi^2} + \frac{m}{\xi} \cdot \frac{d\phi_1}{d\xi} + \lambda_1^2 \xi^{n-m} \phi_1 = 0, \dots \dots \dots (3)'$$

where $\lambda_1 = \lambda \xi_1^{-(n-m)/2}$
 ξ_1 is the length of the entrance or the run in the ship's hull,

The solutions of (3)' are presented by Bessel functions of the first kind¹⁵⁾ as follows, provided $\mu/(\mu+\nu)$ takes nonintegral,

$$\psi_1 = A_1 \xi^\mu J_{-\mu/(\mu+\nu)}\left(\frac{\lambda_1}{\mu+\nu} \xi^{\mu+\nu}\right) + B_1 \xi^\mu J_{\mu/(\mu+\nu)}\left(\frac{\lambda_1}{\mu+\nu} \xi^{\mu+\nu}\right), \dots \quad (6)$$

where, $\mu = (1-m)/2$, $\nu = (1+n)/2$,

it is true in the limits of $m-n < 2$.

On the other hand, the solutions of (4) is easily obtained as well known form,

$$\psi_2 = A_2 \sin \lambda \xi + B_2 \cos \lambda \xi. \dots \dots \dots (7)$$

The characteristic value λ in (6) and (7) is computed by the boundary conditions at the end of the beam, at the end of the parallel body and the midpoint of the length of the beam or the midship section of the hull, whose last condition in the even number node is different from the odd number one. Thus, the equation of determination of λ is shown as the following form

$$\frac{J_{-\mu/(\mu+\nu)}\left(\frac{\lambda}{\mu+\nu} \xi_1\right)}{J_{-\{\mu/(\mu+\nu)\}+1}\left(\frac{\lambda}{\mu+\nu} \xi_1\right)} + \frac{\sin \lambda \xi_1 + B' \cos \lambda \xi_1}{\cos \lambda \xi_1 - B' \sin \lambda \xi_1} = 0, \dots \dots \dots (8)$$

where $B' = -\tan \lambda/2$ for odd number node,
 $B' = \cot \lambda/2$ for even number node.

The curves presented in Fig. 1 were computed with the following numerical values of m and etc.

for full loaded cond.		for ballast cond.	
m	$-\frac{\mu}{\mu+\nu}$	m	$-\frac{\mu}{\mu+\nu}$
1	0	1	0
1.875	1/3	2	1/2
3	2/3	3	1

The following tables of the Bessel functions were used in the present computation of the characteristic values.

1. K. Hayashi, "Tafeln der Besselschen, Theta, Kugel, und anderer Funktion," Springer, 1930.
2. Columbia University Press, "Tables of Bessel Function of Fractional Order"—Vol. I. 1949.

(Received July 3, 1955)

¹⁵⁾ Watson, Theory of Bessel Function, 1922.