Maxwell’s Demon: More about PHYS

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We discuss more about the PHYS part of the digest on Maxwell's demon (https://hdl.handle.net/2324/7157421).

We have applied Clausius's inequality to the composite of the system and the demon in the digest so that the actual physical processes at the system or at the demon are hidden in the discussion. In this note we describe the measurement process focusing on the demon and the control process focusing on the system in a physical manner.

Measurement

In the measurement process the demon plays the role of a measurement apparatus where demon's state j is interpreted as the pointer of the apparatus. Since the apparatus is in the contact with a reservoir, its time-evolution is stochastic. In other words, we are considering an information transfer through a noisy channel from the system to the demon.

The measurement apparatus interacts with the system and its time-evolution depends on system's state i. If system's state is i, the entropy of the demon changes from $H_i^0 = -\sum_j q_{j/i}^0 \log q_{j/i}^0$ to $H_i = -\sum_j q_{j/i} \log q_{j/i}$. Here $P_{ij}^0 = q_{j/i}^0 \cdot p_i$ for $\hat{P}_0 = \{P_{ij}^0\}$ and $P_{ij} = q_{j/i} \cdot p_i$ for $\hat{P} = \{P_{ij}\}$. We have assumed that the measurement can be carried out without disturbing the system so that $\vec{p}$ is unchanged. Clausius's inequality tells

$$H_i - H_i^0 \geq \beta Q_i$$

where Q_i is the heat supplied to the apparatus from the reservoir.

Averaging this inequality over $\vec{p}$ we obtain

$$H(\hat{P}/\vec{p}) - H(\hat{P}_0/\vec{p}) \geq \beta Q_{meas}$$

with $Q_{meas} \equiv \sum_i p_i Q_i$. By the measurement the probability distribution of the pointer changes from $\vec{q}_0 = (q_1^0, q_2^0, q_3^0, \cdots)$ to $\vec{q} = (q_1, q_2, q_3, \cdots)$. Since $H(\hat{P}_0/\vec{p}) = H(\vec{q}_0)$ and $H(\hat{P}/\vec{p}) = H(\vec{q}) - I(\hat{P})$ under the assumption for our measurement process, we obtain

$$H(\vec{q}) - H(\vec{q}_0) - I(\hat{P}) \geq \beta Q_{meas}.$$

Control

In the control process the demon operates on the system. The operation depends on the result of the measurement j. Since the system is in the contact with the reservoir, its time-evolution is stochastic. If the measured value is j, the entropy of the system changes from $H_j = -\sum_i p_{i/j} \log p_{i/j}$ to $H_j' = -\sum_i p_{i/j}' \log p_{i/j}'$. Here $P_{ij} = p_{i/j} \cdot q_j$ for $\hat{P} = \{P_{ij}\}$ and $P_{ij}' = p_{i/j}' \cdot q_j$ for $\hat{P}' = \{P_{ij}'\}$. We have assumed that $\vec{q}$ is unchanged in the control process. Clausius's inequality tells

$$H_j' - H_j \geq \beta Q_j$$

where Q_j is the heat supplied to the system from the reservoir.

Averaging this inequality over $\vec{q}$ we obtain

$$H(\hat{P}'/\vec{q}) - H(\hat{P}/\vec{q}) \geq \beta Q_{cont}$$

with $Q_{cont} \equiv \sum_j q_j Q_j$. By the control the probability distribution of the system changes from $\vec{p} = (p_1, p_2, p_3, \cdots)$ to $\vec{p}' = (p_1', p_2', p_3', \cdots)$. Since $H(\hat{P}'/\vec{q}) = H(\vec{p}')$ and $H(\hat{P}/\vec{q}) = H(\vec{p}) - I(\hat{P})$ under the assumption for our control process, we obtain

$$H(\vec{p}') - H(\vec{p}) + I(\hat{P}) \geq \beta Q_{cont}.$$