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ON THE DESIGN OF FRAMES UNDER CONSIDERATION OF YIELD SECTIONS

By Fukuhei TAKABEYA

This note deals with the special design of frames already subjected to the stresses after yielding.

There will be noted the author's opinion such that for the materials near the yield sections there must be considered different values of elastic modulus from the other parts of the structure; and the relations between the coefficient of end moments and that of capacity loads are investigated.

1. Introduction. The author investigated formerly[1] from both experimental and theoretical standpoints, which story should be most severely damaged in a tall building frame under the action of seismic disturbances, especially by the horizontal movement of the foundation bed. The author divided at that time the state of the damage into two classes: damage of the first order and damage of the second order. The former was taken to mean such that structures do not suffer from injury to the main frames but the cracks appear merely in the concrete walls. This failure of walls in the damage of the first order tends to occur in the story where column deflection has the maximum value and this story belongs most probably to the second or third layer in the ordinary building frames.

Damage of the second order is such a case as that when not only are the walls shaken down but columns and girders are also destroyed.

In such a severe damage, failures in the main frames seemed to occur at the points where the fibre stresses and shearing stresses are at their maximums as obtained by ordinary statical calculation, and even if the free vibration period of a building frame is smaller than that of the earthquake, damage seemed to extend to girders in the upper stories as well as to the fixed ends of the lowest columns, depending upon the stiffness ratio of girders and columns.

About ten years afterward, the author had a chance to inspect the damages of the ferro-concrete buildings of Nagasaki University destroyed by an atomic bomb and wondered then if there were the similarity of the state of the damages. The destroyed positions and features which the bomb had given were of something similar to those of the experimental seismic investigations had furnished.

[1] F. Takabeya and T. Sakai. Experimental Investigations on the Weakest Point in the Resistance of Tall Building Frames against Earthquake. Memoirs of Faculty of Engineering, Hokkaido University. Vol. 5. No. 1. November, 1938.

These ferro-concrete building frames exist having many elastic-plastic girders and columns as well as many yield hinges which might have transformed the structure into the one whose degree of redundancies has been reduced.

The actual building frames of Nagasaki University are in use at present after having given some concrete grouting to the yield sections at both ends of the girders of the first, second and third stories and lower ends of some columns near foundation beds.

This paper shows that such a building frame which is statically indeterminate, having cracks at both ends of girders furnishes a new problem to be investigated.

For materials near cracks or yield parts of the structure, it seems to have the different value of elastic modulus from those of the other part of the structure and the elastic-plastic modulus is to be supposed as a known quantity. For the known value of such elastic-plastic modulus, the capacity loads against yielding might be also determined theoretically.

Bent with Yield Section

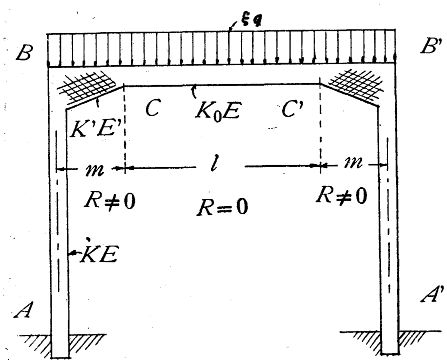

FIG. 1.

2. Frames of Yield Hinges. For the bent given in Fig. 1, parts $B-C$ and $B'-C'$ are supposed to be elastic and plastic, having values of elastic modulus E', while those of the other parts of the bent are assumed to keep the initial value of elastic modulus E, even after loadings to induce the yield hinges. By the well known Slope Deflection Method, we obtain $2EK$ times of changes in the slope of the tangent to the elastic curve at B and C respectively:

$$\varphi_b = 2EK\theta_b = \frac{\xi q}{12\gamma}\left[2m^2(3e\rho + 4\rho_0) + 6ml(e\rho + \rho_0) + l^2 e\rho\right] \tag{1}$$

and

$$\varphi_c = 2EK\theta_c = \frac{\xi q}{12\gamma}\left[2m^2(3e\rho + 4) + 6ml(e\rho + 2) + l^2(e\rho + 4)\right] \tag{2}$$

where we denote: $K = I_{ab}/l_{ab}$, in which I_{ab} = moment of inertia of the section of the member AB; l_{ab} = length of the member AB and

$$\left.\begin{aligned} &\rho_0 = K_0/K, \quad \rho = K'/K, \quad e = E'/E \\ &\gamma = e\rho(\rho_0 + 2) + 4\rho_0 \end{aligned}\right\} \tag{3}$$

For the term of deflection angle, i.e. $-6EK$ times of the deflection angle R for the part $B-C$, we get:

$$\Psi = -6EKR = -\frac{\xi q}{4\gamma}\left[m^2\left(6 + 6e\,\rho + 5\rho_0 + 4\frac{\rho_0}{e\,\rho}\right)\right.$$

$$\left. + m\,l\left(8 + 6e\,\rho + 4\rho_0 + \frac{4\rho_0}{e\,\rho}\right) + l^2(2 + e\,\rho)\right] \tag{4}$$

The joint moment at the end B of column $A\,B$ is expressed as a function of the changes in the slopes φ_b and φ_c as well as of the deflection angle Ψ as in the following:

$$M_{ba} = -M_{bc} = \frac{\xi q}{6\gamma}\left[2m^2(3e\,\rho + 4\rho_0) + 6m\,l\,(e\,\rho + \rho_0) + l^2\,e\,\rho\right] \tag{5}$$

At the fixed point A of column $A\,B$, equation (1) gives:

$$M_{ab} = 2EK\,\theta_b = \frac{\xi q}{12\gamma}\left[2m^2(3e\,\rho + 4\rho_0) + 6m\,l\,(e\,\rho + \rho_0) + l^2\,e\,\rho\right] \tag{6}$$

The joint moment at the point C is expressed as a function of φ_c and is given as follows:

$$M_{cc'} = -M_{cb} = \frac{\xi q}{6\gamma}\left[m^2\,\rho_0\,(4 + 3e\,\rho) + 3m\,l\,\rho_0\,(2 + e\,\rho) - l^2\,e\,\rho\right] \tag{7}$$

With respect to problems of common elastic frames subjected to a load q, we substitute $e = 1$ and $\xi = 1$ in equations (5) to (7) and obtain:

$$M_{ab}^{\circ} = \frac{q}{12\mu}\left[2m^2(3\rho + 4\rho_0) + 6m\,l\,(\rho + \rho_0) + l^2\rho\right] \tag{8}$$

where

$$\mu = \rho\,(2 + \rho_0) + 4\rho_0 \tag{9}$$

$$M_{ba}^{\circ} = -M_{bc}^{\circ} = \frac{q}{6\mu}\left[2m^2(3\rho + 4\rho_0) + 6m\,l\,(\rho + \rho_0) + l^2\rho\right] \tag{10}$$

$$M_{cc'}^{\circ} = -M_{cb}^{\circ} = \frac{q}{6\mu}\left[m^2\,\rho_0\,(4 + 3\rho) + 3m\,l\,\rho_0\,(2 + \rho) - l^2\,\rho\right] \tag{11}$$

For the elastic-plastic state, the author assumes that the end moment M_{bc}° of a common elastic frame given by equation (10) is effective merely by α $(\alpha < 1)$ and censequently takes $\alpha\,M_{bc}^{\circ}$ for it; while on the other hand the expression of M_{bc} by equation (5) gives a function of elastic-plastic modulus as well as a capacity load $\xi\,q$.

The coefficient ξ is to be then taken less than 1 and is required to be found to satisfy the following criterion:

$$\alpha\,M_{bc}^{\circ} = M_{bc} \tag{12}$$

From equation (12) we determine the relation between α and ξ as follows:

$$\frac{\alpha}{\xi} = \frac{\mu}{\gamma}\left[\frac{2m^2(3e\,\rho + 4\rho_0) + 6m\,l\,(e\,\rho + \rho_0) + l^2\,e\,\rho}{2m^2(3\rho + 4\rho_0) + 6m\,l\,(\rho + \rho_0) + l^2\,\rho}\right] \tag{13}$$

As a numerical calculation, given: $K = K_0 = K'/2$, $m = l/7$, $e = E'/E = 1/5$, we obtain: $\rho_0 = 1$, $\rho = 2$ and from equation (5)

$$M_{bc} = -0.0581\,\xi\, q\, l^2 \qquad \text{(a)}$$

If $E' = E$ or $e = 1$, equation (10) gives:

$$M_{bc}^{\circ} = -0.083\, q\, l^2 \qquad \text{(b)}$$

which is nothing but the bending moment at the point B for the initial state, i.e. the moment before the elastic-plastic state.

From equations (a) and (b) one notices that the moment decreases absolutely in its magnitude from $-0.083\, q\, l^2$ to $-0.0581\, q\, l^2$ $(\xi = 1)$.

Caused by the effect of elastic-plastic state, the moment at that yield sections seems to decrease the magnitude.

For the sake of safety, we consider then a certain percent β of the moment $-0.0581\, q\, l^2$, namely $-0.0581\, \beta\, q\, l^2$ $(\beta < 1)$ considered. This gives the same result as assumed to be resistant to a smaller load ξq $(\xi < 1)$; that is to say, the value $-0.0581\, \xi\, q\, l^2$ is here to be compared with the moment $-0.083\, \alpha\, q\, l^2$ $(\alpha < 1)$ which is α times of the moment magnitude before the elastic-plastic conditions, considering the following equation to be satisfied:

$$-0.083\, \alpha\, q\, l^2 = -0.0581\, \xi\, q\, l^2$$

This equation gives;

$$\frac{\alpha}{\xi} = \frac{0.0581}{0.083} = 0.7 \qquad \text{(c)}$$

If there is assumed: $\alpha = 5/8$ in the same problem above treated, then equation (c) gives $\xi = 0.89$ and the capacity load is given by $0.89\, q$. If assumed $\alpha = 1/2$, we obtain from equation (c): $\xi = 0.71$.

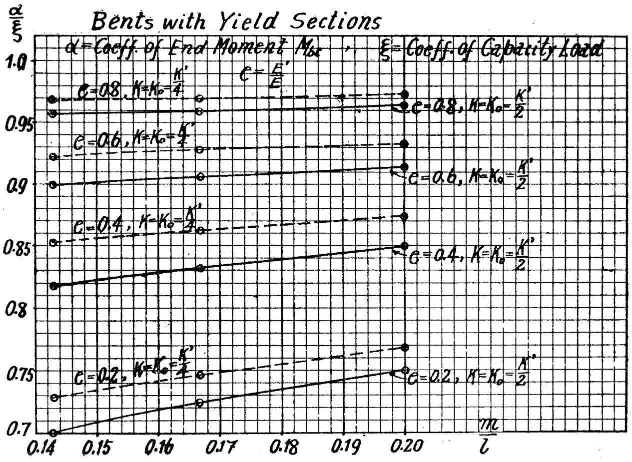

FIG. 2.

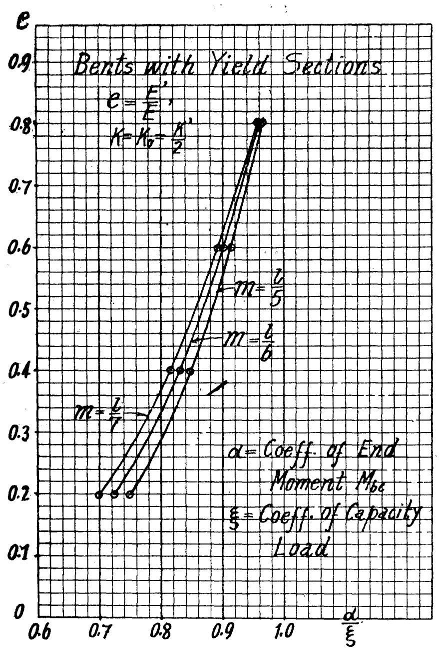

FIG. 3.

TABLE I. Bents with Yield Sections. Calculation Results of α/ξ.
$K = K_0 = K'/2$ $\qquad$ $e = E'/E$

e	$m=l/5$	$m=l/6$	$m=l/7$
1/5	0.75	0.723	0.70
2/5	0.848	0.831	0.817
3/5	0 914	0.905	0.897
4/5	0.963	0.959	0.956

TABLE II. Bents with Yield Sections. Calculation Results of α/ξ.
$K = K_0 = K'/3$ $\qquad$ $e = E'/E$

e	$m=l/5$	$m=l/6$	$m=l/7$
1/5	0.756	0.732	0.711
2/5	0.860	0.846	0.834
3/5	0.925	0.917	0.911
4/5	0.968	0.965	0.962

In a similar way, numerical calculations were performed based upon the method proposed; these calculation results are tabulated in Tables I ~ III and illustrated in Figs. 2 and 3.

Fig. 2 shows the relation between α/ξ and m/l; there are illustrated eight different kinds of curved lines, of which, for example, the first line below in the figure shows the case of $e = 0.2$, $K = K_0 = K'/2$, while the second line there shows the case of $e = 0.2$, $K = K_0 = K'/4$; from these two different lines we can find the intermediate case of $e = 0.2$, $K = K_0 = K'/3$; in a similar way, from the third and fourth ones there will be traced the case of $e = 0.4$, $K = K_0 = K'/3$.

Similarly, in Fig. 3 we see the relation between α/ξ and e (e shows the ratio of E'/E), there are shown three different cases: (1) $m = l/5$, $K = K_0 = K'/2$; (2) $m = l/6$, $K = K_0 = K'/2$; (3) $m = l/7$, $K = K_0 = K'/2$; from these lines we can trace other many lines of intermediate cases likewise.

These two figures give values of α/ξ and assuming the value of α, we can work out the coefficient of capacity load required.

3. Summary and Conclusions. In this note there are treated bent designs that consider different values of elastic modulus for materials near yield sections of a structure and the capacity load against the yielding are investigated.

For the bent shown in Fig. 1, assuming $K = K_0 = K'/2$, $e = E'/E = 1/5$ and length $m = l/7$, one obtains: $\alpha/\xi = 0.70$ for the moment M_{bc}°, in which α denotes the co-

TABLE III. Bents with Yield Sections.
Calculation Results of α/ξ.
$K = K_0 = K'/4 \qquad e = E'/E$

e	$m=l/5$	$m=l/6$	$m=l/7$
1/5	0.768	0.747	0.728
2/5	0.874	0.862	0.852
3/5	0.934	0.928	0.922
4/5	0.973	0.970	0.968

efficient of utilization for the end moment and ξ the coefficient of a capacity load; the load is allowed to the limit value ξq, in order to be resistant to the moment αM_{bc}°.

If $\alpha = 5/8$ is assumed, the value $\alpha/\xi = 0.70$ from Table I gives $\xi = 0.89$ and the capacity load is $0.89\,q$. Keeping the length $m = l/7$ further, when $e = E'/E = 2/5$, gives: $\alpha/\xi = 0.817$ for the moment M_{bc}°. If $\alpha = 3/4$, we obtain $\xi = 0.92$ therefrom and the capacity load is $0.92\,q$. Here we have to notice that for the estimation of value α we must increase α with the increase of e; while if we should not increase the value α, in spite of increasing e, it might result such that one considers smaller value ξ for the capacity load required.

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