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https://doi.org/10.5109/7153280

出版情報：Reports of Research Institute for Applied Mechanics. 1 (1), pp.17-32, 1952-01. 九州大学応用力学研究所
バージョン：
権利関係：

Reports of Research Institute for Applied Mechanics
Vol. 1, No. 1, 1952

ON THE FORMS OF HEN'S EGGS

By Jun-ichi OKABE

The force acting on an egg in its passage down the oviduct is classified into three components, viz. (i) the pressure from the wall of the oviduct dilated forcibly by the egg, (ii) the thrust due to its peristaltic motion and (iii) the friction between the egg and the tube. The elastic pressure acts on the egg all over the surface tending to keep it in a definite form, while the thrust pushes it forward against the friction. Contours of eggs are constructed mathematically by means of the condition of the minimum resistance in the passage. A possibility is proved theoretically that the egg goes down the pointed end foremost in accordance with the observations.

1. Introduction. Professor Tange concluded after many observations that hen's eggs, although exceptionally a few of them come out of the mother's body the blunt end foremost, are laid without exception the pointed end foremost in the oviduct through which they go down under its secretion of albumen.(1) In the note which follows the form of eggs in the oviduct will be discussed theoretically together with some considerations of force that they are subject to.

2. Simplifications of the problem. (i) The egg is assumed to be of a constant volume while it is in motion. This assumption may appear quite misleading at first sight, because then the gradual growth of the egg in the oviduct by the secretion of albumen would not be considered at all. But we can divide the whole process of an egg's motion in the oviduct into small intervals of time, during each of which its volume may be regarded as practically constant while displaced for some distance, and a small quantity of albumen secreted during this interval will adhere to this accumulation working as lubricant.

(ii) The slow advance of an egg by the peristalsis of the oviduct will be simplified to a mean steady motion :— let us suppose that the egg is subject to stationary pressure within the oviduct and that the resultant force drives it forward with constant speed against the friction of the tube; the motion being very slow we may safely neglect the effects of windings of the path. The rotative motion of the egg about its own axis, although

(1) Tange, M. and Mimura, H., Observations on the Egg Laying in Fowls (in Japanese), *Dôbutu Gaku Zassi* (*Zool. Mag.*) 43, pp. 450 ~ 458 (1931). As to these inversions which were observed to happen with the relative frequency of about 1/4, the authors pointed out a possibility in the *uterus* after a solid shell has enveloped an egg,

this may be very effective to make a body of exact axial symmetry, seems unimportant to the formation of a so-called *egg-shape* and will be left out of consideration. The friction, on the other hand, between the egg and the wall is assumed to be like that between solid bodies, i. e. proportional to the normal component of pressure on the contact surface and independent of their relative speed. Obviously the fact is that the friction of very complex mechanism comes into play between the albumen and the soft tissue of the oviduct. However, being impossible to describe all the details mathematically since we know very little about the muscular physiology of the oviduct and considering that the egg is kept at any rate in a definite form, we may simplify the problem, without much spoiling the nature of it, by introducing this assumption.

(iii) The effect of co-existence of the yolk and the white, the *heterogeneity* of an egg in other words, and the surface tension of albumen are both ignored. The former does not evidently play any important rôle in this problem, and about the latter we can conclude that no surface tension can take place because the surface of the egg is always wetted by the albumen incessantly secreted by the wall of the oviduct.

(iv) Then, where does the force originate and how does it act that keeps an egg in a definite form and drives it forward? In order to answer this question let us imagine a model: a solid ball contained in a soft and extensible tube. We can push the ball forward by, say, narrowing the tube behind it with our fingers. Then the force acting on the solid consists of three components: (1) the thrust produced by the fingers, (2) the frictional resistance from the wall and (3) the reversing force of the elastic wall dilated forcibly by the ball. If the motion is very slow, the thrust is regarded to be balanced by the friction, while on the other hand the elastic force of the tube acts on the body all over the surface independently of the former two. Although, of cource, an egg not completely formed is not a solid but is made of jelly-like substance, we can as well classify the force from the oviduct into the thrust produced by the peristaltic motion, the frictional resistance and the reversing force of the wall.

Let us next consider the cohesion of albumen. It was pointed out that the cohesive power of albumen of the inner and the middle portion of an egg is quite sufficient to maintain the form of its own moderately well when we dissect a hen and take out an egg with no shell membrane.[1] But if we assume here that the chief agent determining the form of an egg is the interactions between the egg and the oviduct, then our attention must be necessarily concentrated on the force that the tube is exerting on the egg. The white will adhere about the yolk under this pressure *much* where the duct is easily dilated and *little* where it is not, will unite with the accumulation and will become jelly-like together. If we reason in this way, the most important factor in maintaining the egg in a definite form will be the reversing force due to elasticity of the dilated

(1) Tange and Mimura, *ibid.*

oviduct. Under this pressure, however, the egg's substance may be treated theoretically as a moderately hard lump of jelly owing to its cohesive power. Thus the cohesion of albumen will not appear explicitly in the coming calculations, but it must be noticed that it is postulated implicitly behind the theory.

Incidentally it is convenient here to summarize the discussions by Professor D. W. Thompson. His theory relates, however, as a matter of fact to the process after the shell-membrane has already enveloped an egg as it passes down a portion of the oviduct called the *isthmus*. We are afraid he supposed that the form would be indefinite before the membrane is formed (albumen would be so fluid, in other words), but in reality in the oviduct eggs were sometimes observed having no membrane but shaped distinctly already into the so-called egg-form with a pointed end and a blunt one.(1)

He writes as follows:(2) "Along the walls of the oviduct pass peristaltic waves of contraction. These muscular contractions may be described as the contractions of successive annuli of muscles, giving annular pressure to successive portions of the egg; they drive the egg forward against the frictional resistance of the tube, while tending at the same time to distort its form. From the nature and direction of motion of the peristaltic wave the pressure will be greatest somewhere behind the middle of the egg, and the simple result follows that the *anterior end of the egg becomes the broader and the posterior narrower*.(3) But that form once acquired, the egg, consisting of an extensible membrane filled with an incompressible fluid, may remain in equilibrium both as regards form and position within the tube, even after that excess of pressure on the posterior part is relieved. For, denoting by p_n the normal component of the external pressure at the point where r and r' are the radii of curvature, by T the tension of the envelope and P the internal fluid pressure, the equation of the surface is

$$p_n + T\left(\frac{1}{r} + \frac{1}{r'}\right) = P, \tag{2.1}$$

which shows that a normal pressure no greater and (within certain limits) actually less acting on the posterior part than on the anterior part of the egg will be sufficient to maintain the fluid in equilibrium. These procedures will be repeated until at last the shell is secreted about the egg, the latter having meanwhile passed on into a wide portion of the oviducal tube, called the *uterus*. Here the egg assumes its permenent form, here it becomes rigid. ..."

But we can state against him. First, as mentioned above, the fact is that the egg acquires its form before the membrane is secreted. This cannot be explained by his theory. Secondly, it is true that the excess of pressure due to the peristalsis of the oviduct exists on the posterior part

(1) Tange and Mimura, *ibid.*

(2) Thompson, D. W., *Growth and Form.* Cambridge (1917), pp. 652 ~ 669. Quoted with slight alterations.

(3) This statement is contrary to the observed fact, cf. § 1.

of the egg, but this does not necessarily mean that it distorts the form making the posterior part narrower. Such a deformation would take place actually if the egg would be very fluid, but since it is a moderately hard lump of jelly, the egg, instead of receiving this distortion, will advance as a whole escaping the grasp of the annular muscles. This is the case all the more because the contractions of the annular muscles will be no doubt very slow. Thirdly, is egg-substance really in equilibrium? The peristaltic motion is of course periodic, the motion of the egg therefore must be more or less of this nature. So we cannot help doubting but that the condition of equilibrium would be so essential in this problem as to determine the form of the egg for itself.

(v) The reversing force of the elastic wall and the excess of pressure on the posterior part of an egg due to the peristaltic motion, both of these pressures are assumed to act along the normal on the surface of the egg.

(vi) Lastly, we assume that the form of an egg is determined by the condition of the minimum resistance. According to the preceding discussions it will be quite natural to assume that the form of the egg would be determined by the interactions between the egg and the oviduct. Of various interactions conceivable we choose this condition as the most important: *the condition of the easiest passage.* Of course it is not likely that this one condition would be sufficient to determine all the details completely. The form will be subject naturally to various other conditions, physiological or of individual variety. But difficult as it is to point out these veiled conditions of less importance, let us actually construct the contour mathematically by means of this condition only, disregarding all others for the time being. And if we obtain the form successfully which is torelably well like a natural egg, the preceding considerations will be verified to a certain degree and possibility will be suggested of reducing such a complex physiological phenomenon into a simple problem of mathematics through some appropriate assumptions.

3. Mathematical Preliminaries. For mathematical convenience the solution will be constructed in two steps. In the first step we will find out a family of the forms of minimum resistance, in which l, the length of the egg, is contained as a parameter. And in the next step, of all these figures the most favourable value of l will be determined.

The axis of the egg lies along the x-axis between 0 and l, cf. Figure 1. The y-axis being taken perpendicular to it, the volume of the egg is given by

$$V = \int_0^l \pi y^2 \, dx.$$

On introducing non-dimensional lengths $\xi = x/l$ and $\eta = y/l$, we have

$$\pi l^3 \int_0^1 \eta^2 \, d\xi = V.$$

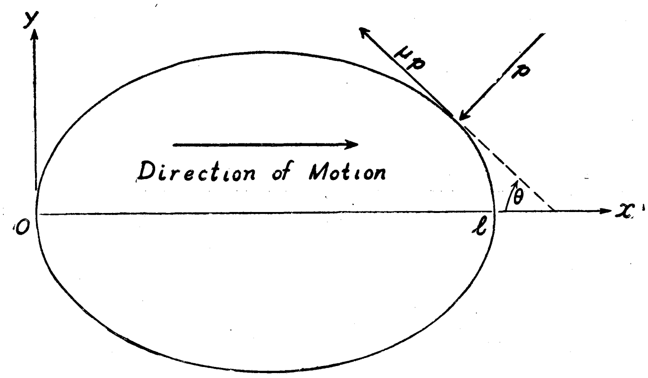


FIG. 1.

Further, in terms of λ defined by

$$\lambda = l\,(V/\pi)^{-1/3},$$

it becomes

$$\lambda^3 \int_0^1 \eta^2 \, d\xi = 1. \tag{3.1}$$

R, the total resistance of the egg, is given by

$$R = \iint \mu p \cos\theta \, dS,$$

where p is the normal component of pressure from the oviduct, μ the coefficient of friction, θ the angle indicated in the figure, and the integration is extended all over the surface whose one element is denoted by dS. But this expression can be rewritten in more accessible form

$$R = \int_0^l \mu p(x) \, 2\pi y \, dx = 2\pi\mu \left(\frac{V}{\pi}\right)^{2/3} \lambda^2 \int_0^1 \tilde{p}(\xi)\, \eta \, d\xi, \tag{3.2}$$

where $\tilde{p}(\xi) \equiv p(x)$. We have, therefore, to determine η so that the integral (3.2) may be the minimum under the constraint (3.1). This is equivalent, however, to minimize the integral

$$\int_0^1 \left\{\lambda^2 \tilde{p}(\xi)\eta - k\lambda^3\eta^2\right\} d\xi \tag{3.3}$$

under the same constraint, k being the Lagrange's indeterminate multiplier. But $\tilde{p}(\xi)$ has been left unspecified as yet. Very little being known of the muscular physiology of the oviduct, we must at this moment content ourselves with taking the model distribution simplest as possible. As stated in § 2, pressure may be classified into two parts: the reversing force of the dilated oviduct and the thrust due to its peristaltic motion.

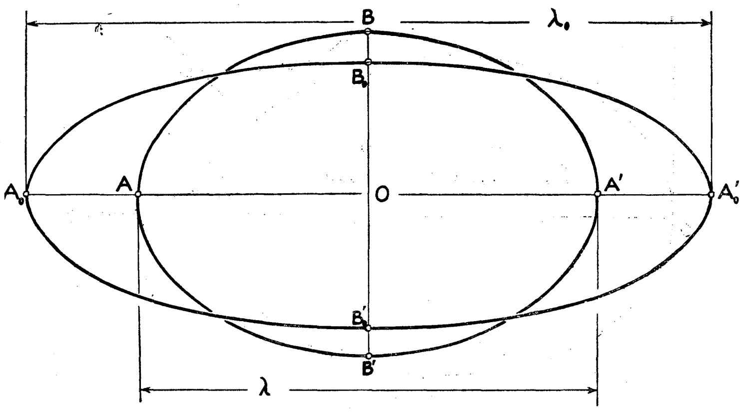


FIG. 2.

(i) The reversing force acts on the body pressing it all over the surface. Now we can imagine a certain form which is rather hypothetical: the form, when the egg assumes it the reversing pressure will be uniform everywhere on the surface. This figure may be defined otherwise as the form of an egg when it would remain unmoved from the beginning in the oviduct under the uniform pressure, and so the fore- and after-part of that form will be necessarily symmetrical. But in reality the egg in its passage cannot remain in this figure but is deformed more or less owing to the friction from the wall: the actual egg which is pushed forward against the resistance will be shorter in length in the direction of its axis when compared with this hypothetical form. (On the contrary, if the egg is subject to no resistance, no thrust is necessary and consequently no contraction will be caused.) The reversing pressure over the actual egg will be therefore no longer uniform, but additional pressure will take place on those parts which have protruded beyond that hypothetical figure, while on the other hand some pressure will be relieved over the dented surface tending to restore equilibrium. Then, what is the form of an egg when it is under the uniform pressure? Let us assume it simply as an ellipsoid of revolution.

We shall consider in more detail the elastic reversing pressure p_1 when the configuration of equilibrium A_0 B_0 A_0' B_0', whose length being l_0 or non-dimensionally $\lambda_0 = l_0(V/\pi)^{-1/3}$ is deformed into another ellipsoid of revolution A B A′ B′, whose length is l or non-dimensionally λ,[1] cf. Figure 2. From the condition of the constant volume, it must be that

[1] Strictly speaking, ABA′B′ to be considered here must be an actual form of an egg and not an ellipsoid of revolution. But as the reversing force we mean that part of pressure which is symmetrical with respect to $\xi = 1/2$ (and accordingly which will be rather independent of asymmetry of the form), so we may safely approximate the form with an ellipsoid of revolution for the sake of simplicity in the calculation.

$$B_0B'_0 = \sqrt{\frac{6}{\lambda_0}}, \quad \text{and} \quad BB' = \sqrt{\frac{6}{\lambda}}.$$

When $\lambda = \lambda_0$, p_1 must have a constant value a, say, but if the figure is deformed giving rise to the contraction $(\lambda - \lambda_0)$ along AA′ accompanied by the elongation $\{(6/\lambda)^{1/2} - (6/\lambda_0)^{1/2}\}$ along BB′, additional pressure takes place. We assume here that the additional pressure thus brought about at a point to be proportional to the displacement (i. e. the elongation or the contraction) of this point, and that this proportional factor varies from point to point. For simplicity, we put p_1 in the form proportional to $(6/\lambda_0)^{1/2}(\lambda - \lambda_0)$ at A and A′ and to $\lambda_0\{(6/\lambda)^{1/2} - (6/\lambda_0)^{1/2}\}$ at B and B′. The two multipliers $(6/\lambda_0)^{1/2}$ and λ_0 embody a simple expedient to describe the polarity of the oviduct: in our case λ_0 being greater than $(6/\lambda_0)^{1/2}$, the egg will be subject to greater pressure (positive or negative) on B and B′ than on A and A′ when the displacements at these points are the same, in other words the tube can be dilated more easily in the direction of AA′ than BB′, the easiness being assumed to be proportional to the ratio of the lengths of the major and the minor axes of the figure of equilibrium. For the points intermediate between A and B we put it in the form proportional to

$$\lambda_0\left(\sqrt{\frac{6}{\lambda}} - \sqrt{\frac{6}{\lambda_0}}\right) + 4\left(\xi - \frac{1}{2}\right)^2 \left\{\sqrt{\frac{6}{\lambda_0}}(\lambda - \lambda_0) - \lambda_0\left(\sqrt{\frac{6}{\lambda}} - \sqrt{\frac{6}{\lambda_0}}\right)\right\}:$$

a parabolic distribution symmetrical with respect to $\xi = 1/2$. In conclusion p_1 is given by

$$p_1 = a\left[1 + \omega\{f(\lambda, \lambda_0) + 4g(\lambda, \lambda_0)(\xi - 1/2)^2\}\right], \tag{3.4}$$

where

$$f(\lambda, \lambda_0) = \lambda_0\left(\sqrt{\frac{6}{\lambda}} - \sqrt{\frac{6}{\lambda_0}}\right),$$

$$g(\lambda, \lambda_0) = \sqrt{\frac{6}{\lambda_0}}(\lambda - \lambda_0) - \lambda_0\left(\sqrt{\frac{6}{\lambda}} - \sqrt{\frac{6}{\lambda_0}}\right),$$

and $\omega = b/a$. a and b are the constants, presumably dependent on the nature of the tissue of the oviduct as well as on the volume of the egg but they are assumed to remain independent of λ. The method is reserved to the later section how to determine the value of λ_0, but it must be noticed here in advance that, as was mentioned before, λ_0 will be larger more or less than the final length λ.

(ii) The thrust is produced by the peristaltic motion of the oviduct. As pointed out by Thompson this peristaltic pressure p_2 must be greater on the posterior part of the egg than on the anterior; let us take the simplest function

$$p_2 = c(1 - \xi) \qquad (c > 0) \tag{3.5}$$

to express this distribution; c being naturally a function of λ and λ_0, and when we find it is convenient to make it clear we shall write $c(\lambda, \lambda_0)$. The functional form of c can be determined as follows. We assumed in (ii) of §2 that the egg moves forward under the balance of the external forces acting upon it. The condition of equilibrium of these forces is

$$\int_0^l p(x)\, 2\pi y \frac{dy}{dx} dx \text{ (Thrust)} = \int_0^l \mu p(x)\, 2\pi y dx \text{ (Resistance)}. \quad (3.6)$$

Strictly speaking, we must substitute for y on both sides of this equation the function of the contour we are now striving for, but we may safely approximate it with the ellipsoid of revolution whose length and the volume are l and V respectively, viz.

$$y = (6/\lambda)^{1/2} (V/\pi)^{1/3} \xi^{1/2} (1-\xi)^{1/2},$$

or

$$\eta = (6/\lambda)^{1/2} \lambda^{-1} \xi^{1/2} (1-\xi)^{1/2}. \quad (3.7)$$

In (3.6) taking account of the relation

$$\int_0^1 p_1 \eta \frac{d\eta}{d\xi} d\xi = 0$$

in virtue of symmetry of p_1 and η with respect to $\xi = 1/2$, and for p_1, p_2 and η availing ourselves of their values given by (3.4), (3.5) and (3.7) respectively, we get

$$\frac{c}{\lambda}\left(\frac{3}{2\lambda}\right)^{1/2} \int_0^1 (1-3\xi+2\xi^2) d\xi = \mu \int_0^1 a \Big[\xi^{1/2}(1-\xi)^{1/2}$$

$$+ \omega \Big\{\Big\{ f \xi^{1/2}(1-\xi)^{1/2} + 4g \Big\{ \xi^{5/2}(1-\xi)^{1/2} - \xi^{3/2}(1-\xi)^{1/2}$$

$$+ \frac{1}{4} \xi^{1/2}(1-\xi)^{1/2} \Big\} \Big\}\Big\} \Big] d\xi + \mu \int_0^1 c\, \xi^{1/2}(1-\xi)^{3/2} d\xi \ .$$

Further, making use of the B-functions (*the Eulerian Integral of the First Kind*)[1] defined by

$$\int_0^1 \xi^{p-1}(1-\xi)^{q-1} d\xi = B(p, q) \qquad (p, q > 0)\ ,$$

the above equation can be rewritten symbolically as follows:

$$\frac{c}{\lambda}\sqrt{\frac{3}{2\lambda}}\ \frac{1}{6} = \mu a \Big[B\left(\frac{3}{2}, \frac{3}{2}\right) + \omega \Big\{\Big\{ f B\left(\frac{3}{2}, \frac{3}{2}\right)$$

$$+ 4g \Big\{ B\left(\frac{7}{2}, \frac{3}{2}\right) - B\left(\frac{5}{2}, \frac{3}{2}\right) + \frac{1}{4} B\left(\frac{3}{2}, \frac{3}{2}\right) \Big\} \Big\}\Big\} \Big]$$

$$+ \mu c\, B\left(\frac{3}{2}, \frac{5}{2}\right). \quad (3.8)$$

(1) Whittaker, E. T. and Watson, G. N., *A Course of Modern Analysis*. 4th Edition, Cambridge (1927), p. 253.

The necessary B-functions can be evaluated as follows:

$$B\left(\frac{3}{2},\frac{3}{2}\right)=\frac{\pi}{8},\ B\left(\frac{5}{2},\frac{3}{2}\right)=B\left(\frac{3}{2},\frac{5}{2}\right)=\frac{\pi}{16},$$

$$B\left(\frac{7}{2},\frac{3}{2}\right)=\frac{5\pi}{128}.$$

Introducing these values into (3.8), we get finally

$$p_3=c(\lambda,\lambda_0)(1-\xi)=\frac{\pi\mu a\left\{1+\omega\left(f+\frac{1}{4}g\right)\right\}}{\left(\frac{8}{3}\right)^{1/2}\lambda^{-3/2}-\frac{1}{2}\pi\mu}(1-\xi). \qquad (3.9)$$

Combining (3.4) and (3.9), we can arrive at the complete expression of the pressure distribution $\tilde{p}(\xi)$ delivered from the oviduct to the egg. Or we may put it into another form

$$\tilde{p}(\xi)=a(L+M\xi+N\xi^2), \qquad (3.10)$$

where

$$\left.\begin{aligned} L&=1+\omega\left\{f(\lambda,\lambda_0)+g(\lambda,\lambda_0)\right\}+\frac{c(\lambda,\lambda_0)}{a},\\ M&=-\left\{4\omega\, g(\lambda,\lambda_0)+\frac{c(\lambda,\lambda_0)}{a}\right\},\\ \text{and}\quad N&=4\omega\, g(\lambda,\lambda_0). \end{aligned}\right\} \qquad (3.11)$$

4. Determination of η. Since we have finished the mathematical preliminaries in the preceding section, we are now in a position to construct the contour of the egg. Our problem is one of the special examples in the calculus of variation because none of the derivatives of η are contained in the integral (3.3) which is to be minimized.

Let us put η in the form

$$\eta=\xi^{1/2}(1-\xi)^{1/2}(\alpha+\beta\xi); \qquad (4.1)$$

$\alpha\xi^{1/2}(1-\xi)^{1/2}$ represents an ellipsoid of revolution whose major axis lies between $\xi=0$ and 1, the minor axis being α, and the term of β on the other hand can be interpreted as the deviation from it. For the time being λ can be regarded as a constant in our computation, so the integral to be minimized is

$$I\equiv\int_0^1[\tilde{p}(\xi)\eta-\kappa'\eta^2]\,d\xi$$

multiplied by a constant, where $\tilde{p}(\xi)$ and η are given by (3.10) and (4.1) respectively; κ' stands for $\kappa\lambda$. We have, on writing in terms of B-functions,

$$\begin{aligned} I=a\Big[&L\alpha B\left(\frac{3}{2},\frac{3}{2}\right)+(M\alpha+L\beta)B\left(\frac{5}{2},\frac{3}{2}\right)\\ &+(N\alpha+M\beta)B\left(\frac{7}{2},\frac{3}{2}\right)+N\beta B\left(\frac{9}{2},\frac{3}{2}\right)\\ &-\kappa'\left\{\alpha^2B(2,2)+2\alpha\beta B(3,2)+\beta^2B(4,2)\right\}\Big]. \end{aligned}$$

Those functions which have not yet been evaluated in the preceding section are

$$B\left(\frac{9}{2}, \frac{3}{2}\right) = \frac{7\pi}{256}, \; B(2, 2) = \frac{1}{6}, \; B(3, 2) = \frac{1}{12}, \text{ and } B(4, 2) = \frac{1}{20}.$$

α, β and κ' are determined as usual by the equations

$$\partial I/\partial\alpha = \partial I/\partial\beta = 0$$

together with the constraint (3.1).

Thus in conclusion we obtain

$$\left.\begin{aligned} \alpha &= \frac{3}{8}\left(L - \frac{1}{8}M + \frac{5}{16}N\right)\kappa_0 , \\ \beta &= \frac{15}{32}(M + N)\kappa_0 , \\ \text{where } \kappa_0 &\equiv \frac{\pi a}{\kappa'} = \left\{\frac{3}{128}\lambda^3\left(L^2 + LM + \frac{21}{64}M^2 + \frac{15}{32}MN \right.\right. \\ &\qquad \left.\left. + \frac{45}{256}N^2 + \frac{5}{8}NL\right)\right\}^{-1/2} . \end{aligned}\right\} \tag{4.2}$$

As is shown by (3.11), L, M, and N are the functions of λ, and (4.2) above obtained defines the family of possible contours of the egg leaving λ unspecified. Our next step is therefore to find out the most favourable value of λ.

On account of the assumption of equilibrium of the external forces (3.6), we may be open to the same criticism as was mentioned in (iv) of § 2 where we summarized the theory of Thompson. But this condition which was introduced in order to determine the function $c\,(\lambda, \lambda_0)$ is not so essential in our discussion as his assumption (2.1) was, for even if we abandon (3.6), c must be positive at any rate which means β must be negative, cf. (3.11), thus necessarily the egg goes down the pointed end foremost: this is, as we shall see later, consistent with our conclusion. But no such further details as discussed in the next section will be clarified if we leave the value of c unspecified. The following, although it seems somewhat artificial, is one tentative to find out some simple relations among a number of parameters introduced and left untouched so far (c, λ_0, ω, μ, etc.).

5. Determination of η (*continued*). Properly speaking, the length of an egg must be determined by the following procedure: R (the total resistance) being given by (3.2), on substituting for $\tilde{p}(\xi)$ and η their values in (3.10), (3.11) and (4.1), (4.2) respectively, we can obtain R as a known function of λ. The most favourable value of λ (let us write λ' for shortness) can be found by the condition that it is the minimum point of R. But as a matter of fact this calculation is very troublesome so an approximate alternative will be taken.

The condition of equilibrium of forces (3.6) gives

$$R = \text{Thrust} = \int_0^l p(x) 2\pi y \frac{dy}{dx} dx = 2\pi\left(\frac{V}{\pi}\right)^{2/3} \lambda^2 \int_0^1 \tilde{p}(\xi)\eta \frac{d\eta}{d\xi} d\xi .$$

Strictly speaking, of course, (4.1) and (4.2) must be used for this η too, but the same approximation as (3.7) will be legitimate on the assumption that asymmetry between the anterior and the posterior part of the egg is not so marked. Then the integral of p_1 will be cancelled as before and we get

$$R = \pi \left(\frac{V}{\pi}\right)^{2/3} \frac{c(\lambda, \lambda_0)}{\lambda} .$$

Leaving unnecessary constants out of consideration, the required value of λ (i. e. λ') is the minimum point of

$$\frac{c(\lambda, \lambda_0)}{\lambda} = \frac{\pi}{8} \mu a \frac{1 + \omega(f + g/4)}{24^{-1/2}\lambda^{-1/2} - \pi\mu\lambda/16} . \tag{5.1}$$

In particular when the coefficient of friction is vanishingly small, we have

$$\frac{c}{\lambda} = \left(\frac{3}{8}\right)^{1/2} \pi\mu a \lambda^{1/2} \left\{1 + \omega \left(f + \frac{1}{4} g\right)\right. . \tag{5.2}$$

We assume here that when the friction is vanishingly small, the egg will maintain the form which it takes under uniform pressure a without being distorted while it is in motion, in other words that λ_0 will be the minimum point of (5.2). For, when μ vanishes, so does the resistance and by the assumption of equilibrium of forces the thrust vanishes too, no such effects of motion therefore as explained in the concluding paragraph of (i) of § 3 take place, and further by putting $\mu = 0$ in (3.11) and (4.2) we find $M = -N$ i. e. $\beta = 0$, thus the egg will be an ellipsoid of revolution; all of these suggest us that it is natural to assume that the egg will not be deformed from the original form so long as the coefficient of friction is vanishingly small. Retaining only the essential part of the right hand side of (5.2),

$$\varphi(\lambda) \equiv \lambda^{1/2} \{1 + \omega(f + g/4)\} ,$$

and putting $d\varphi/d\lambda = 0$ at $\lambda = \lambda_0$, we obtain

$$\omega = 4 (6\lambda_0)^{-1/2} .$$

We can easily verify that λ_0 is really the minimum point and not the maximum. Making use of ω thus determined in (5.1), the possible length of the egg in the general case when μ is not zero can be defined as the minimum point of the function

$$\psi(\lambda) \equiv \frac{(\lambda/\lambda_0) + 3\{(\lambda/\lambda_0)^{-1/2} - 1\}}{24^{-1/2}\lambda^{-1/2} - \pi\mu\lambda/16} .$$

Hence the equation satisfied by λ' is

$$\mu = \frac{96^{1/2}(1 - \lambda'/\lambda_0)}{3\pi\lambda'(3\lambda_0^{1/2} - 2\lambda'^{1/2})} . \tag{5.3}$$

We can prove as well that λ' is really the minimum point and not the maximum. The general behaviour of ψ is shown in Figure 3, in which the branch B has no physical meanings. The pole of ψ exists at the point

$$\lambda = (32/3)^{1/3}\pi^{-2/3}\mu^{-2/3} ,$$

and on introducing (5.3) into the denominator of ψ, we can verify that λ' lies on the branch A.

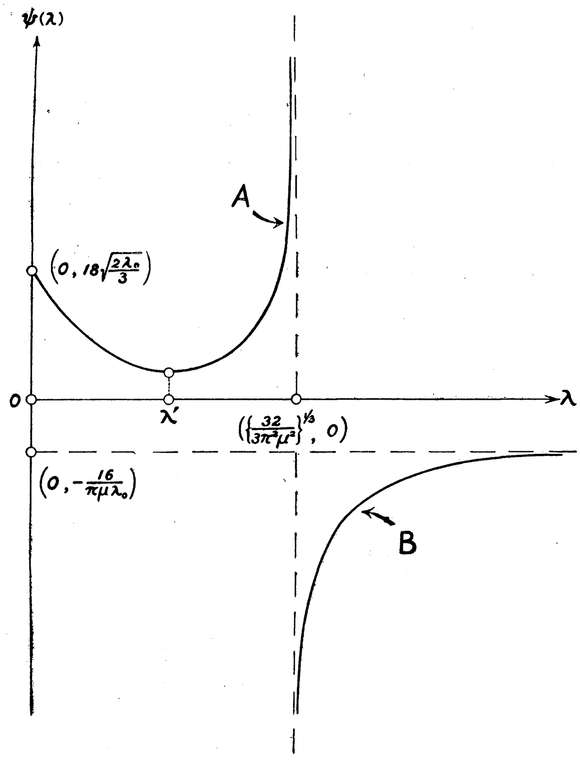


FIG. 3.

Examples:——assuming the values of $\lambda_0 = 3.0$ and $\lambda' = 2.2$, we obtain from (5.3) $\mu = 0.0565$. Substituting these values in (5.2), we find

$$\frac{c}{\lambda} = 0.533 ,$$

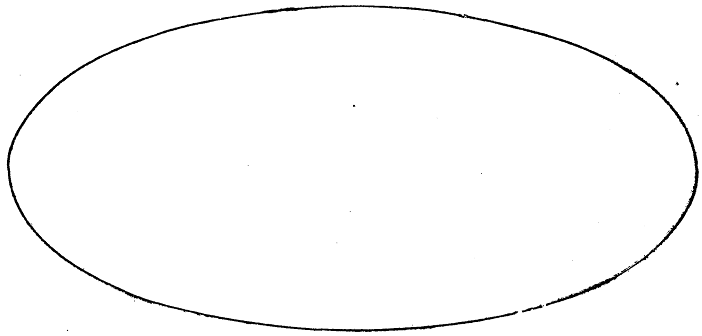
and by (3.11),

$$L = 0.467 , \quad M = 6.417 , \quad N = -6.951 .$$

Finally by (4.2),

$$\alpha = 0.913 , \quad \beta = -0.331 , \quad \kappa_0 = 1.325 .$$

In Figures 4 and 5 the original ellipsoid of revolution $\lambda_0 = 3.0$ and its final form $\lambda' = 2.2$ are reproduced. It is concluded therefore, at least a possibility has been proved theoretically, that an egg goes down ***the pointed end foremost.***



$\lambda_0 = 3.0$

FIG. 4.

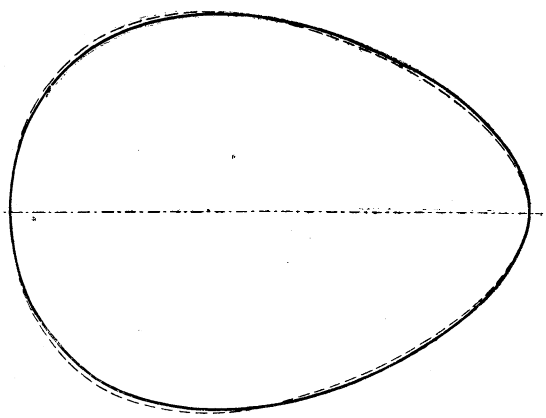


FIG. 5.

———— $\mu = 0.0565,\ \lambda_0 = 3.0,\ \lambda = 2.2$

- - - - - $\mu = 0.0707,\ \lambda_0 = 5.0,\ \lambda = 2.2$

In the interesting book cited before Thompson writes "…We may accordingly presume that the more pointed eggs are those that are large relatively to the oviduct through which they have to pass, or in other words, are those which are subject to the greatest pressure while being forced along.…" In illustration of this conclusion he enumerated the facts that the eggs of the guillemots, curlews, sandpipers, phalaropes and terns are large relatively to the parent birds and that the first eggs laid by a young pullet are usually smaller and at the same time much more nearly spherical than the later ones. In order to ascertain whether our conclusions are consistent with these examples, we take another rather extreme value: $\lambda_0 = 5.0$,

$\lambda' = 2.2$, and consequently we find $\mu = 0.0707$. By the larger value of λ_0, we may consider, an egg which is large relatively to the oviduct is implied. On taking the same procedure as before we get the following results:

$$\alpha = 0.952 , \qquad \beta = -0.413 , \qquad \kappa_0 = 0.787 .$$

the contour is shown in Figure 5 with a broken line.

Acknowledgements. In conclusion the author wishes to acknowledge his indebtedness to Professor Tange of the School of Agriculture and Professor Yamada of the Research Institute. Professor Tange suggested this topic to the author and gave him biological knowledges as well as many valuable advices. On the other hand the essential part of the physical ideas is due to Professor Yamada. If it had not been for their encouragements, this note would not have been completed.

(Received November 14, 1951)

ERRATA

Liquid Rise in a Small Tube

By Jun-ichi OKABE and Michio OHJI

(Reports of Research Institute for Fluid Engineering, Vol. 7, No. 1, 1950)

Page 20, line 7, for "facts" read "fact."

Page 23, line 6 and 7 from the bottom, for "w_0" read "w_0'."

On the Waves of Ships

By Jun-ichi OKABE and Tatsuo JINNAKA

(Reports of Research Institute for Fluid Engineering, Vol. 7, No. 1, 1950)

Page 46, line 1 in the foot-note, add a comma after "Ships."

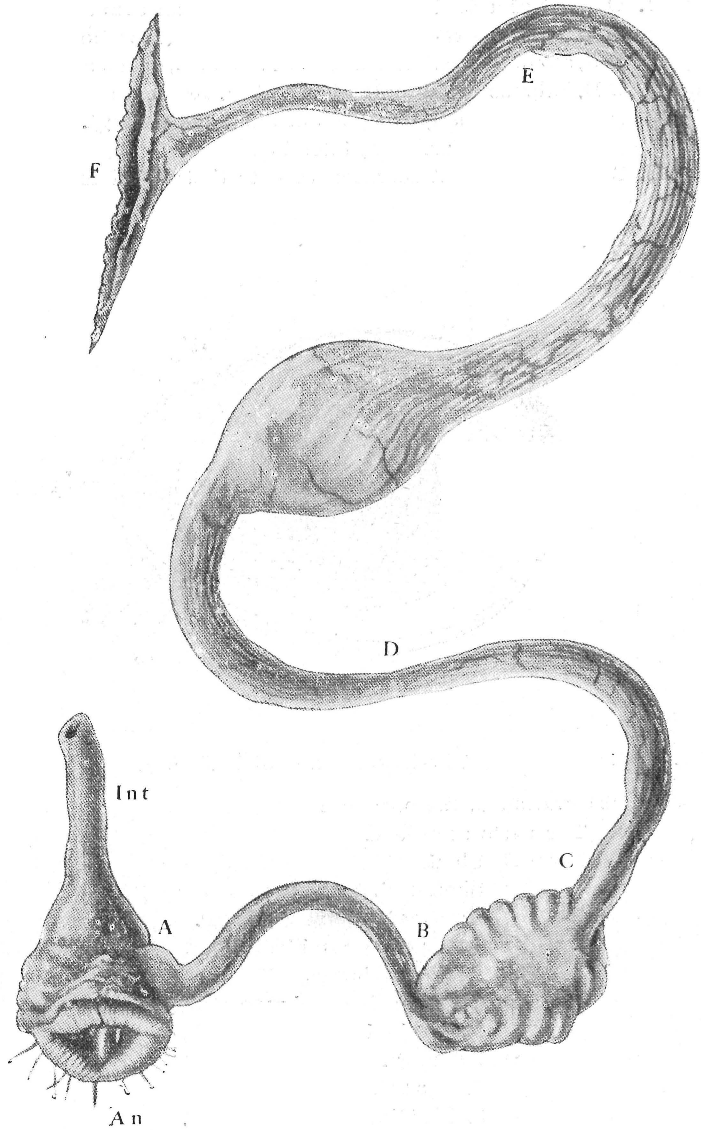
Page 46, line 4 in the foot-note, add a comma after "*London.*"

Page 60, line 9 and 8 from the bottom, for "through out" read "throughout."

OKAJIMAS FOLIA ANATOMICA JAPONICA, BD. XVII.

PLATE I.

I



II

H. Mimura.

Explanation of Plate I.

I. Oviduct of a Leghorn female containing an ovum in the region of albumen portion. E-F, infundibulum (secretion: chalazae). D-E, albumen portion (ca. 50% of albumen, dense and jelly-like). C-D, isthmus (shell membrane and ca. 10% of albumen, watery). B-C, uterus (shell, 40 ~ 50% of albumen, watery). A-B, vagina (——). An., anus. Int., intestine.

II. Schematic oviduct corresponding to that which is drawn in I.

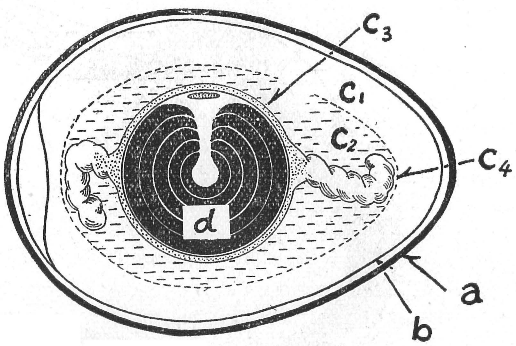


Plate II.

The Examination of hen's egg.

I. Observation of the structure.

The parts of an egg.

(a) Shell.
- Shell cuticle (bloom).
- Intermediate spongy layer.
- Inner or mammillary layer.

(b) Shell-membrane.
- Outer shell membrane (thick).
- Inner shell membrane (thin).

(c) Albumen.
1. Outer watery portion.
2. Middle jelly-like portion.
3. Inner dense portion.
4. Chalazae.

(d) Yolk.
- Vitelline membrane.
- Yellow and white yolk.
- Blastoderm.

(By courtesy of Professor Tange)