Orthotropic Semi-infinite Plate with a Hole

Higuchi, Masakazu
Research Institute for Applied Mechanics, Kyushu University

https://doi.org/10.5109/7157915

出版情報：Reports of Research Institute for Applied Mechanics. 2 (7), pp.161-163, 1953-09. 九州大学応用力学研究所

Reports of Research Institute for Applied Mechanics
Vol. II, No. 7 September, 1953

Orthotropic Semi-infinite Plate with a Hole

By Masakazu HIGUCHI

The two-dimensional boundary value problem of the orthotropic strip having an elliptic hole was dealt in my previous paper.[1] The similar method can be applied to the problem of the semi-infinite orthotropic plate having an elliptic hole. For a mere semi-infinite plate of orthotropic elasticity, a number of solutions have been given and another solution of the same problem is, of course, unnecessary. In spite of it I duplicate here the solution, because it is necessary as a preliminary step for my calculation.

The function (say, *stress function*[2]) with which the stresses of the orthotropic semi-infinite plate are computed is

$$\left.\begin{aligned} Z_r''(z_r) &= F_r''(z_r) + G_r''(z_r) \\ \text{and} \quad F_r''(z_r) &= \frac{1}{2\pi i\,(k_r - k_s)} \int_{-\infty}^{\infty} \frac{k_s\, p_y(S) - i\, p_x(S) + 2k_s \,\mathrm{Re} - 2i\, \mathrm{Im}}{S - (z_r + i\, k_r B)}\, dS , \end{aligned}\right\} \quad (1)$$

where

$$\mathrm{Re} \equiv \text{real part of } \left[G_r''(z_r) + G_s''(z_s)\right]_{x=S,\, y=0}$$

$$\mathrm{Im} \equiv \text{imaginary part of } \left[k_r\, G_r''(z_r) + k_s\, G_s''(z_s)\right]_{x=S,\, y=0}$$

and $G_r''(z_r)$ etc. are the known *stress functions* which may have singular points on the region of the plate.

We can also obtain the solution in the form

$$F_r(z_r) = \int_0^{\infty} f_r(\gamma)\, \exp(i\, \gamma\, z_r / M_r)\, \frac{d\gamma}{\gamma^2}$$

when the straight boundary of the plate is not parallel to either of the axes of symmetry of orthotropic elasticity, provided that $M_r \equiv \cos\phi + i\, k_r \sin\phi$.

Now, representing an elliptic hole of the plate with $x^2/a^2 + y^2/b^2 = 1$ and using another variable ζ_r, which is given by the relation $z_r = \alpha_r \zeta_r + \beta_r/\zeta_r$ and $2\alpha_r \equiv a + k_r b$, $2\beta_r \equiv a - k_r b$, we can follow the method used in the previous paper and obtain

$$f_r(\gamma) = e^{-k_r B \gamma}\, \frac{i\, \boldsymbol{P}_x(\gamma) - k_s \boldsymbol{P}_y(\gamma)}{k_r - k_s},$$

$$H_r''(\zeta_r) = \frac{1}{k_r - k_s}\left\{ g_r'(\zeta_r) + Q_r'(\zeta_r) - i \sum_{m=1}^{\infty} \frac{m\, h_{rm}}{(-\zeta_r)^{m+1}} \right\} \frac{d\zeta_r}{dz_r}$$

$$\boldsymbol{P}_x \equiv \frac{1}{\pi} \int_{-\infty}^{\infty} \mathrm{Im}\,\{k_r G_r'' + k_s G_s'' + k_r H_r'' + k_s H_s''\}\, e^{-i\gamma S}\, dS + \frac{1}{2\pi} \int_{-\infty}^{\infty} p_x(S)\, e^{-i\gamma S}\, dS$$

---

[1] Rep. Res. Inst. for App. Mech., Vol. I, No. 1.

[2] Notations are referred to Ditto. Vol. II, No. 6.

$$P_y \equiv \frac{1}{\pi}\int_{-\infty}^{\infty} \mathrm{Re}\{G_r'' + G_s'' + H_r'' + H_s''\}\, e^{-i\gamma S}\, dS + \frac{1}{2\pi}\int_{-\infty}^{\infty} p_y(S)\, e^{-i\gamma S}\, dS$$

$$g_r(\zeta_r) \equiv \frac{1}{2\pi i}\oint_{\Gamma} \{[(k_r - k_s)\, G_r' - (k_r + k_s)\, \overline{G_r'} - 2k_s\, \overline{G_s'}]_A^C\} \frac{d\omega}{\omega - \zeta_r}$$

$$Q_r(\zeta_r) \equiv \frac{1}{2\pi i}\oint_{\Gamma} [\int_A^{C(\omega)} \{k_s\, q_y(C) - i\, q_x(C)\}\, dC] \frac{d\omega}{\omega - \zeta_r}$$

$$h_{rm} \equiv \int_0^{\infty} [\{(k_r - k_s)\, \mu_r^{-2m} f_r(\gamma) + (k_r + k_s)\overline{f_r(\gamma)}\}\mu_r^m J_m(\lambda_r \gamma) + 2k_s\, \mu_s^m J_m(\lambda_s \gamma)\overline{f_s(\gamma)}] \frac{d\gamma}{\gamma}$$

and the *stress function*

$$Z_r''(z_r) = F_r''(z_r) + G_r''(z_r) + H_r''(z_r)\,. \tag{2}$$

This does not give the solution in a closed form, but we are lead to the linear, simultaneous integral equations with respect to $f_r(\gamma)$ and $f_s(\gamma)$, or otherwise, the first-order, simultaneous equations having the infinite number of unknown constants h_{rm}, h_{sm} $(m = 1, 2, \cdots)$.

Subsidiary relations are derived in particular when σ_x and σ_y are symmetrical about the axis y;

$$\overline{f_r(\gamma)} = f_r(\gamma)\,, \quad \overline{h_{rm}} = (-)^m\, h_{rm}\,. \tag{3}$$

On the other hand, when τ_{xy} is symmetrical about the axis y

$$\overline{f_r(\gamma)} = -f_r(\gamma)\,, \quad \overline{h_{rm}} = (-)^{m+1}\, h_{rm}\,. \tag{4}$$

Numerical calculation is rather cumbersome, though not so much as in the case of the previous paper. We take first the example in which case both the periphery of the hole and the straight edge of the plate are free and the plate is under uniform tension p_0 along the axis x, namely $G_r'' = -p_0/2(k_r^2 - k_s^2)$. The following equations determine the approximate values of h_{rm}, h_{sm} successively.

$$\left.\begin{aligned} h_{rm} &= C_{rm} + \sum_{n=1}^{\infty} (-1)^{n+1} K_{r,\,mn}\, h_{rn} + \sum_{n=1}^{\infty} (-1)^{n+1} L_{r,\,mn}\, h_{sn}\,, \\ h_{sm} &= C_{sm} + \sum_{n=1}^{\infty} (-1)^{n+1} K_{s,\,mn}\, h_{sn} + \sum_{n=1}^{\infty} (-1)^{n+1} L_{s,\,mn}\, h_{rn}\,. \end{aligned}\right\} \tag{5}$$

where

$$\left.\begin{aligned} C_{rm} &\equiv \frac{i\, b\, p_0}{2}\{\delta_{mr}\, J(\lambda_r, m;\, \lambda_r, 1) + \varphi_{mr}\, J(\lambda_r, m;\, \lambda_s, 1) \\ &\qquad + \varepsilon_{mr}\, J(\lambda_s, m;\, \lambda_s, 1) + \psi_m\, J(\lambda_s, m;\, \lambda_r, 1)\}\,, \\ K_{r,\,mn} &\equiv n\,\{\delta_{mr}\, J(\lambda_r, m;\, \lambda_r, n) + \psi_m\, J(\lambda_s, m;\, \lambda_r, n)\}\,, \\ L_{r,\,mn} &\equiv n\,\{\varphi_{mr}\, J(\lambda_r, m;\, \lambda_s, n) + \varepsilon_{mr}\, J(\lambda_s, m;\, \lambda_s, n)\}\,, \end{aligned}\right\} \tag{6}$$

in which

$$\delta_{mr} \equiv -(k_r + k_s)\,\frac{k_r + k_s + (k_r - k_s)\, \mu_r^{-2m}}{(k_r - k_s)^2}\,,$$

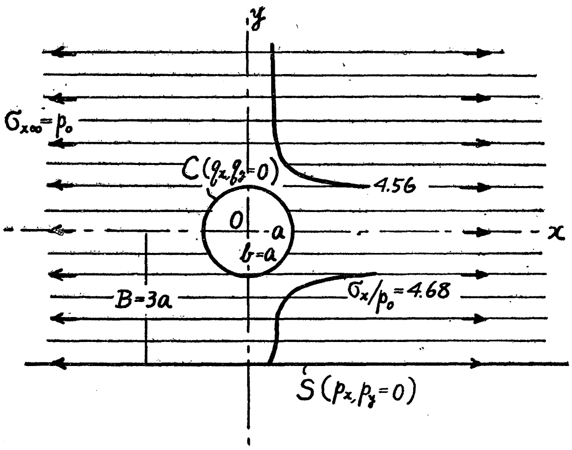
$$\varphi_{mr} \equiv 2k_s\,\frac{k_r + k_s + (k_r - k_s)\, \mu_r^{-2m}}{(k_r - k_s)^2}\,,$$

$$\varepsilon_{mr} \equiv -2k_s \frac{k_r + k_s}{(k_r - k_s)^2}, \quad \psi_m \equiv \frac{4k_r k_s}{(k_r - k_s)^2}$$

and

$$J(\lambda_r, m;\ \lambda_s, n) \equiv \mu_r{}^m \mu_s{}^n \int_0^\infty e^{-(k_r+k_s)B\gamma} J_m(\lambda_r \gamma) J_n(\lambda_s \gamma) \frac{d\gamma}{\gamma}.$$

J_m denotes the m-order Bessel's function.



Stress concentration of the semi-infinite plate of Compreg having a hole under uniform tension.
$E_x = 1{,}794\ \mathrm{kg/mm^2}$ $E_y = 366.6''$ $G_{xy} = 219.9''$
$\nu_{xy} = -0.497$ ($k_r = 2.530$, $k_s = 0.874$)

Similarly, we are lead to the same equation as (5) in the other cases the plate is free on the boundaries and under uniform shearing force p_0, namly $G_r'' = i\,p_0/4k_r$ or under bending force as $G_r'' = z_r/\{2ik_r B(k_r^2 - k_s^2)\}$ though the expression (6) with which the coefficients of (5) are to be determined is varied.

On calculating numerically, the numerical values of $J(\lambda_r, m;\ \lambda_s, n)$ in (6) are necessary for $m, n = 1 \sim 5$, or more when the hole is closely located to the straight edge. In the example shown in the annexed figure, the values of $J(\lambda_r, m;\ \lambda_s, n)$ are calculated directly only for $m, n = 4, 5$ and, for $m, n = 1, 2, 3$, indirectly obtained being referred to the easily known recurrence formulae of $J(\lambda_r, m;\ \lambda_s, n)$ by use of the values for $m, n = 4, 5$, while the real part and imaginary part of H_r'' are obtained graphically.

(Received June 30, 1953)