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RESEARCHES ON PERIPHERAL PUMP

By Yasutoshi SENOO

The peripheral pump is widely used for a small type pump because of its suitable performance as a small type pump and the simplicity of its construction. The theory of its pumping action, however, has not been clarified yet and the way of its design has not been established. As the pump is small and is easily constructed, a designer can find a suitable design by trial, utilizing the characteristics of many pumps made for examination. Certainly there has been previous research work done on the pump, but it has been only fragmental and could not be easily consolidated. Some theoretical researches were carried out by a group of researchers[1], and the performances of a pump as shown by these theories were compared with the actual performances of the pump. The results show that the theories are not complete.

The author carried out experimental researches on the characteristics of a pump as a whole, as well as pressure distribution inside the pump; thus the outline of the pumping action of the impeller was made clear. As the flow of liquid driven by the impeller has an especially close relationship with turbulent flow, the author adopted a new theory which was something like a classical theory of turbulence, and established theoretical equations which satisfied many experimental data previously done. Using these equations we can consolidate almost all the researches on peripheral pump. In addition to these, the author carried out the experimental and theoretical researches on the mechanism of pumping action which he had assumed for the analysis of the pump characteristics. He not only verified that the assumption he had used was suitable, but also found the cause of cavitation which was obstructive to good performance of the pump.

As a peripheral pump is used for discharging a little quantity of liquid against rather high pressure, the characteristics are influenced greatly by internal leakage. The author also studied this subject both theoretically and experimentally. In addition to these, he carried out quite a few experimental researches and made clear the influences of other factors which could not be clarified by his theory.

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1. Introduction. A peripheral pump has many names. Sometimes it is called a regenerative pump, a turbine pump,[2] and a friction pump. In Japan it is usually called a Westco pump. I am not sure why these names are used, but each of them seems to show a part of the characteristic features. Many books[2] [3] explain that the liquid passed through the vanes to the periphery of the impeller is directed back into the root of other vanes of the impeller and additional energy is imperted to the liquid. This recirculation through the large number of vanes keeps up, and a multi-stage effect is produced with only a single impeller. It is this reason that this kind of pump is named a regenerative pump.

However, if the spaces between the vanes of the impeller are the main passage-ways of liquid, the discharge of the pump will be influenced greatly by the area of the space and the number of vanes. However, according to the author's experimental data, the maximum discharge, which is obtained at zero total head is hardly influenced by the number of vanes changed from 32 to 120 and the area of the space. Therefore, it is not reasonable to think that the space between the vanes is the passage-way of liquid.

Certainly the liquid between the vanes is thrown by a centrifugal force into the pumping passage[4] between the impeller and the casing. This liquid will mix with the liquid in the passage and will transmit the momentum of the impeller to the liquid in the pumping passage. Therefore, the vanes do not serve as the device for increasing pressure but as the device for conveying the momentum of the impeller to liquid. Accordingly the performance of the pump will be decided by the momentum being conveyed to the liquid in the passage.

2. The pumps and the devices used for experimental researches.

A peripheral pump is a kind of pump which utilizes the flow of liquid that revolves together with a disk—impeller—in a casing. For example, in Figure 1, the pumping passage formed between the casing and the impeller is blocked by a partition at one part of the periphery, and a suction nozzle and a discharge nozzle are set on both sides of this partition. Liquid sucked through the suction nozzle is driven by the impeller and flows through the pumping passage against the pressure gradient, then it is discharged through the discharge nozzle.

In carring out this research, two kinds of pumps are chiefly used. One of them is the pump shown in Figure 1, which is constructed accurately and has two clearances c of as wide as only 0.04 mm between the impeller and the casing. This pump is chiefly used for the research on the internal leakage through clearances. The other pump shown in Figure 2 has a convenient construction for the reformation which is necessary for carrying

[2] F. A. Kristal and F. A. Annett: Pumps

[3] C. Pfleiderer: Die Kreisel Pumpen

[4] The space between the casing and the periphery of the impeller. This connects the suction nozzle with the discharge nozzle round most part of the periphery of the impeller.

out the researches, and the most parts of this study are carried out using this pump.

The pump is driven by a D.C. shunt motor at various speeds of from 1500 to 4000 rpm, and input power is measured with a geared dynamometer which has accuracy of 0.001 mkg. Total head of the pump is controlled with a sluice valve inserted in the discharge pipe line. Suction pressure and low discharge pressure are measured with mercury manometers, and high discharge pressure is measured with some calibrated pressure gauges.

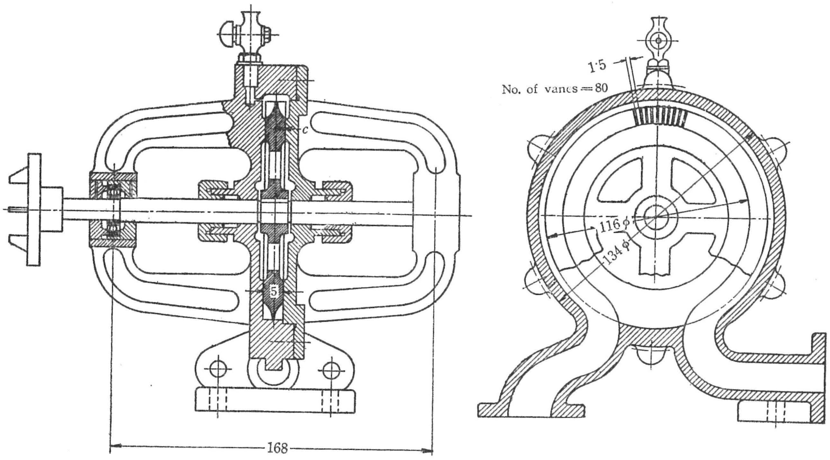


FIG. 1
Construction of No. 1 peripheral pump.

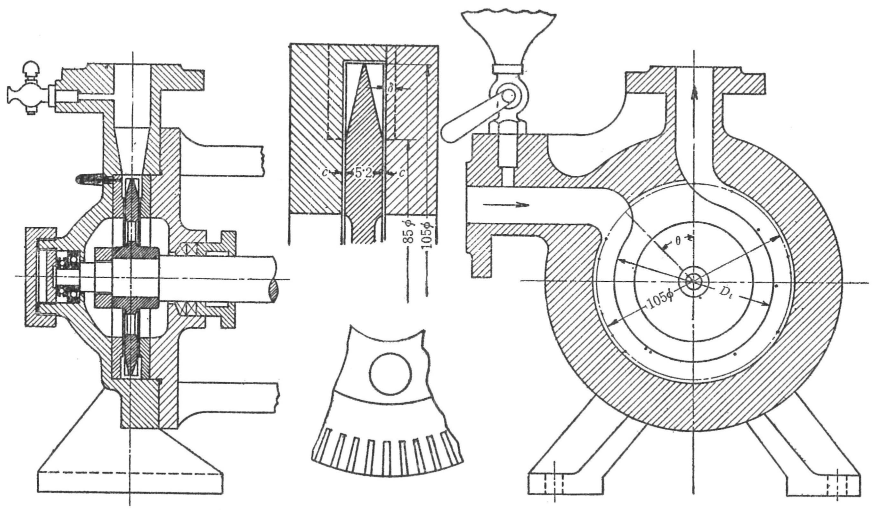


FIG. 2
Construction of No. 2 peripheral pump.

TABLE 1.

Chief dimensions of the pumps used by the author.

Experiment	Pump	Pumping passage: Cross-sectional area A, cm^2	Pumping passage: Mean depth δ, cm	Clearance c, cm	Number of vanes Z	Reference Figure
Exp. 1	No. 2	0.54	0.199	0.015	72	16, 32
Exp. 2	No. 2	0.98	0.327	0.035	92	5, 18, 19, 32
Exp. 3	No. 2	0.98	0.387	0.035	72	13, 18, 19, 32
Exp. 4	No. 2	0.98	0.327	0.035	52	18, 19, 32
Exp. 5	No. 2	0.98	0.327	0.035	32	18, 19, 32
Exp. 6	No. 2	0.54	0.199	0.022	72	16
Exp. 7	No. 2	0.98	0.327	0.015	72	16
Exp. 8	No. 2	1.17	0.36	0.005	60	6, 12, 16, 28, 30
Exp. 9	No. 2	1.17	0.36	0.035	72	16
Exp. 10	No. 2	0.98	0.36	0.035 (20° forward inclined)	92	Omission; cf. 22
Exp. 11	No. 2	0.98	0.36	0.035 (20° backward inclined)	92	Omission; cf. 22
Exp. 12	No. 2	0.98	0.36	0.035	0	24
Exp. 13	No. 2	1.17	0.36	0.005	120	20
Exp. 14	No. 1	0.74	0.287	0.004	80	7, 8, 9, 11, 14, 15, 16, 31, 32
Exp. 15	No. 1	0.74	0.287	0.008	80	8, 9, 11, 15, 16, 31
Exp. 16	No. 1	0.74	0.287	0.012	80	8, 9, 11, 15, 16, 31
Exp. 17	No. 1	0.74	0.287	0.016	80	8, 9, 11, 15, 16, 31
Exp. 18	No. 1	0.74	0.287	0.020	80	8, 9, 11, 15, 16, 31
Exp. 19	No. 1	0.74	0.287	0.024	80	8, 9, 11, 15, 16, 31
Exp. 20	No. 1	0.74	0.287	0.028	80	8, 9, 10, 11, 15, 16, 31
Exp. 21	No. 1	0.74	0.287	0.036	80	8, 9, 10, 11, 15, 16, 31

The revolving speed of a pump is measured with a counter and a tachometer, and if deviation from an expected speed is recognized, the speed is adjusted at once. The discharge of a pump is measured with a weighing tunk. All of these values are prepared and measured so as to keep the error of less than one per cent.

We carefully prevent the leakage of air into water at the parts where pressure of liquid is lower than atmospheric pressure. The stuffing boxes where the shaft pierces through the casing are sealed with water. Leakage of air into a pump seems to be judged by whether the discharged liquid contains air, but when cavitation occures in a pump the discharged liquid always contains a little quantity of air. Air is recognized in the discharged liquid even if the entire pump and the pipe line are submerged in water. This air must be what has been dissolved at a low pressure from the water, which has resolved a certain amount of air at atmospheric pressure. Therefore, the air in the discharged liquid does not always mean leakage of air into the system. There is no way by which the experimental device is verified as perfectly air-tight.

Improvement of the theoretical consideration requires different kinds of experimental data to verify the findings. Therefore, all the experiments are carried out in close relationship with the theoretical researches, and often the use of strange devices are required. For example, as the theory shows the discharge at zero and negative head is very important for the analysis of a pump, another pump and a by-pass line are inserted in the suction pipe line of the objective pump and the inserted pump charges the liquid of proper pressure into the suction pipe line.

In order to make clear the mechanism of cavitation in a pump, quite a few number of holes of 0.6 mm diameter are drilled on the wall of the pumping passage along the periphery of the impeller, through which various pressures in the pumping passage are measured. The measured pressure distribution shows the status of increase of pressure in the pumping passage. That pressure distribution near the suction nozzle serves a very important suggestion not only for analysis of the mechanism of the pumping action but also for reduction of cavitation which will be explained later. In carrying out the cavitation test of a pump, special attention is piaced on the prevention of air leakage. However, low pressure in the suction pipe is induced by the resistance at the sluice valve, and so a little quantity of air may resolve from water due to the local low presure just behind the valve. It is difficult to say that the condition of water in this case is completely the same as that at a high suction hight. However, the results obtained are quite regular and reasonable, and some pumps tested in this device show very good suction ability. Therefore, there is no influence of the resolved air or, if there is, it is not very effective to reduce the suction ability of a pump. To measure such low pressure, special attention is stressed to exclude resolved air in the measuring apparatus, lest it should become the cause of vapor lock or some other troubles.

3. Flow in the pumping passage. The liquid in the pumping passage is driven by the vanes of the impeller. The mechanism is as follows. The liquid between the vanes moves at the same angular velocity as that of the impeller, and is driven tangentially to the periphery of the impeller by the centrifugal force. The liquid leaving the periphery of the impeller is forced into the liquid in the pumping passage and conveys its momentum to the surrounding liquid. At the same time a volume of the liquid in the pumping passage flows into the space between the root of the vanes, and thus a secondary flow is induced. The secondary flow conveys momentum between the liquid in the pumping passage and the impeller. That is, the liquid in the pumping passage is given the peripheral velocity, on the other hand the impeller receives a force against its revolution. In order to analyse the status of flow under such a condition, it is necessary to make clear the mechanism of transportation of momentum in the liquid in the pumping passage.

One of the most important parts of the theory of turbulence is to make clear the mechanism of transportation of momentum in a flow, and the results obtained show rather simple and reliable rules. Of course the condition of flow in the pumping passage, our major concern, is different from that in a pipe or a channel about which the theory of turbulence is developed, because in our case the source of turbulence is the vanes of the impeller and not the gradient of velocity. However, the mechanism of transportation of momentum in liquid will be the same in both cases, for it is a motion between two volumes of liquids and is not remarkably influenced by the source of turbulence. Such being the case let us develop our consideration under the assumption that the secondary flow in the pumping passage is a kind of turbulence, and a properly modified momentum transfer theory is applicable to the main flow in the pumping passage.

The curvature of the pumping passage, however small it may be, plays the most important role for the induction of the secondary flow in the pumping passage. Therefore, it can not be neglected as far as concerns the mechanism and the intensity of turbulence. If the intensity of turbulence is properly estimated considering the curvature, however, the curvature of the passage will not influence the condition of flow very much except that pressure is different along the radius. This is so because the radius of curvature is rather large compared with the depth of the pumping passage. Now we regard the pumping passage as a straight channel composed of two parallel plates, one of which is a fixed wall and the other moves with a velocity U inducing turbulence; i.e. sending momentum to the liquid in the passage.

In the case mentioned above, we may use the theory of mixture length only if the value of the turbulent viscosity is properly estimated, thus the following equation is induced

$$\tau = \rho\, l\, v\, du/dy\,, \tag{1}$$

where

y = distance from the fixed wall,
l = mixture length,
u = time mean value of the velocity parallel to the wall at y,
v = component of the turbulent velocity perpendicular to the wall at y,
τ = frictional force working on unit area of a surface parallel to the wall at y,
ρ = density of liquid.

In general, l, u, v and τ are functions of y.

In the above equation, the magnitude of velocity component v is influenced by the mechanism of turbulence, and we must not use the value estimated in a pipe flow. In the major part of a cross-section of the pumping passage velocity component v is chiefly induced by the secondary flow produced by the impeller, but the influence of the secondary flow will decay near the fixed wall and it is not clear whether the turblent velocity there is also dominated by the impeller.

In order to make clear the mechanism of the secondary flow or the turbulence in the pumping passage, we must study the source of turbulence. The detail of the study will be explained in the later. The result shows that the turbulence induced by the impeller is stronger than that induced by the velocity gradient, even near the fixed wall. Therefore, as far as concerns a peripheral pump of usual type, the turbulence in the pumping passage is induced by the impeller throughout the passage.

As the velocity of liquid in the passage is smaller than the peripheral velocity U of the impeller, the centrifugal force working on the liquid in the passage is weaker than that between the vanes of the impeller. The difference of these forces accelerates the liquid between the vanes radially, and a secondary flow occurs. In this case each of the centrifugal forces is nearly propotional to the square of the peripheral velocity U, and the velocity of the secondary flow or the turbulent velocity perpendicular to the wall v is nearly proportional to the square root of the difference of the forces induced by the centrifugal force. Therefore the turbulent velocity v is nearly proportional to the peripheral velocity U of the impeller.

As the mixing action is obstructed by the fixed wall, mixture length l will be zero on the wall and increases proportionally to the distance from the wall just as in a pipe flow and a channel flow, because the mixture length does not seem to be influenced remarkably by the character of turbulence. However, the moving wall or the impeller will not pose as an obstructor, because it is the source of the secondary flow, and the liquid having momentum is projected from the moving wall. We may think, therefore, that the mixture length is maximum on the moving wall and decreases linearly to zero on the fixed wall. Therefore the turbulent viscosity $\rho l v$ is proportional to $\rho U y/\delta$ where δ is the distance between the two walls. If the distance between the two walls increases, the restriction on the turbulence in the passage will diminish. Roughly speaking mixture length will be proportional to the distance δ. On the other hand the turbulent velocity v is chiefly influenced by the radius of curvature of the impeller and the number and the shape of the vanes, and it will not be influenced by the distance δ. Therefore, the turbulent viscosity $\rho l v$ is propotional to $\rho U y/\delta \times \delta = \rho U y$. Thus, there is a relation

$$\tau = \rho\, l\, v \cdot du/dy = \kappa\, \rho\, y\, U\, du/dy \tag{1'}$$

where κ is a constant which shows the intensity of turbulence in the passage and it is influenced by the shape and the number of the vanes and the curvature of the passage. According to the equation (1′) the velocity gradient becomes infinity on the fixed wall, because turbulent viscosity is zero there. However, the molecular viscosity is dominant there and an adjustment of equation (1′) is necessary. Generally speaking the mixture length theory is not applicable near a wall, because there is a laminar sublayer on the wall owing to the molecular viscosity. Between a laminar sublayer and a turbulent region there is a transient layer. However, as the laminar sublayer and the transient layer are very thin, we may simply assume that the turbulent viscosity on the fixed wall is the same as the molecular viscosity. This condition is easily formulated by transfering the co-ordinate, and the equation (1′) changes as follows:

$$\left.\begin{aligned} \tau &= \kappa\, \rho\, y\, U\, du/dy\,, \\ \mu &= \kappa\, \rho\, \varepsilon\, U, \end{aligned}\right. \tag{2}$$

where $y = \varepsilon$ means the fixed wall and $y = \varepsilon + \delta$ means the moving wall.

4. Relationship between the head and the discharge. Assuming that the curvature of the pumping passage is negligible and that the mean flow is parallel to the walls and has not the velocity component perpendicular to the walls, the pressure p is a function of x alone which is the ordinate parallel to the walls. There is a following relationship between forces working on the liquid,

$$d\tau/dy = dp/dx\,.$$

This equation is integrated under the boundary condition $\tau = \tau_0$ at $y = \varepsilon$. The solution is

$$\tau = (dp/dx)(y - \varepsilon) + \tau_0\,. \tag{3}$$

Using equations (1) and (2),

$$\rho\, \kappa\, y\, U\, du/dy = (dp/dx)(y - \varepsilon) + \tau_0\,,$$

$$u(y) = \left\{ dp/dx \left(y - \varepsilon - \varepsilon \ln \frac{y}{\varepsilon} \right) + \tau_0 \ln \frac{y}{\varepsilon} \right\} \Big/ \rho\, \kappa\, U. \tag{4}$$

Since the boundary condition is $u = U$ at $y = \varepsilon + \delta$,

$$\tau_0 = \left[\rho\, \kappa\, U^2 - \frac{dp}{dx} \left\{ \delta - \varepsilon \ln \left(1 + \frac{\delta}{\varepsilon} \right) \right\} \right] \Big/ \ln \left(1 + \frac{\delta}{\varepsilon} \right). \tag{5}$$

The frictional force working on the moving wall is

$$\tau_\delta = \tau_0 + \delta \frac{dp}{dx} = \left[\rho \kappa U^2 - \frac{dp}{dx}\left\{\delta - (\varepsilon + \delta)\ \ln\left(1 + \frac{\delta}{\varepsilon}\right)\right\}\right] \Big/ \ln\left(1 + \frac{\delta}{\varepsilon}\right). \quad (6)$$

Denoting the rate of flow per unit breadth by q, and integrating the equation (4) from ε to $\varepsilon + \delta$, we have the following equation:

$$q = \int_\varepsilon^{\varepsilon+\delta} u\, dy = [0.5\, \delta^2\, dp/dx + (\tau_0 - \varepsilon\, dp/dx)\{(\varepsilon + \delta) \ln(1 + \delta/\varepsilon) - \delta\}]/\rho \kappa U.$$

Neglecting ε^2 since $\varepsilon \ll \delta$, the approximate value of q is represented by

$$q = U\delta\left(1 + \frac{\varepsilon}{\delta} - \frac{1}{\ln(1 + \delta/\varepsilon)}\right) - \frac{\delta^2}{\rho \kappa U} \frac{dp}{dx}\left(\frac{1}{2} - \frac{\varepsilon}{\delta} - \frac{1}{\ln(1 + \delta/\varepsilon)}\right). \quad (7)$$

The first term of this equation is the rate of flow under zero pressure gradient, and the second term is the quantity which flows back under a pressure gradient dp/dx.

As q is the rate of flow per unit breadth, we must multiply q by the effective breadth of the pumping passage, in order to estimate the discharge of a pump. Since the passage is three dimensional, it is difficult to estimate the effective breadth accurately. Thus, let us separate the passage into three parts and treat each of them as a two dimensional passage. In this case, denoting each depth by δ_i and each effective breadth by b_i, it is desirable that these values satisfy the relationship $\sum b_i \delta_i = A$. As the fixed wall of the passage is longer than the moving wall, there are some regions shown by hatching in Figure 3 where the pattern of flow assumed in the theory is not appropriate. Roughly speaking, however, as the turbulence in these regions is not remarkably different from that of the other region and the influence of the walls δ_1 and δ_2 are not important, we may include them in some of the three parts as shown in Figure 3.

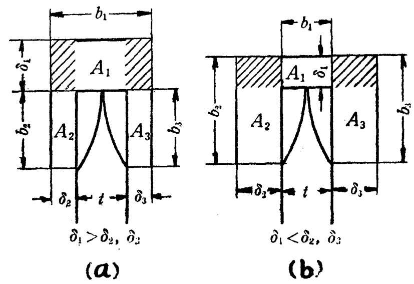


FIG. 3

Method of how to separate the three-dimensional pumping passage into three two-dimensional pumping passages.

Each of the three passages must have the same pressure distribution along the periphery of the impeller. Therefore the discharge of the pump Q is indicated as the sum of the quantity of liquid flowing through the three passages at the same pressure gradient dp/dx.

$$Q = \int q\, db = \sum_{i=1}^{3} q_i b_i = \sum_{i=1}^{3} \left[b_i U_i \delta_i \left(1 + \frac{\varepsilon}{\delta_i} - \frac{1}{\ln(1 + \delta_i/\varepsilon)}\right) - \frac{b_i \delta_i^2}{\rho \kappa U_i} \frac{dp}{dx}\left(\frac{1}{2} + \frac{\varepsilon}{\delta_i} - \frac{1}{\ln(1 + \delta_i/\varepsilon)}\right)\right]. \quad (8)$$

In this equation as ε is very small compared with δ_i, the variation of ε/δ_i is negligible compared with 1.0 even if δ_i may vary. The value $\ln(1+\delta_i/\varepsilon)$ is influenced only slightly by a variation of δ_i. Therefore, in equation (8) even if the value of δ_i in the parentheses is not accurate, it will hardly influence the value of the discharge Q. Accordingly we may use average value δ instead of each δ_i in the parentheses.

For the sake of convenience we use average values of b, U and δ, thus simplify the equation. For this purpose the following two conditions must be satisfied:

$$\sum_{i=1}^{3} b_i U_i \delta_i = b U \delta , \quad \sum_{i=1}^{3} \frac{b_i \delta_i^2}{U_i} = \frac{b \delta^2}{U} .$$

As the variables are b, U and δ, one freedom remains. Then we adopt another condition $\sum_{i=1}^{3} b_i \delta_i = A = b \delta$ to determine these values. These values are estimated easily by the following equations:

$$U = \sum_{i=1}^{3} b_i U_i \delta_i / A ,$$

$$\delta = (U/A) \sum_{i=1}^{3} b_i \delta_i^2 / U_i ,$$

$$b = A/\delta .$$

According to these equations the average value of δ may be different from the actual value of δ when these three δ_i's are the same, although the difference is usually less than one per cent. Using these values, the discharge of the pump Q is shown by the following equation:

$$Q = A U\left(1 + \frac{\varepsilon}{\delta} - \frac{1}{\ln(1+\delta/\varepsilon)}\right) - \frac{A \delta}{\rho \kappa U} \frac{dp}{dx}\left(\frac{1}{2} + \frac{\varepsilon}{\delta} - \frac{1}{\ln(1+\delta/\varepsilon)}\right) . \quad (10)$$

This equation is easily reformed into the following equation:

$$\frac{dp}{dx} = \frac{\rho \kappa U}{\delta}\left[U\left\{1 + \frac{\varepsilon}{\delta} - \frac{1}{\ln(1+\delta/\varepsilon)}\right\} - \frac{Q}{A}\right] \Big/ \left\{\frac{1}{2} + \frac{\varepsilon}{\delta} - \frac{1}{\ln(1+\delta/\varepsilon)}\right\} . \quad (11)$$

The total head H is obtained by integrating the pressure gradient along the total length of the pumping passage.

$$H = \int (1/\rho g)\, dp/dx \cdot dx , \quad (12)$$

where g is the gravity.

Equation (11) is applicable only for a uniform flow and it must not be directly adopted for the passage near the suction and the discharge nozzles. In order to estimate theoretically the influence of the suction and the discharge nozzles it is necessary to clarify the conditions of the flow and the turbulence there. As it is very difficult to clarify these conditions, the author estimates the influence experimentally and utilizes the results for integrating equation (12), as shown in the next section.

5. Influences of the suction nozzle. In order to clarify the influences of the suction and discharge nozzles, the author measures pressure at 15 holes along the pumping passage and clarifies the performance of the impeller at each point. One of the results is shown in Figure 4. In this figure the full lines connecting ○ show the pressure at the tip of the vanes of the impeller and the broken lines connecting △ show the pressure at the root of the vanes. The thick lines show the data carried out at the condition $c = 0.14$ mm and the fine lines show the data at the condition $c = 0.32$ mm where c is the clearance between the impeller and the casing as shown in Figure 1. In the case of the discharge $Q = 0$ the pressure begins to rise at the suction nozzle, but in the condition $Q = 0.255$ l/s the pressure gradient near the suction nozzle is more gentle than that at the remaining passage. This influence of the suction nozzle becomes more remarkable as the discharge increases, and the pressure at a certain point of the pumping passage becomes even lower than the suction pressure. In the figure, the condition $Q = 0.435$ l/s is an example of such cases and the minimum pressure is nearly -10 m Aq. If the minimum pressure in the pumping passage decreases to the vapor pressure of the liquid, cavitation will occur there. In the case of cavitation, despite a decrease of the discharge pressure, the discharge does not increase further, while the length of the passage filled with the bubbly liquid becomes longer.

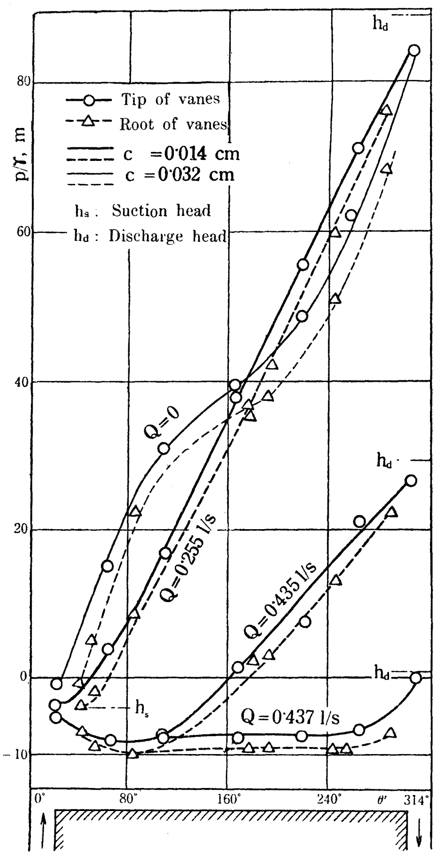


FIG. 4.
Pressure distribution along the pumping passage.

All of these are easily inferred according to the mechanism of turbulence explained in section 3. If any external force does not work on the liquid in the space between the vanes of the impeller, the liquid will be projeced at a tangent to the impeller. A volume of the liquid begins to be projected just after it passes through the partition. Therefore, that volume of the liquid will flow for a considerable length along the periphery of the impeller, before it arrives at the fixed wall and then flows through both side parts of the pumping passage into some roots of the vanes conveying its momentum to the surrounding liquid. Before that volume of the liquid conveys its momentum to the surrounding liquid throughout a cross-section of the pumping passage, the liquid in the pumping passage will not be given the momentum of the impeller. Therefore, the turbulence is hardly induced by the impeller near the suction nozzle.

However, the position where the liquid throughout the cross-section receives the momentum from the impeller is a function of the pressure gradient. That is, a steep pressure gradient forces the liquid in the pumping passage to flow backward, accordingly the volume of the liquid projected from the impeller conveys its momentum to the surrounding liquid throughout a cross-section before it flows far from the suction nozzle. When the pressure gradient is small, the turbulence does not fully develop near the suction nozzle. Therefore, the passage near the nozzle does not work as an effective pumping passage, that is, although its shape is the same as that of the pumping passage, it can be compared to a simple passage as far as concerns the pumping action.

The cross-sectional area of the suction pipe and the suction nozzle is wide enough and the velocity of the liquid is very small. While, the cross-sectional area of the passage above mentioned is very small and the liquid flows speedily. As the liquid is not yet accelerated by the impeller and has not increased its total energy there, the pressure decreases proportionally to the square of the discharge, when the liquid flows into the passage. After the liquid flows against a small pressure gradient for a considerable length which is named the entry length, the status of flow becomes its final condition. The theory above mentioned is applicable only to this final condition.

The entry length appears to be proportional to the discharge. The pressure distributions in Figure 4 verify the above presumption qualitatively, but it is not convenient for a quantitative research. In order to infer the influences quanitatively, let us assume that the influences are the following two factors:

(1) There is a pressure decrease at the entry of the pumping passage.
(2) There is a certain length at the entry which does not contribute to increase the discharge pressure.

The equation for the performance of a pump should be adjusted by these two factors and the amount of these influences will be deduced from the experimental data of pumps.

6. Influence of the discharge nozzle. Ideally speaking the kinetic energy of liquid in the pumping passage changes to pressure energy when the liquid is discharged into the discharge nozzle, because the cross-sectional area of the discharge nozzle is much larger than that of the pumping passage. In conventional pumps, they are connected directly with each other, so the kinetic energy of liquid in the pumping passage hardly changes to pressure energy in the discharge nozzle. Additionally, generally speaking, it is very difficult to change kinetic energy into pressure energy, if total energy is not uniform in a cross-section of a passage. Therefore, the discharge pressure hardly increases at the discharge nozzle, and the kinetic energy of liquid in the pumping passage goes to decay there. Of course this energy has been produced at the entry of the pumping passage partly by a decrease of pressure and partly by the impeller. If the kinetic energy completely decays at the discharge nozzle, the pressure in the discharge nozzle is the same as that at the end of the pumping passage. In this case the impeller produces the discharge pressure as well as the kinetic energy in the pumping passage which will be lost at the discharge nozzle. If the kinetic energy is recovered in the discharge nozzle, the discharge pressure increases by the kinetic energy without requiring any additional driving power.

In order to evaluate the driving power of a pump, it is necessary to estimate the magnitude of the kinetic energy in the pumping passage which will be shown under several conditions. Using equations (4) and (5) the velocity in the pumping passage is

$$u(y)=\frac{dp/dx}{\rho\,\kappa\,U}\left\{y-\varepsilon-\delta\frac{\ln(y/\varepsilon)}{\ln(1+\delta/\varepsilon)}\right\}+U\frac{\ln(y/\varepsilon)}{\ln(1+\delta/\varepsilon)}. \qquad (13)$$

A new coefficient k[5)] is introduced, where $q=k\,U\delta\left\{1+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}\right\}$. Equation (7) becomes

$$\frac{dp}{dx}\frac{1}{\rho\,\kappa\,U}=\frac{U}{\delta}(1-k)\frac{1+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}}{0.5+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}}.$$

Therefore, equation (13) becomes

$$u(y)=U(1-k)\left\{\frac{y-\varepsilon}{\delta}-\frac{\ln y/\varepsilon}{\ln(1+\delta/\varepsilon)}\right\}\frac{1+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}}{0.5+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}}+U\frac{\ln y/\varepsilon}{\ln(1+\delta/\varepsilon)}.$$

When the discharge pressure increases and the pressure gradient becomes larger than a certain value, the liquid flows backward locally in the pumping passage. The critical status occurs at the condition $du/dy=0$ at $y=\varepsilon$, and the critical values of the pressure gradient and the discharge are

$$\left(\frac{dx}{dp}\right)_{cri}=\frac{\rho\,\kappa\,U^2}{\delta-\varepsilon\,\ln(1+\delta/\varepsilon)},$$

$$q_{cri}=U\delta\left\{1+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}-\frac{0.5+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}}{1-\frac{\varepsilon}{\delta}\ln(1+\delta/\varepsilon)}\right\}.$$

For usual pumps $q_{cri}\fallingdotseq 0.5\,U\delta\fallingdotseq 0.6\,U\delta\left(1+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}\right)$.

5) k is the ratio of rate of flow versus maximum rate of flow under zero pressure gradient.

The rate of kinetic energy which flows through a unit breadth at a unit time is

$$\int_{\varepsilon}^{\delta+\varepsilon} \rho\,|u(y)|^3\,dy = \int \{A(y-\varepsilon) + B\,\ln(y/\varepsilon)\}^3\,dy\,,$$

where

$$A = \frac{U}{\delta}(1-k)\frac{1+\dfrac{\varepsilon}{\delta}-\dfrac{1}{\ln(1+\delta/\varepsilon)}}{0.5+\dfrac{\varepsilon}{\delta}-\dfrac{1}{\ln(1+\delta/\varepsilon)}}\,,\qquad B = \frac{U-\delta A}{\ln(1+\delta/\varepsilon)}\,.$$

We must calculate the above equation only in the range where u is positive, because the kinetic energy of the backward flow is given by the pressure gradient and is not given by the impeller. When $k=1$, i.e. there is no pressure gradient, the rate of kinetic energy is $0.665\,\rho\,\delta\,U^3/2$; when $k=0.59$ i.e. the critical status above mentioned, it is $0.250\,\rho\,\delta\,U^3/2$; when $k=0.25$, it is $0.154\,\rho\,\delta\,U^3/2$; when $k=0$ it is $0.113\,\rho\,\delta\,U^3/2$.

The rate of energy passing through a unit breadth at a unit time decreases together with the discharge, but the energy per unit quantity of liquid, i.e. the mean kinetic energy or the velocity head, is nearly constant. Its magnitude is roughly speaking equal to the velocity head of liquid flowing at a velocity of 80–90% of the peripheral velocity of the impeller.

7. Head capacity performance of a pump. Assuming that the influences of the suction and the discharge nozzles are the two factors mentioned in section 5, the total head of a pump H is described as follows:

$$H = \int_{L} (1/\rho g)\,dp/dx \cdot dx = \{(L-L')/\rho g\}(dp/dx)_0 - h\,,$$

where

$(dp/dx)_0$ = the pressure gradient at the final flow, described in equation (11),
L = the total length of the pumping passage,
L' = the ineffective length at the entry,
h = the amount of the pressure decrease at the entry,
g = the gravity,
ρ = the density of the liquid.

Using equations (11) and (12), the head capacity performance of a pump is explained as follows:

$$\frac{H+h}{L-L'} = \frac{\kappa U}{g\delta}\left\{U\left(1+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}\right)-\frac{Q}{i\,A}\right\}\Big/\left\{\frac{1}{2}+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}\right\}, \quad (14)$$

where i is a correction factor used for the cross-sectional area of the pumping passage and in many cases it is about 1.00. In engineering unit

$$H+h = \frac{L-L'}{95\times 10^3}\,\frac{\kappa U}{\delta}\,\frac{\left\{U\left(1+\dfrac{\varepsilon}{\delta}-\dfrac{0.4343}{\log(1+\delta/\varepsilon)}\right)-\dfrac{Q}{i\,A}\times 10^3\right\}}{\left\{\dfrac{1}{2}+\dfrac{\varepsilon}{\delta}-\dfrac{0.4343}{\log(1+\delta/\varepsilon)}\right\}}\,. \quad (14')$$

In this equation, the value of $1+\frac{\varepsilon}{\delta}-\frac{0.4343}{\log(1+\delta/\varepsilon)}$ is slightly influenced by the dimension of a pump and the number of revolutions of the impeller, but in many cases it is about 0.85. Accordingly we may use the following equation in order to estimate the approximate performance of a pump.

$$H+h=\frac{(L-L')\,\kappa\,U}{33\times 10^3\,\delta}\{0.85\,U-(Q/i\,A)\times 10^3\} \qquad (14'')$$

The meaning and the unit of the symbols used in the foregoing equations are as follows:

H = the total head of a pump m,
h = the amount of the pressure decrease at the entry m,
Q = the discharge of a pump l/s,
L = the total length of the pumping passage cm,
L' = the ineffective length at the entry cm,
A = the cross-sectional area of the pumping passage cm^2,
δ = the mean depth of the pumping passage, described in equation (9) cm,
U = the mean velocity of the vanes of the impeller described in equation (9) cm/s,
$\varepsilon = \nu/\kappa\,U$,
ν = the kinematic viscosity cm^2/s,
κ = the coefficient of turbulence,
i = the correction coefficient of the cross-sectional area of the pumping passage, usually $i = 1.00$.

It is most convenient to deduce the values of h and L' from the performances of many pumps. The value of h estimated by this method will be examined[6] by the data of cavitation test whether it is reasonable. It will be verified later that the influence of the suction nozzle is remarkable and even a little difference in the shape of the suction nozzle has a considerable effect on the characteristics of the pump. Therefore the values of h and L' of many pumps are not completely the same respectively. However, these values are approximately shown by the following equations for many pumps already manufactured:

$$L'=11300\,Q/A\,U\left\{1+\frac{\varepsilon}{\delta}-\frac{0.4343}{\log(1+\delta/\varepsilon)}\right\}\fallingdotseq 13300\,Q/A\,U, \qquad (15)$$

$$h=j\frac{1}{2g}\left(\frac{Q}{A}\right)^2\times 10^3\fallingdotseq(1.18\sim 1.37)\frac{1}{2g}\left(\frac{Q}{A}\right)^2\times 10^3\fallingdotseq(6.0\sim 7.0)\left(\frac{Q}{A}\right)^2\times 10^3. \qquad (16)$$

The meaning and the unit of the symbols used here are the same as those in the equations (14') and (14''). L' is a length which shows the influence

[6] h is not exactly the same as the pressure decrease estimated from cavitation tests. However, there is a closs relationship which will be explained in section 17.

of the entry. It may be larger by increasing the cross-sectional area of the pumping passage or by increasing the diameter of the impeller. In these cases a longer passage may be required before the liquid in the pumping passage receives enough momentum from the impeller to develop a complete pumping action. However, the author's experimental data reveal such a tendency is not recognized. If the cross-sectional area of a pumping passage and the diameter of an impeller are not the same as those in his experiments, the value of L' may be different from that above mentioned.

The amount of the pressure decrease at the entry h is a little larger than the mean velocity head. This is reasonable because there is a little loss of head at the entry.

If the pressure in the pumping passage is equal to the vapor pressure of the liquid at that temperature, the liquid begins to vaporize and the status of flow in the passage is quite different from that of a normal condition. Since the vapor pressure is the lowest pressure that can be attained, the maximum quantity of liquid which will enter into the pumping passage is limited by the allowable pressure decrease. On the other hand, if the impeller works effectively, the pressure becomes higher than the vapor pressure and there is no longer any cavitation. Accordingly, the pressure gradient in the pumping passage, where the impeller effectively works, is a certain value decided by equation (11). Therefore, if the discharge pressure is low enough and the discharge is limited by cavitation, the effective length of pumping action is much shorter than the geometric length of the pumping passage, and in the remaining passage the bubbly liquid flows at the vapor pressure of the liquid without creating pumping action.

This is shown in Figure 4. The discharge pressure at $Q = 0.437$ l/s is much lower than that at $Q = 0.435$ l/s but both discharges are almost equal to each other, because the discharges are limited by cavitation in both cases. Therefore the pressure gradients in the effective passage are nearly equal to each other and the effective length in the case of $Q = 0.437$ l/s is much shorter than that of $Q = 0.435$ l/s. If we adopt the ineffective length of the passage L_{cav} in case of cavitation instead of L', we may use the equation (14) for every condition including cavitation. However, the length L_{cav} increases as the discharge pressure diminishes, so it has not any important meaning for estimating the characteristics of a pump. This fact shows that any discharge pressure lower than a certain value is attainable at a certain value of discharge, only if the discharge is the critical value of cavitation.

Figure 5 shows an example of the characteristics of the pump shown in Figure 2. The shape of the pumping passage is shown on the right side of the diagram, and the chief dimensions are calculated from the shape. The points in the diagram are the experimental values. The value of κ is assumed so that the predicted characteristics[7] satisfy the experimental data as well as possible; the value is decided by a few trials. The predicted

[7] There is a simple diagrammatic method to calculate the characteristic equation. It will be explained in another paper.

characteristics are shown in the diagram as full lines. The value of κ is very small compared with those of other cases because of the remarkable internal leakage.

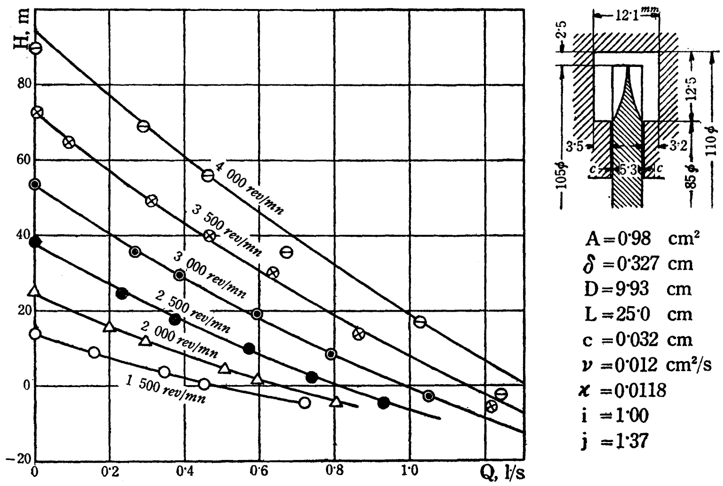


FIG. 5.
Head-capacity characteristics of No. 2 pump.

In carrying out this experiment another pump is inserted in the suction pipe line, and it charges liquid of high pressure to the pump under consideration, so that the characteristics of the pump at negative total head is clarified. Using these experimental data we can directly evaluate the magnitude of the pressure decrease at the suction nozzle, because the negative total head at the discharge $Q = i\,A\,U\left\{1 + \frac{\varepsilon}{\delta} - \frac{0.4343}{\log(1+\delta/\varepsilon)}\right\} \times 10^{-3}$ is equal to the pressure decrease. In the cases of high speed revolutions of 3500 and 4000 rpm, the experimental data deviate from the theoretical lines when the discharge is large. This seems to be due to cavitation, because the pressure in the suction pipe line is not higher than 7 m.

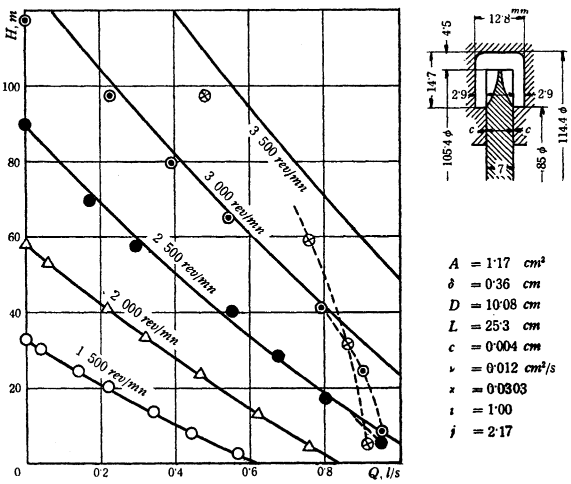


FIG. 6.
Head-capacity characteristics of No. 2 pump with smaller leakage-clearances and a larger pumping passage.

Figure 6 shows another example. In this case the shapes and the dimensions of the pumping passage and the impeller are different from those in the foregoing case, and the clearance c is much smaller. The theoretical characteristics satisfy the experimental data fairly well at a variable rate ranging from 1500 to 2500 rpm. In the case of high speed and high total head, experimental data are smaller than theoretical values. This tendency is slightly recognized in many other instances and remarkably so when the depth of the pumping passage δ is large. This seems to be true because in the case of high pressure gradient and large value of δ, the status of turbulence in the pumping passage may be different from that assumed in the foregoing theory.

The experimental data deviate from the full lines, at $Q \geqq 0.9$ l/s. This fact is easily verified quantitatively by the consideration of cavitation. The value of j used in this case appears to be too large, but this value is verified to be correct by the research on cavitation of this pump.

Figure 7 shows the characteristics of the pump indicated in Figure 1. The strength of the construction prohibits the head being higher than 70 m. The experimental data deviate slightly from the theory at high pressure. This seems to be due to the foregoing reason as well as to the increase of leakage through the clearance which is increased by the deformation at a high pressure.

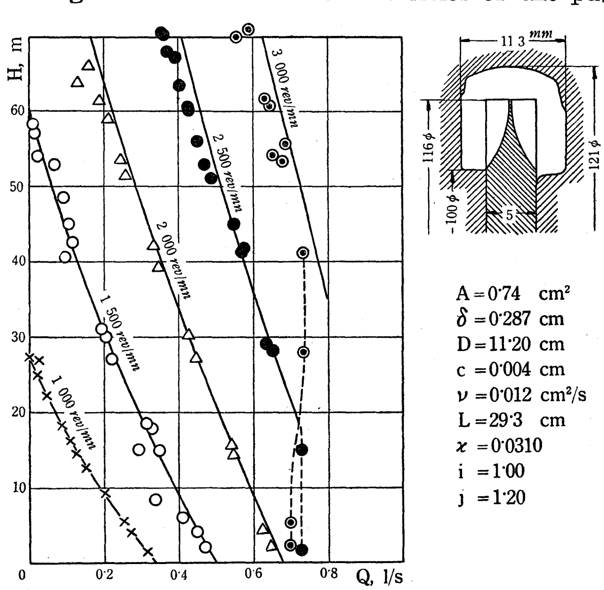


Fig. 7.
Head-capacity characteristics of No. 1 pump.

The surface of the pumping passage is not smooth; i.e. it is roughly finished by a machine but at some parts the casting surface remains. Therefore the cross-sectional area of the pumping passage A is not accurately kept constant along the length of the passage, and the value of A used in the calculation is the arithmetic mean of the cross-sectional areas at several sections of the passage.

This theory has been applied to several other pumps, the characteristics of which were experimentally clarified by a group of engineers. The result of each case shows that if proper value for κ is estimated, the theory satisfies the experimental data comparatively well when i and j are about 1.0.

8. Intensity and characteristics of turbulence in the pumping passage.

Many experimental data verify that the theory is applicable for prediction of characteristics of any peripheral pump. This means that the assumption used in the theory is not far from the truth. Section 8 includes a more detailed discussion of the turbulence in the pumping passage. The author explained in a foregoing section that the turbulence induced by the impeller is much stronger than that induced by the velocity gradient. In this section a numerical comparison will be made in regard to the two kinds of turbulence induced by the impeller and by the velocity gradient in the pumping passage.

According to Prandtl's theory of turbulence, in ordinary turbulence there is the following relation between frictional force τ and velocity gradient du/dy:

$$\tau = \rho\, l^2 \left|\frac{du}{dy}\right| \frac{du}{dy} = \rho\, k^2 y^2 \left|\frac{du}{dy}\right| \frac{du}{dy}, \tag{17}$$

where l is the mixture length which increases in proportion to the distance from the wall and it is shown by equation $l = ky$ in which k is a proportional constant. The turbulent viscosity is shown by $\rho k^2 y^2 |du/dy|$. In order to calculate the turbulent viscosity, we must solve the equations of motion considering the equation (17). This calculation is very complicated[8)].

For the sake of simplicity we assume that the velocity gradient is the same as equation (13) and calculate the turbulent viscosity using equation (17). Accordingly, the turbulent viscosity μ_P is shown as follows:

$$\mu_P = \rho\, k^2 y^2 |du/dy| = \rho k^2 y \{y\, |du/dy|\},$$

where

$$y\frac{du}{dy} = \frac{dp/dx}{\rho\,\kappa\, U}(y-\varepsilon) + \frac{U}{\ln(1+\delta/\varepsilon)} - \frac{(dp/dx)\,\delta}{\rho\,\kappa\, U \ln(1+\delta/\varepsilon)} + \frac{(dp/dx)\,\delta}{\rho\,\kappa\, U}$$
$$= \frac{dp}{dx}\frac{y}{\rho\,\kappa\, U}\left\{1 - \frac{\delta}{y}\,\frac{1}{\ln(1+\delta/\varepsilon)}\right\} + \frac{U}{\ln(1+\delta/\varepsilon)}. \tag{18}$$

According to the authorities the proportional constant k is about 0.2 in the case of flow through a pipe or a parallel channel.

On the other hand, according to the theory of turbulence induced by an impeller, the turbulent viscosity μ_I is $\mu_I = \rho\,\kappa\, y\, U$, where mean value of κ is about 0.03.

To simplify matters let us compare these two types of turbulent viscosity with each other, assuming the two extreme cases; one is zero pressure gradient and the other is no discharge.

According to the modified Prandtl's theory, the turbulent viscosity at zero pressure gradient region is $\mu_P = \rho\, k^2 y \{y\, |du/dy|\} = \rho\, k^2 y \{U/\ln(1+\delta/\varepsilon)\}$. Since $1/\ln(1+\delta/\varepsilon)$ is, in most cases, about 0.15, the turbulent viscosity becomes approximately $\mu_P = \rho\, k^2 y^2 |du/dy| \doteqdot 0.006\, \rho\, U y$. Conversely, according to the theory of turbulence induced by the impeller, the turbulent viscosity

8) Miyazu solved this equation under a special simple condition. It is of no use for the present purpose, but it shows how it is complicated to solve this equation. A. Miyazu: Theory of Westco pump. Transactions of the Japan Society of Mechanical Engineering. Vol. 5, No. 18, 1939.

at zero pressure gradient region is $\mu_I = \rho\,\kappa\, y\, U = 0.03\,\rho\, U\, y$. In the case of zero pressure gradient, therefore, the intensity of turbulence induced by the impeller is about five times as strong as that induced by the velocity gradient throughout the pumping passage.

If the turbulence is induced by the velocity gradient in the pumping passage, the surface of the impeller or the moving wall does not pose as the source of turbulence but poses only as a wall. For that reason the mixture length may be short not only near the fixed wall but also near the moving wall. Accordingly as concerns the turbulence near the moving wall, the turbulence induced by the velocity gradient is much weaker than that induced by the impeller.

In the case of zero discharge i.e. attainable maximum pressure gradient, the pressure gradient is

$$\left(\frac{dp}{dx}\right)_{q=0} = \frac{\rho\,\kappa\, U^2}{\delta}\left\{1 + \frac{\varepsilon}{\delta} - \frac{1}{\ln(1+\delta/\varepsilon)}\right\}\Big/\left\{\frac{1}{2} + \frac{\varepsilon}{\delta} - \frac{1}{\ln(1+\delta/\varepsilon)}\right\}.$$

Applying this to equation (18)

$$\left(y\frac{du}{dy}\right)_{q=0} = \frac{U}{\delta}\,\frac{1+\dfrac{\varepsilon}{\delta} - \dfrac{1}{\ln(1+\delta/\varepsilon)}}{\dfrac{1}{2}+\dfrac{\varepsilon}{\delta} - \dfrac{1}{\ln(1+\delta/\varepsilon)}}\left\{y - \frac{\delta}{\ln(1+\delta/\varepsilon)}\right\} - \frac{U}{\ln(1+\delta/\varepsilon)}.$$

According to Prandtl's theory the turbulent viscosity is as follows:

$$\begin{aligned}\mu_P &= \rho\, k^2 y^2 \frac{du}{dy}\\ &= 0.04\,\rho\, y\, U\left\{\frac{1}{\ln(1+\delta/\varepsilon)} + \frac{1+\dfrac{\varepsilon}{\delta} - \dfrac{1}{\ln(1+\delta/\varepsilon)}}{\dfrac{1}{2}+\dfrac{\varepsilon}{\delta} - \dfrac{1}{\ln(1+\delta/\varepsilon)}} \times \left(\frac{y}{\delta} - \frac{1}{\ln(1+\delta/\varepsilon)}\right)\right\}.\end{aligned}$$

In general $1/\ln(1+\delta/\varepsilon)$ is about 0.15, and the turbulent viscosity becomes approximately as follows:

$$\mu_P = \rho\, k^2 y^2 \frac{du}{dy} = 0.04\,\rho\, y\, U\left(2.43\frac{y}{\delta} - 0.22\right).$$

On the other hand according to the theory of turbulence induced by the impeller, the turbulent viscosity is $\mu_I = \rho\,\kappa\, y\, U = 0.03\,\rho\, y\, U$.

In the range $y/\delta < 0.4$, the turbulence induced by the impeller is stronger than that induced by the velocity gradient. In the latter instance, as was previously explained, the mixture length near the moving wall may be proportional to the distance from the moving wall $(\delta - y)$ instead of y. Accordingly the turbulence of the former is stronger than the latter even in the range $y/\delta > 0.55$. In general, the turbulence induced by the impeller is stronger or at least nearly equal to that which is induced by the velocity gradient.

Consequently, even if we neglect the influence of velocity gradient and consider only the fact that the turbulence is induced by the impeller alone, it will not mislead our consideration.

The author tested the characteristics of the pump with different kinds of impellers which will be shown in forth-coming articles. These experiments show that the number and the shape of the vanes of the impeller have great influences on the characteristics of the pump. This means that the turbulence in the pumping passage is chiefly induced by the impeller and not by the velocity gradient in the passage. In addition to these, he also tested the characteristics of the pump with a disk instead of an impeller. Since the disk has no vanes, it is natural that the turbulence is not induced by the vanes of the impeller but is induced by the velocity gradient. The characteristics obtained are quite different from those of a peripheral pump as shown in the next paragraphs.

If the discharge pressure is lower than the suction pressure by the pressure decrease at the entry, there is no pressure gradient along the length of the pumping passage. The experimental data of the pump with the disk show that the discharge at zero pressure gradient is one half the product of the cross-sectional area of the pumping passage times the peripheral velocity of the impeller. Authorities state that if there is a channel made of two parallel plates and one of them moves along itself, the mean velocity of the liquid in the channel at zero pressure gradient is one half of the velocity of the wall, whether the flow is laminar or turbulent. The characteristics of the pump with the disk satisfy this relationship. i.e. Accordingly the flow must be either laminar flow or turbulent flow of ordinary type. If it were laminar flow, theoretical calculation shows, the maximum head at 4000 rpm should be 0.45 m, but experimental datum shows that it is actually 6 m. This means that the flow in the pumping passage is the turbulent flow of ordinary type.

It is difficult to analyse this flow accurately and to estimate the intensity of turbulence in the passage. But if equation (14′) is adopted to analyse experimental data, the coefficient of turbulence κ is estimated. The value obtained is 0.0007 which is about 1/40 of a usual value of $\kappa = 0.03$. Then, it is apparent that the turbulence in the pumping passage of an ordinary pump is induced by the impeller, and any influence of the turbulence induced by the velocity gradient is negligible.

9. Internal leakage. The discharge nozzle and the suction nozzle are located close to each other on the periphery of the impeller, and a partition wall checks the flow against the pressure difference between these two nozzles. The impeller moves through the partition wall from the discharge nozzle to the suction nozzle, and there is a clearance between the partition wall and the impeller, through which liquid flows from the discharge nozzle to the suction nozzle.

Because the pumping passage is enclosed by the revolving impeller and the stationary casing, they create two clearances on both sides of the impeller, which connect the pumping passage with the central chamber. Through these clearances, liquid flows between the pumping passage and the central chamber.

Generally speaking a peripheral pump discharges a rather small quantity of liquid against a rather high pressure, so internal leakage is apt to occur, influencing the performance of the pump a great deal.

As explained above there are two kinds of internal leakage.

(1) Leakage through the partition wall. A volume of the liquid which has attained the discharge pressure leaks through the partition wall into the suction nozzle. Here it is mixed up with the liquid which flows from the suction pipe. Therefore, the sum of the discharge and the leakage which permeates the partition flows through the pumping passage and is given energy by the impeller.

(2) Leakage between the pumping passage and the central chamber of the pump. Liquid leaks into the pumping passage of a lower pressure from the central chamber. Accordingly, the quantity of liquid increases along the length of the passage. In the pumping passage of a higher pressure, the liquid leaks from the pumping passage into the central chamber and the quantity of liquid flowing through the pumping passage gradually decreases. Accordingly, the quantity of liquid which flows through the pumping passage is not constant but varies along the length. Correctly speaking we must not apply equation (14) to this case, which has been established under the assumption that the rate of flow is constant in length of the pumping passage. However, the turbulence induced by the impeller does not seem to be influenced by the leakage, and so equation (11) is applicable to the flow in the pumping passage even if there is leakage.

In order that the quantity of flow Q in equation (11) is the true quantity of flow at the considering section, it is not the discharge Q_0 but the sum of the suction quantity and the total amount of the leakage along the length of the pumping passage from the suction nozzle to the considering section. This quantity is explained by $Q_0+\int q\,dx$, because the suction quantity is equal to the discharge.

In the above equation, q is the quantity of liquid which leaks into the pumping passage per unit length; x is the length measured from the entry along the pumping passage. Accordingly, the following equation shows the relation between the quantity of flow and the pressure gradient:

$$\frac{dh}{dx}=\frac{\kappa_0 U}{g\delta A}\,\frac{AU\left\{1+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}\right\}-(Q_0+\int q\,dx)}{\frac{1}{2}+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}}. \tag{19}$$

In the above equation κ_0 is the coefficient of true turbulence and the subscript 0 is used in order to distinguish the coefficient of true turbulence from that of apparent turbulence κ which is influenced by the internal leakage. This fact will be elaborated upon at a later date. In the above equation, the rate of leakage q is dependent upon; 1. the difference between the pressure in the pumping passage and in the central chamber, 2. the resistance (frictional and otherwise), 3. the cross-sectional area through which the liquid leaks. Accordingly the rate of leakage depends upon the pressure gradient dh/dx. Therefore, it is very difficult to solve the above equation accurately. In order to solve this equation, three assumptions are adopted:

(1) In the pumping passage the pressure increases linearly from the suction nozzle to the discharge nozzle.

According to experimental data, in a situation with much discharge, turbulence does not rise rapidly near the entry. The pressure gradient at that point is less than that at another point, and sometimes the pressure at the entry is lower than the suction pressure. However, the internal leakage becomes important only when the discharge pressure is high. Fortunately, the length influenced by the entry becomes shorter as the discharge pressure increases, so the influence of the entry is not important for internal leakage.

When the internal leakage is very large, the quantity of flow which passes through the middle part of the pumping passage is much ampler than that passing through both ends of the pumping passage, and so pressure does not increase linearly along the passage but increases as the thin full line in Figure 4 illustrates. However, it will be verified later that the pressure distribution in the pumping passage does not remarkably influence the internal leakage or the characteristics of the pump. Accordingly the assumption mentioned above is allowable.

(2) Although the impeller is revolving, the rate of flow through the clearance is approximately equal to that through the clearance of the same dimensions which is constructed with two stationary parallel plates, if the radial pressure difference due to the centrifugal force is properly considered.

In many cases Reynolds number of the leakage is small and the flow is laminar. It has been verified that if the flow is laminar, the rate of flow is hardly influenced by rotation of one of the walls which enclose the clearance. In the case of turbulent flow, authorities analysed and verified that the error due to revolution of a wall is not a great percentage. The influence of the radial pressure difference due to the centrifugal force will be canceled by assuming that the pressure of the central chamber is what will be explained in the next article instead of its true value.

(3) The pressure in the central chamber of the pump is the mean value of the suction pressure and the discharge pressure, if the radial pressure difference due to the revolution of the impeller is disregarded.

Generally speaking there are the following relations between the pressure difference Δh and the rate of leakage per unit length q:

$$q = c\,v\,, \quad \Delta h = a\,v^2 + b\,v\,, \tag{20}$$

where Δh is the pressure difference between both ends of the leakage-passage, c is the dimension of the clearance through which the liquid leaks, a and b are coefficients of resistance which are determined by the form of the clearance, and v is the mean velocity of the liquid leaking through the clearance. According to the foregoing assumptions, the distribution of the pressure difference Δh along the pumping passage is shown by the following equation:

$$\Delta h = (H/2)\{1-(2x/L)\}, \tag{21}$$

where H is the total head of the pump, x is the length of the pumping passage measured from the suction nozzle or the entry of the pumping passage, and L is the geometric total length of the pumping passage. In order to evaluate the value of $\int q\,dx$ easily, let us analyse the following two extreme examples.

One is the case in which the clearance is so thin that the resistance due to the viscosity in the clearance is considerably great, and the entry and exit losses are negligible compared with the frictional resistance, i.e. a is zero in equation (20). Accordingly

$$v = \Delta h/b = (H/2b)\{1-(2x/L)\},$$

$$\int q\,dx = \frac{cH}{2b}\int\left(1-\frac{2x}{L}\right)dx = \frac{cH}{2b}\left(x-\frac{x^2}{L}\right).$$

Applying this equation to equation (19) and integrating it from the entry to the exit, the following equation is obtained:

$$H = \frac{\rho\kappa_0 UL}{g\delta A}\cdot\frac{AU\left\{1+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}\right\}-Q_0-\frac{H}{2b}\frac{cL}{6}}{\frac{1}{2}+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}}, \tag{22}$$

where $\int_L dh = H.$

The other is the case in which the clearance is so wide that the laminar frictional resistance in the clearance is negligible compared with the entry and exit losses, or there is no frictional resistance due to laminar flow but due to turbulent flow. In this case b vanishes in equation (20) and the following equations are obtained:

$$v = \sqrt{\frac{\Delta h}{a}} = \sqrt{\frac{H}{2a}\left(1-\frac{2x}{L}\right)},$$

$$\int q\,dx = \sqrt{\frac{H}{2a}}\,c\int\sqrt{1-\frac{2x}{L}}\,dx = \sqrt{\frac{H}{2a}}\,c\,\frac{L}{3}\left\{1-\left(1-\frac{2x}{L}\right)^{3/2}\right\}.$$

Applying this equation to equation (19) and integrating it from the entry to the exit, the following equation is obtained:

$$H=\frac{\rho\kappa_0 UL}{g\delta A}\cdot\frac{AU\left\{1+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}\right\}-Q_0-\sqrt{\frac{H}{2a}}\frac{cL}{5}}{\frac{1}{2}+\frac{\varepsilon}{\delta}-\frac{1}{\ln(1+\delta/\varepsilon)}}, \qquad (23)$$

where $\int_L dh=H$.

Comparing equations (22) and (23) with equation (14), it is clear that the discharge of a pump which is influenced by the leakage in the pumping passage is less than that of an ideal pump of no leakage by the last term of the numerator; that is, a certain amount of the discharge leaks radially through a certain equivalent cross-sectional area at the pressure difference of one half of the total head of the pump. The equivalent cross-sectional area for the leakage is the product of the clearance c and $1/5\sim1/6$ of the length of the pumping passage. In practice both a and b are not zero, so the equivalent length of the clearance is between 1/6 and 1/5 of the geometric length, and the smaller clearance is accompanied with the shorter length.

When the clearance c is very large, the pressure distribution in the pumping passage is not a straight line but a curve shown by thin lines in Figure 4. Therefore, one of the assumptions mentioned above should be examined. In order to calculate the equivalent length of the clearance at such kind of pressure distribution, for the sake of simplicity, let us assume that the distribution of the pressure difference between the pumping passage and the central chamber is described by the following equations:

$$\Delta h=H/2\left(1-\frac{2x}{L}\right)^2 \qquad \text{at } x=0\sim L/2,$$

$$\Delta h=-H/2\left(1-\frac{2x}{L}\right)^2 \qquad \text{at } x=L/2\sim L.$$

These equations extremely exaggerate the influence of the internal leakage. In practice, however large the clearance may be, the pressure increases more straightly along the pumping passage.

In the case mentioned above, since the clearance is very large, the resistance against the leakage through the clearance is nearly proportional to the square of the mean velocity of leakage v. Therefore there are the following equations at $x=0\sim L/2$:

$$v=\sqrt{\frac{H}{2a}}\left(1-\frac{2x}{L}\right),$$

$$\int q\,dx=c\sqrt{\frac{H}{2a}}\int\left(1-\frac{2x}{L}\right)dx=c\sqrt{\frac{H}{2a}}\left(x-\frac{x^2}{L}\right).$$

Applying this relation to equation (19), let us integrate it from $x=0$ to $x=L/2$. The total head of the pump is twice the head obtained in this integration, because the status of flow at $x = L/2 \sim L$ is symmetrical to that at $x = 0 \sim L/2$. The total head obtained is the same as equation (23) except that the last term of the numerator is $\sqrt{\frac{H}{2a}}\frac{cL}{6}$ instead of $\sqrt{\frac{H}{2a}}\frac{cL}{5}$.

If the resistance is proportional to the velocity of the leakage v, although such case hardly occurs, the equivalent length of the area for the leakage is $L/8$ instead of $L/6$ in equation (22). After all, the influence of the pressure distribution is not remarkable. Additionally the practical pressure distribution is nearly straight. Thus the equation (22) and (23) are not far from the truth.

Now we must estimate the proportional constant a and b in equation (20). As explained before, the influence of revolution of the impeller is negligible, so the condition of the radial flow is nearly the same as that which passes through the clearance of ringshape. In the case of a peripheral pump of usual type, as the radius of curvature is large enough compared with the length of the leakage-passage through the clearance, the influence of the curvature is not important. Accordingly this phenomenon can be compared to the flow through two parallel walls.

If it is laminar flow, there is the relation $\Delta h_1 = (12\, m\, \nu / g\, c^2)\, \bar{v}$, where ν is the kinematic viscosity of the liquid, m is the length of the leakage-passage through the clearance, $\bar{v}$ is the mean velocity and Δh_1 is the loss of head due to the frictional resistance. In this case the velocity head Δh_2 is shown by $\Delta h_2 = 1.54(\bar{v}^2/2\, g)$, and most of this energy is wasted at the exit of the clearance. At the entry of the clearance, as there is a strong secondary flow which may obstruct the flow of the liquid into the clearance, the loss of head at the entry Δh_3 is rather large. As many experimental data show that the loss is about 3/4 of the velocity head, the loss of head at the entry Δh_3 is $\Delta h_3 = 0.75\, \Delta h_2 = 1.16\, (\bar{v}^2/2g)$. Accordingly the value of a and b in equation (20) are $a = 2.7/2\, g$ and $b = 12\, m\, \nu / c^2\, g$ respectively. These equations are applicable for the leakage between the pumping passage and the central chamber as well as the leakage between the discharge nozzle and the suction nozzle.

The performance of a pump in which the liquid leaks internally is shown by a modification of the ideal performance of the same pump of no leakage. That is, the discharge of a pump, in which the liquid leaks internally, is estimated as a part of the discharge of the ideal pump; the remainder of the discharge leaks from the discharge nozzle to the suction nozzle internally. The quantity of the leakage was explained above. That is, the leakage between the discharge nozzle and the suction nozzle is a part of this; the other is the leakage between the pumping passage and the central chamber, which has been reevaluated as the equivalent leakage between the discharge nozzle and the suction nozzle. Therefore the discharge of a practical pump at a certain discharge pressure is less than the ideal discharge of the same pump of no leakage by the equivalent leakage Q', where Q' is shown as follows:

$$
\begin{aligned}
Q' &= c\,l_1\,\bar{v}_1 + c\,l_2\,\bar{v}_2, \\
(1.35/g)\,\bar{v}_1^2 + (12\,m_1\,\nu/c^2 g)\,v &= H, \\
(1.35/g)\,\bar{v}_2^2 + (12\,m_2\,\nu/c^2 g)\,v &= H/2.
\end{aligned} \tag{24}
$$

In the above equations the subscript 1 means the leakage between the discharge nozzle and the suction nozzle, the subscript 2 means the leakage between the pumping passage and the central chamber. For example, $l_1 c$ and $l_2 c$ are equivalent area for the leakage respectively. The length l_1 is obviously estimated as $(b_2 + b_3 + t)$ in Figure 3(a). On the other hand, the length of the pumping passage adjacent to the central chamber is $\pi D_i(1-\theta^\circ/360^\circ)$ in Figure 2, and as explained before, the equivalent length for the leakage is 1/5 to 1/6 of this length. As the liquid leaks on both sides of the impeller, l_2 is twice of this equivalent length.

In usual cases, the flow through a clearance is laminar. If the clearance is about $c = 0.3$ mm, however, Reynolds number is so large that the flow may be turbulent. As explained before, even if the flow is turbulent, the influence of the impeller's rotation is negligible, so the quantity of the leakage can be easily estimated. The coefficient of friction λ in a pipe flow is applicable in this case only if the hydraulic mean depth is adopted as the representative dimension of the passage. In these cases the value of Reynolds number is 1000 to 2000; thus the value of λ is about 0.01. As the hydraulic mean depth of the clearance of a peripheral pump is half the dimension of the clearance c, the pressure drop Δh_1 due to friction in the passage is shown as follows:

$$
\Delta h_1 = \lambda \left(m \Big/ \frac{c}{2} \right) \frac{\bar{v}^2}{2g} = 0.02 \frac{m}{c} \frac{\bar{v}^2}{2g},
$$

where m is the length of the leakage-passage. As the value of m is about 1.0 to 3.5 cm, and c is about 0.03 cm, the above equation becomes $\Delta h_1 \doteqdot 0.7\,v^2/2g$.

If the flow is turbulent, the velocity is rather uniform and the kinetic energy is nearly equal to the velocity head of the mean velocity. Assuming that the loss of head at the entry and the exit of the passage is about 1.75 times the velocity head, the total loss of head is about $2.5(\bar{v}^2/2g)$ which is nearly the same as the first term of equation (24). Needless to say the loss of head due to turbulent flow is larger than that due to laminar flow, so the second term of equation (24) is smaller than $0.7(\bar{v}^2/2g)$. For example, assuming that the dimensions of the pump are the values mentioned above and that the total head is 20 m, the second term is about $0.2(\bar{v}^2/2g)$. The coefficient of this term diminishes as the total head increases. Accordingly, in usual pumps even if the flow is turbulent we may use the equation (24) approximately, in which we may either neglect or adopt the second term, for the term is not important.

In order to verify the theory above mentioned, the author carried out a series of experimental tests using the pump shown in Figure 1. He changed the value of the clearance c eight times from 0.04 mm to 0.36 mm and clarified the influence of the clearance experimentally, and then compared them with the theory above mentioned.

In order to change the clearance, paper liners were used and the thickness was measured through the change of the distance of both casings. The impeller was carefully set at the center of the casings using a dial-gauge. Therefore the increase of the clearance is about one half of the thickness of the liners. Furthermore according to the theory above mentioned, even if the impeller is not set at the center of the casings, the internal leakage is hardly influenced.

The pump of each clearance was tested at 1000, 1500, 2000, 2500 and 3000 rpm, and its normal performance as well as cavitation performance were investigated.

Figure 8 shows the performances of pumps of different clearances at 2500 rpm and 1000 rpm. These performances show that the shut off pressure of a large clearance pump is only one fourth of that of a small clearance pump. Therefore, it is clear that without clarifying the influence of the internal leakage through the clearance the performance of a pump is hardly understood.

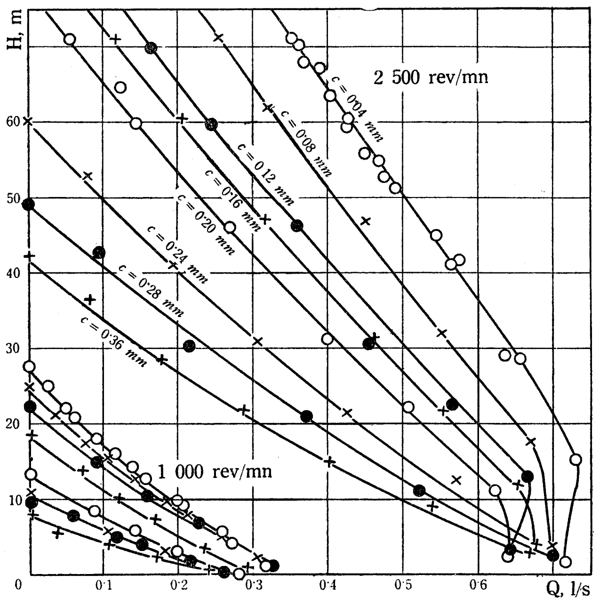


FIG. 8.
Head-capacity characteristics of pumps with different clearances.

In order to compare these experimental data with the theoretically expected performance of these pumps, the ideal performances of the pump of no leakage must be anticipated beforehand. The difference between the discharge of the ideal pump and the experimental discharge at various total heads, is calculated and shown on the figure of the performances of the ideal pump. This value is the equivalent leakage of the pump, and it is compared with the value calculated by the theory explained before.

Figure 9 shows the results of these. In this figure different marks indicate the equivalent-leakages calculated through the experimental data, and full lines pass through the origin show the theoretical equivalent-leakage. These experimental data are hardly influenced by the speed of the impeller and assemble together, according to the clearance, on each theoretical line. Some of the experimental data do not coincide with the theoretical lines, but in general we can use the theory for the estimation of the leakage

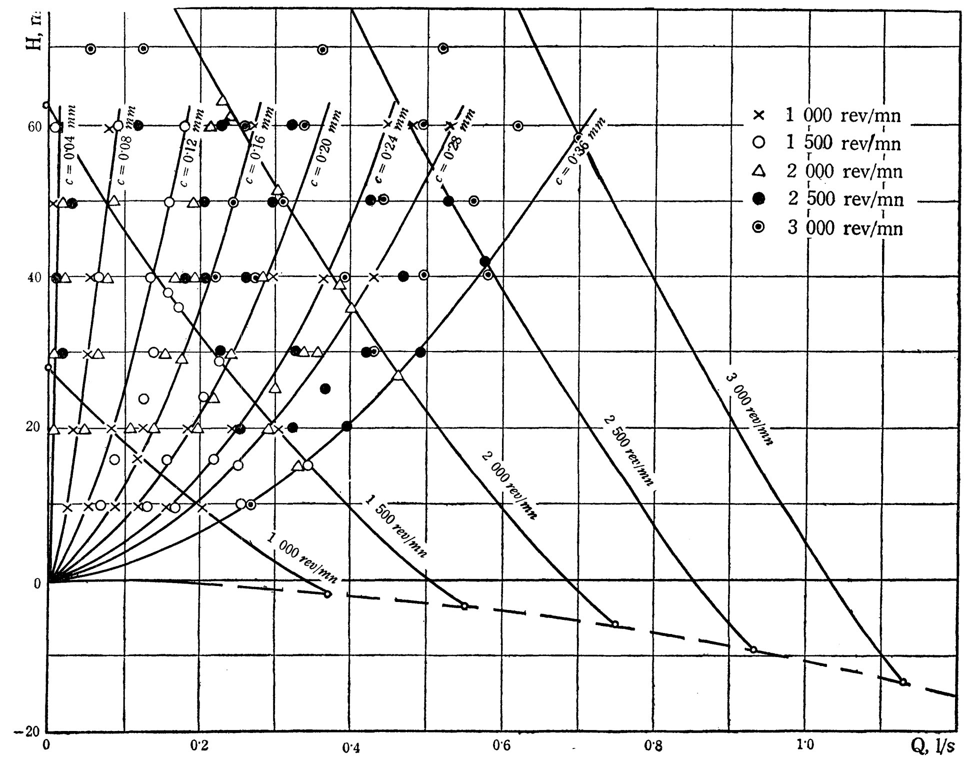


FIG. 9.

Theoretical and experimental relationship between internal leakage and head.

at any value of the clearance. These data shows that the performance of a peripheral pump of 0.04 mm clearance is hardly inferior to the ideal pump, but if the clearance is larger than 0.04 mm the performance of the pump becomes poorer as the clearance increases.

According to the foregoing description we can theoretically estimate the performance of a pump with a certain value of clearance by subtracting the equivalent leakage from the performance of the ideal pump at various pressures. But accurately speaking, if the total head is low, the influence of the entry which has been neglected in the thoery becomes important, and the theoretically estimated performance of the pump will not coincide with the actual performance of the pump. The broken line in Figure 9 shows the pressure decrease at the entry of the pumping passage. In order that the theoretically estimated performance is correct, the tolal head should be considerably larger than the amount of the pressure decrease which is shown by the broken line.

We can estimate the performance of a pump by the method mentioned above, but there is still another simpler way. The shut off pressure of a pump with a certain value of clearance is equal to the pressure at which

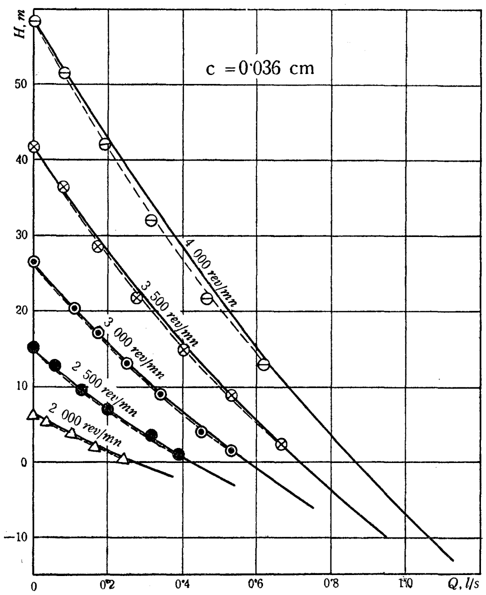


FIG. 10.
Comparison among theoretical, simplified theoretical and experimental characteristics of No. 1 pump with 0.036 cm clearance.

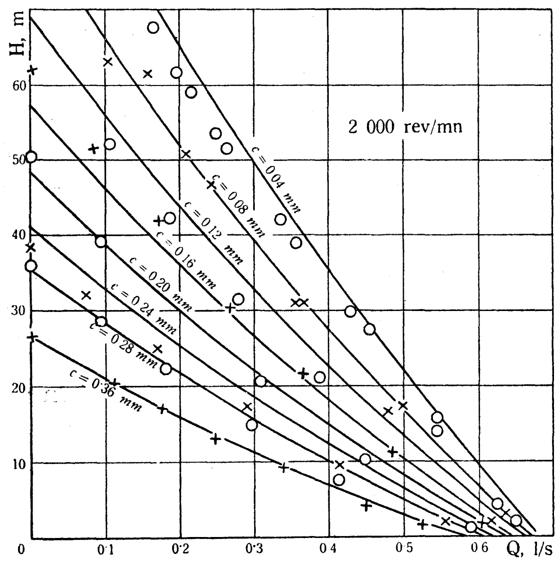


FIG. 11.
Head-capacity characteristics of No. 1 pump with various clearances.

the equivalent-leakage coincides with the discharge of the ideal pump. The intersecting point of the equivalent leakage line and the performance line of the ideal pump further illustrates this. Now, let us introduce an apparent turbulent coefficient κ, which is estimated as the product of κ_0 and the ratio of the actual shut off pressure to the ideal shut off pressure of no leakage. Adopting an apparent turbulent coefficent κ instead of κ_0 in equation (14′), the performance of a pump with a certain clearance will be shown by this equation without considering the internal leakage anymore. At least the highest attainable pressure obtained by this equation is correct. On the other hand, the maximum discharge at zero pressure gradient is hardly influenced by any little change of the turbulent coefficient; additionally liquid does not leak internally at zero pressure gradient. Therefore the maximum discharge calculated by the method above mentioned will be correct. Accordingly the performance of a pump as a whole will be shown by this method fairly well.

Figure 10 shows an example in the case of $c = 0.36$ mm. In this figure the broken lines show the performances which are calculated as the difference between the ideal discharge and the internal leakage. As explained before, this method is not applicable for low pressure. The full lines are the performances calculated by the above mentioned method. These performances coincide with the experimental data well. If the equivalent turbulent coefficient κ is adopted instead of the true turbulent coefficient κ_0, the performance of a pump with a certain value of clearance, is easily estimated by the equation, which has been induced for estimating the performance of an ideal pump with no clearance. The performances shown in Figures 5, 6 and 7 have been calculated without any consideration of internal leakage, adopting proper equivalent turbulent coefficient κ in each case. Therefore, all of these verify that the method explained above is applicable to these cases.

Figure 11 shows the theoretical performances estimated by the method mentioned above along with the experimental data of the pump at various values of the clearance.

Accurately speaking, in carrying out the calculation of equation (14), κ utilized as $\varepsilon = \nu/U\kappa$ in equation (14) should be κ_0. However, both $1+\varepsilon/\delta$ and $\ln(1+\delta/\varepsilon)$ are not influenced remarkably by any little change of ε. Therefore we may adopt either κ or κ_0 in calculating the value ε. Sometimes the value of the clearance is not shown accurately; therefore it is difficult to calculate the value κ_0. Accordingly in this thesis κ is adopted instead of κ_0 to calculate the value of ε.

10. The torque and the total head. In order to discharge the liquid against a pressure gradient, the impeller must revolve against the following resistances: the first is the mechanical torque at the stuffing boxes and the bearings; the second is the disk frictional torque due to the revolution of the shaft and the impeller in the liquid; the third is the resistance due to the pressure difference between the discharge nozzle and the suction nozzle. Since a peripheral pump is a small type pump, the mechanical torque occupies a considerable percentage of the total torque. The mechanical torque is influenced by the construction, the status of maintenance and some other factors, so it is not considered here.

The pressure working on the front surface of the vanes of the impeller may be different from that on the back surface. In the theory mentioned above, however, the force working on the surface of the vanes can be compared to the frictional force working on the periphery of the impeller. Accordingly we may consider that the force working on the impeller is only the frictional force, defying the pressure difference on the vanes. Therefore, the torque which is required to drive the impeller is calculated by integrating the product of the tangential force and the radius throughout the surface of the impeller.

The frictional force working on the surface of the impeller in the pumping passage was shown in Section 4, but the frictional force near the entry of the pumping passage seems to be difficult to calculate, because the status of flow at that point is quite different from that at any other point.

Equation (6) shows that the frictional force is composed of two parts; one, the frictional force due to the rotation of the impeller as shown in the first term of the equation, and the other, the force due to the pressure gradient in the pumping passage as shown in the second term of the equation. Since the turbulence does not develop completely near the entry, the coefficient of turbulence κ in the first term may be smaller than the ordinary value at that place which is not so long. Since the second term of the equation is influenced by the pressure gradient alone, the integrated value of the second term depends only upon the total head of pump H. Accordingly if the special status near the entry of the pumping passage is properly considered, this equation appears to be applicable therewith.

According to section 6, the mean kinetic energy of the liquid in the pumping passage is $(0.7\sim 0.9\,U)^2/2\,g$ which decays at the exit of the pumping passage. Since the kinetic energy and that part of the pressure energy which is wasted are originally supplied by the impeller, it is forced to exert an excessive amount of work in order to compensate for these losses of energy. Since the entry and the exit losses are the function of the kinetic energy, all of the losses are shown by $\lambda U^2/2g$, where λ is a constant and nearly "one". Consequently, we may use $\rho g\{H+(\lambda U^2/2g)\}$ instead of $\int_L dp/dx \cdot dx$ in equation (6).

According to the above consideration, the torque of the impeller is not formally influenced by the pressure decrease at the entry. The decrease of pressure at the entry is due to the increase of kinetic energy of liquid, which has been considered in the equation as the loss of head at the exit. This equation will be checked in later section by experiment.

The torque which is required to drive the liquid in the pumping passage is as follows:

$$T_p = \int_b \int_L \frac{D}{2}\tau_\delta \, dx \, db$$

$$= \frac{D}{2} b L \frac{\rho \kappa_0 U^2}{\ln(1+\delta/\varepsilon)} + \frac{D}{4}\rho \delta b \lambda \psi U^2 \left\{1 + \frac{\varepsilon}{\delta} - \frac{1}{\ln(1+\delta/\varepsilon)}\right\}$$

$$+ \frac{D}{2}\rho g b \delta \psi H \left\{1 + \frac{\varepsilon}{\delta} - \frac{1}{\ln(1+\delta/\varepsilon)}\right\}. \quad (26)$$

In the above equation, the first term is the frictional torque due to the turbulence, which is proportional to the turbulent coefficient κ_0. In this case, needless to say, the turbulent coefficient must be the true turbulent coefficient κ_0 instead of the apparent turbulent coefficient κ. The first term does not depend upon the head and the discharge but only depends upon the peripheral velocity of the impeller; it can be compared to a disk frictional torque in the pumping passage. The other terms of Equation (26) are the torques against the pressure difference. That is, the second term is the torque against the losses of head, and the third term is the torque against the total head of the pump. These two terms have a common correction coefficient ψ. Experimental data show that the value of ψ is nearly "one"; thus the theory mentioned above is fairly correct. The value of λ in the second term is a coefficient which shows the losses of head at the entry and the exit. Since most part of the loss occurs at the exit, the value of λ, nearly one, can be diminished by improving the form of the exit of the pumping passage.

If the form of the exit is improved, a part of the kinetic energy changes into the pressure head. The total head H increases in accordance with the diminution of the value of λ, but the driving torque of the pump will not be influenced by a reformation of the exit. Accordingly, a proper reformation of the exit will increase the head of the pump without increasing the driving power at the same discharge; thus the efficiency of the pump will improve.

As the whole impeller is immersed in liquid and revolved, a disk-frictional resistance is induced on both sides of the impeller. Since the induced disk-frictional resistance is a usual one, the value is easily estimated by theoretical and empirical formulae previously deduced.

According to the theory of Karman, the disk-frictional torque T is shown by the following formula, assuming that the Reynolds number is larger than 5×10^5:

$$T_f = 0.021\, d^3 u^2 (\rho/2)/R^{0.2}, \tag{27}$$

where d is the diameter of the disk, u is the peripheral velocity of the disk, R is ud/ν, and ν is the kinematic viscosity of the liquid. More accurate research was later published by Schultz and Grunow.[9] We can estimate the disk-frictional force using any of these formulae.

As equation (27) shows, the disk-frictional torque is nearly proportional to the square of the peripheral velocity and to the cube of the diameter of the disk, because it is influenced only slightly by Reynolds number. On the other hand the first term of equation (26) is proportional to the square of the peripheral velocity and to the cube of the diameter, because L and b are nearly proportional to the diameter. Since the first term of equation (26) has the same character as equation (27), the effect of the disk-friction may be included in the first term of equation (26) adopting a proportional coefficient ξ in the first term. In this case ξ is shown by the equation

$$\xi = 1 + \{2\, T_f \ln (1 + \delta/\varepsilon)/Db\, L\, \rho\, \kappa_0\, U^2\}. \tag{28}$$

Many examples show that the value of ξ is about 1.05 to 1.10.

The total torque acting on the shaft of a pump is as follows:

9) Schultz-Grunow: Der Reibungs Widerstand rotierender Scheibe in Gehäusen: Z. A. M. M. Heft 4, 1935.

$$T = T_m + \xi \frac{D}{2}\frac{A}{\delta} L \frac{\rho \kappa_0 U^2}{\ln(1+\delta/\varepsilon)}$$

$$+ \rho \psi g A \frac{D}{2}\left(\lambda \frac{U^2}{2g} + H\right)\left\{1 + \frac{\varepsilon}{\delta} - \frac{1}{\ln(1+\delta/\varepsilon)}\right\}. \qquad (29)$$

In engineering units

$$T = T_m + 0.2215 \times 10^{-3} \xi \frac{D A L \rho \kappa_0 U^2}{\delta \log(1+\delta/\varepsilon)}$$

$$+ 50 \psi \rho D A (5.1 \lambda U^2 \times 10^{-6} + H)\left\{1 + \frac{\varepsilon}{\delta} - \frac{0.4343}{\log(1+\delta/\varepsilon)}\right\}. \qquad (29')$$

Each symbol in this equation is the same as that in equation (14″). The new symbols T and T_m are the total torque in cm-g and mechanical torque in cm-g respectively. λ is the coefficient mentioned above and the value is nearly one; ψ is a correction coefficient and the value is also nearly one.

In equation (29) the second term means the frictional force working on the surface of the impeller. Since the frictional force is not influenced by the leakage, the coefficient of turbulence should be the coefficient of true turbulence κ_0 instead of the coefficient of apparent turbulence κ. That is, since the apparent turbulent coefficient κ was introduced to simplify the calculation of the pressure discharge performance of a pump, it would not be applicable in the evaluation of the torque performance of the pump. However, in many cases, it is difficult to estimate the coefficient of true turbulence κ_0. Therefore, the coefficient of apparent turbulence κ is sometimes abopted instead of κ_0 for the sake of simplicity. This is allowable, if the influence of the adoption of the apparent turbulent coefficient κ is conpensated by a modification of another coefficient; e.g. ξ in the second term or λ in the third term is assumed to be larger than its true value.

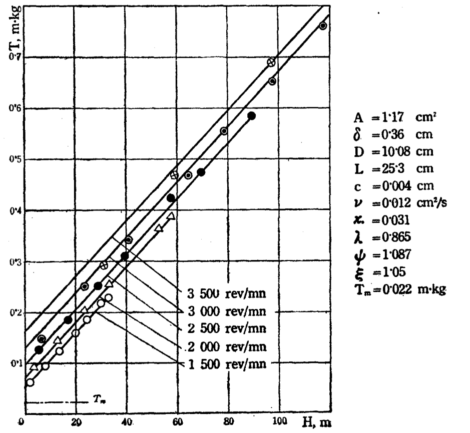


Fig. 12.
Head-torque characteristics of No. 2 pump.

However, since this research has the purpose of determining the general characteristics of a peripheral pump, this convenient method is not applied but the accurate method is used in the calculations shown in Figures 12, 13 and 14.

Figure 12 illustrates the torque performance of the pump, the pressure-discharge characteristics of which are shown in Figure 6. The full lines are the values calculated by equation (29′) where the mechanical torque $T_m = 2.2 \times 10^3$ g-cm,

$\lambda = 0.865$, $\psi = 1.087$, and $\xi = 1.05$. The mechanical torque was measured experimentally driving the impeller in the dry casing of the pump. Accurately speaking, in this case the impeller drove air, and so the mechanical torque might be estimated a little larger. Additionally, the mechanical torque might be influenced by the humidity of the stuff.

The equation (29′) shows that the torque at zero head is proportional to the square of the peripheral velocity of the impeller, if cavitation does not occur in the pump. Now we assume that the torque at zero head consists of two kinds, one independent of the number of revolutions per minute and the other proportional to the square of the number of revolutions per minute. The former being the mechanical torque. Accordingly, we can estimate the value of mechanical torque, analysing the torque at zero head at various number of revolutions per minute. In every case the mechanical torque estimated by this method coincides closely with the experimental value mentioned above.

Figure 6 shows that there is cavitation in the pumping passage when the number of revolutions per minute is large and the head is low. Some data at 3500 rpm in Figure 12 indicate the experimental data under cavitation; these do not coincide with the theoretical values at ordinary condition as shown by the full lines in Figure 12. In the case of cavitation, since the liquid in the pumping passage contains a great number of bubbles and the mean density of the fluid in the pumping passage is smaller than that at ordinary condition, the torque shown by equation (29′) may accordingly be smaller. These data show that the torque at cavitation is less than that at ordinary condition at the same total head. However, the rate of decrease of the discharge due to cavitation is larger than that of torque, and so the efficiency of the pump diminishes.

Figure 13 shows the torque performance of a pump. Its head-capacity characteristics are shown in Figure 5. This figure contains the performance of the pump at negative head, which shows that equation (29′) is applicable even in the case of negative head. In this figure the inclination of the experimental data at 4000 rpm is slightly different from others. The remarkable point in this figure is that the value of ψ is only 0.88; much smaller than 1.087 in Figure 12. Smaller value of ψ means less torque. However, the value of ψ has a close relationship with the internal leakage, which will be mentioned later. That is, smaller value of ψ

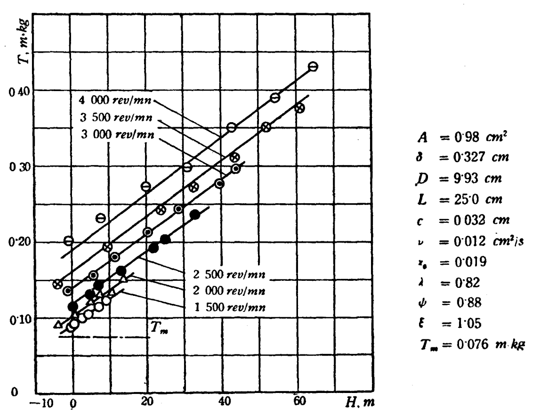


FIG. 13.
Head-torque characteristics of No. 2 pump.

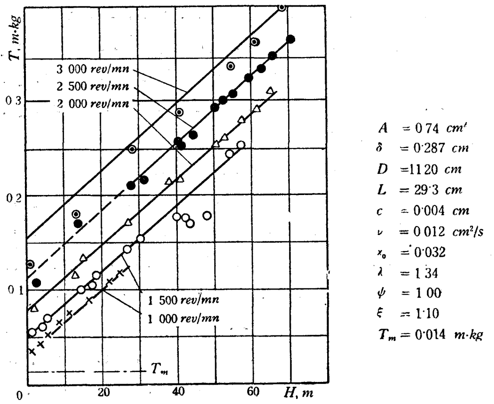


FIG. 14.
Head-torque characteristics of No. 1 pump.

does not always mean a greater efficiency, but usually means considerable leakage or a smaller value of the coefficient of apparent turbulence κ.

Figure 14 shows the experimental data of the pump shown in Figure 1. The head-capacity characteristics are shown in Figure 7.

There are many experimental data which show the head-capacity performance of a pump, but few of them show accurate input power. The author gathered some experimental data and applied equation (29). The results show that the equation is applicable in these cases.

11. Correction coefficient for torque. In Section 9, the author studied the influence of the clearance c, between the casing and the impeller, on the head-capacity performance of a pump both experimentally and theoretically. The head-torque performances obtained in these experiments are shown in Figure 15, which illustrate that the internal leakage influences the torque-head performance slightly: that is, the torque at zero head is hardly influenced by the internal leakage; however, the rate of increase of the torque due to increase of the head diminishes a little as the clearance increases.

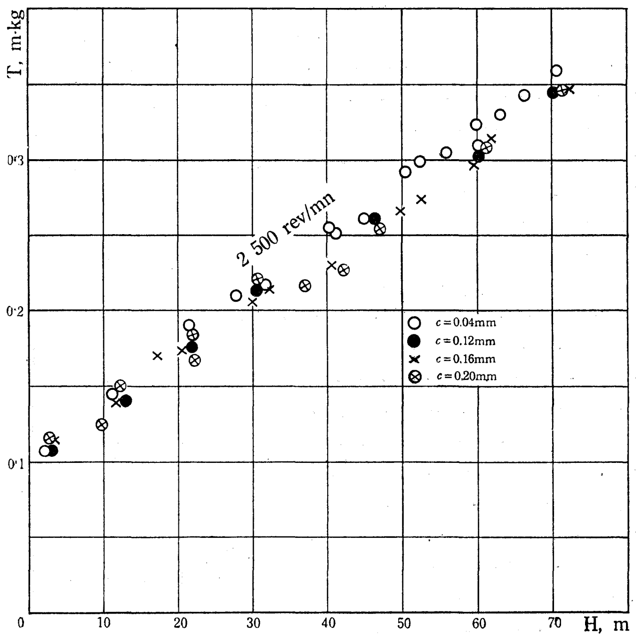


FIG. 15.
Effect of clearance on head-torque characteristics.

If the clearance between the casing and the impeller is large, there is a remarkable internal leakage between the pumping passage and the central chamber, which changes the rate of flow in the pumping passage along the length. Owing to the change of the rate of flow, the pressure distribution along the pumping passage becomes a complicated one such as the thin line in Figure 4 instead of a straight one. Such a pressure distribution seems to be the cause for changing the rate of increase of the torque against unit increase of the head. However, as equation (29) illustrates, the torque is decided only by the pressure difference between the entry and the exit and the number of revolutions per minute; i.e. the torque does not depend upon the status of the pressure gradient in the pumping passage.

However, Figure 15 shows that the relationship between the torque and the total head is slightly influenced by the clearance or the internal leakage. This seems to be due to the internal leakage between the discharge nozzle and the suction nozzl. The liquid, which is accelerated by the pressure difference between the discharge pressure and the suction pressure, flows from the discharge nozzle to the suction nozzle through the partition. As the velocity is much larger than the peripheral velocity of the impeller, the impeller is accelerated by the liquid. Since the velocity of the liquid increases owing to the increases of the pressure difference and the clearance, the deviation of the driving torque from the ideal situation becomes remarkable as the clearance and the pressure difference increase. Therefore, the relationship between the torque and the head is slightly influenced by the dimension of the clearance as shown in Figure 15.

The author has not analysed this problem theoretically but has only studied it experimentally. These influences are represented by the value of ψ in equation (29′). Figure 16 shows the relationship between ψ and the coefficient of apparent turbulence κ which represents the internal leakage. The circles and the broken lines indicate the results of the experimental data of the pump shown in Figure 2. The crosses and the full line indicate the results of the experimental data of the pump shown in Figure 1. These data show that the value of ψ is not decided by the value of κ alone. Needless to say, the coefficient of apparent turbulence κ is influenced by not only the quantity of the internal leakage, but also many other factors; including the coefficient of the true turbulence, the cross sectional area of the pumping passage and the condition of the partition. Therefore it is not profitable to find a general relationship

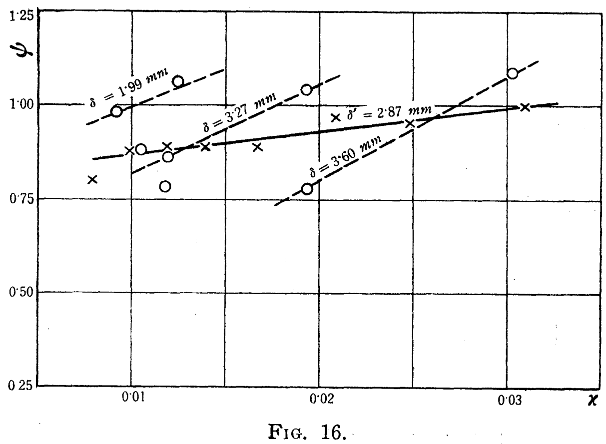
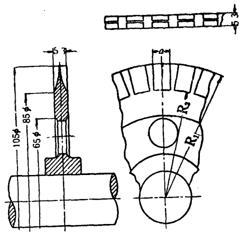


FIG. 16.
Effect of apparent turbulence κ on coefficient ψ.

between the value of ψ and the coefficient of apparent turbulence κ for different types of peripheral pumps. However, these data indicates that if the other conditions remain the same, the value of ψ decreases as the internal leakage increases. They also show that the value of ψ decreases as the depth of the pumping passage δ increases assuming that the coefficient of apparent turbulence κ is the same. In general, a larger value of δ means a larger cross-sectional area of the pumping passage. In the case of larger cross-sectional area, the performance of the pump is not influenced by the internal leakage remarkably; i.e. the value of κ is larger. Conversely if the coefficient κ is the same, larger value of δ means greater leakage; as the result the value of ψ is smaller.

According to the above explanation, the internal leakage decreases the value of ψ or the driving power of the pump. Therefore, it seems as if the internal leakage increases the efficiency of the pump. However, the internal leakage remarkably decreases the discharge of the pump; accordingly it decreases also the efficiency of the pump, in spite of the decrease of the driving power.



Impeller	No. of vanes, z	a, mm	$\frac{za(R_1-R_2)}{\pi(R_1^2-R_2^2)}$
No. 1	92	1·5	0·46
No. 2	72	3·0	0·72
No. 3	52	4·0	0·70
No. 4	32	7·0	0·75

FIG. 17.
Shapes and dimensions of impellers.

12. Structural relationship between the shape of the impeller and the intensity of turbulence. Since the turbulence in the pumping passage is induced by an impeller, it is clear that the coefficient of true turbulence κ_0 is chiefly decided by the shape of the impeller. In order to make clear the influence quantitatively, the author tested the performance of the pump, shown in Figure 2, exchanging four different types of impellers. In Figure 17 the main dimensions of the impellers are shown, while Figure 18 illustrates various examples of the performances of the pump with these impellers. The chief factors, which influence the intensity of the turbulence in the pumping passage, appear to be the number of vanes, curvature of the bottom of the space between the vanes, and the distance between every two adjacent vanes. In this research

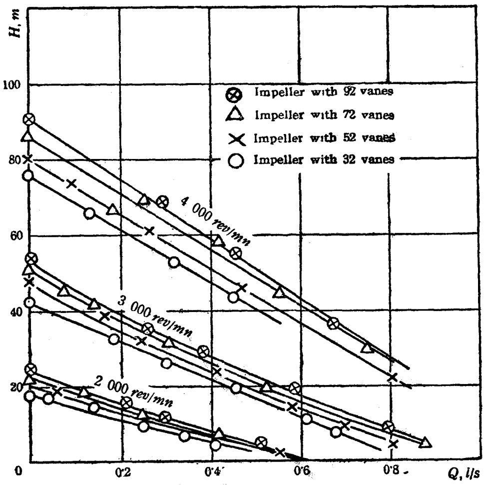


FIG. 18.
Effect of number of vanes on head-capacity characteristics.

the curvature is kept constant and the total sum of the spaces between vanes is kept approximately constant except in the case of 92 vanes. In this case if the total sum of the spaces was kept constant, the thickness of the vane would become too thin to manufacture. Since the vane has a proper thickness, the total area of the spaces is a little smaller than the other cases. A clearance of 0.35 mm between the impeller and the casing is maintained in each case. Although this value is too large to analyse the detail of the characteristics of the pump, the approximate characteristics of the pump at zero leakage will be estimated by the theory above mentioned.

Figure 18 illustrates various performances of the pump utilizing these impellers. If the coefficient of turbulence is properly assumed, the theoretical equation satisfies all of these performances. Although the head-capacity performance of the pump is influenced by the shape of the impeller, the efficiency-capacity performance of the pump remains unchanged. This fact illustrates that the theoretical equations mentioned above are applicable in these cases and niether of the values of the coefficients in the equation, except the coefficient of turbulence κ_0, is influenced by the shape of the impeller.

Figure 19 shows the relationship between the coefficient of turbulence and the number of vanes of the impeller. The ordinate shows the ratio of the coefficient of turbulence of the impeller in question to that of the impeller which has 92 vanes. The chain line shows the relationship between the number of vanes and the apparent turbulence, and the full line shows that between the number of vanes and the true turbulence. The broken line shows the relationship estimated from the experimental data carried out by Terada and Nanya who used a pump of very small clearance. Concerning the impellers with 60 and 72 vanes, their experience is nearly equal to that of the present author.

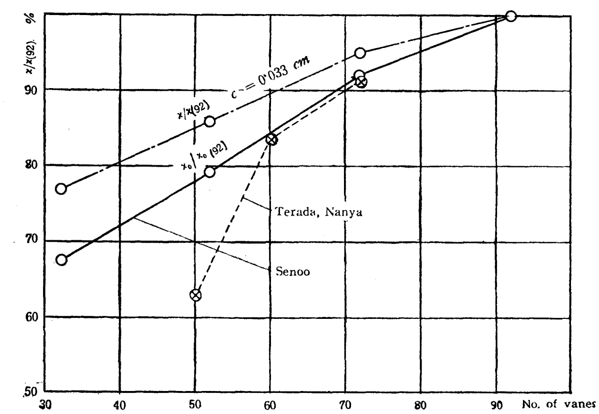


Fig. 19.
Effect of number of vanes on the coefficient of turbulence κ.

In their case, however, the coefficient of turbulence of the impeller with 50 vanes is much smaller, and the efficiency of the pump is very low. Needless to say it is clear that the relationship between the intensity of turbulence and the number of vanes shown by the full line will not be kept down to any small number of vanes. If the number of vanes is less than a certain value, the intensity of turbulence will be very weak, therefore the theory and most of the equations mentioned above will not be applicable. The author supposes that the critical number of vanes is less than 32 in the author's case and it is between 50 and 60 in Terada and Nanya's case. It is not clear why the critical value is different in these two cases; it is perhaps due to the fact that the critical value depends upon the cross-sectional area of the pumping passage, the total area of the spaces between the vanes and so on. Their paper does not report the detail of the shape and the form of the impeller. There is no other paper which reports the influence of the impeller. Therefore it is difficult to make the problem clear.

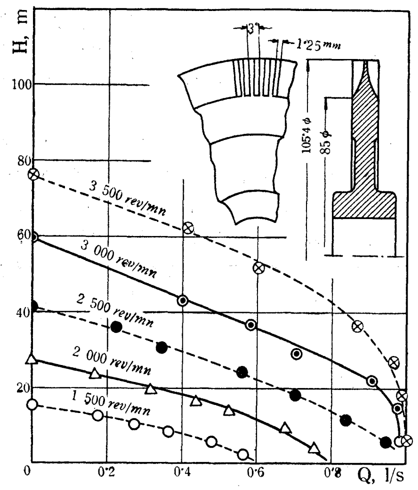


FIG. 20.
Head-capacity characteristics of the pump with 120 vanes' impeller: the space between the vanes is very small.

According to Figure 19, if the number of vanes increases and the total area of the spaces between the vanes remains constant, the intensity of turbulence increases linearly; on the other hand if the total area of the spaces diminishes, the intensity of turbulence diminishes. Accordingly an optimum number of vanes creates the highest intensity of turbulence in the pumping passage. In the case of an impeller of about 100 mm diameter, the optimum number of vanes is about 70 or 80 and the space between the adjacent vanes should be as large as possible.

Figure 20 illustrates the performance of the pump which has an impeller with 120 vanes. In this case, the space between the vanes is very small, because each vane has to have a certain thickness. Therefore, there is not large enongh quantity of liquid which flows through the space between the vanes and conveys the momentum from the impeller to the liquid in the pumping passage. In addition to this, since the motion of the liquid between two adjacent vanes is restricted by the frictional force on the surface of the vanes, the liquid is projected with a slow radial velocity. As the result, the momentum of the liquid will not arrive to the deepest part of the pumping passage, and the character of the turbulence will be different from that of the other cases. Because of this, the head-capacity performance of the pump may be convex instead of concave, and the highest attainable head is rather low. The detail will be explained later.

Mii carried out a series of experiments using many impellers of 80 mm diameter and made clear the influences of the shapes of the vanes and the impeller. Figure 21 illustrates the various shapes of the vanes used. The result of the experiments indicates that the performances of the pumps with the impellers shown in (A) and (B) are hardly different from each other.

But in the case of (C) the performance of the pump is worse than the other two cases; i.e. the coefficient of turbulence is about 70 % of those of the other two cases. In addition, the vanes are not connected to each other by a rim, so they are weak. Therefore, the vane should be thick enough. Consequently the number of vanes diminishes. As the result the coefficient of turbulence will further decrease.

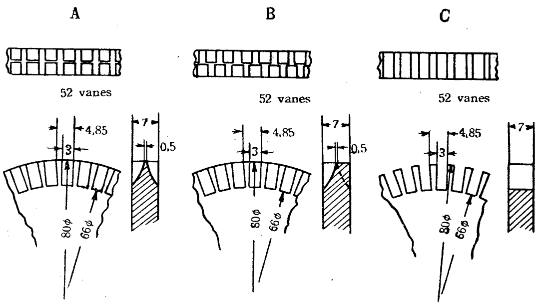


FIG. 21.
Three types of impellers.

He also carried out experiments using the impellers which did not have radial vanes but straight vanes of a 20° forward inclination. The performance of one of them is nearly equal to the performance which will be created by an impeller of radial vanes. But the performances of most of them are quite different from those of the pump with an impeller of radial vanes. Figure 22 illustrates various examples of these performances. In this figure the broken line indicates the performance of a pump with an impeller of radial vanes.

The author also conducted such kind of experiments using two impellers having the vanes with an inclination of 20° forward and backward respectively. According to his experimental data, the total heads created by these impellers are lower than that created by the radial vane impeller type at any rate of discharge. Generally speaking at a large rate of discharge the performance of an inclined vane impeller is hardly worse. In some cases it is better than that of a radial vane impeller. However, the total head at a small rate of discharge is greatly lower than that of a radial vane impeller.

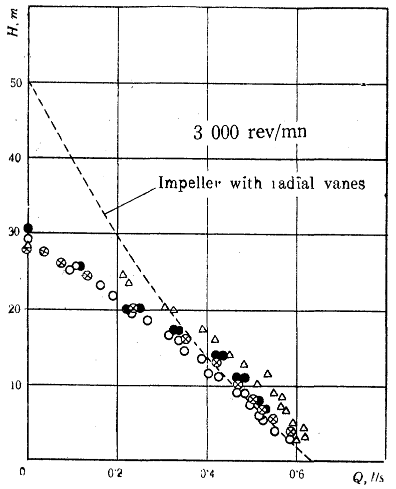


FIG. 22.
Head-capacity characteristics of pumps with impellers having straight vanes of 20° forward inclination.

The relationship between the driving power and the total head is nearly equal to that of the radial vane impeller. Since an inclined vane impeller can not create a high total head, the driving power does not increase extremely even when the discharge pipe line is closed by an unforeseen accident. This is a strong point of an inclined vane impeller. However, the efficiency of a pump of this type is lower than the type with a radial vane impeller at a high total head, because of its lesser discharge.

According to the theory which explains the condition of flow in the pumping passage, the turbulence in the pumping passage is induced by the momentum of liquid which is projected from the space between the vanes of the impeller. The intensity of turbulence, therefore, depends upon the quantity and the condition of the projected liquid. Needless to say, the best direction to project a volume of liquid from an inner part of a revolving disk is the radial direction in relation to the disk. Therefore, the intensity of turbulence is greater in the case of the radial vane impeller.

Figure 23 shows an example of the performance tests of a pump at two different speeds which were carried out by Iguchi. The performance of the pump at the lower speed is not strange, but the performance at the higher speed is quite different from that of an ordinary peripheral pump. In the pressure range lower than 34 m, the rate of discharge does not increase so effectively as usual due to the decrease of the total head. This appears to be due to cavitation, although the suction pressure is not explained by him. The three experimental data at 34 m, 43 m and 52 m show the normal performance of the pump which is expected comparing with the performance at 1450 rpm. However, the total heads does not increase effectively when the rate of discharge decreases furthermore. Consequently, the shape of the performance curve is convex. This kind of performance is often recognized in the performance of a pump which has a rather deep pumping passage.

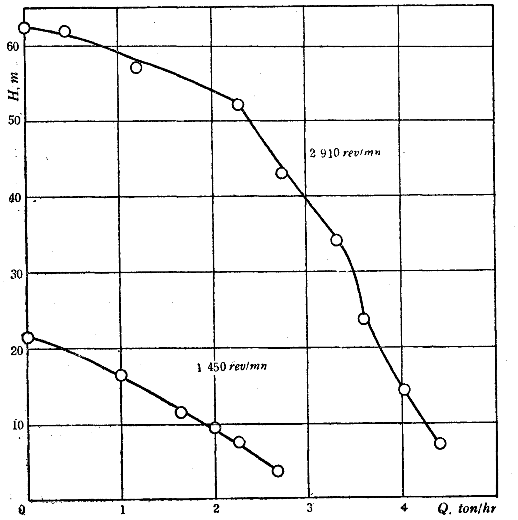


FIG. 23.
Head-capacity characteristics of a pump with a pumping passage of large cross-sectional area.

If the turbulence fully develops near the surface of the impeller but does not develop near the fixed wall, the shut off total head decreases and the maximum rate of discharge increases. This will be numerically shown in the following section. Accordingly, if the state of turbulence varies depending upon the total head, e.g. the turbulence induced by the impeller does not spread effectively to the fixed wall when the total head is high, but it spreads more effectively as the total head decreases, a convex performance curve will be obtained as shown by Figures 20 and 23. Conversely, although the vanes of the impeller are not radial, if the depth of the pumping passage is not very deep, the turbulence induced by the impeller spreads to the fixed wall. Accordingly, the performance of the pump looks quite like that of a usual peripheral pump with a radial vane impeller.

Although there is almost no practical meaning, the author tested the performance of a pump which has a simple disk instead of an impeller. Figure 24 shows the results. In this case since there is no mechanism which creates the turbulence mentioned in Section 3, the turbulence in the pumping passage can be compared to an ordinary turbulence in a pipe or a channel. If the turbulence is an ordinary one and there is no pressure gradient along the passage, the mean velocity of the liquid in the passage will be just one half of the peripheral velocity of the disk. This seems to be logical and this is verified theoretically by Miyazu and experimentally by Taylor and Wattendorf.[10]

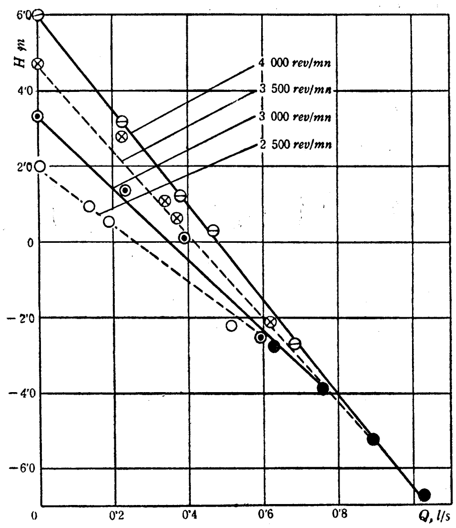


FIG. 24.
Head-capacity characteristics of a pump with a disk instead of an impeller.

In Figure 24 the black circles indicate the rate of discharge at which the mean velocity of the liquid is one half of the peripheral velocity of the disk. If the description mentioned above is accurate, there is no pressure gradient in the pumping passage; that is, the negative total head shown by these black circles should be equal to the pressure decrease at the entry of the pumping passage. The pressure decrease shown by these circles is 1.27 times the velocity head of the flow through the pumping passage. Considering the loss of head at the entry and the velocity head which is given to the liquid at the entry of the pumping passage, this pressure decrease is reasonable.

The shut off total head nearly increases in proportion to the square of the peripheral velocity of the disk. This shut off total head is about one sixteenth of that of the peripheral pump and is about fourteen times the shut off total head which is calculated under the assumption of laminar flow.

All of these show that the turbulence in this case is an ordinany turbulence, but the turbulence in a peripheral pump is a special turbulence induced by an impeller.

[10] Goldstein: Modern development in fluid dynamics. p. 385. Proc. Roy. Soc. A. 151 (1935) p. 494 & 157 (1936) p. 546.

13. The performance of a pump in which turbulence does not fully spread. As explained in the foregoing section, let us estimate the performance of a pump assuming that the turbulence induced by the impeller does not spread effectively to the fixed wall of the pumping passage.

In the theory mentioned in Section 3, the author assumed that the turbulent viscosity at the fixed wall is equal to the molecular viscosity of the liquid and the turbulent viscosity increases proportionally to the distance from the fixed wall. That is,

$$\tau = \rho \kappa y U(du/dy), \tag{30}$$

$$\mu = \rho \kappa \varepsilon U. \tag{31}$$

The result obtained is the following equation:

$$q = U\delta\left\{1 + \frac{\varepsilon}{\delta} - \frac{1}{\ln(1+\delta/\varepsilon)}\right\} - \frac{\delta^2}{\rho \kappa U}\frac{dp}{dx}\left\{\frac{1}{2} + \frac{\varepsilon}{\delta} - \frac{1}{\ln(1+\delta/\varepsilon)}\right\}. \tag{32}$$

Considering that the turbulence is not effective at the fixed wall, we will assume the following conditions: the turbulent viscosity at the surface of the impeller is the same as that assumed in Section 3; the viscosity at the fixed wall is the same as the molecular viscosity; the turbulent viscosity in the pumping passage varies proportionally to the square of the distance from the fixed wall instead of in proportion to the distance itself.

Using the same symbols as Section 3, these conditions are shown by the following equations:

$$\tau = \rho \kappa (y/\delta)^2 \delta U(du/dy), \tag{33}$$

$$\mu = \rho \kappa (\varepsilon/\delta)^2 \delta U. \tag{34}$$

There is the following relationship between the frictional force and the pressure gradient:

$$\tau = (y - \varepsilon)(dp/dx) + \tau_0. \tag{35}$$

From equations (33) and (35), the following relationship is induced:

$$\frac{du}{dy} = \frac{\delta}{\rho \kappa U}\left\{\left(\frac{1}{y} - \frac{\varepsilon}{y^2}\right)\frac{dp}{dx} + \frac{\tau_0}{y^2}\right\}.$$

By integrating this equation, it is changed into

$$u = \frac{\delta}{\rho \kappa U}\left\{\left(\ln y + \frac{\varepsilon}{y}\right)\frac{dp}{dx} - \frac{\tau_0}{y}\right\} + C_1.$$

The integration constant C_1 is decided by the boundary condition $u = 0$ at $y = \varepsilon$; and the velocity u is shown by the equation

$$u = \frac{\delta}{\rho \kappa U}\left\{\left(\ln \frac{y}{\varepsilon} + \frac{\varepsilon - y}{y}\right)\frac{dp}{dx} + \tau_0\left(\frac{1}{\varepsilon} - \frac{1}{y}\right)\right\}. \tag{36}$$

Applying another boundary condition $u = U$ at $y \doteq \varepsilon + \delta$ in this equation, the frictional force working on the fixed wall τ_0 is decided as follows:

$$\tau_0 = \left\{\frac{\rho\,\kappa\,U^2}{\delta} - \frac{dp}{dx}\left(\ln\frac{\varepsilon+\delta}{\varepsilon} - \frac{\delta}{\varepsilon+\delta}\right)\right\}\frac{\varepsilon(\varepsilon+\delta)}{\delta}. \tag{37}$$

The rate of flow per unit breadth q is obtained by integrating the velocity u from $y=\varepsilon$ to $y=\varepsilon+\delta$.

$$q = \int_{\varepsilon}^{\varepsilon+\delta} u\,dy = \frac{\delta}{\rho\,\kappa\,U}\left\{(y\ \ln y - y - y\ \ln\varepsilon + \varepsilon\ \ln y - y)\frac{dp}{dx} + \tau_0\left(-\ln y + \frac{y}{\varepsilon}\right)\right\}$$

$$= U\delta\frac{\varepsilon}{\delta}\left(1+\frac{\varepsilon}{\delta}\right)\left(\frac{\varepsilon}{\delta} - \ln\frac{\varepsilon+\delta}{\delta}\right) + \frac{\delta^2}{\rho\,\kappa\,U}\frac{dp}{dx}\left[\left(1+\frac{2\,\varepsilon}{\delta}\right)\ln\frac{\varepsilon+\delta}{\varepsilon} - 2 - \frac{\varepsilon}{\delta}\left(1+\frac{\varepsilon}{\delta}\right)\right.$$
$$\left.\times\left\{\left(\ln\frac{\varepsilon+\delta}{\varepsilon}\right)^2 - \left(\frac{\delta}{\varepsilon+\delta}+\frac{\delta}{\varepsilon}\right)\ln\frac{\varepsilon+\delta}{\varepsilon} + \frac{\delta^2}{\varepsilon(\varepsilon+\delta)}\right\}\right]. \tag{38}$$

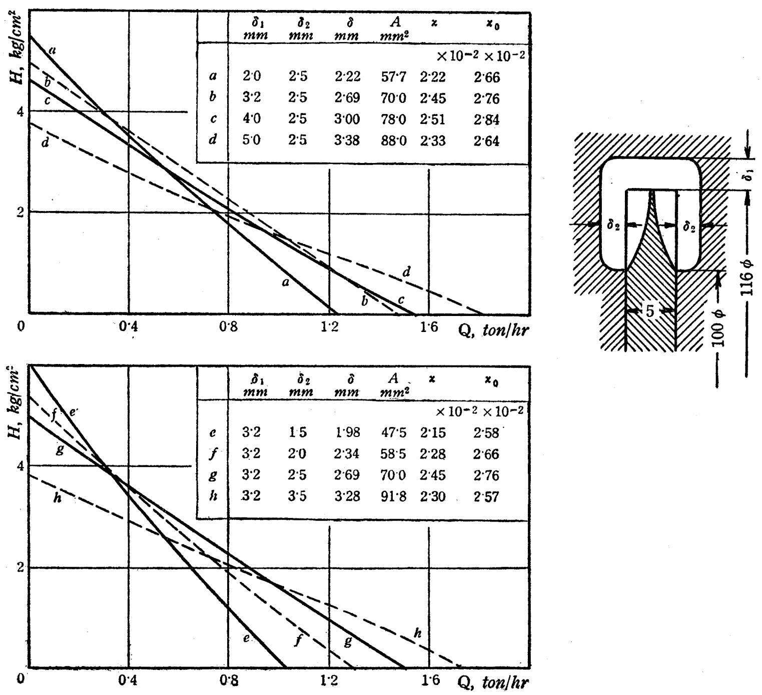
The first term of equation (38) corresponds to the first term of equation (32), and it shows the rate of flow at zero pressure gradient. The value of the first term of equation (32) is about 0.85 $U\delta$ for an ordinary peripheral pump. On the other hand the first term of equation (38) is about 0.93 $U\delta$. The second term of equation (32) is about $0.35\frac{\delta^2}{\rho\,\kappa\,U}\frac{dp}{dx}$, and that of equation (38) is about $0.67\frac{\delta^2}{\rho\,\kappa\,U}\frac{dp}{dx}$: this term shows the rate of reverse flow. Accordingly, in the case of equation (38) the maximum rate of flow is about 10 % larger than that of equation (32), and the rate of reverse flow due to the pressure gradient is about twice as large as that of equation (32). Thus, the shut off total head of equation (38) is only one half of that of equation (32).

If the turbulent viscosity is proportional to y^n where $1<n<2$, the maximum rate of flow and the shut off total head are respectively between these two cases. Accordingly, if the turbulence does not spread effectively to the interior of the pumping passage when the pressure gradient increases, i.e. if the value of n increases together with the pressure gradient, the performance curve will be convex as Figures 20, 22 and 23 show.

14. The influences of the dimension and the shape of the pumping passage on the coefficient of turbulence. There are several papers which have treated the influences of the dimension and the shape of the pumping passage experimentally. The theoretical equations introduced in this paper show the influences of the cross-sectional area A and the mean depth δ of the pumping passage. However it is a question whether these equations show the influences of these factors completely. Even if the equation itself is quite correct, it may be possible that the experimental performance of the pump does not coincide with what will be expected by the equation, because the coefficient of turbulence κ_0 itself may also be influenced by these factors.

If a volume of liquid is projected out of the spaces between the vanes into the pumping passage due to revolution of the impeller and it induces a secondary flow, the shape and the dimension of the pumping passage will have some influence on the secondary flow; that is, the intensity of turbulence κ_0 may be influenced by these factors. On the other hand, if the momentum of the impeller is conveyed to the adjacent liquid in the pumping passage and the conveyed momentum is further transfered to the interior liquid and thus the momentum of the impeller spreads throughout the passage, the shape and the dimension will hardly influence the intensity of turbulence. However, even in this case there may be a limit of δ under which the theory is applicable.

In the foregoing section some examples were shown, which did not obey the theory especially when the depth of the pumping passage was large. They suggest that there is a limit of depth δ. At the same time, it is doubtful whether the theory is accurately applicable when the value of δ is smaller than the limit. Using the experimental data published by some researchers, the author made clear the influences of these factors on the coefficient of turbulence κ.



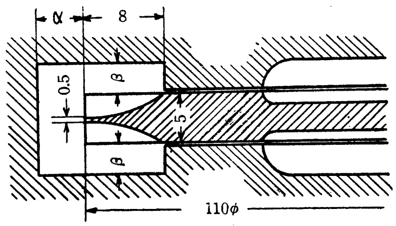
	δ_1 mm	δ_2 mm	δ mm	A mm²	κ $\times 10^{-2}$	κ_0 $\times 10^{-2}$
a	2·0	2·5	2·22	57·7	2·22	2·66
b	3·2	2·5	2·69	70·0	2·45	2·76
c	4·0	2·5	3·00	78·0	2·51	2·84
d	5·0	2·5	3·38	88·0	2·33	2·64

	δ_1 mm	δ_2 mm	δ mm	A mm²	κ $\times 10^{-2}$	κ_0 $\times 10^{-2}$
e	3·2	1·5	1·98	47·5	2·15	2·58
f	3·2	2·0	2·34	58·5	2·28	2·66
g	3·2	2·5	2·69	70·0	2·45	2·76
h	3·2	3·5	3·28	91·8	2·30	2·57

FIG. 25.
Effect of the shape and the dimensions of the pumping passage on the coefficient of turbulence κ.

In order to generalize the result obtained, the coefficient of true turbulence κ_0 should be considered instead of the apparent turbulence κ. Because, if the clearance for the internal leakage is kept constant and the cross-sectional area is increased, the influence of the internal leakage will become less important and the coefficient of apparent turbulence will increase.

Figure 25 shows the result of a series of experiments carried out by Fuziwara.[11] The values of κ and κ_0 were calculated by the present author: in this case the clearance for the internal leakage was assumed 0.03 mm according to Fujiwara's description. This table shows that there is a certain relationship between κ_0 and δ, and that κ_0 attains the maximum value at about $\delta = 3.00$ mm. However in the range of $\delta = 1.98 \sim 3.38$ mm, the minimum value of κ_0 is about 93 % of the maximum value. Accordingly, unless the experiment is carried out carefully and accurately, the tendency will be covered by the error of experiment.



1800 rpm

	α mm	β mm	δ mm	A mm²	D mm	H_{max} m	κ	Difference from the mean value %
No. 1	3	2	2.44	57	107.0	68	2.19×10^{-2}	+5
No. 2	5	2	3.68	77	109.6	46	2.18	+5
No. 3	7	2	5.18	95	102.0	33	2.05	−1
No. 4	3	3	3.02	81	106.7	46	1.91	−8
No. 5	5	3	4.02	103	109.0	35	1.83	−12
No. 6	7	3	5.36	125	111.2	33	2.18	+5
No. 7	3	5	4.28	125	106.0	38	2.29	+10
No. 8	5	5	5.02	155	108.2	30	2.00	−4
No. 9	7	5	6.10	185	110.7	28	2.10	+1

FIG. 26.
Effect of the shape and the dimensions of the pumping passage on the coefficient of turbulence κ.

Figure 26 shows another example of such experiments which was carried out by Terada and Nanya.[12] Unfortunately they did not describe

[11] Fujiwara: Hitachi Hyoron, Vol. 30, No. 3 (1948) p. 104.
[12] Terada and Nanya: Hitachi Hyoron, Vol. 25, No. 5 (1942) p. 268.

the dimension of the clearance for the internal leakage. They said that they tried to keep the clearance as small as possible, but they found that the clearance was not kept constant throughout their experiments. The maximum value of the coefficient of apparent turbulence is 0.022 and the minimum value is 0.019, and any definite tendency is not recognized between the coefficient of turbulence κ and the depth of the pumping passage δ.

As these data show, the influence of the cross-sectional area and the depth of the pumping passage on the intensity of turbulence is not important compared with the influence of the number of vanes of the impeller.

15. Influence of the molecular viscosity of liquid. Since all of the experiments were carried out using water, it is not clear whether the theory is applicable to a viscous liquid. Although the theory has been constructed considering the influence of the laminar sub-lager due to the molecular viscosity, the intensity of turbulence itself may also be influenced by the molecular viscosity. Especially if the Reynolds number is smaller than a certain value, the viscosity will decay the turbulence in the pumping passage and the state of flow will be quite different from that at a larger Reynolds number. This problem is not only theoretically interesting but also important, practically speaking, because a peripheral pump has come to be used for pumping oil of various viscosities. Some catalogs of a peripheral pump describe that the performance of a pump is very poor at a viscosity of higher than 500 SSU.

The author carried out a series of experiments using a oil at different viscosities. In this case different viscosities were realized by changing the temperature. Since the liquid is viscous, the laminar disk-frictional

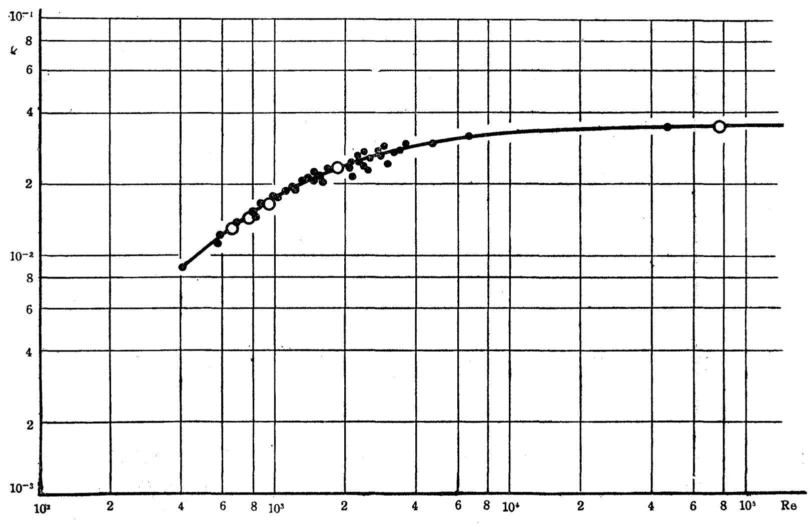


FIG. 27.
Effect of Reynolds number $U\delta/\nu$ on the coefficient of turbulence κ.

force at the clearances is very large, but the internal leakage is not important any more. The detail of the experiment will be explained in another paper; this report is confined to the influence of the viscosity on the intensity of turbulence.

He changed the viscosity of the oil from 0.653 to 0.156 cm^2/sec by changing the temperature from 15° C to 40° C. The speed of revolution was changed from 1500 to 3500 rpm. Consequently, Reynolds number $U\delta/\nu$ varied from 400 to 7000. He tested also the performance of the pump using water. In this case, Reynolds number was about 45000. Applying the experimental data to the theoretical equation, the author estimated the value of the coefficient of turbulence κ. Reynolds number seems to have a very important influence on the intensity of turbulence: the relationship between κ and $U\delta/\nu$ is shown in Figure 27. Most of the values calculated from the experimental data are on a smooth curve which illustrates that a decrease in $U\delta/\nu$ is accompanied with a reduction of κ. This seems to be reasonable because the turbulence will be reduced by the viscosity of liquid.

A series of experimental data was shown by Abramson who had tested a peripheral pump using water and four kinds of oil. The viscosities of these oils were 410 SSU (0.85 cm^2/s), 604 SSU (1.26 cm^2/s), 900 SSU (1.90 cm^2/s) and 1074 SSU (2.20 cm^2/s). Although the detail of the dimensions of the pump was not described, they were estimated inversely so that the experimental data using water satisfied the theoretical characteristic equations. The viscosity is rather high but the Reynolds number is not very small because of the large dimension of the pump. The relationship between κ and $U\delta/\nu$ calculated from these data is shown in Figure 27 by white circles which coincide with those of the author's experiments.

The author received some experimental data from Fujiwara, who studied the influence of the viscosity on the performance of a pump changing the temperature of water from 19 to 83°C. According to his experimental data, the shut off total head increases slightly as the viscosity increases. This tendency appears to be contradictory to the tendency mentioned above. However, it is reasonable. That is, in his experiments, the largest viscosity is about thrice as large as the smallest viscosity, but Figure 27 shows that the Reynolds numbers of his experiments are too large to change the value of κ. Additionally according to the theoretical characteristic equations, if the coefficient of turbulence κ remains constant, the shut off total head increases due to an increase of the molecular viscosity. Table 2 shows the comparison of the theoretical and experimental data. Since the experimental shut off total head does not vary smoothly as the temperature of the oil changes from 39° C to 45° C, the difference between the theoretical and the experimental values also has a discontinuous point. The author suspects that this is due to a slight change of the condition of the pump or the apparatus. If it is so, the theoretical values coincide with the experimental values fairly closely in the all range of the experiment.

The variation of the maximum discharge due to the change of the viscosity is shown theoretically by $\ln(1+\delta/\varepsilon)$, which has the same tendency as

TABLE 2.
Effect of the temperature of water on the shut off total head.

Water			Shut off total head		
Temperature °C	Specific gravity g/cm³	Kinematic viscosity cm²/s	Theoretical kg/cm²	Experimental kg/cm²	Difference kg/cm²
19	0.998	10.39×10^{-3}	4.70	4.70	0
29	0.996	8.24	4.65	4.65	0
39	0.993	6.76	4.60	4.60	0
45	0.990	6.11	4.58	4.48	0.10
59	0.984	4.84	4.52	4.40	0.12
69	0.978	4.21	4.46	4.35	0·11
83	0.970	3.56	4.41	4.30	0.11

the experimental data but does not coincide with these quantitatively. The maximum discharge is remarkably influenced by the pressure decrease at the entry of the pumping passage. Therefore, the quantitative discrepancy seems to be due to an increase of the pressure decrease, which is caused by an increase of the viscosity.

16. Efficiency of a pump. Since the total head decreases approximately in proportion to the increase of the discharge, the water horse-power attains the maximum value at one half of the maximum discharge. On the other hand the input horse-power decreases as the discharge increases, because the torque which is required to drive the pump increases in proportion to the total head. Accordingly the efficiency of a pump, which is the ratio of the water horse-power to the input horse power, attains the maximum value at a discharge larger than one half of the maximum discharge. Figure 28 shows some examples of the relationships between the efficiency and the discharge. As this figure shows, the efficiency of a pump is high only at a limited range of discharge. When the discharge of this pump is larger than 0.8 l/s, the efficiency decreases discontinuously because of cavitation which is illustrated in Figure 6.

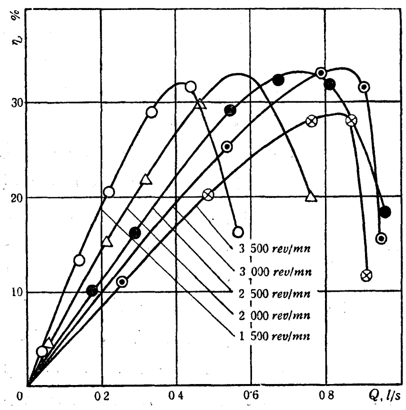


Fig. 28.
Efficiency of a pump at various speeds.

The efficiency of a pump is easily estimated, if the relationship between the discharge, the head, the torque and the speed of revolutions are clarified. Since the relationship between the head, the discharge and the speed of revolution and the relationship between the torque, the head and the speed of revolution have been theoretically clarified, the relationship between the efficiency and these factors will be obtained, and it will have the same reliability as the other relationships have.

Using this relationship we shall be able to find some factors which will influence the efficiency of a pump. Especially it is very interesting to estimate the maximum attainable efficiency and the minimum Reynolds number which is practically available.

In order to simplify the equation, the following new symbols are introduced:

$\theta = \varepsilon/\delta$,
$\alpha = 1/\ln\{1+(1/\theta)\}$,
$Q_0 = AU(1+\theta-\alpha)$, discharge at zero pressure gradient,
$q = Q/AU$, rate of discharge,
$q_0 = Q_0/AU = (1+\theta-\alpha)$, maximum rate of discharge.

Using equations (14′) and (29), the efficiency of a pump η is shown by the following equation:

$$\eta = \rho g Q H/\omega T$$
$$= \frac{\rho g\{(L-L')\kappa U(q_0-q)/(0.5+\theta-\alpha)-gh\delta/U\}}{\dfrac{2\delta T_m}{DA}+\xi L\rho\kappa_0\alpha U^2+0.5\delta\lambda\psi\rho U^2 q_0+\dfrac{\psi\rho(L-L')\kappa U^2 q_0(q_0-q)}{0.5+\theta-\alpha}-\psi\rho g\delta q_0 h}. \tag{39}$$

Both the second and the third term of the denominator are proportional to the square of the peripheral velocity and do not depend upon the rate of flow q. Thus, the third term may be included in the second term, if a new symbol ξ_1 is adopted instead of ξ in the second term; where ξ_1 is

$$\xi_1 = \xi + \{\delta\lambda\psi(1+\theta-\alpha)/2L\kappa_0\alpha\}. \tag{40}$$

The second term of equation (40) is decided by the dimensions and the shape of a pump, and the value is about one or one and a half.

Equation (39) shows that η is a function of the rate of discharge q. The maximum value of η is obtained when a rate of discharge q satisfies the equation $d\eta/dq = 0$. Equations (15) and (16) illustrate that L' and h vary with the rate of discharge q. Accordingly it is very difficult to find the rate of discharge q which satisfies the equation $d\eta/dq = 0$.

In this paper, for the sake of simplicity, the last term of the denominator in equation (39) is disregarded, because it is very small compared with other terms. If it is necessary, the influence of the disregarded term in the denominator will be recovered by assuming properly smaller values than their true values for T_m and ξ_1 in the denominator. Additionally the author assumed that $H/H+h$ and L' are independent of the rate of discharge q.

Using these assumptions, the rate of discharge which satisfies the equation $d\eta/dq=0$ is easily estimated. This rate of discharge is different from that which accurately satisfies the equation $d\eta/dq=0$ without any assumption. However, there is a certain relationship between these two kinds of rate of discharge. The latter is almost 0.87 time the former. Thus, the correct maximum efficiency will be calculated applying the corrected rate of discharge to equation (39).

Although the rate of discharge, which satisfies the equation $d\eta/dq=0$ under the assumptions above mentioned, does not accurately satisfy the equation $d\eta/dq=0$, the value of $d\eta/dq$ at that rate of discharge is very small. Consequently, even if the rate of discharge under the assumption is adopted for the calculation of the efficiency, the result obtained will hardly be different from the true maximum efficiency. The difference was numerically estimated and verified that it was very small. Accordingly, for the sake of convenience, the maximum efficiency will be discussed under the assumption above mentioned.

As explained before, the rate of discharge at the maximum efficiency $q(\eta_{max})$ is the value of q which satisfies the equation $d\eta/dq=0$. The result of the calculation is

$$q(\eta_{max}) = q_0 + \beta - \sqrt{\beta^2 + q_0\beta}\,, \tag{41}$$

where

$$\beta = \frac{0.5+\theta-\alpha}{1+\theta-\alpha}\,\frac{L}{L-L'}\left(\xi_1\,\alpha\,\frac{\kappa_0}{\kappa} + \frac{2\,T_m\,\delta}{D\,A\,\rho\,\kappa\,L\,U^2}\right). \tag{42}$$

Introducing a new parameter

$$\gamma = \frac{\beta}{1+\theta-\alpha} = \frac{0.5+\theta-\alpha}{(1+\theta-\alpha)^2}\,\frac{L}{L-L'}\left(\xi_1\,\alpha\,\frac{\kappa_0}{\kappa} + \frac{2\,T_m\,\delta}{D\,A\,\rho\,\kappa\,L\,U^2}\right), \tag{43}$$

the maximum efficiency is shown by the following equation:

$$\eta_{max} = (H/H+h)(1+2\gamma - 2\sqrt{\gamma^2+\gamma})\,. \tag{44}$$

If the second term in the parentheses of equation (43) is included in the first term by reevaluating the value of ξ_1, the maximum efficiency of a pump is decided by $H/H+h$, $L/L-L'$, ξ_1, κ_0/κ and $\kappa\,Re$, in which Re is Reynolds number $Re=U\delta/\nu$: because, in the above equation α is a function of θ alone, and $1/\theta = U\delta\,\rho\,\kappa_0/\mu = \kappa_0\,Re$. Figure 29 shows some of these relationships among η_{max}, $\kappa\,Re$ and ξ_1 assuming $L/L-L'=1$ and $H/H+h=1$. Since $H/H+h<1$, as a matter of fact, the maximum efficiency decreases in proportion to $H/H+h$, as equation (44) shows. The term $L/(L-L')$ has the same influence upon γ and η as ξ_1 has, if the influence of T_m is included in ξ_1. Accordingly we may disregard the term $L/L-L'$ if we assume a new symbol $\xi_1'=\xi_1(L/L-L')$ instead of ξ_1.

The value shown by the line $\xi_1=1$ is not attainable, unless the mechanical friction and the disk-friction are zero as well as the pressure decrease at the entry of the pumping passage and the loss of head at the exit are excluded.

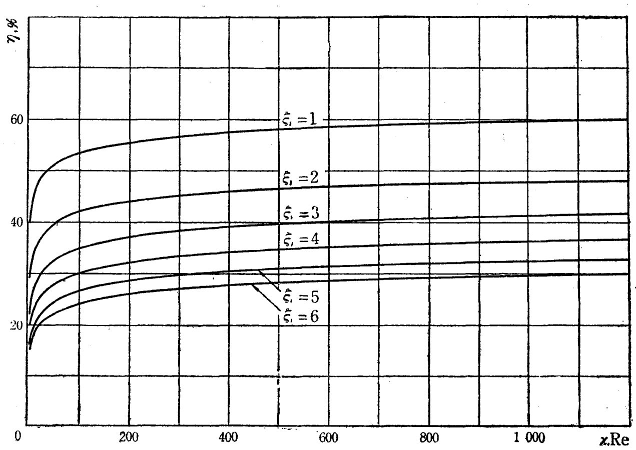


FIG. 29.
Effect of $\kappa_0 U \delta / \nu$ and ξ_1 on efficiency.

If the velocity head is recovered to pressure head at the exit of the pumping passage, the value of λ in equation (40) or the value of ξ_1 decreases. Concerning conventional pumps, λ is not small; consequently, the value of ξ_1 is large: this is one of the chief causes of low efficiency. Such being the case, it is hardly possible to attain a efficiency which is higher than the line $\xi_1 = 2$. Unless some proper shape is adopted for the entry of the pumping passage, $H/H+h$ is about 0.95 and $L/L-L'$ is about $1.3 \sim 1.5$ due to the influence of the entry. Accordingly it will be very difficult to attain a efficiency which is higher than the line $\xi_1 = 3.0$, even if there is no internal leakage at all in the pump.

The internal leakage makes the coefficient of apparent turbulent viscosity κ smaller than that of true turbulent viscosity κ_0. Due to the internal leakage, β or γ increases, as equation (43) shows, as if ξ_1 changes to $\xi_1 \kappa_0/\kappa$. In addition to this, since the head H decreases due to the internal leakage, the ratio $H/H+h$ decreases. This also indirectly becomes a small part of the cause of decreasing the efficiency. This being the case, occasionally the value of ξ_1 of a conventional pump is about five or six.

The above consideration shows that the chief causes which diminish the efficiency of a pump are; (1) loss of head at the exit of the pumping passage, (2) loss of head at the entry of the pumping passage, (3) pressure decrease at the entry of the pumping passage, (4) ineffective length at the entry of the pumping passage, (5) interal leakage. Most of these have the same influence on the efficiency as the increase of ξ_1 has. In order to make a peripheral pump of a good efficiency, it is necessary to design proper shapes for the entry and the exit of the pumping passage as well as to make the clearances between the casing and the impeller small.

Figure 29 illustrates two important facts: one, the maximum efficiency of a pump is described as a function of the product of Reynolds number and the coefficient of turbulence κRe; the other, the maximum efficiency of a pump is very small if the product is less than about twenty.

In many cases the diameter of a peripheral pump is about 100 mm, the mean depth of the pumping passage is 2 to 5 mm, and a pump is used for pumping water; thus the value of κRe is between 300 and 3000. Consequently the maximum efficiency does not vary remarkably due to the change of κRe. When a peripheral pump is used for oil, however, since oil is much more viscous than water, the value of κRe sometimes becomes smaller than twenty. As explained before, some catalogs describe that the highest viscosity which can be handled by this type of pump is about 400 or 500 SSU (0.87 or 1.10 cm^2/s). One of the reason for the foregoing statement is that the intensity of turbulence becomes too small because of the small value of Reynolds number and another reason is the poor efficiency of the pump which was previously explained. The only way to use a peripheral pump for handling a viscous liquid is to make Reynolds number as large as possible by adopting a high peripheral velocity and a large value of the depth of the pumping passage.

17. Cavitation. When a peripheral pump is working at a normal condition, the discharge increases as the total head diminishes. However, in some cases especially when the number of revolutions per minute is large, the discharge does not increase even when the total head diminishes. This seems to be due to cavitation which grows at a low pressure region. As explained before, since pressure decreases at the entry of the pumping passage, if the pressure becomes the vapor pressure of the liquid at that temperature, the liquid begins to vaporize.

In Figure 4 the curves of $Q = 0.435$ l/s and of $Q = 0.437$ l/s have nearly equal pressure gradient at the end of the pumping passage. This relationship between discharge and pressure gradient is expected from the theoretical equation (11). However, since the discharge head is very low in the case of $Q = 0.437$ l/s, the effective length, at which the pressure increases, is very short; i.e. pressure gradient is not recognized at the remaining part of the pumping passage where the pressure is about -10 m water column.

Since the rate of flow is constant or at least nearly constant at any section of the pumping passage, the pressure gradient should be constant only if the impeller effectively works in the pumping passage. Accordingly, the fact that there is no pressure gradient at a certain part of the pumping passage means that the impeller does not effectively work there. Since the pressure at that point is nearly equal to the vapor pressure, there must be many bubbles in the liquid which obstruct the transfer of the momentum from the impeller to the liquid.

When the liquid flows into the pumping passage the liquid is accelerated by the negative pressure gradient at the entry; that is, the minimum pressure should be lower than the suction pressure. The difference is equal to the sum of the velocity head and the loss of head at the entry. Since the minimum pressure is limited by the vapor pressure, the maximum rate of flow into the pumping passage is decided by the suction pressure. When discharge pressure decreases and the discharge increases to a certain value, the minimum pressure at the entry of the pumping passage goes down to the vapor pressure. Since the minimum pressure can not decrease anymore, the rate of flow which decides the pressure gradient in the effective length does not increase, even if the discharge pressure further decreases.

Consequently the effetive length diminishes and in the remaining part of the pumping passage the liquid flows together with the bubbles of vapor without increasing the pressure.

Such being the case, if the decrease of the discharge pressure is not accompanied by the increase of the discharge as usual, we may think that cavitation has grown in the pumping passage. In this case the difference between the suction pressure and the vapor pressure at that temperature appears to be equal to the pressure decrease at the entry of the pumping passage.

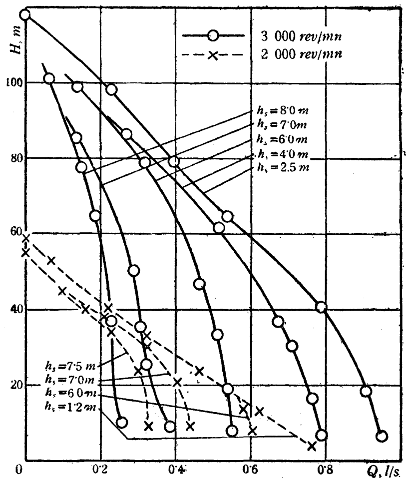


Fig. 30.
Head-capacity characteristics at condition of cavitation.

Figure 30 shows the head-capacity performances at various suction pressures which are materialized by obstructing the flow in the suction pipe with a sluice valve. These experimental data show that at a certain suction pressure, as the number of revolutions per minute decreases, the maximum discharge increases, although the difference is not great. Many other experimental data show, however, that the maximum discharge at a certain suction head is not influenced by the number of revolutions per minute.

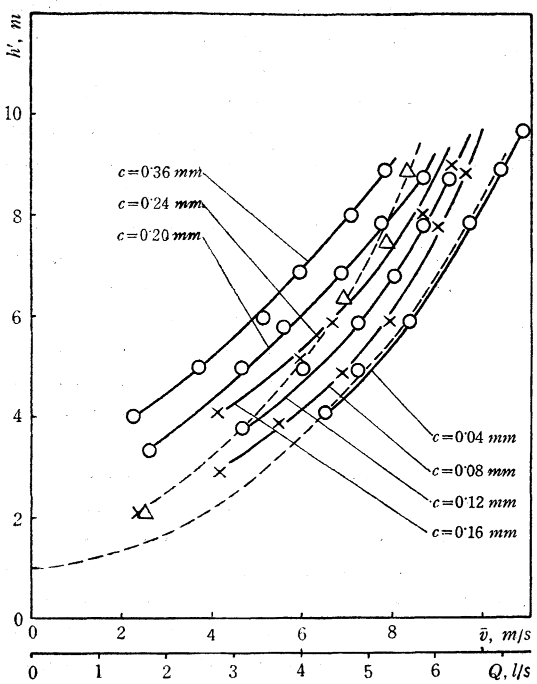


Fig. 31.
Effect of clearance on the pressure decrease at the entry of the pumping passage.

Since the pressure decrease at the entry of the pumping passage is decided by the rate of flow at that point, the critical discharge will be small when the internal leakage is large. When there is internal leakage, the rate of flow at the entry of the pumping passage equals the sum of the discharge and a part of the internal leakage which adds to the main flow before it flows into the entry.

That part of the internal leakage consists of two parts: one, the internal leakage through the partion between the discharge nozzle and the suction nozzle, the other, a part of the leakage which comes from the central chamber. Accordingly the critical discharge diminishes due to the leakage which is approximately one half to two thirds of the total internal leakage of the pump.

In order to verify this, a series of experiments is carried out by changing the speed of revolution and the dimensions of the clearance. The pressure decrease in the pumping passage is estimated by the following equation, and the relationship between the decrease h' and the discharge is shown in Figure 31. In this figure the clearance c is the parameter.

$$h' = H_a - H_v - h_s,$$

where h' : pressure decrease in the pumping passage in m Aq.,
H_a: atmospheric pressure in m Aq.,
H_v: vapor pressure of the liquid at that temperature in m Aq.,
h_s : absolute static pressure just before the entry in m Aq.

Since the state of cavitation is not very stable, the experimental data scatter. However, it is clearly seen that the discharge diminishes as the clearance increases at a constant suction head, and the difference is about 1/2 to 2/3 of the equivalent leakage calculated before. This coincides with the fact that the sum of the amount of the leakage through the partition and a part of the leakage, which comes from the central chamber of the pump to the entry of the pumping passage, is about 1/2 to 2/3 of the equivalent leakage.

The experimental data of $c = 0.04$ mm coincide with the broken line which indicates that pressure decrease h' varies in proportion to the square of the rate of flow or the discharge Q. This line shows that the pressure decrease at $Q = 0$ is not zero but a certain amount of about 1.0 m Aq. It was verified in Section 9 that there is little internal leakage in the case $c = 0.04$ mm. Therefore the pressure decrease at $Q = 0$ does not mean that which is due to the internal leakage, but means the pressure decrease which is not influenced by the flow.

Since the impeller revolves even when there is no discharge, the liquid flows partly forward and partly backward; thus there is a pressure difference along a radius due to the centrifugal force. Since the suction nozzle is connected with the periphery of the pumping passage, the suction pressure is higher than the lowest pressure at the smallest radius. If bubbles are created at the root of the vanes where the pressure is at its lowest ebb, the bubbles obstruct the exchange of the momentum between the impeller and the liquid in the pumping passage. Accordingly the condition of cavitation is decided by the minimum pressure in the pumping passage.

Figure 32 shows the relationships between the mean velocity in the pumping passage and the pressure decrease which are calculated from the cavitation tests of pumps with different shaped pumping passages. Each of these relationships is shown by the equation

$$h' = h_0 + k\,(v^2/2\,g)\,,$$

where h_0 is the pressure difference along the radius due to the centrifugal force and is about 1 m Aq. in these experiments. The coefficient k is about 1.47 to 1.92 in these cases. Needless to say the value of k is chiefly decided by the shape of the pumping passage entrance, but these experimental data show that the value is also influenced by the intensity of turbulence or the value of κ_0. That is, four impellers, each with a different number of vanes, are inserted in a pump casing one after another while the cavitation test is carried out. The result shows that the value of k is largest at the minimum value of κ_0 or that impeller with the minimum number of vanes.

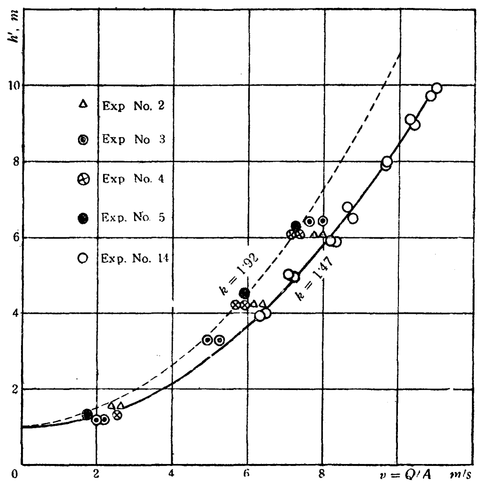


Fig. 32.
Pressure decrease at the entry of the pumping passage of various pumps.

The value of the coefficient j, which showed the relationship between the discharge and the pressure decrease at the entry, was estimated in Section 7 from the performance of the pump. For each pump, the value of j is smaller than the value of k. For example, concerning the pump shown in Figure 1, the value of j is 1.20 as shown in Figure 7 while the value of k is 1.47.

It is recognized experimentally that the amount of pressure decrease at the entry of the pumping passage becomes large, in spite of a constant discharge, when cavitation is created by diminishing the suction pressure. Figure 33 shows the pressure distribution in the pumping passage just before and after cavitation. Although the suction pressures differ only slightly from each other, the minimum pressures created in the pumping passage are different by 1.0 m Aq. This seems to be the cause of the difference between the values of j and k.

It is not clarified yet why the amount of pressure decrease becomes larger when cavitation occurs. When cavitation occurs the liquid must flow into the pumping passage together with bubbles, the author supposes, therefore the mean velocity is larger than that at a normal condition, although the mean density of the mixture is smaller. The velocity head of the mixture is proportional to the square of the velocity and to the density of the mixture. Accordingly, if the quantity of liquid is kept

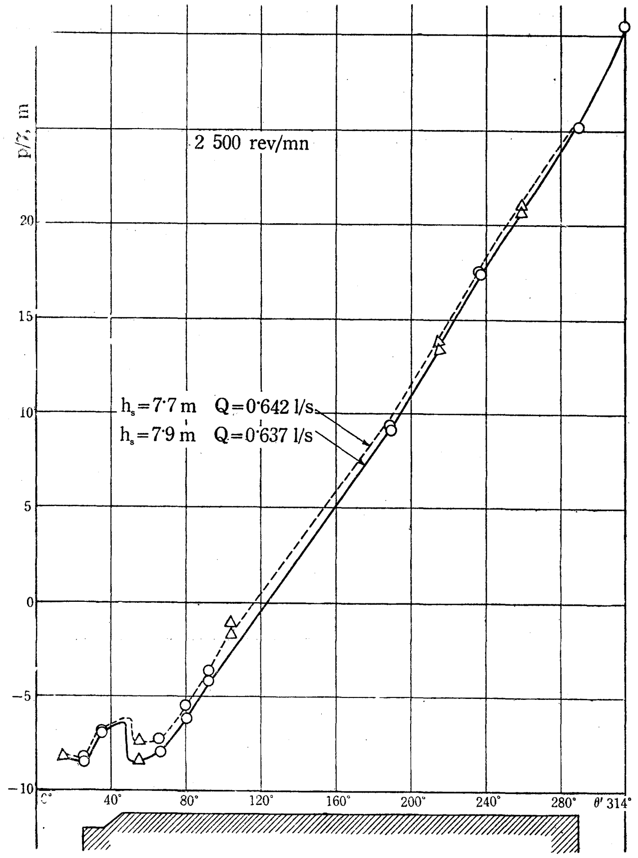


FIG. 33.
Pressure distributions along the pumping passage just before and after cavitation.

constant, the velocity head of the mixture is larger than that of the liquid alone; i.e. the pressure decrease at the entry of the pumping passage will increase due to cavitation.

The performance lines in Figure 30 show that, even in the case of cavitation, an increase of the total head is accompanied by a slight decrease of the discharge. It appears to be due to internal leakage, since internal leakage becomes considerably large as the total head increases.

Figure 30 illustrates also that the performance at rather high suction head is worse than that at an ordinary condition, even if the discharge is less than the critical value for cavitaion. This may be due to the extra decrease of pressure at the entry of the pumping passage. That is, when the suction head is high, the dissolved air in the liquid resolves itself into air, and the mixture of liquid and air increases its volume. This mixture will have a similar influence on the decrease of pressure at the entry as does the mixture of cavitation. Accordingly the total head may be lower than that at normal condition even if the discharge is less than the critical discharge mentioned above, providing the suction head remains high.

In some cases, even if the suction head is not high, the performance of a peripheral pump does not coincide with that at normal condition. This is chiefly due to the air contained in the liquid or the air which leaks into the pump through the stuffing box. If the surface of the liquid in the basin is disturbed by the supplied liquid or vortices, air may be mixed with the liquid and flows into the pump. If the total head is low, the pressure at the stuffing box is apt to become vacuum, which sucks air into the pump through the stuffing box. Accordingly the liquid is mixed with air before it flows into the pumping passage. In general, if the liquid contains air, the volume of the air increases at the entry of the pump because of the low pressure, and the air obstructs the pumping action of the impeller. In this case the performance of the pump is similar to that at cavitation explained before, but this performance is easily improved by preventing air from mixing with the liquid.

18. Conclusion. In order to clarify the mechanism of the pumping action and the performances of a peripheral pump, the author studied them both theoretically and experimentally. For analysing the pumping action, he constructed the theoretical equations assuming a proper turbulence in the pumping passage. He then compared the equations with the experimental data and studied the difference. By investigating the difference he found that the cause of the difference is the influence of the entry. Correcting the theoretical equations by a consideration about the influence of the entry, he established the equations which show the performances of the pump. These equations (14) and (26) are applicable for many cases.

He studied internal leakage and theoretically estimated the influence of leakage on the performance of a pump. The result was compared with the experimental data which were observed under various distances of the clearance. He also carried out different kinds of experiments, some of which verified the theoretically established equations and some of which served as correction factors to improve the equations.

He suggested that the removal of the influence of the entry would improve the performance. The researches on this problem and the influence of the viscosity will be reported at a later date.

The chief and concrete results clarified by this research were as follows:

(1) The liquid in the pumping passage is driven by the momentum of the liquid which is projected by a centrifugal force from the spaces between the vanes of the impeller. This is the mechanism of the pumping action of a peripheral pump.

(2) The status of the exchange of momentum between a volume of liquid and the surrounding liquid is formularized by a kind of mixture length theory. In this theory, the intensity of turbulence is proportional to the peripheral velocity of the impeller, and the mixture length increases in proportion to the distance from the fixed wall or the inner surface of the casing.

(3) When the rate of flow is great, the pumping passage near the entry does not work effectively and the pressure there decreases. The decrease of the pressure spoils the suction ability as well as the performance of a pump at normal condition.

(4) An equation is induced which shows the relationship among the discharge, the head, the speed of revolution and the main dimensions of the pump. According to this equation, the discharge at any total head

is the difference between the rate of flow at zero pressure gradient and the rate of flow which is retarded by the total head.

(5) An equation is induced which shows the relationship between the torque, the head, the speed of revolution and the main dimensions of the pump. According to this equation, the torque is made of two parts; one proportional to the total head, the other proportional to the square of the speed of revolution which is somewhat like a disk frictional torque.

(6) A pump sometimes creates a bad performance at high speed of revolution. This is due to cavitation which develops in the pumping passage. The author studied the mechanism and determined the critical condition quantitatively which was verified by some experimental data.

(7) When cavitation grows, the pressure at the entry of the pumping passage decreases more remarkably than it does under normal condition. This was inferred by the analysis of the performances of various pumps and verified by the pressure distribution in the pumping passage.

(8) The author studied the influence of the internal leakage on the performance of a pump theoretically, which was verified by a series of experimental tests.

(9) The influence of the internal leakage on the performance of a pump can be compared to the reduction of the turbulence in the pumping passage. Accordingly, the apparent turbulence is adopted which simplifies the estimation of the performance of a pump with an internal leakage as if there is no leakage. This method is very convenient for a design of a pump.

(10) The intensity of the turbulence in the pumping passage is proportional to the peripheral velocity of the impeller and the mean depth of the pumping passage; it is also influenced by the number and the shape of the vanes of the impeller, but it is hardly influenced by the shape of the pumping passage.

(11) The kinematic viscosity of liquid was changed from 0.01 to 2.10 cm^2/s, when the Reynolds number varied from 400 to 4500. The theoretical equation for the head-capacity performance is applicable for many of these experiments, while the intensity of the turbulence diminishes as the Reynolds number decreases.

(12) Each factor which influences the efficiency of pump is clarified. The highest attainable value of the maximum efficiency is about 60% at best, but the efficiency greatly diminishes as the Reynolds number decreases to less than about twenty. A peripheral pump is hardly used at a Reynolds number less than this value owing to it's lack of efficiency.

(13) Several coefficients which are contained in the performance equations have been quantitatively decided by many experiments.

The present study was undertaken under the advice of Professor Kasai and the experiments have been conducted under his kind guidance. As to the theory made use of in this work, the writer is largely indebted to Professor Kurihara, who has helped the writer with useful suggestions. The writer also expresses his deep appreciation for the staff of the Hydraulic Laboratory of Kyushu University who kindly helped the writer in his experiments and for those students who worked with him.

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