Supplementary Data for the Theory of the Seepage Through an Earth Dam: I

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Supplementary Data for the Theory of the Seepage Through an Earth Dam (I)

By Michio OHJI

In his previous paper on the seepage of ground water through an earth dam [1], the present author derived new approximate formulas in a fairly simple way. But as was noticed there, further numerical examples would be desirable, inasmuch as the degree of approximation introduced in this theory (that is, the error associated with the "linearity assumption") is mathematically somewhat vague.

Now after the kind suggestion of Professor K. Shinohara the author has had an opportunity to fulfil such a requirement partly, refering to the recent paper of P. Huard de la Marre [2] where a number of experimental data have been reproduced. He refined the method of electrical models successfully and applied it to the various cases of an earth dam. In the present note a comparison will be made between his measurements and the author's approximate formulas for the trapezoidal cases in particular.

In the beginning the following remarks should be mentioned:

(i) The measurements were carried out, making use of homogeneous electro-conductive paper.

(ii) The errors of measurements of the total flux were checked beforehand for the rectangular case and found to be less than a few per cent.

(iii) The experiments concern the cases of a symmetrical trapezoid ($\alpha=\beta$) only.

(iv) All of the notations used here are consistent with those in the paper [1] (Fig. 1).

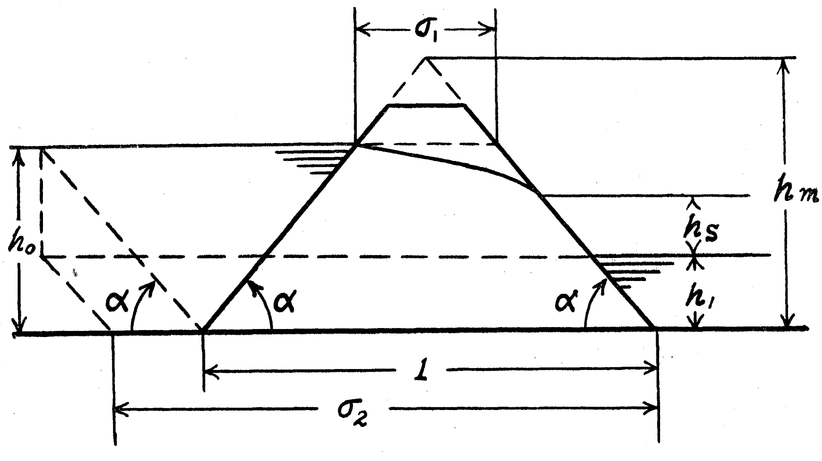


FIG. 1. Schematic sketch of a symmetrical dam (in non-dimensional scale).

(v) The formulas now in question are

$$q = \frac{a}{1+\sigma_1}(\pm\sqrt{1+b}-1), \quad \text{for} \quad a \gtrless 0$$

$$= \sigma_1\sqrt{\frac{\sigma_1+\sigma_2}{1+\sigma_1}}, \quad \text{for} \quad a = 0$$

$$h_S = \frac{q\,\Delta h}{q+\sigma_1},$$

where
$$a = \frac{1}{2}[\sigma_1(\sigma_1+\sigma_2)-(h_0{}^2-h_1{}^2)],$$

$$b = \frac{\sigma_1(1+\sigma_1)(h_0{}^2-h_1{}^2)}{a^2},$$

$$\sigma_1 = 1-2\,h_0 \cot\alpha, \quad \sigma_2 = 1+(\Delta h)\cot\alpha,$$

(Eqs. (4.7) — (4.9), p. 8 of [1], but $\alpha=\beta$ here).

The results of comparison are summarized in the subsequent sets of tables and figures.

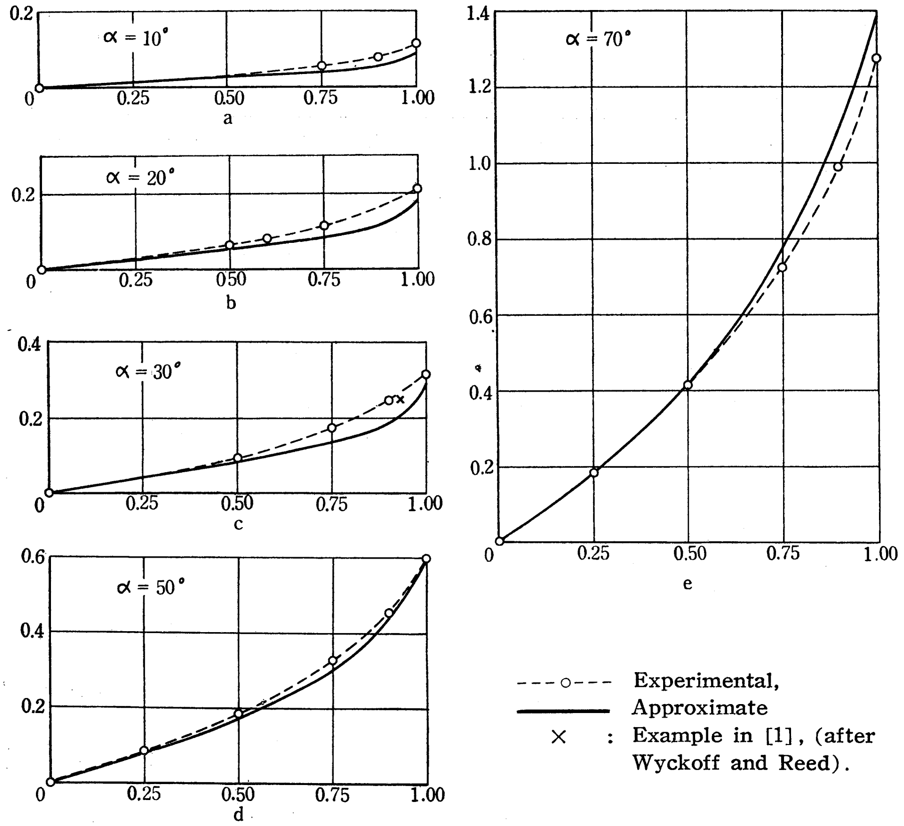
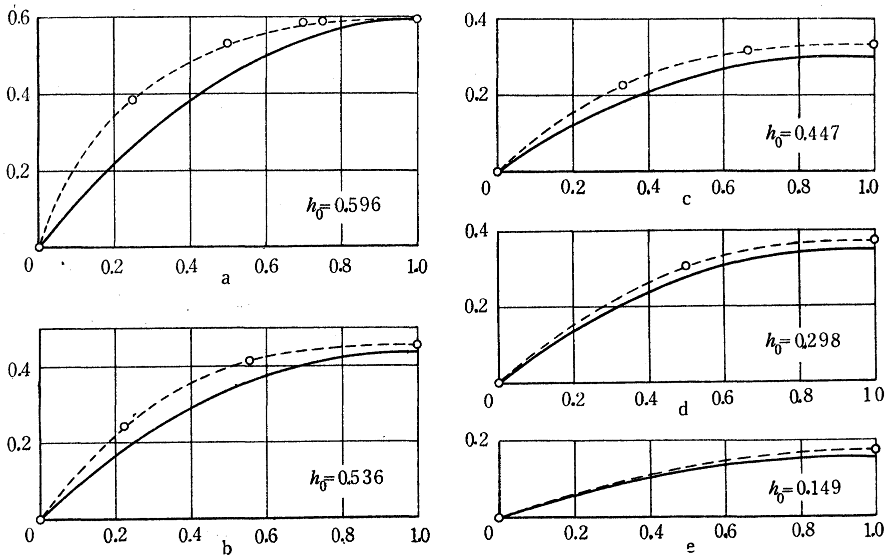


FIG. 2. Plot of q/h_0 (ordinate) against h_0/h_m (abscissa) for various values of α, where $h_1=0$. (cf. TABLE 1).

TABLE 1. Results for various inclinations of the faces and inflow-levels at zero outflow-level. (cf. Fig 2).

α	h_0	h_0/h_m*	I: q/h_0 (exp.)	II: q/h_0 (approx.)	ξ=I/II
10°	0.088	1.0	0.116	0.088	1.32
	0.079	0.90	0.0804	0.0514	1.56
	0.066	0.75	0.0556	0.0406	1.37
20°	0.182	1.0	0.210	0.182	1.15
	0.137	0.75	0.1145	0.0852	1.34
	0.109	0.60	0.0810	0.0643	1.26
	0.091	0.50	0.0620	0.0521	1.19
30°	0.289	1.0	0.311	0.289	1.08
	0.260	0.90	0.244	0.185	1.32
	0.217	0.75	0.175	0.137	1.28
	0.144	0.50	0.096	0.0836	1.15
50°	0.596	1.0	0.596	0.596	1.00
	0.536	0.90	0.451	0.434	1.04
	0.447	0.75	0.330	0.299	1.10
	0.298	0.50	0.185	0.1726	1.07
	0.149	0.25	0.0852	0.0796	1.07
70°	1.372	1.0	1.271	1.372	0.93
	1.235	0.90	0.986	1.092	0.90
	1.029	0.75	0.721	0.767	0.94
	0.686	0.50	0.414	0.413	1.00
	0.343	0.25	0.182	0.184	0.99

* $h_m = 1/2 \tan\alpha$, or the maximum inflow-level for the given value of α. (see Fig. 1).
The values of h_S are not shown in [2].



---o--- Experimental, ———— Approximate

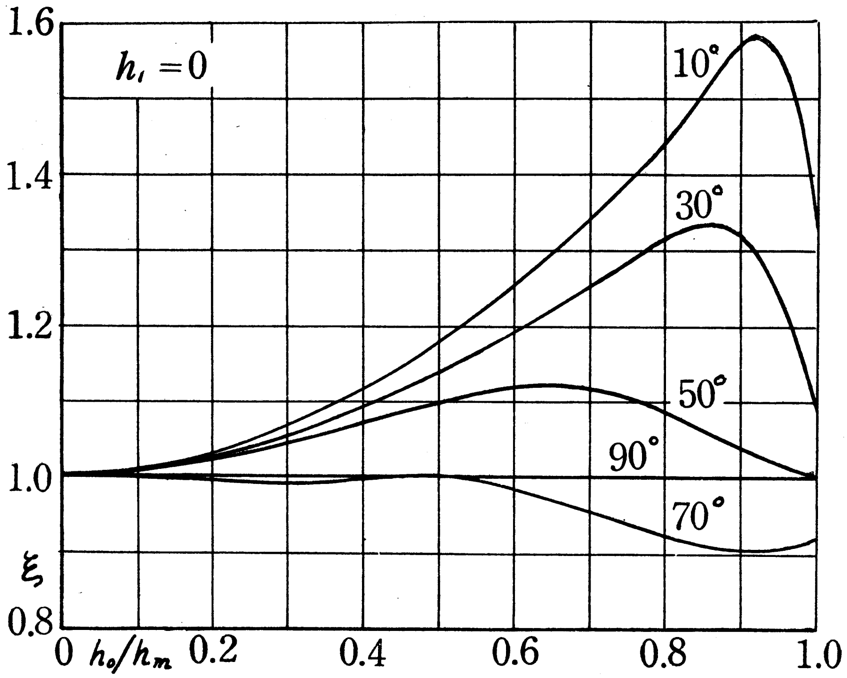
FIG. 3. Plot of q/h_0 (ordinate) against $\Delta h/h_0$ (abscissa) for various values of h_0, where $\alpha = 50°$. (cf. TABLE 2).

TABLE 2. Results for various inflow- and outflow-levels at $\alpha = \beta = 50°$. (cf. Fig. 3).

h_0	h_1	h_1/h_0	I: q/h_0 (exp.)	II: q/h_0 (approx.)	ξ=I/II	h_S* (exp.)	h_S (approx.)
0.596	0	0	0.596	0.596	1.00	—	—
	0.119	0.25	0 588	0.558	1.05	—	—
	0.298	0.50	0.531	0.449	1.18	—	—
	0.447	0.75	0.383	0.261	1.47	—	—
0.536	0	0	0.451	0.434	1.04	0.326	0.376
	0.238	4/9	0.414	0.360	1.15	0.142	0.196
	0.417	7/9	0.245	0.182	1.35	0.031	0.062
0.447	0	0	0.330	0.299	1.10	0.196	0.159
	0.149	1/3	0.313	0.280	1.12	0.075	0.099
	0.298	2/3	0.222	0.182	1.22	0.022	0.037
0.298	0	0	0.185	0.1726	1.07	0.092	0.028
	0.149	0.50	0.154	0.1390	1.11	0.013	0.014
0.149	0	0	0.0852	0.0796	1.07	0.025	0.002

* The values in this column are obtained from Fig. 7 of [2].

In order to show the general aspects of the errors associated with computations of total flux, Fig's 4 and 5 are written after suitable inter- and extrapolation of these data. There for convenience sake the values of $\xi = q(\text{exp.})/q(\text{approx.})$, or the correction-multiplier which is to be applied to the approximate value are plotted.



$\xi = q(\text{exp.})/q(\text{approx.})$

FIG. 4. ξ-curves, corresponding to Fig. 2. h_0/h_m is ordinate and α is a parameter.

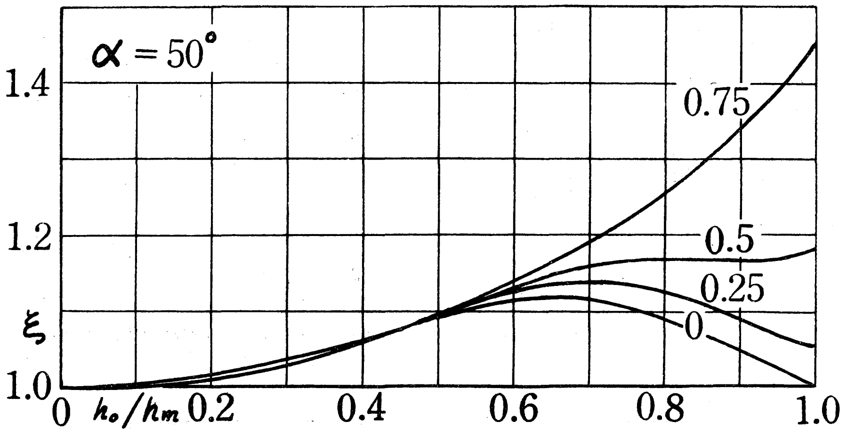


FIG. 5. ξ-curves, corresponding to Fig. 3.
h_0/h_m is ordinate and h_1/h_0 is a parameter.

It is seen in general that the greater the angles of inclination are, and the smaller the outflow-level is, the less becomes the error of computation. For the height of seepage surface, on the other hand, the examples presented here are still not sufficient to draw such curves. But undoubtedly the approximation is much poorer here than in the case of the total flux: this tendency appears in the rectangular case, too, and is supposed to be an essential character of our present method of approximation. The detailed discussion, however, has not yet been established.

At the conclusion of this note, the author would like to add a few words concerning the Dupuit-Forchheimer's formula for the total flux of a rectangular dam. That is, the legitimacy of this formula, which was somewhat emphasized by the present author (p. 5 of [1]), had already been pointed out in Huard de la Marre's paper [2]. Indeed, he writes "Cette formule, connue sous le nom de Dupuit-Forchheimer, peut se démontrer rigoureusement par une application de la formule de Green, sans introduire d'autres hypotheses que Φ est harmonique." ([2], p. 195, footnote. Φ means, of course, the velocity-potential).

References

[1] Ohji, M., A Contribution to the Theory of the Seepage through an Earth Dam, *Rep. Res. Inst. App. Mech.*, Vol. III, No. 9 (1954) pp. 1-10.

[2] Huard de la Marre, Pierre, Nouvelles méthodes pour le calcul expérimental des écoulements dans les massifs poreux, *La Houille Branche*, N° spécial *A*/1953, pp. 193-207.

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