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ON THE BUCKLING OF CURVED RECTANGULAR PLATES WITH CLAMPED EDGES UNDER UNIFORM SHEAR*

By

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Abstract

The buckling of thin curved rectangular plate, with clamped edges and with finite dimensions, under the action of shearing force has been investigated in this paper.

In practice, it is seen in thin walled structures with stringers and frames, such as gas-tanks, wagons, motor-cars, etc. In these structures, we are sometimes enough if we take into account only the buckling of a surface element bounded by stringers and frames.

Using the energy method, and assuming the buckled form in suitable trigonometric functions, the general solution has been obtained in comparatively simple form.

Numerical calculation of it has been shown in figures.

1. Introduction.

The object of the present paper is to obtain the buckling load of the curved rectangular plate under uniform shearing force acting on its clamped edges

In practice, it is seen in thin walled structures with stringers and frames. When the buckling of the skin of gas-tanks, wagons, motor-cars, etc. is considered, we are sometimes enough if we take into account only

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the buckling of a surface element bounded by stringers and frames. In this paper, as the boundaries were assumed to be clamped, the results are applied to the structures whose wall is fixed to the rigid stringers and frames by welding or rivetting in practical problems.

Same problem was treated by T.E. Schunck in order to estimate the safety wind-pressure for gas-tank. But, as he could not obtain the general solution, he showed a method of successive approximation and obtained the buckling load for one example.

D.M.A. Leggett treated the problem for the infinitely long, curved strip ($l=\infty$) under the both boundary conditions, simply supported and clamped. But, as he made some simplifications in problem by assuming that the curvature is small, his results can be applied only for slightly curved plates.

A. Kromn solved the same problem with Leggett and his results are applicable also to the largely curved plate with clamped edges. In practical structures the ratio l/b takes an arbitrary value, and when the results by Leggett and Kromm cannot be applied, we have not been able to estimate the buckling load. This paper gives more general solution than former papers.

On the rectangular flat plate, S. Timoshenko gave a solution for simply supported one, and our result gives for clamped condition, too.

2. Analysis.

It is very difficult to obtain the exact solution from the differential equation, therefore the energy method was used. Taking the coordinates as in Fig. 1, and considering the limit of stability to be the initial state, then the increase of strain energy by additional displacements is given by

$$V=\int_0^l\int_0^a\int_{-h/2}^{h/2}\left(\int_0^{\varepsilon_x}\sigma_x\,d\varepsilon_x+\int_0^{\varepsilon_\theta}\sigma_\theta\,d\varepsilon_\theta+\int_0^{\gamma}\tau d\gamma\right)\left(1-\frac{z}{r}\right)rdzd\theta dx \tag{1}$$

where

l: length of the plate in x-direction,

α: central angle of the curved plate,

σ_x, σ_θ, τ: normal stresses in x- and y-directions respectively and shearing stress,

ε_x, ε_θ, γ: normal strains in x- and y-directions respectively and shearing strain,

r: radius of curvature of the curved plate,

h: thickness of the curved plate.

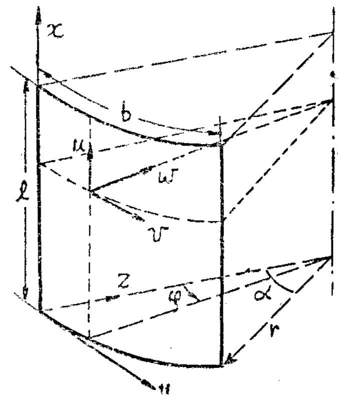

Fig. 1

When we consider the buckling in elastic region, the ratio z/r can be neglected against 1 in usual structures.

For convenience, we write as follows

$$\sigma_x = \sigma_{x1} + \sigma_{x2}, \quad \sigma_\theta = \sigma_{\theta 1} + \sigma_{\theta 2}, \quad \tau = \tau_1 + \tau_2$$

where the suffix 1 means the values at the limit of stability and the suffix 2 the variation of them respectively.

In this problem $\sigma_{x1} = \sigma_{\theta 1} = 0$, therefore

$$V = \int_0^l \int_0^a \int_{-h/2}^{h/2} \tau_1 \gamma r dz d\theta dx + \frac{1}{2} \int_0^l \int_0^a \int_{-h/2}^{h/2} (\sigma_{x2}\varepsilon_x + \sigma_{\theta 2}\varepsilon_\theta + \tau_2 \gamma) r dz d\theta dx. \quad (2)$$

Before the buckling of the plate, the shearing force acting uniformly along the plate-edges distributes in the interior of the plate uniformly.

If we denote with N_{xy} the shearing force in y-direction per unit length of the section of the plate perpendicular to x-axis, then $\tau_1 = N_{xy}/h$. Moreover, if we assume that the additional stresses remain within the limit of elasticity, the relations between stresses and strains are

$$\left\{ \begin{aligned} \sigma_{x2} &= \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_\theta) \\ \sigma_{\theta 2} &= \frac{E}{1-\nu^2}(\varepsilon_\theta + \nu\varepsilon_x) \\ \tau_2 &= \frac{E}{2(1+\nu)}\gamma \end{aligned} \right. \quad (3)$$

where E: Young's modulus, ν: Poisson's ratio.

Therefore, substituting (3) in (2), we obtain

$$V=\frac{N_{xy}}{h}\int_0^l\int_0^a\int_{-h/2}^{h/2}\gamma r dz d\theta dx$$
$$+\frac{E}{2(1-\nu^2)}\int_0^l\int_0^a\int_{-h/2}^{h/2}\left[(\varepsilon_x+\varepsilon_\theta)^2-2(1-\nu)\left(\varepsilon_x\varepsilon_\theta-\frac{\gamma^2}{4}\right)\right]rdzd\theta dx. \tag{4}$$

If the strain components ε_x, ε_θ, and γ are expressed by the components of displacement u, v, and w (u, v, and w denote the x-, y-, and z-components of the displacement in the middle plane of the plate),

$$\begin{cases}\gamma=\gamma_1+\gamma_1'-2z\chi_{x\theta}\\ \varepsilon_x=\varepsilon_1-z\chi_x\\ \varepsilon_\theta=\varepsilon_2-z\chi_\theta\end{cases} \tag{5}$$

where

$$\begin{cases}\gamma_1=\dfrac{1}{r}\dfrac{\partial u}{\partial\theta}+\dfrac{\partial v}{\partial x}, \quad \gamma'=\dfrac{1}{r}\dfrac{\partial v}{\partial x}\left(\dfrac{\partial v}{\partial\theta}-w\right)+\dfrac{1}{r}\dfrac{\partial w}{\partial x}\left(\dfrac{\partial w}{\partial\theta}+w\right),\\ \chi_{x\theta}=\dfrac{1}{r}\left(\dfrac{\partial v}{\partial x}+\dfrac{\partial^2 w}{\partial x\partial\theta}\right), \quad \varepsilon_1=\dfrac{\partial u}{\partial x}, \quad \chi_x=\dfrac{\partial^2 w}{\partial x^2}\\ \varepsilon_2=\dfrac{1}{r}\dfrac{\partial v}{\partial\theta}-\dfrac{w}{r}, \quad \chi_\theta=\dfrac{1}{r^2}\left(\dfrac{\partial v}{\partial\theta}+\dfrac{\partial^2 w}{\partial\theta^2}\right).\end{cases} \tag{6}$$

τ_2 is very small compared with τ_1, therefore, γ_1' in the second term of (4) can be neglected.

In order to use the principle of virtual work, considering the total potential energy Q, the work done by the external force cancels with strain energy by γ_1, therefore, Q can be expressed by the second order terms of displacements and of their derivertives as follows:

$$Q=\frac{K}{2}\int_0^l\int_0^a\left[(\varepsilon_1+\varepsilon_2)^2-2(1-\nu)\left(\varepsilon_1\varepsilon_2-\frac{\gamma^2}{4}\right)\right]rd\theta dx$$
$$+\frac{D}{2}\int_0^l\int_0^a\left[(\chi_x+\chi_\theta)^2-2(1-\nu)\left(\chi_x\chi_\theta-\frac{\chi_{x\theta}^2}{4}\right)\right]rd\theta dx+N_{xy}\int_0^l\int_0^a\gamma_1' rd\theta dx \tag{7}$$

where

$$K=\frac{Eh}{1-\nu^2}, \quad D=\frac{Eh^3}{12(1-\nu^2)}.$$

When the plate exceeds the limit of stability, there occur the additional stresses. Accordingly, the first term of (7) expresses the work by the additional stresses and the strains of the middle surface. The second term expresses the work by the additional stresses and the bending of the plate, and the third the difference of the work by the initial stress and the strain of the middle surface and the work by the external force.

Now, we must assume the solution suitably. Kromm obtained the solution by infinite series of trigonometric functions. Schunck assumed the solution to be expressed by $f(x) \cdot \varphi(y)$, and firstly gave some suitable expression only for $f(x)$ so as to satisfy the boundary conditions at $x =$ const. Substituting it in the differential equation about y derived from $\delta Q = 0$, he obtained the corresponding expression for $\varphi(y)$. Of which the first approximation is the $\varphi(y)$ which contains the minimum value of N_{xy}. Where N_{xy} are determined from the boundary conditions for $\varphi(y)$ at $y =$ const. Substituting the $\varphi(y)$ in the differential equation about x derived from $\delta Q = 0$, he obtained the second approximation of $f(x)$ in the same way. He repeated the above process successively to the desired degree of approximation. But, their calculations are very troublsome. If the expression of the solution is suitable, the calculation will not be so complex. Observing the buckling of plates and shells by shearing force, we know that the waves occur in oblique direction and approximately in parallel each other. Therefore, considering the boundary conditions (clamped), we choose the displacements as follows.

$$\begin{cases} u = A\cos\left(m\pi\dfrac{x}{l} - n\pi\dfrac{\theta}{\alpha}\right)\sin^2\pi\dfrac{x}{l}\cdot\sin^2\pi\dfrac{\theta}{\alpha} \\ v = B\cos\left(m\pi\dfrac{x}{l} - n\pi\dfrac{\theta}{\alpha}\right)\sin^2\pi\dfrac{x}{l}\cdot\sin^2\pi\dfrac{\theta}{\alpha} \\ w = C\sin\left(m\pi\dfrac{x}{l} - n\pi\dfrac{\theta}{\alpha}\right)\sin^2\pi\dfrac{x}{l}\cdot\sin^2\pi\dfrac{\theta}{\alpha} \end{cases} \tag{8}$$

We can see that the displacements (8) are approximately those of the actually buckled curved plate: the expression of w means that the nodal lines occur in the direction which makes the angle $\tan^{-1}\dfrac{r\theta}{x} = \tan^{-1}\left(\dfrac{r\alpha}{n}\Big/\dfrac{l}{m}\right)$ with x-axis and that the number of waves is $m/2$ in x-direction and $n/2$

in θ-direction. $\sin^2\pi\frac{x}{l}$ and $\sin^2\pi\frac{\theta}{\alpha}$ means respectively that the displacements and the inclinations at the boundaries vanish.

Substituting (8) in (7), and assuming m and n to be any integer, we can easily perform the integrations about x and θ, and we obtain

$$Q=\frac{3\pi^2}{128}\beta K\Big[\frac{A^2}{2}\Big(\mathbf{b}-\mathbf{a}\frac{N_{xy}}{K}\Big)-AB\cdot\mathbf{c}+\frac{B^2}{2}\Big\{\mathbf{d}+\frac{1}{12}\Big(\frac{h}{r}\Big)^2\mathbf{e}-\mathbf{a}\frac{N_{xy}}{K}\Big\}$$
$$-BC\Big(\frac{b}{\pi r}\Big)\Big\{\mathbf{f}+\frac{\pi^2}{12}\Big(\frac{h}{b}\Big)^2\mathbf{g}-\mathbf{h}\frac{N_{xy}}{K}\Big\}+\frac{C^2}{2}\Big\{3\Big(\frac{b}{\pi r}\Big)^2+\frac{\pi^2}{12}\Big(\frac{h}{b}\Big)^2\mathbf{i}-\mathbf{a}\frac{N_{xy}}{K}\Big\}$$
$$+CA\Big(\frac{b}{\pi r}\Big)\mathbf{j}\Big] \tag{9}$$

where

$$\left\{\begin{aligned}
\mathbf{a}&=6mn\frac{1}{\beta}\\
\mathbf{b}&=(3m^2+4)\frac{1}{\beta^2}+\frac{1-\nu}{2}(3n^2+4)\\
\mathbf{c}&=\frac{3(1+\nu)}{2}mn\frac{1}{\beta}\\
\mathbf{d}&=\frac{1-\nu}{2}(3m^2+4)\frac{1}{\beta^2}+(3n^2+4)\\
\mathbf{e}&=2(1-\nu)(3m^2+4)\frac{1}{\beta^2}+(3n^2+4)\\
\mathbf{f}&=3n\\
\mathbf{g}&=\Big\{(2-\nu)(3m^2+4)\frac{1}{\beta^2}+(3n^2+12)\Big\}n\\
\mathbf{h}&=6m\frac{1}{\beta}\\
\mathbf{i}&=(3m^4+24m^2+16)\frac{1}{\beta^4}+\frac{2}{3}(3m^2+4)(3n^2+4)\frac{1}{\beta^2}+(3n^4+24n^2+16)\\
\mathbf{j}&=3\nu m\frac{1}{\beta}\\
\beta&=\frac{l}{b}.
\end{aligned}\right. \tag{10}$$

According to the principle of virtual work, the condition $\delta Q=0$ must be satisfied. In this case, this condition is satisfied when

$$\frac{\partial Q}{\partial A}=0\,,\quad \frac{\partial Q}{\partial B}=0\,,\quad \frac{\partial Q}{\partial C}=0\,. \tag{11}$$

Therefore, substituting (9) in (11), we obtain

$$\left\{\begin{aligned}
&A\left(\mathbf{b}-\mathbf{a}\frac{N_{xy}}{K}\right)-B\cdot\mathbf{c}+C\left(\frac{b}{\pi r}\right)\mathbf{j}=0\\
&-A\cdot\mathbf{c}+B\left\{\mathbf{d}+\frac{1}{12}\left(\frac{h}{r}\right)^2\mathbf{e}-\mathbf{a}\frac{N_{xy}}{K}\right\}-C\left(\frac{b}{\pi r}\right)\left\{\mathbf{f}+\frac{\pi^2}{12}\left(\frac{h}{b}\right)^2\mathbf{g}-\mathbf{h}\frac{N_{xy}}{K}\right\}=0\\
&A\left(\frac{b}{\pi r}\right)\mathbf{j}-B\left(\frac{b}{\pi r}\right)\left\{\mathbf{f}+\frac{\pi^2}{12}\left(\frac{h}{b}\right)^2\mathbf{g}-\mathbf{h}\frac{N_{xy}}{K}\right\}\\
&\qquad\qquad+C\left\{3\left(\frac{b}{\pi r}\right)^2+\frac{\pi^2}{12}\left(\frac{h}{b}\right)^2\mathbf{i}-\mathbf{a}\frac{N_{xy}}{K}\right\}=0
\end{aligned}\right. \tag{12}$$

The coefficient of B and that of C in the second equation of (12) can be simplified by the neglection of small terms with $(h/r)^2$ and $(h/b)^2$ respectively, as they are small compared with the other terms.

Performing above simplification, and eliminating A, B and C from (12) we obtain the following equation to determine the buckling load

$$\begin{vmatrix}
\mathbf{b}-\mathbf{a}\frac{N_{xy}}{K} & -\mathbf{c} & \left(\frac{b}{\pi r}\right)\mathbf{j}\\
-\mathbf{c} & \mathbf{d}-\mathbf{a}\frac{N_{xy}}{K} & -\left(\frac{b}{\pi r}\right)\left(\mathbf{f}-\mathbf{h}\frac{N_{xy}}{K}\right)\\
\left(\frac{b}{\pi r}\right)\mathbf{j} & -\left(\frac{b}{\pi r}\right)\left(\mathbf{f}-\mathbf{h}\frac{N_{xy}}{K}\right) & 3\left(\frac{b}{\pi r}\right)^2+\frac{\pi^2}{12}\left(\frac{h}{b}\right)^2\mathbf{i}-\mathbf{a}\frac{N_{xy}}{K}
\end{vmatrix}=0\,. \tag{13}$$

Expanding (13), we obtain

$$\begin{aligned}
&\left\{-\mathbf{a}^3+\mathbf{a}\mathbf{h}^2\left(\frac{b}{\pi r}\right)^2\right\}\left(\frac{N_{xy}}{K}\right)^3+\left\{\left(\frac{b}{\pi r}\right)^2(3\mathbf{a}^2-\mathbf{b}\mathbf{h}^2-2\mathbf{a}\mathbf{f}\mathbf{h})\right.\\
&\left.+\frac{\pi^2}{12}\mathbf{a}^2\mathbf{i}\left(\frac{h}{b}\right)^2+\mathbf{a}^2(\mathbf{b}+\mathbf{d})\right\}\left(\frac{N_{xy}}{K}\right)^2\\
&+\left\{\left(\frac{b}{\pi r}\right)^2(-2\mathbf{c}\mathbf{h}\mathbf{j}+2\mathbf{b}\mathbf{f}\mathbf{h}-3\mathbf{a}\mathbf{b}-3\mathbf{a}\mathbf{d}+\mathbf{a}\mathbf{j}^2+\mathbf{a}\mathbf{f}^2)\right.\\
&\left.-\frac{\pi^2}{12}\left(\frac{h}{b}\right)^2\mathbf{a}(\mathbf{b}+\mathbf{d})\mathbf{i}+\mathbf{a}\mathbf{c}^2-\mathbf{a}\mathbf{b}\mathbf{d}\right\}\frac{N_{xy}}{K}
\end{aligned}$$

$$+\left\{\left(\frac{b}{\pi r}\right)^2(2\mathbf{cfj}-3\mathbf{c}^2+3\mathbf{bd}-\mathbf{dj}^2-\mathbf{bf}^2)+\frac{\pi^2}{12}\left(\frac{h}{b}\right)^2(-\mathbf{c}^2\mathbf{i}+\mathbf{bdi})\right\}=0\,. \qquad (14)$$

Substituting the given dimensions of the curved plate in the above equation and choosing suitable values for m and n, we obtain some solutions for N_{xy}. The buckling load is the smallest one of them. But this equation is too complicated to search out the minimum value of N_{xy} by several combination of m and n. Therefore, under the following consideration, we simplify (14). For the first step, we consider the flat plate ($r=\infty$). In this case, $b/\pi r=0$. Therefore, (14) becomes

$$-\mathbf{a}^3\left(\frac{N_{xy}}{K}\right)^3+\left\{\frac{\pi^2}{12}\left(\frac{h}{b}\right)^2\mathbf{a}^2\mathbf{i}+\mathbf{a}^2(\mathbf{b}+\mathbf{d})\right\}\left(\frac{N_{xy}}{K}\right)^2$$

$$-\left\{\frac{\pi^2}{12}\left(\frac{h}{b}\right)^2\mathbf{a}(\mathbf{b}+\mathbf{d})\mathbf{i}-\mathbf{ac}^2+\mathbf{abd}\right\}\frac{N_{xy}}{K}+\frac{\pi^2}{12}\left(\frac{h}{b}\right)^2(-\mathbf{c}^2\mathbf{i}+\mathbf{bdi})=0\,. \qquad (15)$$

In the coefficient of N_{xy}/K, the terms with the infinitesimal factor $(h/b)^2$ can be neglected against the other terms.

While, even the maximum shearing stress in usual materials is very small compared with $E/(1-\nu^2)$, and especially in the elastic region, $N_{xy}/K = \tau\Big/\frac{E}{1-\nu^2}$ is very small magnitude. Therefore, in (15), the first and the second terms can be neglected.

As the result of above simplification, we obtain the buckling load of the flat plate in a very simple form as follows.

$$N_{xy}=K\cdot\frac{\pi^2}{12}\left(\frac{h}{b}\right)^2\cdot\frac{\mathbf{i}}{\mathbf{a}} \qquad (16)$$

Now, on the curved plate, the similar neglection of the first and the second terms of (14) can be performed.

In the equation, moreover, we neglect the terms with the infinitesimal factor $(h/b)^2$ and the terms with $(b/\pi r)^2$, as they are considered to be comparatively small, by the estimation of the values of m and n obtained from the numerical calculation of (16) for given β on the flat plate. Then it becomes

$$N_{xy}=\frac{K}{\mathbf{a}}\left\{\left(\frac{b}{\pi r}\right)^2\left(3+\frac{\mathbf{dj}^2+\mathbf{bf}^2-2\mathbf{cfj}}{\mathbf{c}^2-\mathbf{bd}}\right)+\frac{\pi^2}{12}\left(\frac{h}{b}\right)^2\mathbf{i}\right\}. \qquad (17)$$

Substituting (10) in (17), the general expression for the shearing buckling load of the curved rectangular plate with clamped edges is obtained as follows.

$$N_{xy}=\frac{D\pi^2}{b^2}\cdot\frac{1}{6mn\frac{1}{\beta}}\Bigg[\Big\{(3m^4+24m^2+16)\frac{1}{\beta^4}+\frac{2}{3}(3m^2+4)(3n^2+4)\frac{1}{\beta^2}$$

$$+(3n^4+24n^2+16)\Big\}+\frac{36}{\pi^4}\left(\frac{b^2}{hr}\right)^2\Bigg\{1-$$

$$\frac{\left\{\frac{1-\nu}{2}(3m^2+4)\frac{1}{\beta^2}+(3n^2+4)\right\}\nu^2m^2\frac{1}{\beta^2}+\left\{(3m^2+4)\frac{1}{\beta^2}+\frac{1-\nu}{2}(3n^2+4)\right\}n^2}{\frac{1}{3}\left\{\frac{1-\nu}{2}(3m^2+4)\frac{1}{\beta^2}+(3n^2+4)\right\}\left\{(3m^2+4)\frac{1}{\beta^2}+\frac{1-\nu}{2}(3n^2+4)\right\}}$$

$$\frac{-3(1+\nu)\nu m^2n^2\frac{1}{\beta^2}}{-\frac{3(1+\nu)^2}{4}m^2n^2\frac{1}{\beta^2}}\Bigg\}\Bigg] \tag{18}$$

The second term of this expression is the term depending on the curvature.

When the dimensions of a plate are given, the actual buckling load is the minimum value of N_{xy} calculated by suitable substitution of m and n. The results obtained by the numerical calculation of (18) for $\nu=0.3$ are shown in Fig. 2 and Fig. 3.

When $b/\pi r$ is larger than 1, the error will become greater. Hence, in these cases, the buckling load should be obtained from (14).

3. Comparison and Consideration of the Results.

a) $l/b=$ finite:

In this case, the data are only Timoshenko's (for the flat plate with simply supported edges) and Schunck's (for the same problem with this paper, but it shows only a numerical example on a plate of special size shown below). Their results are compared with the authors' in Fig. 2.

Schunck obtained the following result for the curved plate ($r=20$ m, $b=5$ m, $l=1.5$ m, $h=0.5$ cm).

$$N_{xy}=158\ \text{kg/cm},\ m=6.25,\ n=1.15.$$

From the authors' formula, it is

$$N_{xy} = 164 \text{ kg/cm}, \quad m = 6, \quad n = 1.$$

The authors' value is larger by only 4% than Schunck's.

b) l/b = infinite:

In this case, the data for clamped edges are Southwell's (for flat plate), Wagner's (experimental formula), and Leggett's. While, those for simply supported edges are Southwell's (for flat plate), Leggett's and Kromm's. Their results are shown in Fig. 2 and 3 with the authors'. For the flat plate, the buckling load obtained by the authors is larger by about 7% than that of Southwell's. The reasons will be as follows: In order to be able to perform the numerical calculation with comparatively little labour

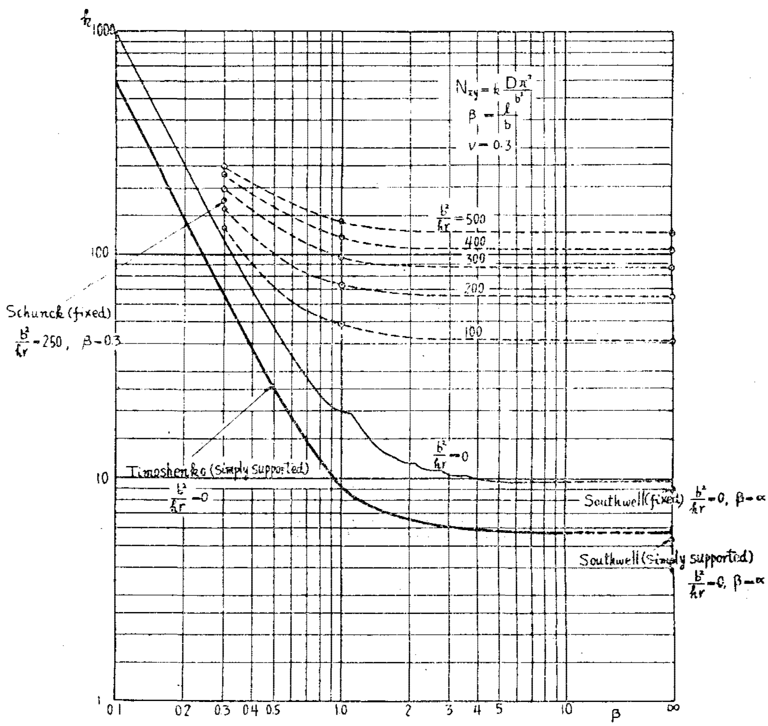

Fig. 2.

the authors represented the displacements not with infinite series but with one term respectively. Accordingly, there exist restrictions that the nodal lines are linear and the number of waves is integral.

On the curved plate, the result of this paper agrees with that of Leggett

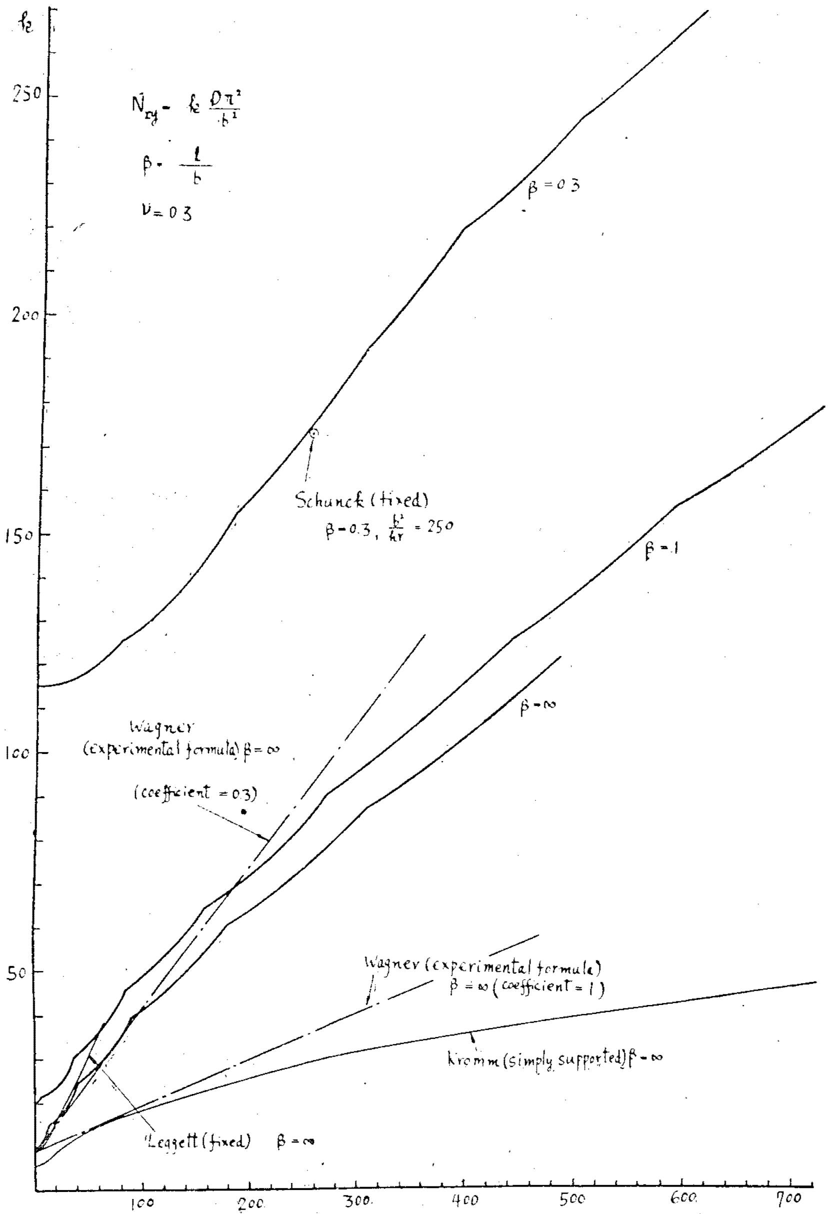

Fig. 3.

for small value of b^2/hr (for the small curvature). But, in Leggett's result there occurs the effect of his assumption that the curvature is small, as Kromm stated on the simply supported case in his paper, for the large value of b^2/hr. Accordingly the authors' result become smaller than that of Leggett. It is reasonable. As the numerical calculation of Leggett had been performed only in the region $b^2/hr = 0$ to $b^2/hr = 15$, the authors extended his curve with more numerical calculation.

On the experimental formula by Wagner there is no detail explanation of his treatise in our hands, and the numerical value of the coefficient in his formula seems to take some different values. We can not know the applicable region of his results. Accordingly it is impossible to discuss our result with his.

4. Conclusions.

The buckling of curved rectangular plates with clamped edges under uniform shear was found in comparatively little labour. Its result is also available for flat plates.

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Bibliography

1) S. Timoshenko: Theory of elastic stability.

2) E. Trefftz: Über die Ableitung der Stabilitätskriterien des elestischen Gleichgewichtes aus der Elastizitätstheorie endlicher Deformationen.
(Proc. 3rd. Int. Congr. Appl. Mech. Vol. III, Stockholm, 1930, p. 44)

3) E. Trefftz: Zur Theorie der Stabilität des elastischen Gleichgewichtes.
(Z.A.M.M. Bd. 13, Heft 2, 1933, S. 160)

4) E. Trefftz: Mathematische Elastizitätstheorie.
(Handbuck der Physik, Bd. IV)

5) A. Kromm: Die Stabilitätsgrenze eines gekrümmten Plattenstreifens bei Beanspruchung durch Schub und Längskräfte.
(Lufo. Bd. 5, 1938, S. 517)

6) T. E. Schunck: Zur Knickfestigkeit schwach gekrummter zylindrischen Schalen.
(Ing. Arcv. Bd. IV, 1933, S. 394)

7) D.M.A. Leggett: The elastic stability of a long and slightly bent rectangular plate under uniform shear.
(Proc. Roy. Soc. London series A, Vol. 162. 1937, p. 62)

8) W. Flügge: Die Stabilität der Kreiszylinderschale.
(Ing. Arcv. Bd. III, 1932, S. 436)

9) O. S. Heck und H. Ebener: Formeln und Berechnungsverfahren für die Festigkeit von Platten- und Schalenkonstruktionen in Flungzeugbau.
(Lufo. Bd, III, 1935, S. 211)

10) H. Wagner: Über Konstruktions- und Berechnungsfragen des Blechbaues.
(Jb. wiss. Ges. Luft. 1928, S. 113, TV)

11) St. Bergmann & H. Reissner: Neuere Probleme aus der Flugzeugtechnik.
(Z. F. M. Bd. 23, 1931, S. 6)

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