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https://doi.org/10.15017/7158047

出版情報：流體工學研究所報告. 6 (2), pp.40-41, 1950-01. 九州大學流體工學研究所

ON TURBULENT VISCOSITY[1)]

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From Vol. 3 No. 3

For Reynolds' stresses in a turbulent flow between two parallel plates the following equations hold[2)]:

$$\left.\begin{aligned}
\frac{D\overline{u'^2}}{Dt} &= 0 = -2\overline{u'v'}\frac{dU}{dy} - 2\overline{u'\frac{\partial p'}{\rho\partial x}} - \frac{d}{dy}\overline{v'(u')^2} + 2\nu\overline{u'\nabla^2 u'} \\
\frac{D\overline{v'^2}}{Dt} &= 0 = \qquad 0 \qquad -2\overline{v'\frac{\partial p'}{\rho\partial y}} - \frac{d}{dy}\overline{v'(v')^2} + 2\nu\overline{v'\nabla^2 v'} \\
\frac{D\overline{w'^2}}{Dt} &= 0 = \qquad 0 \qquad -2\overline{w'\frac{\partial p'}{\rho\partial z}} - \frac{d}{dy}\overline{v'(w')^2} + 2\nu\overline{w'\nabla^2 w'}
\end{aligned}\right\} \tag{1}$$

$$\begin{aligned}
\frac{D\overline{u'v'}}{Dt} = 0 = -\overline{v'^2}\frac{dU}{dy} - \left\{\overline{u'\frac{\partial p'}{\rho\partial y}} + \overline{v'\frac{\partial p'}{\rho\partial x}}\right\} - \frac{d}{dy}\overline{v'(u'v')} \\
+\nu\{\overline{u'\nabla^2 v'} + \overline{v'\nabla^2 u'}\}.
\end{aligned} \tag{2}$$

Adding (1)

$$\begin{aligned}
\frac{D}{Dt}\sum\overline{u'^2} = 0 = -2\overline{u'v'}\frac{dU}{dy} - 2\sum\overline{u'\frac{\partial p'}{\rho\partial x}} - \frac{d}{dy}\overline{v'\sum u'^2} \\
+2\nu\sum\overline{u'\nabla^2 u'}.
\end{aligned} \tag{3}$$

Interpreting the contribution of the pressure fluctuations by (1), (2) and (3) it has been suggested, i) that the turbulent energy is supplied from the mean flow by the random motions of lumps of fluid in such a wise that at first the whole energy is given to the turbulent motion in the direction of the mean flow and then one-thirds of it are equally (when turbulence is nearly istropic) distributed by the action of pressure fluctuations to two other directions, analytically it means $\sum\overline{u'\frac{\partial p'}{\rho\partial x}} = 0$, ii) that the partial means of fluctuations of pressure gradient as the velocity components held constant can be approximately given by

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1) Abbreviated from "Investigations on Turbulence (IV). On Turbulent Viscosity" (in Japanese). Rep. Res. Inst. Fluid Engng. Kyūshū Univ. Vol. **3**. (1947), 18.

2) M. Kurihara, Rep. Res. Inst. Fluid Engng. Kyūshū Univ. **1** (1942) 1~20, its abstract will be seen in this report.

$$\frac{\overline{\partial p'}^p}{\rho \partial x} = a_1 u', \frac{\overline{\partial p'}^p}{\rho \partial y} = a_2 v', \frac{\overline{\partial p'}^p}{\rho \partial z} = a_3 w', \ a_1 > 0, \ a_2, \ a_3 < 0. \tag{4}$$

As to the extent of the correlation $r(P_1, P_2) = \dfrac{\overline{u_1' v_2'}}{\sqrt{\overline{u_1'^2} \cdot \overline{v_2'^2}}}$, which is defined as $\varphi(P_1, P_2) = \dfrac{r(P_1, P_2)}{r(P_1, P_2)}$, a discussion has been given with a result

$$\nu\{\overline{u' \nabla^2 v'} + \overline{v' \nabla^2 u'}\} = -\beta \frac{40\nu}{\lambda^2} \overline{u'v'}, \tag{5}$$

where λ is the so-called diameter of the smallest eddies and β is a constant nearly equal to 1.

By virtue of these relations we get as the fundamental equation for $\overline{u'v'}$ from (3)

$$\frac{a}{3\overline{u'^2}} \frac{dU}{dy} (\overline{u'v'})^2 - \frac{d}{dy} \overline{v'(u'v')} - \beta \frac{40\nu}{\lambda^2} \overline{u'v'} - \overline{v'^2} \frac{dU}{dy} = 0,$$

$$a = 3 \frac{\overline{v'^2} + \overline{w'^2} - \overline{u'^2}}{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}}. \tag{6}$$

For a flow in which its velocity gradient is constant and turbulence is nearly isotropic a relation has been obtained from (4)

$$\frac{l'^2}{\lambda^2} = \frac{\sqrt{3}}{40} \cdot \frac{l' \sqrt{\overline{u'^2}}}{\nu} = 0.0433 \frac{l' \sqrt{\overline{u'^2}}}{\nu}, \tag{7}$$

where l' denotes the mixing length defined as turbulent viscosity $= \frac{1}{3} l' \times$ the root-mean-square speed of turbulence. A comparison with the measurements of the scale of turbulence L by Dryden[1] and his collabolaters which give $\dfrac{L^2}{\lambda^2} = \dfrac{3c_6 \log_{10} e}{15} \dfrac{L\sqrt{\overline{u'^2}}}{\nu} = 0.0436 \dfrac{L\sqrt{\overline{u'^2}}}{\nu}$ seems to confirm our theory.

Neglecting the diffusion term in (6) the theoretical distribution of $\dfrac{\overline{u'^2}}{\overline{v'^2}}$ has been calculated. Its agreement with the measurements[2] is satisfactory.

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1) H. L. Dryden, G. B. Schubauer, W. C. Mock, H. K. Skramsted, N. A. C. A. Tech. Rep. No. 581 (1937).

2) H. C. H. Townend, Proc. Roy. Soc. A. **145** (1934). G. I. Taylor, Proc. Roy. Soc. A, **151** (1935), Statistical theory of turbulence, III.