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https://doi.org/10.5109/7156993

出版情報：Reports of Research Institute for Applied Mechanics. 2 (6), pp.53-64, 1953-06. 九州大学応用力学研究所
バージョン：
権利関係：

Reports of Research Institute for Applied Mechanics
Vol. II, No. 6 June, 1953

ON THE STRESSES DUE TO PRESSURE ON THE PERIPHERY OF THE HOLE IN AN ORTHOTROPIC PLATE

By Masakazu HIGUCHI

With the end to throw light on the peculiarity of the stress produced in the orthotropic plate, more examples of the plane problem are added to those in the previous paper[1]; the solutions are given in the cases when the orthotropic plate having a hole, circular or elliptic, is under the conditions;

a) that a pair of forces is acting at two points on the periphery of the hole,

b) that uniform hydrostatic pressure is prevailing in the hole,

c) that uniform tangential force acts along the periphery of the hole,

d) that a concentrated force acts on a certain point in the plane of the plate,

e) that a concentrated force acts on a point of the periphery of the hole,

f) that distributed pressure are on a certain part of the periphery of the hole,

g) that the circular opening is expanded uniformly, together with the solutions in the cases when the plate has rigid flange, or a rigid plug sticked to the periphery of the hole, and

h) the periphery is turned a certain angle, or

i) the plate is under the uniform shearing force acting on the remote outer boundary of the plate.

The numerical calculations are worked out and the results are shown in the annexed figures.

1. Introduction. In the previous paper[1] the writer investigated the distribution of the stresses in the anisotropic infinite plate having a hole circular or elliptic. The examples given there were concerned with the plate having either a free hole or a rigid plug.

In the present paper, the writer gives a few other examples. Some of them are concerned with the cases where any pressure, concentrated or distributed, is acting on any part of the hole boundary in the plane of the orthotropic plate. The rest are the studies on the stresses produced in the plate of which the opening is elastically enlarged so that the displacing of its circumference may be conformed to a compulsory deformation,

[1] Higuchi, M., Kyushu Univ., Rep. Res. Inst. Elas. Engng., IV, 4. 1947.
Higuchi, M., Kyushu Univ., Rep. Res. Inst. Applied Mechanics, I, 1. 1952.

as well as in the plate under twisting force along the circumference of which is reinforced with a rigid flange.

When the coordinate axes x, y are taken in the plane of the plate so that their directions may coincide with the two axes of symmetry of orthotropic elasticity respectively, we can use as the Airy's stress function the expression

$$\chi(x,y) = Z_r(z_r) + Z_s(z_s) + \mathit{comp.\ conj.},$$

together with $z_r = x + i\,k_r\,y$, $z_r = x + i\,k_s\,y$ and

$$k_r^2\,k_z^2 = \frac{E_x}{E_y}\,, \qquad k_r^2 + k_s^2 = \frac{E_x}{G_{xy}} + 2\nu_{xy}\,, \qquad \text{for generalized plane stress,}$$

$$k_r^2\,k_s^2 = \frac{1-\nu_{yz}\,\nu_{zy}}{1-\nu_{xz}\,\nu_{zx}}\cdot\frac{E_x}{E_y}\,, \qquad k_r^2 + k_s^2 = \frac{1}{1-\nu_{xz}\,\nu_{zx}}\left\{\frac{E_x}{G_{xy}} + 2(\nu_{xy} - \nu_{xz}\,\nu_{zy})\right\}$$

for plane strain.

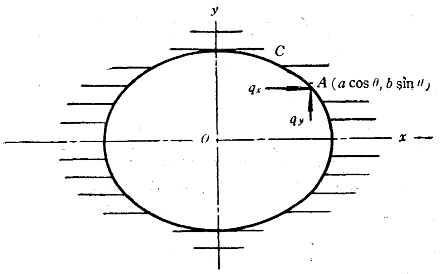

Fig. 1. q_x, q_y Components of an external force acting on the plate per unit depth at A, a point on the circumference C of the hole.

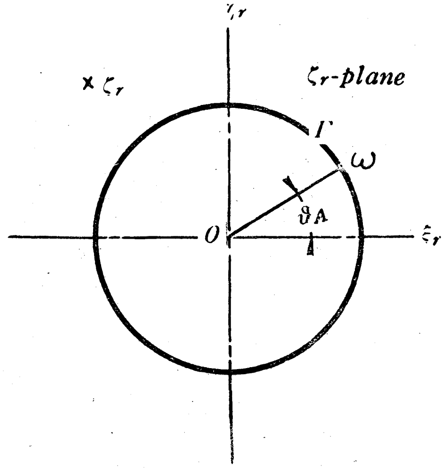

Fig. 2. Transformation of the ellipse C into the unit circle Γ. ω Image of A in x,y-plane $\vartheta_A = \theta$.

Regarding $Z_r(z_r)$ as a function composed of two terms as $Z_r = G_r + H_r$, we shall assume that H_r is regular on our physical region, viz., in the z_r-plane excepting on the region of the hole which is represented with $x^2/a^2 + y^2/b^2 = 1$. G_r may, however, have any singularity in our physical plane. Using the notations

$$2\alpha_r \equiv a + k_r\,b\,, \qquad 2\beta_r \equiv a - k_r\,b$$

and transforming the ellipse $x^2/a^2 + y^2/b^2 = 1$ in the x,y-plane with

$$z_r = \alpha_r \zeta_r + \frac{\beta_r}{\zeta_r}$$

into the unit circle Γ in the ζ_r-plane (Fig. 2), we can apply the Muschelisvili's method and obtain

$$(k_r - k_s) H_r'(\!(\zeta_r)\!) = g_r(\zeta_r) + \mathbf{Q}_r(\zeta_r). \tag{1}$$

Here

$$g_r(\zeta_r) \equiv \frac{1}{2\pi i} \oint_\Gamma [(k_r - k_s) G_r' - (k_r + k_s) \overline{G_r'} - 2k_s \overline{G_s'}]_A^{C(\omega)} \frac{d\omega}{\omega - \zeta_r}, \tag{2}$$

$$\mathbf{Q}_r(\zeta_r) \equiv \frac{1}{2\pi i} \oint_\Gamma \left[\int_A^{C(\omega)} \{k_s q_y(C) - i q_x(C)\} dC \right] \frac{d\omega}{\omega - \zeta_r}, \tag{3}$$

and $H_r'(\!(\zeta_r)\!)$ is used for $dH_r\{z_r(\zeta_r)\}/dz_r$. A prime denotes the differentiation with respect to the complex variable of the function. For instance, G_r' represents dG_r/dz_r, while $\overline{G_r'}$ is $d\overline{G_r}/d\overline{z_r}$. C in the integrals is the length of the curve measured along the periphery of the ellipse from a certain start A. The point $A(x = a \cos\theta_A,\ y = b \sin\theta_A)$ corresponds to the point $e^{i\vartheta_A}(\equiv \omega$, say) on the unit circle Γ in both the ζ_r- and ζ_s-plane. The identity $\theta_A = \vartheta_A$ is derived easily and naturally C is a function of ω.

Referring to another rectangular coordinate axes X, Y, the axis X of which makes an angle ϕ with the axis x, the displacements in the directions of the axes X and Y are generally expressed as

$$\left.\begin{aligned} -2G_{xy} u_X &= (M_r + K_r \overline{M_r}) Z_r' + (M_s + K_s \overline{M_s}) Z_s' + \textit{comp. conj.}, \\ -2G_{xy} v_Y &= (N_r + K_r \overline{N_r}) Z_r' + (N_s + K_s \overline{N_s}) Z_s' + \textit{comp. conj.}, \end{aligned}\right\} \tag{4}$$

respectively, in which

$$K_r = -K_s \equiv (k_r^2 - k_s^2) \frac{G_{xy}}{E_x}$$

for generalized plane stress,

$$K_r = -K_s \equiv (1 - \nu_{xz} \nu_{zx})(k_r^2 - k_s^2) \frac{G_{xy}}{E_x}$$

for plane strain.

When the displacements of the circumference of the opening, $[u_X]_C$ and $[v_Y]_C$, are given, the stress function is calculated from

$$(m_r - m_s) H_r'(\!(\zeta_r)\!) = \mathbf{g}_r(\zeta_r) + \mathbf{U}_r(\zeta_r), \tag{5}$$

where

$$\mathbf{g}_r(\zeta_r) \equiv \frac{1}{2\pi i} \oint_\Gamma \left[\begin{aligned} (m_r - m_s) G_r' - (m_r + m_s) \overline{G_r'} \\ - 2k_s (1 - K_r^2) \overline{G_s'} \end{aligned} \right]_A^{C(\omega)} \frac{d\omega}{\omega - \zeta_r}, \tag{6}$$

$$\mathbf{U}_r(\zeta_r) \equiv \frac{G_{xy}}{\pi} \oint_\Gamma \{(N_s + K_s \overline{N_s}) u_X - (M_s + K_s \overline{M_s}) v_Y\} \frac{d\omega}{\omega - \zeta_r}, \tag{7}$$

and

$$m_r = k_r (1 - K_r)^2, \qquad m_s = k_s(1 - K_s)^2 .$$

In either case the stresses can be evaluated with the formulae

$$\left.\begin{aligned} \sigma_X &= N_r^2 Z_r'' + N_s^2 Z_s'' + comp.\ conj., \\ \sigma_Y &= M_r^2 Z_r'' + M_s^2 Z_s'' + comp.\ conj., \\ \tau_{XY} &= -M_r N_r Z_r'' - M_s N_s Z_s'' - comp.\ conj., \end{aligned}\right\} \tag{8}$$

and

$$M_r = \cos\phi + i\, k_r \sin\phi, \qquad N_r = -\sin\phi + i\, k_r \cos\phi,$$
$$M_s = \cos\phi + i\, k_s \sin\phi, \qquad N_s = -\sin\phi + i\, k_s \cos\phi .$$

Putting $\phi = 0$ in these formulae, we obtain σ_x, σ_y and τ_{xy} naturally, the last two of which attract our particlar attention because most orthotropic materials such as Compreg are found wanting in strength for such stresses.

The above is the résumé of the mathematical part of the previous paper which may possibly be modified in some points of notations.

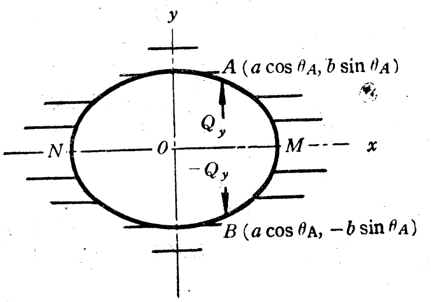

Fig. 3. Plate under a pair of forces directing parallel to the y-axis on the boundary.

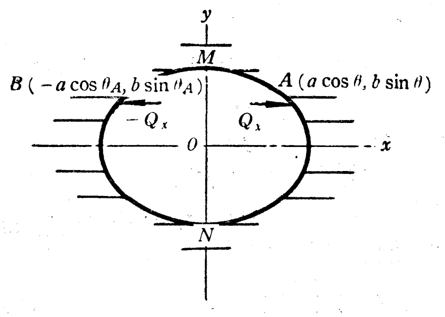

Fig. 4. Plate under a pair of forces directing parallel to the x-axis on the boundary.

2. Solutions of some basic problems.

a) *A pair of concentrated forces.*

When a pair of forces is acting at the points A and B as shown in Fig. 3, we can leave out the function G_r and, consequently, $g_r(\zeta_r)$ vanishes, and the other term $\mathbf{Q}_r(\zeta_r)$ in the expression of $H_r'(\zeta_r)$ is obtained as follows.

$$\int_A^{C(\omega)} \{k_s q_y(C) - i\, q_x(C)\} dC = 0, \qquad A \leq C < B,$$
$$= -k_s Q_y, \qquad B \leq C < A.$$

Therefore

$$(k_r - k_s) H_r' = \mathbf{Q}_r(\zeta_r) = \frac{-k_s Q_y}{2\pi i} \ln \frac{\zeta_r - e^{i\theta_A}}{\zeta_r - e^{-i\theta_A}},$$

or

$$Z_r' = \frac{i\, k_s Q_y}{2\pi (k_r - k_s)} \ln \frac{\zeta_r - e^{i\theta_A}}{\zeta_r - e^{-i\theta_A}}. \tag{9}$$

In the case when the forces make a pair in the direction of the axis x,

$$Z_r' = \frac{Q_x}{2\pi (k_r - k_s)} \ln \frac{\zeta_r - e^{i\theta_A}}{\zeta_r + e^{-i\theta_A}}.$$

However, this is the same case that the coordinate axes in the former case are turned and set so as to make an angle 90° with their initial position. The solution in this case can be, therefore, directly derived from (9).

In the former case, the tensile stresses at the points M and N are

$$\sigma_{y,M} = \left(\frac{1}{k_r} + \frac{1}{k_s}\right) \frac{Q_y}{\pi b} \cot \frac{\theta_A}{2}, \qquad \sigma_{y,N} = \left(\frac{1}{k_r} + \frac{1}{k_s}\right) \frac{Q_y}{\pi b} \tan \frac{\theta_A}{2},$$

while

$$\sigma_{x,M} = (k_r + k_s) \frac{Q_x}{\pi a} \cot \frac{\pi/2 - \theta_A}{2}, \qquad \sigma_{x,N} = (k_r + k_s) \frac{Q_x}{\pi a} \tan \frac{\pi/2 - \theta_A}{2}$$

are the stresses in the latter case (Fig. 4), which are obtained directly from $\sigma_{y,M}$ and $\sigma_{y,N}$ by replacing k_r and k_s with $1/k_r$ and $1/k_s$ respectively.

For the isotropic plate, putting $k_r = k_r = 1$, we obtain

$$\sigma_{y,M} = \frac{2Q_y}{\pi b} \cot \frac{\theta_A}{2} \quad \text{etc.}$$

b) *Uniform hydrostatic pressure on the boundary.*

In the case where is hydrostatic pressure q_0 prevailing in the hole (Fig. 6), inserting

$$q_x(C)\, dC = q_0\, dy = b\, q_0 \cos\theta\, d\theta,$$
$$q_y(C)\, dC = -q_0\, dx = a\, q_0 \sin\theta\, d\theta,$$

in the integrand of (3) together with $e^{i\theta} = \omega$, and evaluating the integral with the residues, we obtain

$$Z_r' = \frac{k_s a - b}{2(k_r - k_s)} \cdot \frac{q_0}{\zeta_r}. \tag{10}$$

The stress σ_x at either end of the major axis of the ellipse is

$$\sigma_y = \left\{-\frac{1}{k_r k_s} + \left(\frac{1}{k_r} + \frac{1}{k_s}\right)\frac{a}{b}\right\} q_0 .$$

This is interesting compared with the stresses of the plate with a hole under uniform tension p_0 at the remote end in the direction of the axis y. The stress at the same point is in this case

$$\sigma_y = \left\{1 + \left(\frac{1}{k_r} + \frac{1}{k_s}\right)\frac{a}{b}\right\} p_0 .$$

When the uniform tensile force is acting parallel to the axis x, on the other hand, the stress at the same point is compressive and

$$\sigma_y = -\frac{1}{k_r k_s} p_0 .$$

Consequently, the fact is that we can arrive at the same solution (10) by superposing three stress functions, viz., the two funcions in the above-mentioned cases and one in the case when a plate having no hole is under uniform hydrostatic compression throughout its region.

The opening of the shape $x^2 + y^2 = a$ is distorted as

$$\frac{x^2}{\{1 + (k_r + k_s - k_r k_s - \nu_{xy}) q_0/E\}^2} + \frac{y^2}{\{1 + (k_r^2 k_s + k_r k_s^2 - k_r k_s - \nu_{xy}) q_0/E\}^2} = a^2 . \tag{11}$$

Since $k_r = k_s = 1$ and $\nu_{xy} = -\nu$ for isotropic materials, we have

$$x^2 + y^2 = a^2 (1 + q_0/2G)^2 ,$$

which is naturally the equation of a circle.

c) *Uniform tangential force on the boundary.*

When the plate has a uniform tangential force on the circumference of its circular hole,

$$dq_x = -a\, q_0 \sin\theta\, d\theta ,$$

$$dq_y = a\, q_0 \cos\theta\, d\theta ,$$

and

$$Z_r' = \frac{1 - k_s}{k_r - k_s} \cdot \frac{i\, a\, q_0}{2\zeta_r} \tag{12}$$

is obtained. The hole is distorted as

$$-\frac{2G_{xy}}{a\, q_0} u_r = \frac{k_r k_s - 1}{k_r - k_s} K_r \sin 2\theta ,$$

$$-\frac{2G_{xy}}{a\, q_0} u_\theta = \frac{2K_r}{k_r - k_s}(\sin^2\theta + k_r k_s \cos^2\theta) + \left(1 - \frac{k_r + k_s}{k_r - k_s} K_r\right) .$$

d) *A concentrated force in the plate.*

Now, with

$$G_r' = C_r \ln (z_r - c_r),$$

where

$$C_r = \frac{1}{8\pi k_r K_r} \{(1 + K_r) Q_x + i k_r (1 - K_r) Q_y\}, \tag{13}$$

and $c_r = x_0 + i k_r y_0$, we can obtain, from (1), the stress function in the case where the plate with a hole is under a concentrated force at the point $(x = x_0, y = y_0)$, the components of which are Q_x and Q_y. If the edge of the hole is free from any external force,

$$\mathbf{Q}_r = 0,$$

and

$$Z_r' = \frac{k_r + k_s}{k_r - k_s} \overline{C_r} \ln \frac{\zeta_r - \omega_{r2}^*}{\zeta_r} + \frac{2k_s}{k_r - k_s} \overline{C_s} \ln \frac{\zeta_r - \omega_{s2}^*}{\zeta_r} + C_r \ln (\zeta_r - \omega_{r1}), \tag{14}$$

where $\overline{C_r}$ and $\overline{C_s}$ are the complex constants conjugate to C_r and C_s given in (13) respectively, and ω_{r2}^* and ω_{s2}^* are determined by the relations

$$\omega_{r2}^* = e^{i\vartheta_r}/\rho_r$$

$$\omega_{s2}^* = e^{i\vartheta_s}/\rho_s$$

together with

$$c_r = \alpha_r \rho_r e^{i\vartheta_r} + \beta_r e^{-i\vartheta_r}/\rho_r, \qquad \rho_r > 1,$$

and

$$c_s = \alpha_s \rho_s e^{i\vartheta_s} + \beta_s e^{-i\vartheta_s}/\rho_s, \qquad \rho_s > 1.$$

e) *A concentrated force on the boundary.*

The procedure to transfer the singular point c_r of load in the solution (14) just on the point of the circumference of the hole leads to the stress function by which the stress state of the plate having a load on the edge of the hole is represented. Considering the case in which the shape of the hole is a circle and the external force is concentrated at the point A $(x = a \cos \theta_A, y = a \sin \theta_A)$ in Fig. 1, we have

$$\omega_{r2}^* \rightarrow e^{i\theta_A}, \qquad \omega_{s2}^* \rightarrow e^{i\theta_A}$$

and

$$Z_r' = \frac{Q_x + i k_s Q_y}{2\pi (k_r - k_s)} \ln \frac{\omega_A - \zeta_r}{\zeta_r} + C_r \ln \zeta_r, \tag{15}$$

where C_r is given by (13).

This is derived, of course, from (1) directly. We should then assume the function $G_r' = C_r \ln \zeta_r$ where $C_r = A_r + i\,B_r$, A_r and B_r being unknown constants, and are to be determined so that the dislocation does not exist anywhere in the plate excepting the inside of the hole. On the other hand, the solution must be the periodic function of θ. These two conditions provide the values of C_r as well as of C_s.

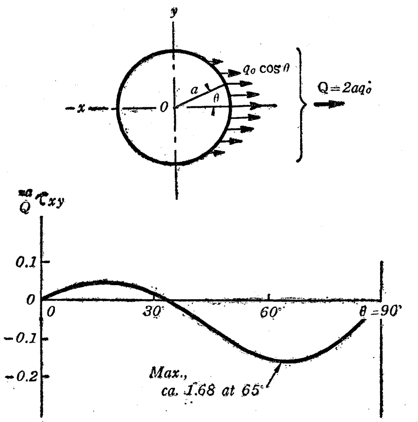

Fig. 5. The shearing stress on the circular boundary of the plate of Com-preg. The case $\phi = 0$ in the Eq. (16) and $k_r = 2.530$, $k_s = 0.874$.

f) *Distributed pressure on the boundary.*

We shall study on the stress state of the plate with a circular hole, on the circumference of which the distributed force is acting and its components are

$$\left.\begin{aligned} dQ_x &= a\,d\theta \cdot q_0 \cos(\theta - \phi) \cdot \cos\phi\,, \\ dQ_y &= a\,d\theta \cdot q_0 \cos(\theta - \phi) \cdot \sin\phi\,, \end{aligned}\right\} \qquad \phi - \frac{\pi}{2} \leq \theta \leq \phi + \frac{\pi}{2}\,,$$

$$dQ_x = dQ_y = 0\,, \qquad \phi + \frac{\pi}{2} < \theta < \phi + \frac{3\pi}{2}\,.$$

The function Z_r' is deduced as

$$Z_r' = \frac{M_r + K_r \overline{M_r}}{4\pi\, k_r K_r}\, a\, q_0 \ln \zeta_r + \frac{a\, q_0 M_s}{2\pi\,(k_r - k_s)}\, I\,, \tag{16}$$

where

$$I = \frac{i}{2} \int^{\zeta_r} \left\{ \left(e^{-i\phi} + \frac{e^{i\phi}}{\zeta_r^{\,2}} \right) \ln \frac{\zeta_r - i\, e^{i\phi}}{\zeta_r + i\, e^{i\phi}} \right\} d\zeta_r - \frac{\pi\, e^{i\phi}}{2\zeta_r} - \ln \zeta_r\,.$$

For $\quad \phi = 90°, \qquad [\sigma_y]_{\substack{x=a\\ y=0}} = \left(\frac{1}{k_r} + \frac{1}{k_s}\right) \frac{q_0}{2} = \left(\frac{1}{k_r} + \frac{1}{k_s}\right) \frac{Q_y}{4b}\,,$

and for $\quad \phi = 0°, \qquad [\sigma_x]_{\substack{x=0\\ y=b}} = (k_r + k_s)\, \frac{q_0}{2} = (k_r + k_s)\, \frac{Q_x}{4a}\,,$

Q_y being the resultant force in the direction of the axis y and equal to $2aq_0$.

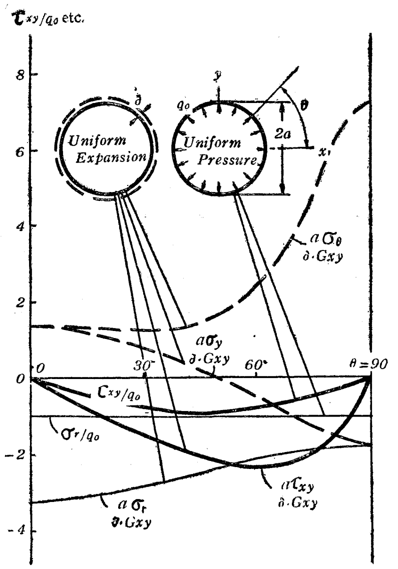

Fig. 6. Comparison of the stresses on the circular boundaries of the plates of Compreg under the uniform expansion of the opening and under the uniform hydrostatic pressure.

Computed with $k_r = 2.530$, $k_s = 0.874$, $E_x = 1{,}794\ \mathrm{kg/mm^2}$, $G_{xy} = 219.9\ \mathrm{kg/mm^2}$ and $\nu_{xy} = -0.497$.

The stresses in the case of *uniform expansion* are given as the ratios to $\delta \cdot G_{xy}/a$.

g) *Uniform expansion of the opening.*

It is interesting to investigate the stress state on the hole boundary when the circular hole is expanded uniformly by δ in radius (Fig. 6). Putting

$$G_r' = 0, \qquad v_Y = 0, \qquad u_X = \delta \ (= \text{const.})$$

in Eq. (5) and evaluating the integral in the equation, we get

$$Z_r' = \frac{\delta \cdot G_{yx}}{m_r - m_s} \{(1 - K_s)\, k_s - (1 + K_s)\} \frac{1}{\zeta_r}. \tag{17}$$

The stress σ_r, being used instead of σ_X, is not constant any more.

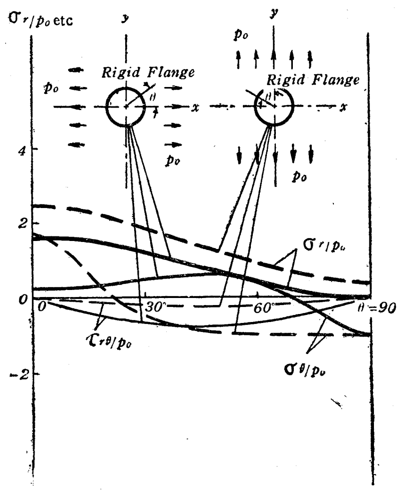

Fig. 7. Numerical examples of the calculation in the writer's previous paper. Plates of Compreg with a circular rigid plug, or a rigid circular flange reinforcing the edge. The elasticity constants are the same as the preceding. It is a noteworthy fact that $\sigma_r > 0$ may happen at $\theta = 90°$, with a minor value though.

h) *Rotation of the periphery with a rigid plug.*
Next, let us consider the case

$$G_r' = 0, \qquad u_X = 0 \quad \text{and} \quad v_Y = \Delta .$$

The solution is obtained as

$$Z_r' = \frac{i\,\Delta \cdot G_{xy}}{m_r - m_s} \}(K_s - 1)\,k_s + (K_s + 1)\} \frac{1}{\zeta_r}. \tag{18}$$

This is the case when the hole boundary is reinforced with a rigid flange and the flange is turned in the plane of the plate by an angle Δ/a rad. round the center of the circular hole.

$$-T = \frac{4\pi\, a\, \Delta \cdot G_{xy}}{m_r - m_s} \{k_r - k_s - (k_r + 1)(k_s + 1)\,K_r\}$$

is the twisting torque necessary for the rotation Δ/a of the flange when the plate is kept from rotating at the outer part remote enough from the hole.

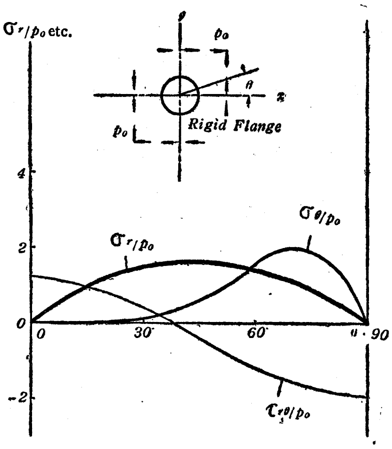

Fig. 8. The stresses on the circular boundary of the plate of Compreg with a sticked plug, or a rigid flange.
Elasticity constants are the same as those in Fig. 6.
p_0 shearing force working uniformly at the remote outer part of the plate.

i) *A rigid plug in the plate under uniform shear.*
Another example of the opening reinforced with a rigid circular flange, or otherwise of the opening with a sticked plug, is the case when the shearing force p_0 working uniformly at the remote outer part of the plate. We have then

$$G_r' = \frac{ip_0}{4k_r} z_r \quad \text{and} \quad u_X = v_Y = 0$$

and therefore

$$Z_r' = \frac{i\,a\,p_0}{2\,(m_r - m_s)} \{(1 - K_s)\,k_s + (1 + K_s)\} \frac{1}{\zeta_r} + \frac{ip_0}{4k_r} z_r . \tag{19}$$

Since the external shearing force is in this case not working symmetrically about the axis of symmetry of the orthotropic elasticity in spite of the symmetrical shape of the opening, we have need of a resisting torque in order to insure $v_Y = 0$. Referring to the equation of moment we get

$$
\begin{aligned}
T &= [z_r Z_r' - Z_r + z_s Z_s' - Z_s + comp.\ conj.]_{ACA} \\
&= \oint_\Gamma z_r \frac{dZ_r'}{d\zeta_r} d\zeta_r + \oint_\Gamma z_s \frac{dZ_s'}{d\zeta_s} d\zeta_s + comp.\ conj. \\
&= \frac{k_r k_s - 1}{m_r - m_s} \cdot 2\pi a^2 p_0 .
\end{aligned}
$$

If the rigid flange is allowed to turn freely as a whole, we must superpose the present solution on that in the preceding example so as to cancell the twisting torque, with the consequence that the stresses will be modified. The rotaion of the rigid flange arises under such circumstance and its magnitude is

$$
\Delta = \frac{a p_0 K_r}{2G_{xy}} \cdot \frac{k_r k_s - 1}{k_r - k_s - (k_r + 1)(k_s + 1) K_r} .
$$

The other remarkable difference from the stress state in the case of the isotropic plate will be recognized from the annexed figures.

Acknowledgements. The writer wishes to express his thanks to Professor T. Isibasi of the Department of Engineering at the Kyushu University for his valuable advices. He also thanks the Grant in Aid for Fundamental Scientific Research of the Ministry of Education in Japan.

(Received March 1, 1952)

ERRATA

On The Two-Dimensional Stress In Aeolotropic Bodies

By Masskazu HIGUCHI

(Reports of Research Institute for Elasticity Engineering, Vol. 4, No. 4, 1947)

Page 57, Equ. (25), for " $= \cos^2\theta + \cdots$," " $= \sin^2\theta + \cdots$" and " $\sin\theta\cos\theta - \cdots$"
read " $= \cos^2\theta - \cdots$," " $= \sin^2\theta - \cdots$" and " $\sin\theta\cos\theta + \cdots$."

〃 58, 〃 (28), for " $(a+k_s b)e^{2i\psi}$" read " $(a+k_s b)\cos 2\psi + i(k_s a + b)\sin 2\psi$."

〃 〃 〃 (29), for " $e^{2i\psi}(a+k_s b)$,"
read " $-2\{(a+k_s b)\cos 2\psi + i(k_s a+b)\sin 2\psi\}$."

〃 59, 〃 (32), for " $i\,p/2$" read " $i\,p/8$."

Calculation Of The Stresses Of The Orthotropic Strip With A Hole

By Masakazu HIGUCHI

(Reports of Research Institute for Applied Mechanics, Vol. 1, No. 1, 1952)

Page 34, line 25, for " υ_x" read " ν_x."

〃 39, 〃 1, for " $-i\sum_{m=0}^{\infty} h_1^{(2m+1)}/\zeta_1^{2m+1}$" read " $-i\sum_{m=0}^{\infty}(-)^m h_1^{(m+1)}/\zeta_1^{m+1}$."

〃 〃 Eq. (10), for " $h_1^{(2m+1)} \equiv 2\int_0^{\infty}[\{(k_1-k_2)\mu_1^{-(2m+1)}+(k_1+k_2)\mu_1^{2m+1}\}\cdots]\frac{d\gamma}{\gamma}$"
read " $h_1^{(m+1)} \equiv \int_{-\infty}^{\infty}[\{(k_1-k_2)\mu_1^{-(m+1)}f_1(\gamma)+(k_1+k_2)\mu_1^{m+1}\overline{f_1(\gamma)}\}J_{m+1}(\lambda_1\gamma)$
$+2k_2\mu_2^{m+1}J_{m+1}(\lambda_2\gamma)\overline{f_2(\gamma)}]\frac{d\gamma}{\gamma}$."

〃 〃 line 5, for " $J_{2m+1}(\lambda_r\gamma)$" read " $J_{m+1}(\lambda_r\gamma)$."

〃 〃 〃 〃 for " $2m+1$" read " $m+1$."

〃 〃 〃 10, for " J_{2m+1}" read " J_{m+1}."

〃 〃 〃 13, for " $\int_C$" read " $\int_\Gamma$."

〃 〃 〃 16, for " $-i\sum_{m=0}^{\infty} h_1^{(2m+1)}/\zeta_1^{2m+1}$" read " $-i\sum_{m=0}^{\infty}(-)^m h_1^{(m+1)}/\zeta_1^{m+1}$."

〃 〃 〃 25, for " $h_1^{(2m+1)}$ and $h_2^{(2m+1)}$," read " $h_1^{(m+1)}$ and $h_2^{(m+1)}$."

〃 45, Table, for " E kg/mm^2 E kg/mm^2 G kg/mm^2"
read " E_x kg/mm^2 E_y kg/mm^2 G_{xy} kg/mm^2."

Besides, it seems that there are not a few mistakes in English usage because of my poor knowledge of English. But, to my regret, it can not be helped now. — M. H.