# THE STRESSES IN THE ORTHOTROPIC SEMI-INFINITE PLATE DUE TO AN RIGID BOLT PRESSED AGAINST ON ITS EDGE

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# THE STRESSES IN THE ORTHOTROPIC SEMI-INFINITE PLATE DUE TO AN RIGID BOLT PRESSED AGAINST ON ITS EDGE

By Masakazu HIGUCHI

In this paper treated are the stresses produced elastically in the orthotropic semi-infinite plate as well as the pressure of contact on the edge, due to a rigid body pressed against. Numerical calculation is worked out in the case where the plate of Compreg is compressed on the edge with a steel bolt and the result is shown in the annexed figures, in which we can recognize the characteristic features peculiar to orthotropic materials in the mechanical aspect. At the same time, we can find the spots subjected to the most unfavorable stress in either case where the edge of the plate is inclined to or coincides with the directions of the axes of symmetry of orthotropic elasticity.

**1. Introduction.** Having been pioneered by H. Hertz, the theory of the pressure between two *isotropic* bodies in contact may be now said one of the well developed branches of the theory of elasticity. Whereas, as far as I know, we have no analytical treatment of the pressure of contact of *anisotropic* material yet. It is the purpose of this paper to study on the two-dimensional contact theory of *orthotropic* elastic material. I have dealt with a special case in which an orthotropic semi-infinite plate such as Compreg is compressed on one part of its straight edge with a steel bolt. Nonetheless, the treatment is considerably general in the sense that it does not put any restriction upon the shape of the compressing body so long as it is rigid enough in comparison with the material of the plate and that it can take the tangential force occurring due to friction or any other cause into account in the form of restraint of displacement along the edge. Moreover, the axis of symmetry of orthotropic elasticity need not be necessarily parallel to the edge of the plate.

From the standpoint of the analytical theory of Mechanics, our problem belongs to the boundary value problem of mixed type; that is to evaluate the distributed pressure on the edge of the plate in contact with a rigid body of the given shape under the conditions that the part of contact of the edge, on loading, deforms according to the shape of the rigid body, and the other point of the edge is free from any constraint, while the extent of the part of contact must be determined according to the resultant of the distributed pressure working on the edge through the rigid body. The stresses produced in the plate are, as a matter of course, obtained from the solution. The distributions of the stresses in the plate are also interesting because the fact is known in the case of isotropic material that the point with the maximum shearing stress is at a certain depth from the edge of contact and the difference of the distributions of the stresses will show the mechanical features peculiar to orthotropic material.

**2. Preliminaries.** Using the complex functions of the modified complex variables $z_r = x + i\,k_r\,y$ and $z_s = x + i\,k_s\,y$ together with

$$k_r^2\,k_s^2 = \frac{E_x}{E_y}\,, \qquad k_r^2 + k_s^2 = \frac{E_x}{G_{xy}} + 2\nu_{xy}\,, \qquad \text{for generalized plane stress,}$$

$$k_r^2\,k_s^2 = \frac{1-\nu_{yz}\,\nu_{zy}}{1-\nu_{xz}\,\nu_{zx}}\cdot\frac{E_x}{E_y}\,, \qquad k_r^2 + k_s^2 = \frac{1}{1-\nu_{xz}\,\nu_{zx}}\left\{\frac{E_x}{G_{xy}} + 2(\nu_{xy} - \nu_{xz}\,\nu_{zy})\right\}\,,$$

for plane strain,

we can express the Airy's stress function in the theory of orthotropic elasticity in the form

$$\chi(x, y) = Z_r(z_r) + Z_s(z_s) + comp.\ conj.$$

Two sets of rectangular coordinate axes are taken as shown in Fig. 1; one of them is set so that the axis of $x$ may coincide with the direction of the axis of symmetry of orthotropic elasticity, and the other so that the axis of $Y$ may coincide with the straight edge of the plate, the angle between the axes of $x$ and $X$ being represented by $\phi$. Then, taking as our physical plane the left half $(X \leq 0)$ of the coordinate plane $XY$, we can obtain the stress function from

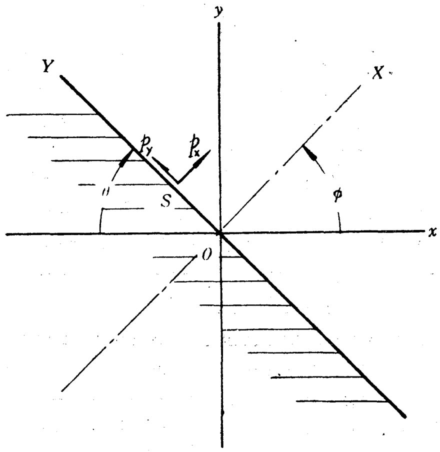

Fig. 1. The coordinate axes and the plate.

$$Z_r''(z_r) = \frac{-1}{2\pi\,(k_r - k_s)\,N_r}\int_{-i\infty}^{i\infty}\frac{M_s\,p_X + N_s\,p_Y}{\mathfrak{w} - \zeta_r}\,d\mathfrak{w} \tag{1}$$

and $Z_s''(z_s)$ of the similar type, the expression of which is obtained by interchanging the suffices $r$, $s$ each other in the expression $Z_r''(z_r)$. Here $Z_r''$ and $Z_s''$ represent the derivatives of $Z_r$ and $Z_s$ with respect to $z_r$ and $z_s$ respectively as

$$Z_r'' \equiv \frac{d^2}{dz_r^2}\,Z_r(z_r)\,, \qquad Z_s'' \equiv \frac{d^2}{dz_s^2}\,Z_s(z_s)\,,$$

and we shall call them simply *stress function.*

$p_X$, $p_Y$ in the integral are the tractions on the edge per unit depth of the plate in the directions of the axes of $X$ and $Y$ respectively, which are generally considered the functions of $S$, the value of coordinate $Y$ on the compressed edge (you could hardly be confused with this notation although we shall sometimes mention the compressed edge itself by $S$).

The complex variable $\zeta_r$ represents an arbitrary point in our physical region (see Fig. 2) and is transformed back into $z_r$ by

$$z_r = -i\, N_r\, \zeta_r. \tag{2}$$

In particular, the value of $\zeta_r$ on the edge is denoted with $\mathfrak{w}$, i.e.,

$$[\zeta_r]_\Sigma = \mathfrak{w},$$

where we can drop the suffix of $\mathfrak{w}$ because

$$[\zeta_r]_\Sigma = [\zeta_s]_\Sigma = i\, Y,$$

as derived from Eq. (2). Here $\zeta_s$ is again connected with $z_s$ by

$$z_s = -i\, N_s\, \zeta_s. \tag{2'}$$

The remaining notations in the expression (1) and in Eqs. (2), (2′) are all related to the angle $\phi$ and

$$M_r = \cos\phi + i\, k_r \sin\phi, \qquad N_r = -\sin\phi + i\, k_r \cos\phi.$$

The similar notations with the suffix $s$ in place of $r$ are used.

$$M_s = \cos\phi + i\, k_s \sin\phi, \qquad N_s = -\sin\phi + i\, k_s \cos\phi.$$

When the extent of the compressed edge is assumed $2b$, we can assign the value

$$-b \leq S \leq b$$

to $S$. Then, $p_X$ and $p_Y$ vanish on the remaining part of the edge and the infinite integral in the expression (1) is reduced to the definite integral

$$\int_{-ib}^{ib} \frac{M_s\, p_X + N_s\, p_Y}{\mathfrak{w} - \zeta_r}\, d\mathfrak{w}. \tag{3}$$

**3. Evaluation of the integral.** Putting

$$-\frac{S}{b} = \cos\Psi, \qquad 0 \leq \Psi \leq \pi. \tag{4}$$

and using the $\Psi$, we can express $p_X$ and $p_Y$, which are the unknown functions of $S$, in the forms of the following series,

$$\left.\begin{aligned} p_X &= \sum_{n=0}^{\infty} A_n \frac{\cos n\Psi}{\sin\Psi}, \\ p_Y &= \sum_{n=0}^{\infty} B_n \frac{\cos n\Psi}{\sin\Psi}, \end{aligned}\right\} \quad (|S| \leq b)$$
$$p_X = p_Y = 0, \qquad (|S| > b) \tag{5}$$

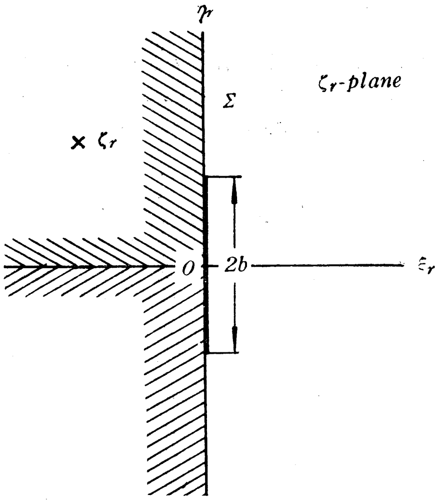


Fig. 2.
Transformation $z_r = -i\, N_r\, \zeta_r$.

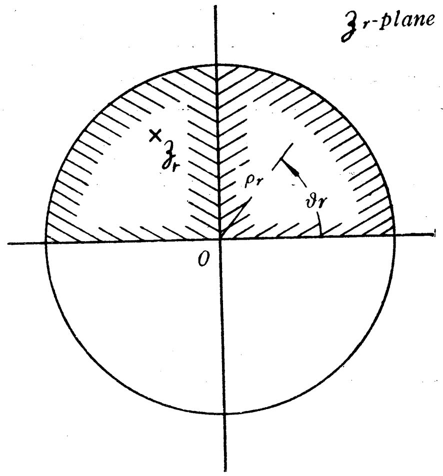


Fig. 3.
Transformation $2i\, \zeta_r / b = \mathfrak{z}_r + 1/\mathfrak{z}_r$.

where $A_n$ and $B_n$ $(n = 0, 1, 2, \cdots)$ are to be determined so as to satisfy the condition that the displacement of the compressed edge of contact must conform to the shape of the rigid body in contact.

Substituting the expressions (5) in the integral (3), we obtain

$$Z_r'' = \frac{A_n M_s + B_n N_s}{2\pi\,(k_r - k_s)\,N_r} \cdot \frac{1}{2}\int_{-\pi}^{\pi} \frac{\cos n\Psi}{\cos\Psi - i\,\zeta_s/b}\, d\Psi \,. \tag{6}$$

In order to evaluate this integral by the residues, we put again

$$\frac{2i\,\zeta_r}{b} = \mathfrak{z}_r + \frac{1}{\mathfrak{z}_r}\,. \tag{7}$$

Then, with

$$\zeta_r = \xi_r + i\,\eta_r\,, \qquad \mathfrak{z}_r = \rho_r\, e^{i\vartheta_r}\,,$$

we get from the transformation (7)

$$\frac{2\xi_r}{b} = \left(\rho_r - \frac{1}{\rho_r}\right)\sin\vartheta_r\,,$$

$$-\frac{2\eta_r}{b} = \left(\rho_r + \frac{1}{\rho_r}\right)\cos\vartheta_r\,.$$

As known from these equations, the unit circle $\Gamma$ in the plane of $\mathfrak{z}_r$ is the image of the segment on the axis of $\eta_r$ on the plane of $\zeta_r$ and the coordinates of the points on the segment are

$$\xi_r = 0\,, \qquad \eta_r = -b\,\cos\vartheta_r\,.$$

Using these values in $-i\,N_r\,\zeta_r = z_r$, we obtain

$$y = -x \cot\phi\,, \qquad (|y| \leq b \cos\phi)$$

or referrring to the coordinates $X$, $Y$

$$X = 0\,, \qquad (|Y| \leq b)\,.$$

The equations, needless to say, represent the line of the edge of the plate. The same result is derived from

$$\frac{2i\,\zeta_s}{b} = \mathfrak{z}_s + \frac{1}{\mathfrak{z}_s}$$

too. That is, the unit circle of the plane of $\mathfrak{z}_s$ also corresponds to the same segment on our physical plane and the identities exist

$$[\vartheta_r]_\Gamma = [\vartheta_s]_\Gamma = \Psi$$

$$[\mathfrak{z}_r]_\Gamma = [\mathfrak{z}_s]_\Gamma \equiv \mathfrak{z}\,, \quad \text{say}$$

and $\mathfrak{z} = e^{i\Psi}$.

Since the real axis on the plane of $\mathfrak{z}_r$ is represented by

$$\vartheta_r = 0\,,$$

it corresponds to $\xi_r = 0$ on the plane of $\zeta_r$, but it is confined only to the part $|\eta_r| > b$ of the axis of $\eta_r$, as known from

$$-\frac{2\eta_r}{b} = \rho_r + \frac{1}{\rho_r}\,.$$

Moreover, the correspondence between the regions of the planes $\zeta_r$ and $\mathfrak{z}_r$ becomes as follows:

| | | | |
|---|---|---|---|
| When | $0 \leq \vartheta_r \leq \frac{\pi}{2}$, | $\xi_r \lesseqgtr 0$ | $\eta_r < 0$; |
| | $\frac{\pi}{2} \leq \vartheta_r \leq \pi$, | " | $\eta_r > 0$; |
| | $\pi \leq \vartheta_r \leq \frac{2}{3}\pi$, | $\xi_r \gtreqless 0$ | $\eta_r > 0$; |
| | $\frac{3}{2}\pi \leq \vartheta_r \leq 2\pi$, | " | $\eta_r < 0$; |

according as $\rho_r \lesseqgtr 1$.

We can now evaluate the integral (6). Using the relation $\mathfrak{z} = e^{i\Psi}$ on $\Gamma$ the integral (6) is transformed into

$$\oint_\Gamma \frac{\mathfrak{z}^n + \mathfrak{z}^{-n}}{(\mathfrak{z} - \mathfrak{z}_1)(\mathfrak{z} - \mathfrak{z}_2)}\, d\mathfrak{z}\,,$$

of which the integrand has three singular points

$$0, \qquad \mathfrak{z}_1 = \frac{i\,\zeta_r}{b} + \sqrt{\left(\frac{i\,\zeta_r}{b}\right)^2 - 1}\,, \qquad \mathfrak{z}_2 = \frac{i\,\zeta_r}{b} - \sqrt{\left(\frac{i\,\zeta_r}{b}\right)^2 - 1}\,.$$

The singular point $\mathfrak{z}_1$, however, is outside the circle $\Gamma$ when $\zeta_r$ is on the left half of the plane. Thus, evaluating the residues we obtain

$$Z_r'' = \frac{-1}{2\,(k_r - k_s)\,N_r} \sum_{n=0}^{\infty} (A_n M_s + B_n N_s) \left\{\frac{i\,\zeta_r}{b} - \sqrt{\left(\frac{i\,\zeta_r}{b}\right)^2 - 1}\right\}^n \Big/ \sqrt{\left(\frac{i\,\zeta_r}{b}\right)^2 - 1}\,. \tag{8}$$

The care must be exercised with reference to the case where $\zeta_r$ is just on $\Gamma$. In such a case as both the points $\mathfrak{z}_1$ and $\mathfrak{z}_2$ are on the circle, you may obtain, their residues cancelling each other, the result given in the usual table of definite integrals. Since the integral is discontinuous on the line $|\eta_r| \leq b$, the result is naturally the mean value of both the integrals on the circle, one of which is the value at the interior point and the other at the exterior point adjacent to the periphery of the circle.

We should, however, adopt the value at the interior point adjacent to the periphery of the circle, the inside of which has been taken as our physical region, and should not take the mean value given in the usual table of definite integrals. Eq. (8) ought to therefore be used always. On the part of contact where $-S/b = \cos\Psi$ for $0 \leq \Psi \leq \pi$, of course, we can simplify more or less the equation as

$$Z_r'' = \frac{i}{2\,(k_r - k_s)\,N_r} \sum_{n=0}^{\infty} (A_n M_s + B_n N_s) \frac{e^{-in\Psi}}{\sin\Psi}\,. \tag{8'}$$

The function $Z_r'$, or $dZ_r(z_r)/dz_r$, necessary for the calculation of displacements (*displacement function,* say) is

$$Z_r' = \frac{-b}{2(k_r - k_s)} \left[ (A_0 M_s + B_0 N_s) \ln\left\{\frac{i\,\zeta_r}{b} - \sqrt{\left(\frac{i\,\zeta_r}{b}\right)^2 - 1}\right\} + \sum_{n=1}^{\infty} \frac{A_n M_s + B_n N_s}{n} \left\{\frac{i\,\zeta_r}{b} - \sqrt{\left(\frac{i\,\zeta_r}{b}\right)^2 - 1}\right\}^n \right]. \tag{9}$$

On the part of contact

$$Z_r' = \frac{-b}{2(k_r - k_s)} \left[ -(A_0 M_s + B_0 N_s)\, i\,\Psi + \sum_{n=1}^{\infty} \frac{A_n M_s + B_n N_s}{n} e^{-in\Psi} \right]. \tag{9'}$$

**4. Expression of displacements.** Leaving out of the displacement and the rotation of the plate as a whole,

$$-2G_{xy}\,u_x = (1 + K_r)(Z_r' + \bar{Z}_r') + (1 + K_s)(Z_s' + \bar{Z}_s')\,,$$
$$-2G_{xy}\,u_y = i\,k_r\,(1 - K_r)(Z_r' - \bar{Z}_r') + i\,k_s\,(1 - K_s)\,(Z_s' - \bar{Z}_s')$$

are the expressions of the displacements in the directions of the axes $x$ and $y$ respectively, where

$$K_r = -K_s \equiv (k_r^2 - k_s^2)\frac{G_{xy}}{E_x},$$

for generalized plane stress,

$$K_r = -K_s \equiv (1-\nu_{xz}\,\nu_{zx})(k_r^2 - k_s^2)\frac{G_{xy}}{E_x},$$

for plane strain.

Referring to the axes $X$ and $Y$, the displacements are

$$-2G_{xy}\,u_X = (M_r + K_r\overline{M}_r)\,Z_r' + (M_s + K_s\overline{M}_s)\,\overline{Z}_s' + comp.\ conj.,$$

$$-2G_{xy}\,v_Y = (N_r + K_r\overline{N}_r)\,Z_r' + (N_s + K_s\overline{N}_s)\,\overline{Z}_s' + comp.\ conj.$$

By use of Eq. (9′) we get the expressions of the displacements of the compressed edge

$$\left.\begin{aligned} 2G_{xy}\,u_X &= \frac{2b\,K_r}{k_r-k_s}\sum_{n=1}^{\infty}\frac{1}{n}\left\{\begin{matrix}(\sin^2\theta + k_r k_s\cos^2\theta)A_n \\ -(1-k_r k_s)\sin\theta\cdot\cos\theta\cdot B_n\end{matrix}\right\}\cos n\Psi \\ &\quad -b\left(1-\frac{k_r+k_s}{k_r-k_s}K_r\right)\left\{B_0\Psi + \sum_{n=1}^{\infty}\frac{1}{n}B_n\sin n\Psi\right\}, \\ 2G_{xy}\,u_Y &= \frac{2b\,K_r}{k_r-k_s}\sum_{n=1}^{\infty}\frac{1}{n}\left\{\begin{matrix}(\cos^2\theta + k_r k_s\sin^2\theta)B_n \\ -(1-k_r k_s)\sin\theta\cdot\cos\theta\cdot A_n\end{matrix}\right\}\cos n\Psi \\ &\quad +b\left(1-\frac{k_r+k_s}{k_r-k_s}K_r\right)\left\{A_0\Psi + \sum_{n=1}^{\infty}\frac{1}{n}A_n\sin n\Psi\right\}, \\ &\qquad 0\leq\Psi\leq\pi. \end{aligned}\right\} \quad (10)$$

which are to conform with the shape of the rigid body given beforehand as the known functions $u_X(S)$ and $v_Y(S)$. Consequently we can determine $A_n$ and $B_n$. In the expressions another notation $\theta$ is used for convenience' sake without using $\phi$ and is the angle between the axes $x$ and $Y$ as shown in Fig. 1. The distributed pressures $p_X(S)$ and $p_Y(S)$ as well as the stresses produced in the plate can be calculated by using $A_n$ and $B_n(n = 0, 1, 2, \cdots)$.

For all that, it may not be simple procedure strictly to solve the problem, because the expressions (10) are not the usual forms of Fourier series. It is rather advisable to seek to obtain the approximate solution with the several top terms. Some examples are shown in the following paragraph.

**5. Some examples.**

a) *The case,* $A_0 \neq 0$ *and all the remaining vanish*:

In the first place we get

$$u_X = 0$$

on the part of the edge of contact, and it corresponds to the case where the rigid beam of width $2b$ is pressed against the edge of the plate.

The distribution of the reacting pressure is

$$p_X = \frac{A_0}{\sin\Psi} = \frac{b\,A_0'}{\sqrt{b^2 - S^2}}\,,$$

or using the total force working through the rigid beam, viz.,

$$P_X = \int_{-b}^{b} p_X\,dS = b\,A_0\,\pi\,, \qquad (11)$$

it is expressed as

$$p_X = \frac{P_X}{\pi\sqrt{b^2 - S^2}} \qquad (12)$$

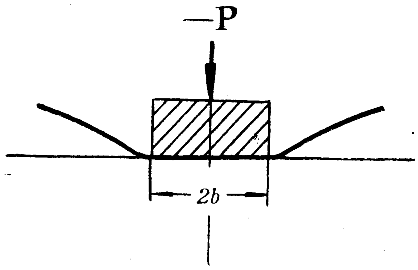

Fig. 4. Plate compressed on its edge with a rigid die of width $2b$.

where the pressure has positive value when directed along the axis $X$.

On the other hand,

$$p_Y = 0\,,$$

that is to say, there is no frictional force along the edge, and the displacement is

$$2G_{xy}\,v_Y = \left(1 - \frac{k_r + k_s}{k_r - k_s}K_r\right)\frac{P_X}{\pi}\cos^{-1}\left(\frac{-S}{b}\right)\,. \qquad (13)$$

If we can expect friction persistent enough to keep the edge from slipping sideways between the surfaces compressed together,

$$v_Y = 0\,, \qquad (|S| \leq b)\,.$$

Consequently $B_n \neq 0$ and from

$$\left(1 - \frac{k_r + k_s}{k_r - k_s}K_r\right)\frac{P_X}{\pi}\Psi = -\frac{2b\,K_r}{k_r - k_s}\sum_{n=1}^{\infty}\frac{1}{n}(\cos^2\theta + k_r k_s \sin^2\theta)\,B_n\cos n\Psi\,, \qquad (14)$$

we can determine the values of $B_n (n = 1, 2, \cdots)$. Then, the distribution of $p_Y$ can be evaluated with (5).

b) *The case, $A_1 \neq 0$ and all other coefficients vanish*:

We have under these conditions

$$2G_{xy}\,u_X = \frac{-2A_1 K_r}{k_r - k_s}(\sin^2\theta + k_r k_s \cos^2\theta)\cdot S$$

$$= \frac{4K_r T}{(k_r - k_s)\,\pi\,b^2}(\sin^2\theta + k_r k_s \cos^2\theta)\cdot S \qquad (15)$$

and

$$p_X = \frac{A_1\cos\Psi}{\sin\Psi} = \frac{2T\,S}{\pi\,b^2\sqrt{b^2 - S^2}} \qquad (16)$$

where $T$, the torque exerted clockwise, is determined as

$$T = \int_{-b}^{b} S \cdot p_X \, dS = -\frac{\pi b^2 A_1}{2} .$$

This is the case illustrated in Fig. 5.

We have infinite pressures at $|S| = b$ in the above two cases as known from Eq. (5). Combining the terms of $A_0$ and $A_2$, or $A_1$ and $A_3$ adequately, we obtain the solutions in the cases where the pressure is finite at the ends of the part of contact.

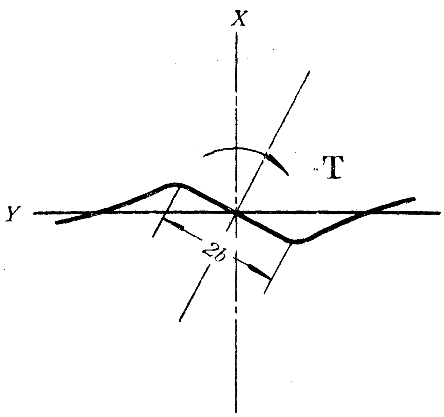

Fig. 5. Plate twisted on its edge.

c) *The case $A_2 = -A_0$ and all the remaining vanish*:

$$2G_{xy} u_X = \frac{b A_2 K_r}{k_r - k_s} (\sin^2 \theta + k_r k_s \cos^2 \theta) \left(\frac{2S^2}{b^2} - 1\right)$$

or leaving out of the term not containing $S$ which is the displacement of the plate as a whole,

$$2G_{xy} u_X = \frac{2b A_2 K_r}{k_r - k_s} (\sin^2 \theta + k_r k_s \cos^2 \theta) \left(\frac{S^2}{b^2}\right) \tag{17}$$

and

$$p_X = -2A_2 \sqrt{1 - S^2/b^2}, \qquad p_Y = 0,$$

$$P_X = \int_{-b}^{b} p_X \, dS = b A_2 \pi .$$

If we approximate the form of the rigid bolt of radius $R$ to such a parabolic curve as Eq. (17),

$$u_X \doteqdot S^2/2R$$

is derived. Consequently, the extent of the compressed part is

$$2b = 2\sqrt{\frac{\sin^2 \theta + k_r k_s \cos^2 \theta}{k_r - k_s} \cdot \frac{2b A_2 K_r R}{G_{xy}}}$$

$$= 2\sqrt{\frac{\sin^2 \theta + k_r k_s \cos^2 \theta}{k_r - k_s} \cdot \frac{2K_r R P_X}{\pi G_{xy}}} , \tag{18}$$

that is, the compressed part of the edge is proportional both to the square roots of the total load $P_X$ and of the radius $R$ of the bolt.

The maximum pressure is at the middle point of the part of contact and

$$p_{X,\max} = 2A_0 = \frac{4}{\pi} p_{X,\text{mean}}$$

$$\doteqdot 1.27 p_{X,\text{mean}} \tag{19}$$

$p_{X,\,\mathrm{mean}}$ being $P_X/2b$.

Using the coefficients

$$A_2 = -A_0 = P_X/\pi b\,, \qquad A_n = 0 \qquad (n = 1, 3, 4, \cdots)$$
$$B_n = 0 \qquad (n = 0, 1, 2, 3, \cdots)$$

and referring to Eq. (8), we can evaluate the stresses

$$\sigma_X = N_r^2 Z_r'' + N_s^2 Z_s'' + comp.\ conj.,$$
$$\sigma_Y = M_r^2 Z_r'' + M_s^2 Z_s'' + comp.\ conj.,$$

and $$\tau_{XY} = -M_r N_r Z_r'' - M_s N_s Z_s'' - comp.\ conj..$$

The distributions of the stresses produced in the plate, in the cases

$$\theta = 0^\circ, \quad (\phi = 90^\circ)$$
$$\theta = 90^\circ, \quad (\phi = 0^\circ)$$

and $$\theta = 45^\circ, \quad (\phi = 45^\circ),$$

are shown in Figs. 7 – 10 perspectively; some of them are compared with those of the isotropic plate in Fig. 11, the broken lines in which are the distributions of the stresses in the isotropic plate due to a concentrated load on the middle point of the edge.

The peak of the shearing stress, $\tau_{xy,\,\max}$, attracting our particular attention, occurs

at $X \doteqdot 0.35b$, $Y \doteqdot \pm 0.9b$ in the case $\theta = 0^\circ$,

at $X \doteqdot 0.6b$, $Y \doteqdot \pm 0.9b$ in the case $\theta = 90^\circ$,

or at $X = 0$, $Y \doteqdot -0.96b$ in the case $\theta = 45^\circ$,

and amounts to nearly

$$\tau_{xy,\,\max} = 0.345\,\frac{2P}{\pi b}, \quad 0.167\,\frac{2P}{\pi b} \quad \text{or} \quad -0.335\,\frac{2P}{\pi b}$$

respectively.

The point where the maximum shearing stress is to occur in the isotropic plate is shifted a little from on the center line as found in Fig. 10 and there it does not take the greatest value although there is surely a peak.

A similar shifting from on the center line is also found in Fig. 9 as to the ridge of the distribution of the normal stress $\sigma_X$. The ridge has the tendency to run along the fibers of the plate as if every fiber were a net perpendicular to the plane of the plate to resist to flowing of water, as it were, poured into the field of the plate from the part of contact.

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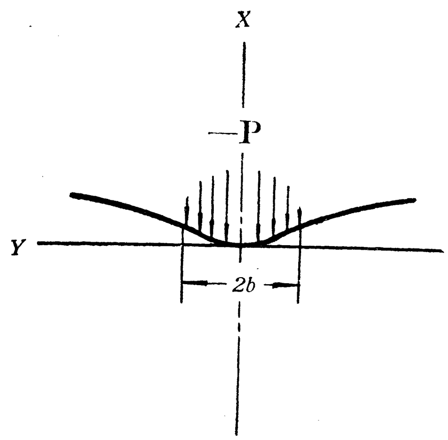


Fig. 6. Plate compressed on its edge with a rigid bolt.

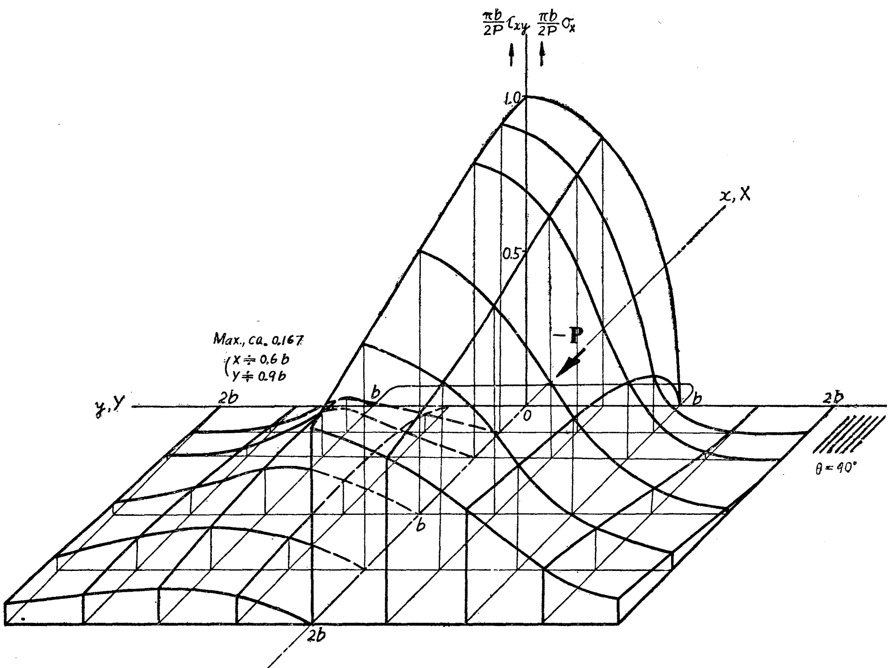


Fig. 7. Perspective view of distributions of generalized plane stresses $\sigma_X$ and $\tau_{xy}$ in Compreg when the edge of the plate is perpendicular to the direction of the fibers.
$E_x = 1{,}794\ \mathrm{kg/mm^2}$, $E_y = 366.6\ \mathrm{kg/mm^2}$, $G_{xy} = 219.9\ \mathrm{kg/mm^2}$, $\nu_{xy} = -0.497$, $k_r = 2.530$, $k_s = 0.874$.

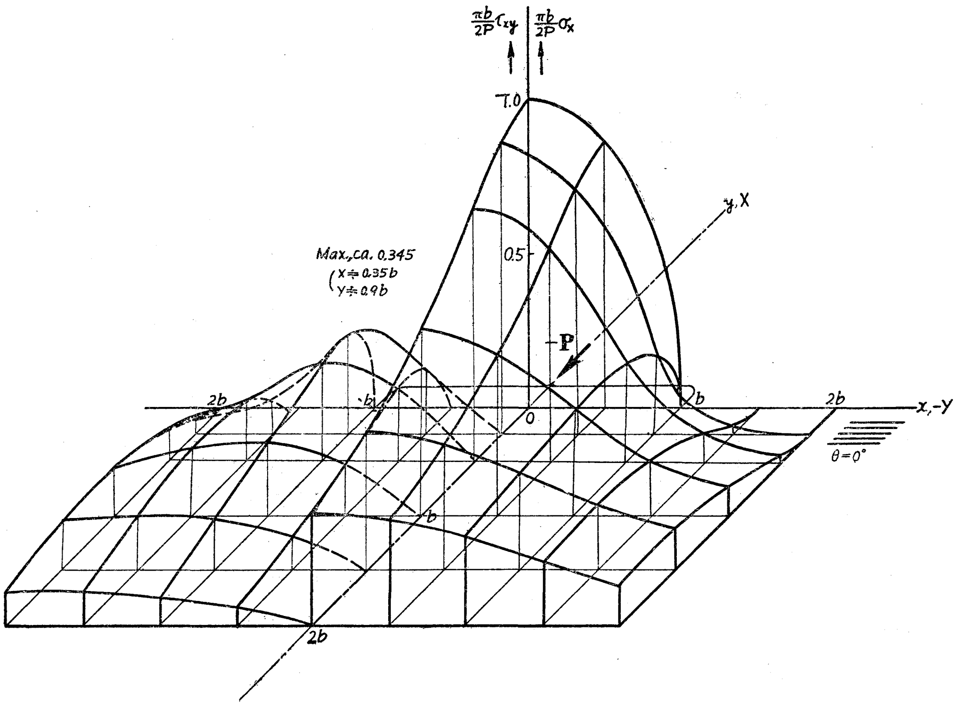


Fig. 8. Perspective view of distributions of generalized plane stresses $\sigma_X$ and $\tau_{xy}$ in Compreg when the edge of the plate is parallel to the direction of the fibers. The elasticity constants are the same as those mentioned in the preceding figure..

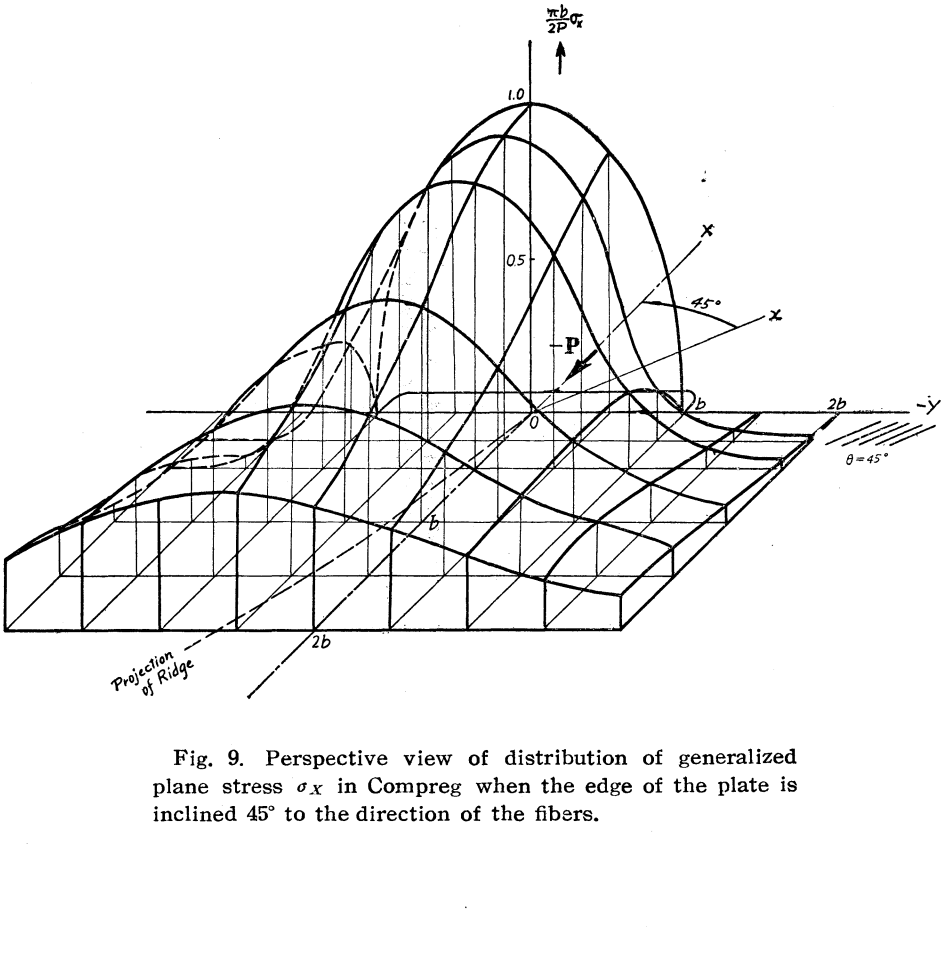


Fig. 9. Perspective view of distribution of generalized plane stress $\sigma_X$ in Compreg when the edge of the plate is inclined 45° to the direction of the fibers.

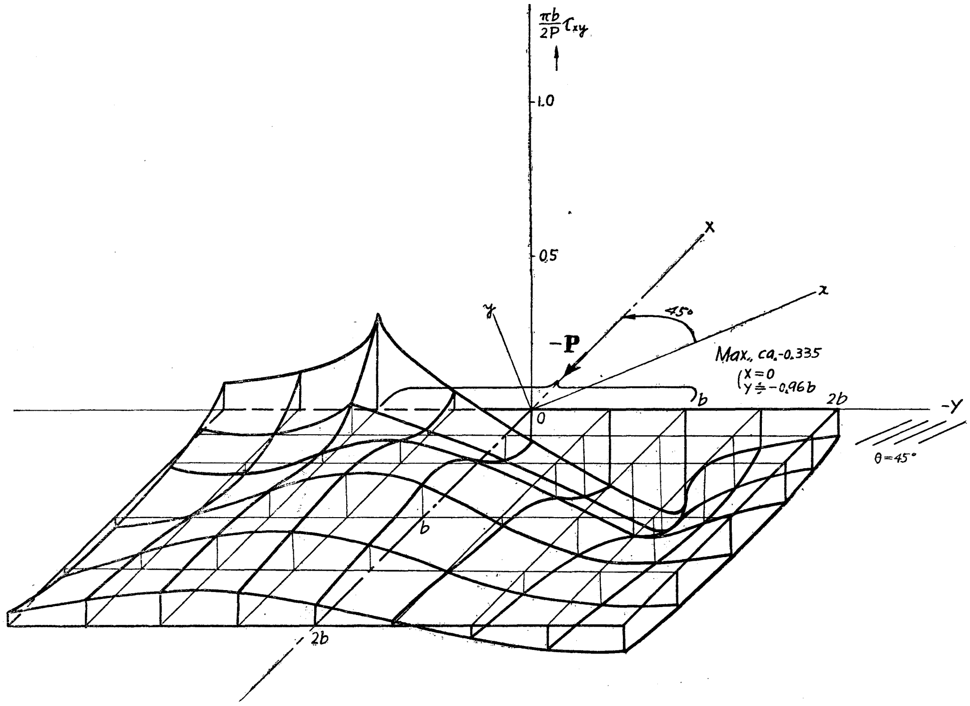


Fig. 10. Perspective view of distribution of generalized plane stress $\tau_{xy}$ in Compreg when the edge of the plate is inclined 45° to the direction of the fibers.

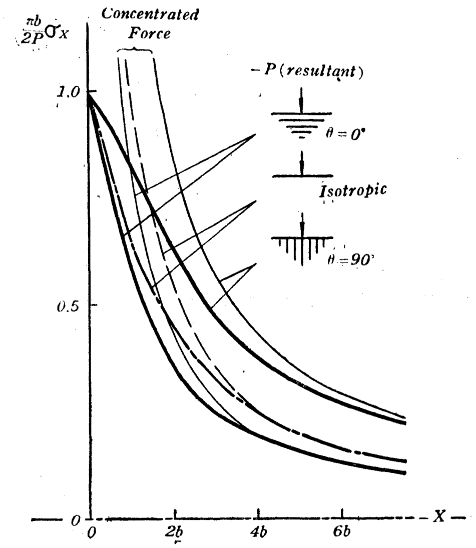


Fig. 11. Diminishing of $\sigma_X$, stress normal to the edge of the plate, along the axis $X$.