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ON THE VELOCITY DISTRIBUTION IN THE OPEN CHANNELS AND RIVERS[1]

By

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The recent theoretical and experimental investigations of turbulent flow in circular pipes or between parallel walls have brought the successful results as to the laws of velocity distributions and frictional registances. On the other hand, the author felt most of the existing studies concerning open channel flow seem to be unreasonable hydrodynamically. Since it is evident that near the boundaries the turbulent field in the both cases are similar, we can treat the open channel flow in the same manner as in the case of the pipes approximately, in spite of a little differences of conditions such as the free surface and the secondary current.

Recently M. Hosoi[2] has analysed his measurements of velocity distributions conducted in the open channel with parllel bars placed laterally in the bed and suggested the rational formula for rough channels. Being unaware of the work of G. Keulegan[3], the present author has inteded to analyse hydrodynamically the most systematic measurements of Bazin[4] which have been considered as the most basic.

The datas used by the autor are Bazin's measurements with a few additive measurements of H. Matsuo[5] concerning smooth open channels and of Torgau and Elbe rivers. The results obtained are as follows:

Firstly, the velocity profiles near the wall in general and throughout the cross-sections in the two dimensional flow show no differences between the pipes and the open channels, that is, we have

$$\frac{u}{u_\tau} = A + 5.75 \log \frac{y}{k}, \tag{1}$$

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[1] Abstract "On the Velocity Distribution in the Open Channels and Rivers" (in Japanese) Rep. Res. Inst. Fluid Engng. Kyūshū Univ. Vol. 4. No. 2. (1947)

[2] M. Hosoi, The study of Civil. Engng. I. Tokyo (1948)

[3] G. Keulegan, Journal Research of National Bureau of Standard. (1938)

[4] Darcy et Bazin; Researches Hydrauliques. (1865)

[5] H. Matsuo; Rep. Inst. Civ. Engng. (1939)

where u_τ is frictional velocity, k the linear dimension of roughness, y the distance measured upward from the bottom and A is a function of the characters of roughness and Reynolds number. However, when side wall effects are so large that the typical flow (i.e. two dimensional flow (1)) are appreciably disturbed in the upper part of flow, it is convenient to assume the parabolic distribution except near the bottom $\left(\frac{y}{y_{\max}} \leqq 0.3\right)$, i.e,

$$\frac{u_{\max}-u}{u_\tau} = \frac{1}{2} \frac{y_{\max} u_\tau}{\varepsilon_m} \left(1-\frac{y}{y_{\max}}\right)^2, \tag{2}$$

where $u_{\max}$ is the maximum velocity, $y_{\max}$ the height at which $u_{\max}$ occurs and ε_m is the mean of eddy viscosity. From the observations of velocity defects we can estimate the eddy Reynolds number $\frac{y_{\max} u_\tau}{\varepsilon_m}$, its value decreasing slightly with roughness and the ratio of depth to width, being maximum 14 for Nikuradse's measurements.[6]

Secondary, we obtain the following formula for mean velocity in the smooth channels after rewriting the experimental results of H. Matsuo[*],

$$\frac{u_m}{u_\tau} = 3.45 + 5.75 \log \frac{u_\tau R}{\nu}, \tag{3}$$

where u_m is the mean velocity, R the hydraulic radius and ν the coefficient of molecular viscosity. (3) shows the almost same result as that of circular pipes for all Reynolds number. As an approximate formular of (3),

$$\left(\frac{1}{\sqrt{2}} \frac{u_\tau}{u_m}\right)^2 = 0.0558 \left(\frac{u_m R}{\nu}\right)^{-\frac{1}{4}}, \tag{4}$$

is valid for Reynolds number less than 2×10^4. (Blausius formula)

Adopting the equivalent roughness, k_s, determined by putting Nikuradse's value $A = 8.48$ in (1), the mean velocity in fully rough flow is well expressed

$$\frac{u_m}{u_\tau} = 6.6 + 5.66 \log \frac{R}{k_s}. \tag{5}$$

Considering the above results, we can obtain some informations on the exponential formulas for the mean velocity. In general, mean velocity may be written by dimensional analysis as follow,

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6) J. Nikuradse; Ver. deutsch. Ing. Forschungsheft. 361. (1933)

*) H. Matsuo; loc. cit.

$$\frac{u_m}{u_\tau} = C\left(\frac{R}{k_s}\right)^{\alpha}\left(\frac{u_\tau k_s}{\nu}\right)^{\beta} = C' R^{\alpha'} I^{\beta'}. \tag{6}$$

As evidently $\beta = 0$ for fully rough flow, α and C are determined by (5) according to the regions of the validity of (6). For instance:

Table. Values of α and C for fully rough flow

	$4 \leqq \frac{R}{k_s} \leqq 20$	$20 \leqq \frac{R}{k_s} \leqq 100$	$100 \leqq \frac{R}{k_s} \leqq 1000$
α	0.25	0.15	0.122
C	0.734	8·98	10.2

It is mentioned here that Manning's formular ($\alpha = \alpha' = \frac{1}{6}$, $\beta = 0$) and Forchheimer's formula ($\alpha = \alpha' = 0.2$, $\beta = 0$) have the adequate indexes as the mean value for the rough flows as seen from the Table.

Equation (6) may be used for the transition region between the smooth law and fully rough law. If we assume monotonous changes of $\frac{u_m}{u_\tau} - 5.66 \log \frac{R}{k_s}$ for this region, confirming with experiments related to roughness of C.F. Colebrook and C.M. White,[7] we can easily find that α' ranges between $\frac{3}{14}$ and the values of α for the fully rough flow as shown above corresponding to relative ronghness considered and β lies between $\frac{1}{14}$ and zero.

[7] C. F. Colebrook and C. M. White: Proc. Roy. Soc., London. A 161. (1937)