

三瀬所長還歴記念論文集（要報）

MISE, Kozaburo

Research Institute for Elasticity Engineering, Kyushu Imperial University

SENDA, Kanae

Research Institute for Elasticity Engineering, Kyushu Imperial University

MATANO, Chujiro

Research Institute for Elasticity Engineering, Kyushu Imperial University

NEGORO, Shosaburo

Research Institute for Elasticity Engineering, Kyushu Imperial University

他

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Hakozaki, Fukuoka.

PROFESSOR KOZABURO MISE, DR. ENG.

DIRECTOR

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Universal Solution of Framed Structures.

By

Kôzaburô MISÉ

(Read March 8, 1946)

The proposed method is an exact and general solution of framed structures. The analysis is equally applicable to the triangular frame and the rectangular frame, and likewise to the indeterminate frame as well as to the determinate frame.

The principle is as follows:

First, find the expressions of the moments $\mathfrak{M}$'s, axial and tangential forces $\mathfrak{N}$'s, $\mathfrak{T}$'s in terms of the elastic displacements δx 's δy 's, $\delta \varphi$'s of the panel points, as in the form

$$\mathfrak{M}_{hi} = 4a_{hi}K_{hi}\delta\varphi_h + 2b_{hi}K_{hi}\delta\varphi_i + c_{hi}\mathfrak{M}_{hi}^f + d_{hi}\mathfrak{M}_{ih}^f - 6e_{hi}K_{hi}\left\{(\delta x_h - \delta x_i)\frac{\sin\alpha_{hi}}{l_{hi}} - (\delta y_h - \delta y_i)\frac{\sin\beta_{hi}}{l_{hi}}\right\}, \quad (1)_1$$

$$\mathfrak{N}_{hi} = L_{hi}\left\{(\delta x_h - \delta x_i)\cos\alpha_{hi} + (\delta y_h - \delta y_i)\cos\beta_{hi} + \varepsilon_{hi}t_{hi}l_{hi}\right\}, \quad (1)_2$$

$$\mathfrak{T}_{hi} = \frac{e_{hi}}{l_{hi}}\left\{6K_{hi}\left[\delta\varphi_h + \delta\varphi_i - 2\left\{(\delta x_h - \delta x_i)\frac{\sin\alpha_{hi}}{l_{hi}} - (\delta y_h - \delta y_i)\frac{\sin\beta_{hi}}{l_{hi}}\right\}\right] + \mathfrak{M}_{hi}^f - \mathfrak{M}_{ih}^f\right\} - \mathfrak{T}_{hi}^0. \quad (1)_3$$

Secondly, applying the conditions of elastic equilibrium to the forces acting at each panel point, we obtain as many equations as there are unknown displacements of the panel points, as follows,

$$\sum_i \mathfrak{M} = 0, \quad \sum_i \mathfrak{M}_{ih} = 0, \quad (\delta\varphi's) \quad (2)_1$$

$$\sum_i \mathfrak{T}_x = 0, \quad \sum_i \mathfrak{N}\cos\alpha - \sum_i \mathfrak{T}\sin\alpha + \mathfrak{P}_i = 0, \quad (\delta x's) \quad (2)_2$$

$$\sum_i \mathfrak{T}_y = 0, \quad \sum_i \mathfrak{N}\cos\beta + \sum_i \mathfrak{T}\sin\beta + \mathfrak{Q}_i = 0. \quad (\delta y's) \quad (2)_3$$

Lastly, solving simultaneously these conditional equations, the displacements and stresses can be determined as linear functions of the given external forces $\mathfrak{P}$'s, $\mathfrak{Q}$'s, $\mathfrak{R}$'s and temperature change t , or generally,

$$\delta x = f_x(\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}, t), \quad \delta y = f_y(\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}, t), \quad \delta \varphi = f_\varphi(\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}, t), \quad (3)$$

$$\mathfrak{M} = F_M(\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}, t), \quad \mathfrak{R} = F_N(\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}, t), \quad \mathfrak{Z} = F_T(\mathfrak{P}, \mathfrak{Q}, \mathfrak{R}, t). \quad (4)$$

Consequently, the influence lines of those stress and strain functions may readily be gained from the general solutions.

The conditional equations are the most general formulas from which the displacements should be determined, and they express the general relations between the $3(m+1)$ displacements at the $(m+1)$ consecutive panel points and the loads at the panel points, m being the number of members which are connected with the panel point under consideration.

Hence the formulas (2) may be called the "Theorem of $3(m+1)$ Displacements". The theorem is entirely general and universally applicable to any framed structures regardless of its number of redundant conditions and no matter what the conditions of loading are.

On the other hand, if it is not necessary to determine the rotation angles of the panel points, it is sometimes convenient to use the Theorem of Four Moments, namely

$$\begin{aligned} & b'_{hi} l'_{hi} \mathfrak{M}_{hi} - 2a'_{hi} l'_{hi} \mathfrak{M}_{ih} + 2a'_{ik} l'_{ik} \mathfrak{M}_{ik} - b'_{ik} l'_{ik} \mathfrak{M}_{ki} \\ & = l'_{hi} \mathfrak{M}_{hi}^f + 2l'_{hi} \mathfrak{M}_{ih}^f + 2l'_{ik} \mathfrak{M}_{ik}^f + l'_{ik} \mathfrak{M}_{ki}^f \\ & + 6EI_c \left[\left\{ (\delta x_h - \delta x_i) \frac{\sin \alpha_{hi}}{l_{hi}} - (\delta y_h - \delta y_i) \frac{\sin \beta_{hi}}{l_{hi}} \right\} \right. \\ & \left. - \left\{ (\delta x_i - \delta x_k) \frac{\sin \alpha_{ik}}{l_{ik}} - (\delta y_i - \delta y_k) \frac{\sin \beta_{ik}}{l_{ik}} \right\} \right]. \quad (5) \end{aligned}$$

Then, applying the theorem at each field, we get as many equations as there are unknown $\mathfrak{M}$'s, and solving the equations simultaneously each end moment will be expressed by the panel point displacements thus

$$\mathfrak{M} = F_M(\delta x, \delta y, \mathfrak{R}). \quad (6)$$

Accordingly, the tangential forces may be expressed as in the form

$$\mathfrak{Z} = F_T(\delta x, \delta y, \mathfrak{R}). \quad (7)$$

Finally, taking the conditional equations $(2)_2$, $(2)_3$, we are able to find the panel point displacements in terms of the given external forces as in the previous method.

In the application of the proposed methods, after dealing with some cases of triangular frames, we will discuss the problems of rectangular frames. In their analyses, also, it will be illustrated to what extent the deformations due to the axial & tangential forces are comparable with that due to the bending moments.

On the Decay of Torsional Vibration of a Wire-and-Rotator.

By

Kanae SENDA

(Received Sept. 15, 1946)

Problems relating to the decay of vibration of an elastic body fixed at some parts of its boundary are much complicated, partly due to variety of causes of the decay and partly due to manifold geometrical and mechanical conditions of the vibrating body. Therefore complete mathematical treatment of such problems is very difficult without some limitations with regard to the kind of decay, the geometrical form of the body and the dynamical conditions on its boundary.

The present writer takes the energy-flow into external body due to propagation of elastic waves caused by the vibration as an origin of the decay and discusses mathematically the problem in the case of torsional vibration of an elastic circular wire ideally welded at its upper end to a semi-infinite elastic body and attached to a rigid rotator at the lower. The results show that the decay due to the above cause is exceedingly feeble and can be neglected in comparison with other kinds of decay. This conclusion has hitherto been believed to be true by every one without any precise reasoning, but we have got now a theoretical verification. Thus the constant of logarithmic decrement of the torsional vibration (its decay is assumed to be due to the above cause alone) can be written as

$$D = \frac{\pi}{24} \frac{a^4}{l} \frac{\sqrt{\rho_1 \mu_1}}{\sin 2\gamma_1 l} \left(\frac{\rho_2}{\mu_2} \right)^{\frac{3}{2}} p^4, \quad \gamma_1 = \sqrt{\frac{\rho_1}{\mu_1}} p,$$

where ρ_1 , μ_1 , a and l denote respectively the density, rigidity, radius and the length of the wire; ρ_2 , μ_2 represent respectively the density and the rigidity of the semi-infinite elastic body; and p means the frequency of the vibration in 2π units of time. If $\gamma_1 l$ is small, D can be rewritten as

$$D = \frac{\pi}{48} \frac{\mu_1}{\mu_2} \frac{a^4}{l} \left(\frac{\rho_2}{\mu_2} \right)^{\frac{3}{2}} p^3$$

The above results have been got mathematically on the assumptions that the frequency of the torsional vibration is not extremely large and the wire is ideally clamped at its upper end to another very large elastic body occupying semi-infinite space, and therefore in other cases appropriate mathematical treatment will be necessary and the conclusion may somewhat be different. The present author wishes to discuss these cases in the near future.

Chujiro MATANO

For the calculation of the coefficient of diffusion D from the experimental data of the concentration c of diffusing substance at the surface of the medium, tables are made in the following two cases. Case 1 is for the medium of the form of infinite plate of thickness l . From the solution of the differential equation $\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$ for $0 \leq x \leq l$, where c is the concentration of the substance at the time t , and at a point x in the medium, under the initial conditions: at $t=0$, $c=C$ for $0 \leq x \leq l$ and $c=0$ for $b \leq x \leq l$ and the boundary conditions: at $x=0$ and $x=l$, $\frac{\partial c}{\partial x} = 0$ the concentration c_0 at the surface $x=l$ is given by

$$\zeta = a + \frac{2}{\pi} \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \sin n\pi a e^{-\left(\frac{n\pi}{2r}\right)}$$

where $\zeta = c_0/C$, $a = b/l$ and $\gamma = 1/2\sqrt{Dt}$, for simplicity. The calculated values of ζ for various values of a and γ are tabulated in Table 1 (abridged from the original paper).

Table 1. Values of ζ for a plate.[illegible]

(continuation)

$\alpha \backslash \gamma$	4.00	4.50	5.00	5.50	6.00	6.50	7.00
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	0000	0000	0000	0000	0000	0000	0000
20	0000	0000	0000	0000	0000	0000	0000
30	0000	0000	0000	0000	0000	0000	0000
40	0004	0001	0000	0000	0000	0000	0000
50	0032	0010	0003	0001	0000	0000	0000
60	0180	0083	0036	0014	0005	0002	0001
70	0741	0466	0281	0163	0091	0048	0025
80	2286	1803	1398	1066	0788	0588	0425
90	5395	4954	4532	4129	3747	3387	3650
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

(For the calculations to four decimal places, $\zeta = a^2$ for $\gamma \leq 0.5$ and to three decimal places, $\zeta = \sqrt{a} [1 - \psi(1 - a\gamma)]$ for $\gamma \leq 7.5$.)

The more detailed tables are shown in the original report.

On a Problem of Bending a Steel Beam by Three Rollers.

By

Shosaburo NEGORO

(Received Sept. 15, 1946)

For the purpose of the Calculation of the bending of a steel beam by three rollers, we assume that the stress-strain curve is replaced as three straight lines, the each straight line corresponding for the elastic part, the yield part and the plastic part respectively. Let us call the element of the beam that is composed of the elastic part only as the elastic beam element and that of both the elastic and the yield parts as the yield beam element, and also, that of all the three parts as the plastic beam element.

Now, in the present problem, if the Origin of the Axis is taken at the contact point of the beam and the middle roller, the x -axis the Axis of the beam, and the y -axis the direction of the weight ($2p$) of the middle roller, the moment can be expressed by the following equations:—

$$\left. \begin{aligned} M &= p\{(L-x) + (y-r)\tan\theta\} \\ x &= \int_0^s \cos\varphi ds, \quad y = y - \int_0^s \sin\varphi ds \\ y &= r(1 - \cos\theta) + \int_0^s \sin\varphi ds, \quad \varphi = \int_0^s \frac{1}{\rho} ds \end{aligned} \right\} \dots\dots(1)$$

in which r is the radius of the supporting rollers at the both ends of the beam, θ the inclination of the beam at the contact point (A) of the beam and the end roller, $2L$ the distance between the centres of the supporting rollers, L' the distance between the point (A) and the y -axis, y the deflection of the central point of the beam, φ the inclination of the axis of the beam, s the length along the beam-axis, s_i the length between the point (A) and the y -axis along the beam-axis and ρ the radius of curvature of the beam-axis.

Next, let us take the section of the beam as the rectangle of the breadth b and of the thickness $H(=2h)$ and assume that the stress-strain curves obtained by the tension and compression tests are symmetry with respect to the origin and that there is no difference of upper and lower yield points.

Then, for each of the above plastic, yield & elastic beam elements, we get the following equations:—

for the plastic beam element,

$$\epsilon_m = h/\rho, \quad \epsilon_{y2} = \xi/\rho, \quad \epsilon_{y1} = \eta/\rho \quad \dots\dots\dots (2_1)$$

$$Eb(\epsilon_{y1}^3 - n\epsilon_{y2}^3)\rho^3 + 3\{Eb(n\epsilon_{y2} - \epsilon_{y1})h^3 + M\}\rho - 2Ebnh^3 = 0 \quad \dots\dots\dots (3_1)$$

where ϵ_m is the surface strain of the cross-section of the beam, (ξ, η) being the distances from the neutral line (z -axis) of the section to the nearest layer of the plastic or yield domain, $(\epsilon_{y1}, \epsilon_{y2})$ the strains at the initial or finished point of the yield part of the stress-strain curves, and nE the inclination of the plastic part of the above stress-strain curves;

for the yield beam element,

$$\epsilon_m = h/\rho, \quad \epsilon_{y1} = \eta/\rho, \quad \dots\dots\dots (2)$$

$$Eb\epsilon_{y1}^3/\rho^2 - 3(Eb\epsilon_{y1}/h^2 - M) = 0 \quad \dots\dots\dots (3)$$

and, for the elastic beam element,

$$\frac{2E}{3\rho}bh^3 = M \quad \dots\dots\dots (3)$$

Now, in this case, we have for the condition at the cross-section, by which the plastic and yield beam element are separated,

$$\sigma_m = \sigma_y, \quad h/\rho = \epsilon_{y2}, \quad \sigma_y bh^2 \left\{ 1 - \frac{1}{3} \left(\frac{\epsilon_{y1}}{\epsilon_{y2}} \right)^2 \right\} = M \quad \dots\dots\dots (4)$$

and for the condition at the cross-section of the yield and plastic beam element

$$\sigma_m = \sigma_y, \quad \rho = h/\epsilon_{y1} = Eh/\sigma_y, \quad M = \frac{2}{3}Eb\epsilon_{y1}h^3 = \frac{2}{3}bh^3\sigma_y \quad \dots\dots\dots (5)$$

in which σ_m is the skin stress and σ_y the yield stress.

And, therefore, the skin stress (σ_m), the radius of the curvature (ρ), and the bending moment M at the above cross-sections are the known values when a material of the beam is given.

Then, from the above mentions, if we solve the simultaneous equations (1), (2) & (3) so as to satisfy the conditions (4) & (5) at the above cross-sections, we can determine all the required Values.

But as it is difficult to solve in mathematical formula, in this report it is treated numerically by the iteration method.

On the Propagation of Spherical Wave of Finite Amplitude

By

Shūjirō KUNII

(Received Sept. 15, 1946)

The laws of propagation of a plane wave of finite amplitude have been treated independently by Airy, Earnshaw and Riemann. The object of this work is to investigate the law of propagation for a spherical wave occurred explosively in a compressible liquid. If we take the centre of the spherical wave as the origin, the equation of continuity is

$$\frac{\partial \log \rho}{\partial t} + u \frac{\partial \log \rho}{\partial r} = -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u) \quad \dots \dots \dots (1)$$

and the Eulerian equation of motion is

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial r}, \quad \dots \dots \dots (2)$$

where u , ρ and p are the radial velocity, the density and the pressure respectively. In our treatment the time-rates of change in u , ρ and p are large compared with the radial gradients of the quantities and u is small, so that the above equations are simplified by the omission of the operator $u \frac{\partial}{\partial r}$. On this assumption (1) and (2) lead to a single equation

$$\frac{\partial^2 x}{\partial t^2} = \frac{1}{\rho_0 r^2} \frac{\partial}{\partial r} \left\{ r^2 e^{-x} \frac{\partial p}{\partial r} \right\}, \quad \dots \dots \dots (3)$$

if we put $\rho = \rho_0 e^x$, where ρ_0 is the constant density in the undisturbed state. We assume that the pressure in the disturbed state depends on ρ and r explicitly, but it takes a constant value p_0 in the undisturbed state. Hence we can put

$$p = p_0 + F(r, x) \quad \dots \dots \dots (4)$$

where F is an unknown function of r and x having the property $F(r, 0) = 0$. Introducing (4) to (3), we get the fundamental equation for x

$$\frac{\partial^2 x}{\partial t^2} = \frac{e^{-x}}{\rho_0} \left\{ \frac{\partial F}{\partial x} \frac{\partial^2 x}{\partial r^2} + \left(\frac{\partial^2 F}{\partial x^2} - \frac{\partial F}{\partial x} \right) \left(\frac{\partial x}{\partial r} \right)^2 \right.$$

$$+ \left(2 \frac{\partial^2 F}{\partial r \partial x} + \frac{2}{r} \frac{\partial F}{\partial x} - \frac{\partial F}{\partial r} \right) \frac{\partial x}{\partial r} + \left(\frac{\partial^2 F}{\partial r^2} + \frac{2}{r} \frac{\partial F}{\partial r} \right) \} . \quad \dots\dots\dots (5)$$

In order to solve this we assume an intermediate integral of the type

$$\frac{\partial x}{\partial t} = \psi(r, x) \frac{\partial x}{\partial r} + \phi(r, x) , \quad \dots\dots\dots (6)$$

where ψ and ϕ are expressible by the series

$$\psi(r, x) = \sum_{n=0}^{\infty} \frac{a_n}{r^n} \quad \dots\dots\dots (7)$$

$$\text{and} \quad \phi(r, x) = \frac{1}{r} \sum_{n=0}^{\infty} \frac{b_n}{r^n} \quad \dots\dots\dots (8)$$

a_0 being a constant and other a 's and b 's being function of x only. Differentiating (6) with respect to t and comparing the resulting equation with (5), we get

$$\left. \begin{aligned} \psi^2 &= \frac{e^{-x}}{\rho_0} \frac{\partial F}{\partial x} \quad \dots\dots\dots \\ 2\psi\psi_x &= \frac{e^{-x}}{\rho_0} \left(\frac{\partial^2 F}{\partial x^2} - \frac{\partial F}{\partial x} \right) \quad \dots\dots\dots \\ \psi\psi_r + \phi\psi_x + 2\psi\phi_x &= \frac{e^{-x}}{\rho_0} \left(2 \frac{\partial^2 F}{\partial r \partial x} + \frac{2}{r} \frac{\partial F}{\partial x} - \frac{\partial F}{\partial r} \right) \quad \dots\dots\dots \\ \phi\phi_r + \phi\phi_x &= \frac{e^{-x}}{\rho_0} \left(\frac{\partial^2 F}{\partial r^2} + \frac{2}{r} \frac{\partial F}{\partial r} \right) , \quad \dots\dots\dots \end{aligned} \right\} \quad (9)$$

where the suffices indicate the partial differentiation. From the first of these equations we obtain

$$F(r, x) = \rho_0 \int_0^x \psi^2(r, \xi) e^{\xi} d\xi . \quad \dots\dots\dots (10)$$

By this relation (10) the second equation of (9) is identically satisfied and the remaining equations of (9) lead to the equations

$$\left. \begin{aligned} \phi\psi_x + 2\psi\phi_x &= 3\psi\phi_r + \frac{2\psi^2}{r} - 2e^{-x} \int_0^x \phi\phi_r e^{\xi} d\xi \quad \dots\dots\dots \\ \phi\phi_r + \phi\phi_x &= 2e^{-x} \int_0^x \left\{ \phi_r^2 + \phi\phi_{rr} + \frac{2}{r} \phi\phi_r \right\} e^{\xi} d\xi . \quad \dots\dots\dots \end{aligned} \right\} \quad (11)$$

Using (7) and (8) to (11), we can find a 's and b 's successively. Thus we obtain

$$\left. \begin{aligned} \psi(r, x) &= a_0 \left\{ 1 - \frac{ax}{3r} \left(1 - \frac{x}{3} + \frac{x^2}{15} - \dots \right) + \frac{a^2}{3r^2} \left(1 + \frac{x}{3} - \dots \right) + \dots \right\} \\ \phi(r, x) &= \frac{a_0 x}{r} \left\{ 1 + \frac{a}{r} \left(1 - \frac{x}{3} + \frac{x^2}{18} - \dots \right) - \frac{2a^2}{3r^2} \left(1 - \frac{2}{3}x + \dots \right) + \dots \right\}, \end{aligned} \right\} \quad (12)$$

where a is an integration constant. Accordingly, our fundamental equation (5) possesses the intermediate integral of the type (6), where ψ and ϕ are given by (12).

To find the general integral of (6), we may solve the auxiliary ordinary differential equations

$$\frac{dt}{1} = \frac{dr}{\psi(r, x)} = \frac{dx}{\phi(r, x)} \quad \dots \dots \dots (13)$$

Neglecting small quantities of the order higher than $\frac{1}{r^3}$ we get the two independent primary integrals

$$\left. \begin{aligned} \frac{rx}{1+a/r} &= C_1 \quad \dots \dots \dots \\ \text{and } r \left\{ 1 - \frac{ax}{3r} + \frac{a^2}{3r^2} \right\} + a_0 t &= C_2, \quad \dots \dots \dots \end{aligned} \right\} \quad (14)$$

where C_1 and C_2 are arbitrary constants. Hence the general integral of (6) are given by

$$\Phi \left(\frac{rx}{1+a/r}, r \left[1 - \frac{ax}{3r} + \frac{a^2}{3r^2} \right] + a_0 t \right) = 0 \quad \dots \dots \dots (15)$$

where Φ expresses an arbitray functional symbol.

By solving this, we finally obtain

$$x = \frac{1}{r} \left(1 + \frac{a}{r} \right) \Psi \left(r \left[1 - \frac{ax}{3r} + \frac{a^2}{3r^2} \right] + a_0 t \right), \quad \dots \dots \dots (16)$$

where Ψ is an arbitrary function of the argument. This is a generalization of the well-known relation

$$x = \frac{1}{r} \Psi(r + a_0 t)$$

for a spherical wave of infinitesimal amplitude. By applying ordinary methods to (16), we can deduce various results concerning the laws of propagation of a spherical wave. Thus we find for example that the progagation velocity of the wave is not a constant value a_0 , but depends on the distance from the centre and on the magnitude of the amyplitude itself.

Stresses in an Orthogonal Aeolotropic Infinite Plate with an Elliptic Hole under Uniform Tension.

By

Masakazu HIGUTI

(Received Sept. 15, 1946)

Without any assumption on rigidity of plates, the author has obtained the Airy's stress function of an orthogonal aeolotropic infinite plate with an elliptic hole under uniform tension σ_n in the direction parallel or perpendicular to that of fibers of the plate by using dual transformation. That is expressed as the sum of three functions:

$$F_1 = \frac{x\sigma_n}{2(k_1 - k_2)} \left\{ \frac{1}{2} (xk_1 - 1) e^{-2a_1 \cos 2\beta_1 - (xk_1 + 1)a_1} \right\},$$

$$F_2 = \frac{x\sigma_n}{2(k_2 - k_1)} \left\{ \frac{1}{2} (xk_2 - 1) e^{-2a_2 \cos 2\beta_2 - (xk_2 + 1)a_2} \right\},$$

$$F_3 = \frac{1}{2} \sigma_n y^2, \quad \text{or} \quad \frac{1}{2} \sigma_n x^2,$$

where the direction of the fibers of the plate is coincided with the x -axis and the elliptic hole is expressed by $x^2/a^2 + y^2/b^2 = 1$ and $x = a/b$. The explanation of curvilinear coordinates (a_1, β_1) and (a_2, β_2) used here, is abridged¹⁾.

From these functions, when the tension is in the direction of the x -axis, i.e., $\sigma_n = [\sigma_x]_{x=\infty}$, the normal stresses acting tangentially on the boundary of the hole are calculated: Namely,

$$-\sigma_n/(k_1 k_2), \quad \text{at the point } (a, 0),$$

$$\sigma_n \{1 + x(k_1 + k_2)\}, \quad \text{at the point } (0, b),$$

where k_1, k_2 are constants relating to the elastic constants of the plate.²⁾

When $\sigma_n = [\sigma_y]_{y=\infty}$,

$$-\sigma_n k_1 k_2, \quad \text{at the point } (a, 0),$$

$$\sigma_n \{1 + x(1/k_1 + 1/k_2)\}, \quad \text{at the point } (0, b).$$

Further, the following results are obtained in the case of a circular

hole: The stress on the boundary of the hole, when $\sigma_n = [\sigma_x]_{x=\infty}$, is

$$\sigma(\beta_0) = \sigma_n + \sigma_1(\beta_0) + \sigma_2(\beta_0),$$

together with

$$\sigma_i(\beta_0) = \sigma_n c_i (\cos^3 2\beta_0 + a_i'' \cos^2 2\beta_0 + a_i' \cos 2\beta_0 + a_i) / (\cos 2\beta_0 + b_i)^3, \quad (i=1, 2),$$

where β_0 is the angular position measured from the x -axis, of which stress is to be calculated and the other notations denote constants containing k_1 , k_2 only. While the stress on the same boundary in the case $\sigma_n = [\sigma_y]_{y=\infty}$, is calculated with, instead of $\sigma_1(\beta_0)$ and $\sigma_2(\beta_0)$,

$$\sigma_1'(\beta_0) = -k_1 \{\sigma_1(\pi/2 - \beta_0)\}, \quad \sigma_2'(\beta_0) = -k_2 \{\sigma_1(\pi/2 - \beta_0)\},$$

respectively.

The stress on the minimum section of the plate is, when $\sigma_n = [\sigma_x]_{x=\infty}$,

$$\sigma(y) = \sigma_n + \sigma_1(y) + \sigma_2(y),$$

and $\sigma_i(y) = (-)^i \sigma_n k_i^2 (\cosh a_i - \sinh a_i) / \{(k_2 - k_1)(\cosh a_i + k_i \sinh a_i)\}$, $(i=1, 2)$,

where the parameters a_1 , a_2 are determined by

$$y = \cosh a_1 + 1/k_1 \sinh a_1 = \cosh a_2 + 1/k_2 \sinh a_2.$$

In the case $\sigma_n = [\sigma_y]_{y=\infty}$,

$$\sigma(x) = \sigma_n + \sigma_1'(x) + \sigma_2'(x),$$

$$\sigma_1'(x) = k_2 \sigma_n (\cosh a_1 - \sinh a_1) / \{(k_2 - k_1)(k_1 \cosh a_1 + \sinh a_1)\},$$

$$\sigma_2'(x) = k_1 \sigma_n (\cosh a_2 - \sinh a_2) / \{(k_1 - k_2)(k_2 \cosh a_2 + \sinh a_2)\},$$

and $x = \cosh a_1 + k_1 \sinh a_1 = \cosh a_2 + k_2 \sinh a_2$.

Reference

- ¹⁾ H. Okubo, Jour. of the Soc. of Mech. Eng., Japan, No. 306, Bd. 45.
- ²⁾ K. Ikeda, Rep. of the Aero. Inst., Tokyo Imp. Univ., No. 259.

Remark: Prof. Ikeda's subsequent work on the same subject as this paper was known by the author when this was in the press. He had solved the problem by another method and published in 1944. In this connection far more general cases are dealt with by the author. It was delivered last year.

On the Slope-Deflection Formulae in Vector Forms for the Three Dimensional Structures.

By

Jiro SUHARA

(Received Sept. 15, 1946)

To simplify calculating, when the stresses are determined, it is common to arrive at an approximate figure assuming a plane frame. But often it is necessary to determine exact stresses of the three dimensional structures, or plane structures loaded in non-common planes.

The three dimensionally stressed structural element is studied on the extension of the slope-deflection formulae and both formulae and conditions of equilibrium on any nodal point are expressed in vector forms.

Consequently the representation of statically indeterminate quantities in space is clarified, and the procedure of tabulating and calculating is much simpler than calculating by usual strain energy method.

On the Hardness of the Superlattice Phase in the Mg-Cd System.

By

Takao SATO

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It is surely recognized by Grube-Schiedt and Dehlinger (Z. anorg. allg. Chem. 194, 190, 223, 1930) that there is the superlattice phase at the composition of Mg_3Cd and $MgCd_3$ in the *Mg-Cd* system.

This superlattice is the only one case between hexagonal close packed lattice and it is not change their hexagonal crystal structure in order-disorder transformation. Their Curie points are $150^{\circ}C$ for Mg_3Cd and $89^{\circ}C$ for $MgCd_3$.

The purpose of this research is to find the relation of the quenching temperature and the hardness in order-disorder transformation. Their degree of order was measured by using of electric resistance.

In the case of the ordered lattice, if you deform it, the disordered arrangement of two atoms, *A* and *B* at least will be increased in their slip plane and the energy of it plane will be increased also, but In the case of the disordered lattice, the arrangement remains as before after the deformation and the energy of the slip plane will be, also, nearly the same as before, so the deformation of the ordered lattice will need much more works than that of the disordered state. The author confirmed this consideration about Mg_3Cd alloy.

On the Approximate Computation of the Critical Load.

By

Hajime NAKAGAWA

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1. Introduction

For the approximate computation of the critical load, the several methods, energy, integral equation, successive approximation process, numerical integration and other special ways, are used. In this paper, the critical load of the long column with varying cross-section in accordance with a simple formula are determined by the three typical approximate methods. Now the approximate results are compared with that obtained by solving the differential equation as general and then the feature and the defect of each approximate method are investigated.

From the 2nd report, it is invented to study on the long column with tapering part of which differential equations can not be solved at present.

2. Result

As an example, the case when it has the following cross-section

$$I = \beta Z^4, \quad Z = b - \frac{b-a}{l}x$$

is investigated, where I =the moment of inertia of cross-section, l =length of column, x =axis passing through the centre of cross-section from the bottom end, β =constant determined by the forms of cross-section, Z =variable representing the dimension in the x cross-section, a, b =constant representing the dimension of cross-section, at the upper and lower ends.

If c_1, c_2 and c_3 are the coefficients of the critical load

$$P = c \frac{EI}{l^2}$$

by the method of. energy, successive approximation process and integral equation respectively and c_0 that obtained by solving the differential equation¹⁾, they take the following results.

a/b	c_0	c_1	c_2	c_3
1/10	0.081	0.174	0.181	0.075
1/5	0.264	0.331	0.391	0.256
1/4	0.377	0.437	0.503	0.348
1/3	0.582	0.626	0.695	0.530
1/2	1.029	1.044	1.099	0.976
2/3	1.500	1.502	1.521	1.461
3/4	1.738	1.741	1.737	1.703
4/5	1.885	1.886	1.868	1.846

3. Conclusions

1. Although in the energy method the elastic line satisfies the boundary conditions, the considerable deviation in the coefficient of the critical load are found by the selection of the elastic line.

2. The successive approximation process seems to give trouble apparently, but even the second approximation gives the suitable result if the integration succeeds.

3. If the Simpson rule be applied in the method of integral equation, this is an advantageous method since it is enough to solve only the determinant of kernel without integration.

4. The energy method gives the coefficient greater than the true value obtained by solving the differential equation. The method of integral equation adopting the Simpson rule gives the coefficient smaller than the true value. Therefore it is easy to determine the true coefficient if the both methods being adopted.

5. The approximate computation is effectual in such case of the simple elastic line as the top end free and the bottom end fixed, or the both ends are freely supported. But it will be difficult for other boundary conditions and will not give the more favourable values for the approximate coefficient.

¹⁾ Japan Mech. Eng. Rep. 1936. Vol. 2. No. 8, P. 305.

On the Mechanism of Disruption.

By

Tetsugoro FUTAGAMI

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As a fundamental study for the mechanism of disruption, the author studied the disruption of a glass rod under a tension, and a glass plate under a concentrated compression.

(I) Increasing the tension exerted on a glass rod, when it reaches a certain value a crack appears in the rod, and spreads over along a cross section of the rod. The surface of the crack is perpendicular to the tension. If the tension is regulated instant to instant to the amount which required to proceed the crack, a cracked surface, the area of which is equal to the cross section, is obtained. Practically, one part of the section is appeared as a plane perpendicular to the tension, and the remaining part is disrupted in a multiple cracked form.

Generally the cracked surface perpendicular to the tension appears in two forms. One of which is a glossy surface and the other is a mat surface. The glossy part appears in the beginning time of disruption and the mat part in the later time. The ratio of the area of the two parts concerns to the amount of tension which exceeds over the necessary amount of the disruption. If the tension is restricted in the necessary and sufficient amount, the cracked surface is glossy all over it.

When a disruption proceeds along a line a glossy surface is produced, and when it proceeds along a surface a mat one is produced, and it occurs in a volume a multiple cracked surface is produced. If the stress is just sufficient amount for disruption, a disruption occurs along a line, and for greater stress, it occurs in other forms.

(II) A crack produced in a glass plate by a concentrated pressure, is a surface of rotation of a logarithmic spiral. In this case, a glossy, mat and multiple cracks are appeared. The shelly crack of quartz or glass is one kind of this cracks appeared in a irregular state.

(III) Next, the following investigation is carried to explain the reason of

the occurrence of the cracks in two forms, that is in the case of tension, a plane perpendicular to the tension, and in the case of pressure, a curved surface to the pressure.

Measuring the strength of glasses for tension, compression and shearing, a Mohr's diagram is constructed. From the diagram, the inclination of cracks for stresses is obtained.

The result is that the inclination of crack for a tension is about 90° to the tension, and the inclination of a crack for compression is about 20° to the compression.

It is clear that a crack of a glass rod under a tension is perpendicular to the tension.

In the case of concentrated compression exerted upon a glass plate, the lines of force are all straight lines which pass through the point of action. A line which has a constant inclination to the straight lines through a point is a logarithmic spiral. Then in a section containing the point of action and normal to the surface of glass plate, the crack is a logarithmic spiral. Therefore, it is also clear, in the case of compression, a conical shaped body, bounded by a surface of revolution of the spiral and a base plane is formed.

When a glass plate is attacked by a ball, bullet or others, similar cone is also formed.