Maxwell’s Demon: A Digest

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Maxwell's Demon: A Digest

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We have discussed Maxwell's demon in the last part of the lecture note on entropy (https://hdl.handle.net/2324/6758968) written in Japanese for undergraduate students. Here we give a short summary of the discussion.

MATH

Let us summarize the basic relation between two probability distributions $\vec{p}$ and $\vec{q}$ in terms of their entropies $H(\vec{p})$ and $H(\vec{q})$.

When $\vec{p}$ and $\vec{q}$ are independent, the mutual relation is depicted as Fig. 1 where the left circle represents $H(\vec{p})$ and the right circle represents $H(\vec{q})$. Here the area of the circle is the value of the entropy. In this independent case the entropy for the joint probability distribution $\hat{P}$ is simply the sum of $H(\vec{p})$ and $H(\vec{q})$:

$$H(\hat{P}) = H(\vec{p}) + H(\vec{q}).$$

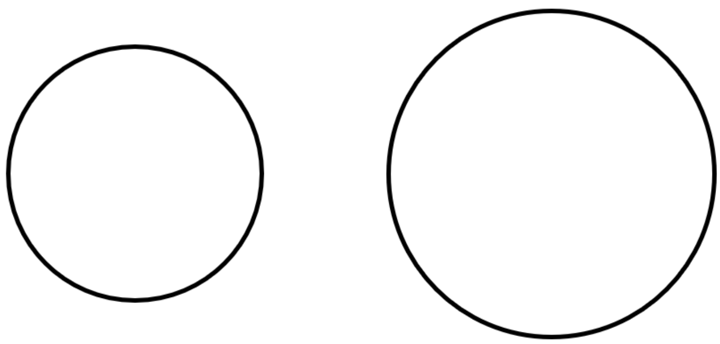

Fig. 1 Independent

When $\vec{p}$ and $\vec{q}$ are dependent, the mutual relation is depicted as Fig. 2 where the area of the overlap region of the two circles is the mutual information $I(\hat{P})$. Now the relation

$$H(\hat{P}) + I(\hat{P}) = H(\vec{p}) + H(\vec{q})$$

holds. Namely, $I(\hat{P})$ represents the degree of the dependence.

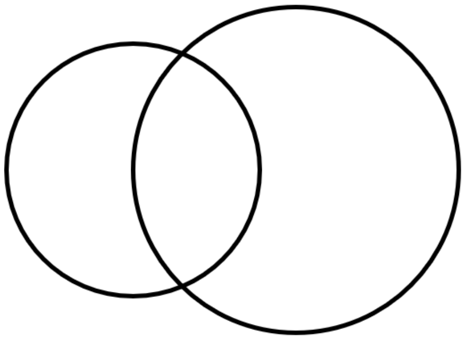

Fig. 2 Dependent

PHYS

First, let us state the second law of thermodynamics by Clausius's inequality

$$\Delta H \geq \beta Q$$

where ΔH is the entropy change for a process under the heat supply Q in the absence of the work ($W = 0$).

Then, let us consider the composite of the system and the demon. The composite is in contact with a reservoir. The activities of the demon are defined as follows. Initially, the demon is decoupled from the system and knows nothing about the system. Next, the demon is coupled to the system and gets the information about the system. Finally, the demon controls the system referring the information. After the control process the demon is decoupled from the system.

In the following we will describe the activities of the demon in terms of the entropy. Since we regard both system and demon as classical systems, we can define the joint probability of the composite, the measurement by the demon can be carried out without disturbing $\vec{p}$ and the control by the demon can be carried out without disturbing $\vec{q}$. (The definitions of $\vec{p}$ and $\vec{q}$ will be given below.)

Initially, the system and the demon are decoupled with independent probability distributions $\vec{p}$ and $\vec{q}_0$. The entropy $H(\hat{P}_0)$ of the joint probability distribution $\hat{P}_0$ for the composite is given by

$$H(\hat{P}_0) = H(\vec{p}) + H(\vec{q}_0).$$

Next, the demon measures the system without disturbing $\vec{p}$ and obtains the information of the system. The probability distribution of the demon changes into $\vec{q}$ reflecting the result of the measurement. At the same time the joint probability distribution for the composite changes into $\hat{P}$. For this change Clausius's inequality tells

$$H(\hat{P}) - H(\hat{P}_0) \geq \beta Q_{meas}$$

where Q_{meas} is the heat supplied to the demon from the reservoir. After the measurement process the system and the demon are coupled so that their probability distributions become dependent:

$$H(\hat{P}) + I(\hat{P}) = H(\vec{p}) + H(\vec{q}).$$

The mutual information $I(\hat{P})$ is interpreted as the information gain of the demon. Since $\vec{p}$ is unchanged by the measurement we obtain

$$H(\vec{q}) - H(\vec{q}_0) - I(\hat{P}) \geq \beta Q_{meas}.$$

Finally, the demon controls the system referring the measurement outcomes. By the control process $\vec{p}$ changes into $\vec{p}'$, while $\vec{q}$ is unchanged. At the same time the joint probability distribution for the composite changes into $\hat{P}'$. For this change Clausius's inequality tells

$$H(\hat{P}') - H(\hat{P}) \geq \beta Q_{cont}$$

where Q_{cont} is the heat supplied to the system from the reservoir. After the control process the demon is decoupled from the system so that their probability distributions become independent:

$$H(\hat{P}') = H(\vec{p}') + H(\vec{q}).$$

Since $\vec{q}$ is unchanged by the control we obtain

$$H(\vec{p}') - H(\vec{p}) + I(\hat{P}) \geq \beta Q_{cont}.$$

If we neglect the demon, we expect the inequality

$$H(\vec{p}') - H(\vec{p}) \geq \beta Q_{cont}$$

for the system. Then, $I(\hat{P})$ apparently violates the second law when we focus on the system. But the second law holds for the composite and we have employed Clausius's inequality for the composite. The mutual information $I(\hat{P})$ is the key to understand Maxwell's demon properly.

For the entire process Clausius's inequality holds:

$$H(\hat{P}') - H(\hat{P}_0) \geq \beta Q$$

without $I(\hat{P})$. Here $Q = Q_{meas} + Q_{cont}$.