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NOTE ON THE HYDRODYNAMICAL ROUGHNESS LENGTH OF A GRASS-FIELD

By Michinori KURIHARA

In this note the author's opinion that the roughness length of grass fields must be determined solely by hydrodynamical conditions independently of the degree of instability of the atmosphere will be described. This opinion will be verified by the several observations by Takeda and by Best.

1. Some fifteen years ago H. U. Sverdrup and O G. Sutton[1] discussed about the validity and significance of the logarithmic law of wind structure near the ground without definite conclusion. The main points of the problem may be summerized as follows: (a) does hold the logarithmic law of wind structure when the temperature gradient differs from the adiabatic, and (b) is it possible to consider the roughness length as a function of the stability ? Sutton was affirmative to these questions. Sverdrup on the contrary was positive in denying them, especially (b), regarding the roughness length as some absolute length associated with the surface.

The question (a) is one of the most important problems in meteorology and offers a fundamental, but very difficult, problem to the theory of turbulence in future. In this note we shall consider merely the second question (b) somewhat in detail.

When we consider the vertical distribution of wind velocity in the layer next to the ground, we may safely regard the turbulent viscosity as the most dominant factor controlling the air flow, the effects of horizontal pressure gradient being neglected. Thus it is obvious that the surface roughness gives effects to the flow by the turbulent field produced by it, namely the turbulent sub-layer[2]. Since, on the other hand, the degree of stability exerts influences upon the flow through Richardson's number R_i, which becomes negligible as the surface is approached, when the wind velocity is not so small, there are no other quantities than hydrodynamical, which directly enter into the problem concerning the turbulent sub-layer and its adjacent thin layer. Accordingly it must be possible for the

1) O. G. Sutton, Quart. J. R. Met. Soc., **62** (1936) 124, **63** (1937) 105. H. U. Sverdrup, ibid., **62** (1936) 461, **65** (1939) 57.

2) M. Kurihara, "On the Resistance Laws in the Transition Region between the Smooth and Rough Pipe Laws". (I), Rep. Res. Inst. Fluid Engng, Kyūshū Univ. **7** (1951) 23.

roughness length to be expressed as a function of only hydrodynamical quantities. Namely, the roughness length of a given grass field must be a function of the typical velocity concerned with the flow near the turbulent sub-layer and its functional form must be determined by solely the geometrical and hydrodynamical characters of the grass itself and its constitutional individual grasses.

Now man observes in fact the roughness length to be a function of the degree of stability, when man applies the logarithmic law for the vertical distribution of measured wind velocity. This may be interpreted as a necessary consequence of the fact that the typical velocity above mentioned depend on not only the velocity outside the boundary layer but also the degree of stability. So that the dependence of the roughness length, thus determined, on the degree of stability cannot be unique and must exhibit a wide scattering as is actually observed. Thus this relation cannot be considered as having grasped the essential feature of the problem.

2. The surface roughness exerts influences upon the flow directly through its drag and indirectly through the additive turbulence produced by itself, inrceasing the viscous drag along the whole surface.

In a turbulent flow through a pipe roughened by solid grains the roughness gives free spaces with at least the same volumes as the individual grains to the moving fluid between the grains. Thus we can without any serious error introduce the turbulent sub-layer to characterize the turbulent field near the roughness grains and the laminar sub-layer to allow the viscous drag of the whole surface (inclusive of the surface of grains) and the small form drag of grains.[1)]

In the case of turbulent flow over a grass field the physical conditions are entirely different. The lateral dimensions of stems or leaves of grasses are small compared with the mean height of the grass and there are so many of these hydrodynamical obstacles in a ground surface of an area with the same linear dimension as the mean hight of grasses, that the flow through the grass must be governed primarily by their resistance, and they can not give the full space covered by them as free space to the turbulence which would be produced by a roughness of solid grains of the same size as the mean height of grasses. Thus, if we compare the turbulent flow over a grass field with that over a roughness of solid grains, we know that, roughly speaking, the flow in the grass corresponds to that in the laminar sub-layer and above them, in both cases, lies the turbulent sub-layer produced by the geometrical irregularities of the roughness surface and lastly follows the upper shear flow in which the turbulent viscosity due to velocity gradient plays a paramount role.

It is to be mentioned here that in the case of a grass field the turbulent viscosity due to small scale turbulence produced by grasses takes the place of molecular viscosity in a pipe flow.

Putting aside the detailed mechanisms of these phenomena we shall refer

1) M. Kurihara, loc. cit,

in this note only the following points: (1) The velocity at the boundary between the turbulent sub-layer and the outer layer, u_0, is intimately connected with drag coefficient of the grass. (2) Since the grasses cannot offer the full space covered by them as free to large scale turbulence, it is reasonable to modify Prandtl's assumption for mixture length and to assume

$$l = K\,(z - \Delta), \tag{1}$$

where z is the height measured from the ground and Δ the height of the virtual wall characteristic to the grass field which would be determined by the configuration of the grass.

Let k be the height of the turbulent sub-layer measured from the virtual wall, then we have the boundary condition for the outer layer:

$$u\,(\Delta + k) = u_0. \tag{2}$$

We now consider the thin layer next to the turbulent sub-layer for which we may safely assume that the influence of stability is negligible. Then we have for the velocity distribution

$$\frac{u}{u_\tau} = \frac{1}{K_0}\log\frac{u_\tau(z-\Delta)}{\nu} + A, \tag{3}$$

where K_0 is the constant K in (1) for adiabatic condition and the other notations are usual.

If we determine the integration constant A by (2), we obtain

$$\frac{u}{u_\tau} = \frac{1}{K_0}\log\frac{z-\Delta}{z_0}\ , \tag{4}$$

where

$$z_0 = \frac{k}{n}\ , \quad \log n = K_0\frac{u_0}{u_\tau}\ . \tag{5}$$

In ordinary condition, excluding very weak wind, we may assume that a fully developed turbulent field is always established in the outer layer. Then k can vary with wind velocity only through the variation of configuration of the grass. Accordingly it seems that k decreases slowly with increasing velocity. On the other hand, because of the smallness of lateral dimensions of leaves or stems of grasses, Reynolds number related to the flow in the grass must be small and lie in such a region, where the drag coefficient of a cylinder or sphere exhibits considerable change. So that it is likely for u_0/u_τ to increase when the velocity increases. The variation of configuration of the grass also favours this tendency. Thus, by equation (5), we may say with all probabilities that z_0 decreases when the wind velocity increases.

Now we can through the above reasoning predict that if we take as a typical velocity u_t the wind velocity at the mean height of the grass and if we calculate the hydrodynamical roughness length z_0 from a series of observational data concerning the vertical distributions of velocities near the surface of a grass field under various conditions of stability and plot z_0 against u_t, then we obtain a smooth curve characteristic to the grass field considered showing decreasing z_0 for increasing u_t independently of the degree of stability. We shall call it the *characteristic roughness curve* of a grass field.

In the following section we shall discuss the above reasoning by some observations.

3. Takeda's Observations. K. Takeda[1] measured wind velocities and temperatures at heights 0.5m. 1.0m, 2m and 5m in the middle of a grass field of an area 1000m × 500m at the base of Mt. Fuji, heights of grasses or bushes being 10cm∼50cm in general. Although only eighteen series of observations were conducted and so the results obtained show no small scattering, his data are avaiable for our purpose.

He applied to the vertical distribution of velocity in layers between $z = 0.5$m to 5m the simplest logarithmic law

$$u = a \log_{10} \frac{z}{z_0} \qquad (6)$$

and obtained the mean values of a's and z_0's. Discussing the relations between degree of stabilities and them, he confirmed the validity of (6).

From our point of view equation (6) corresponds to the case of $\Delta = 0$. No. 2, 4 and 5 of his observations can be considered to be concerned with adiabatic condition because of either the strong wind or the small rate of change of temperature. The mean of these velocity distributons is shown in Fig. 1, which shows clearly $\Delta = 0$.

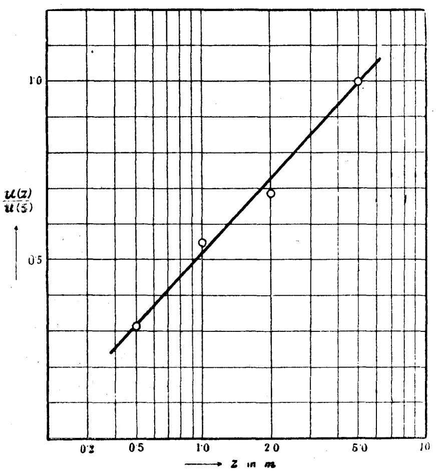

FIG. 1. Adiabatic Velocity Distribution, Takeda's Observations.

The vanishing of Δ seems to mean that as seen from the description of conditions of the field the space is comparatively thinly covered with grasses or leaves, stems and they give an enough free space to large scale turbulences.

Thus we have shown that equation (6) holds in general for the thin layer next to the grass surface. But, if we apply (6) as an approximation to each of consecutive layers in a non-adiabatic atmosphere, we obtain a's which correspond to the local values of R_i and z_0's which are function of z_0 for the lowest layer next to the grass surface (i. e. the true roughness length) and the distribution of R_i in the layer lower than that considered.

Thus, if the effects of stability can be neglected in the lowest layer, the ratio of a to that of the lowest level must start from 1 and then increase or decrease according to the atmospher being stable or unstable when z increases. Using Takeda's data the values of a for three layers

1) K. Takeda, Jour. Met. Soc. Japan, 27 (1949) No. 11, 12.

between $z = 0.5\,\text{m} \sim 1.0\,\text{m}$, $1.0\,\text{m} \sim 2.0\text{m}$ and $2.0\,\text{m} \sim 5.0\,\text{m}$ can be obtained. The results are shown in Fig. 2 and clearly confirm the above consideration.

Next, if we plot the values of z_0's for the above three layers against the wind velocities at the lower boundaries of respective layers, we obtain Fig. 3 a), b) and c). As seem from the figures, at the higher layers the dependency of z_0 on wind velocity is different according to the atmosphere being stable or unstable and the scattering is considerable. This tendency becomes more conspicuous for the higher level and seems to indicate that the apparent roughness length z_0 is a function of not only the wind velocity, but also the degree of stability. As is approached the lowest layer, the difference between the stable and unstable condition and also the scattering of plotted points vanish within observational error and we obtain such a definite relation between the true roughness length and the wind velocity at $z = 0.5\,\text{m}$ that shows a behaviour predicted in the preceding section, namely z_0 decreases as the wind velocity increases (see Fig. 3 a)).

These observational facts may be said to furnish a strong evidence to our reasoning that there is a characteristic roughness curve for a grass field.

For the theoretical treatment of vertical distribution of wind velocity we must use some theory of turbulent

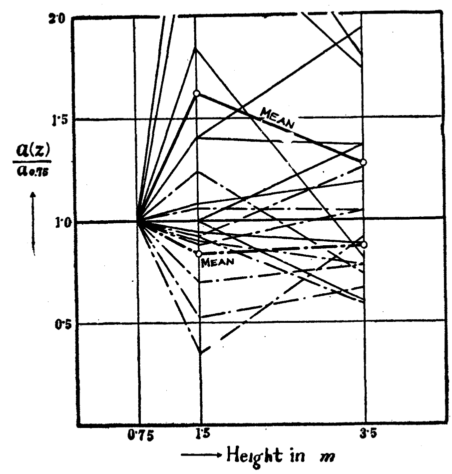

FIG. 2. Variation of a with Height, Takeda's Observations.

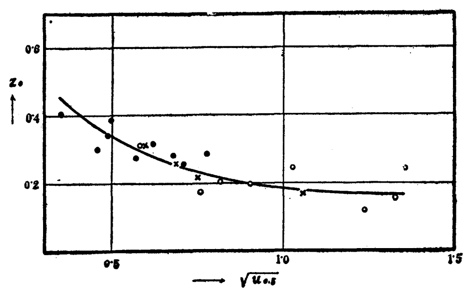

FIG. 3 a). Variation of the Roughness Length (in m) with Velocity, at 0.75m.
• : Stable cases, ○ : Unstable cases

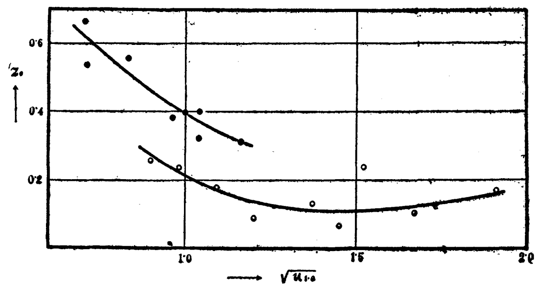

FIG. 3 b). Variation of the Roughness Length (in m) with Velocity, at 1.5 m.

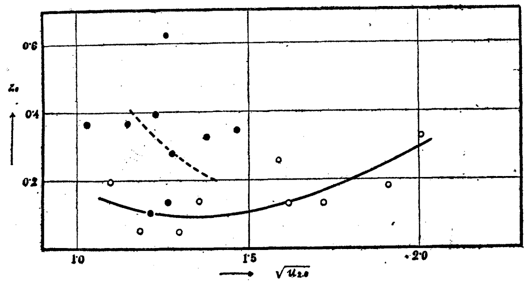

FIG. 3 c). Variation of the Roughness Length (in m) with Velocity, at 3.5m.

atmosphere which takes account of the influences of stability correctly.[1] However equation (4) can be applied as an approximation to comparatively thin layers in which the variations of R_i are small. In this case the roughness length z_0 appears dependent of the degree of stability as seen from Fig. 3 b), c).

4. Best's observations. We now proceed to verify the existence of the characteristic roughness curve of grass fields by the reliable observations of A. C. Best.[2] who took readings at no less than seven heights in the interval 2.5cm to 2 meters above a close cropped grass approximately 1cm high. He gives velocity profiles $U(z)$ (velocity at height z as percentage of velocity at 1m) for unstable, adiabatic and stable conditions:

Values of $U(z)$ for Velocities (at 1m) between 1.5 and 4.0 m/sec.
(Table XVIII in his paper)

z	2.5 cm	5 cm	10 cm	25 cm	50 cm	100 cm	200 cm	
Temperature Gradient								
-3°F/m	42.9	52.0	66.7	81.0	90.3	100	107.1,	(i)
Zero	36.4	49.0	63.0	79.2	90.3	100	111.7,	(ii)
1°F/m	33.9	47.8	60.0	77.1	89.5	100	114.3.	(iii)

According to Best the velocity profile for zero temperature gradient can be represented very closely by

$$U(z) = A + B \log_{10}(z-1), \tag{7}$$

which means that the height of virtual wall Δ is the same order in magnitude as the height of grass because of the closeness of grass on the contrary to the previous case.

If we plot $U(z)$ against $\log_{10}(z-1)$, we obtain for (i), (ii) and (iii) smooth curves shown in Fig. 4 a), b) and c) respectively. Since by our assumption equation (7) holds in the nearest layer to the ground independently of stability, we can easily obtain the hydrodynamical roughness length for each cases, expressing the tangent lines drawn at the left ends of these curves by (7):

$$\text{(i) } z_0 = 0.138\,\text{cm}, \quad \text{(ii) } 0.161\,\text{cm}, \quad \text{(iii) } 0.200\,\text{cm}. \tag{8}$$

1) H. U. Sverdrup used the theory of Rossby and Montgomery in his note (loc. cit.) to analyse Best's observations. But it is easily seen from the values of τ given by him, that his fundamental equation (4) as well as approximate equation (7) fail to be valid far below $z = 100$ cm.

2) A. C. Best, Geophys. Mem., 7 (1935) No. 65.

In order to obtain the typical velocities for the values of z_0 given above, we must first estimate values of velocities at 1 m for each cases, since Best gave velocity at height z as percentage of velocity at 1 m.

This may be easily done by assuming that the relative numbers of experiments for detailed velocity ranges at 2m given in Table XVII of his paper hold also for the other heights. Thus from figures given in the columns for temperature gradient (a) $-3.6° \sim -2.7°$, $-2.7° \sim -1.8°$, (b) $-0.9° \sim 0.0°$, $0.0° \sim 0.9°$ and (c) $0.0° \sim 0.9°$, $0.9° \sim 1.8°$ we obtain mean velocities at 1 m corresponding to the conditions (i), (ii) and (iii) respectively:

$$\left.\begin{array}{l}\text{(i) } u(100) = 2.96\ \text{m/sec}, \\ \text{(ii) } 2.82\ \text{m/sec}, \\ \text{(iii) } 2.35\ \text{m/sec}.\end{array}\right\} \quad (9)$$

If we take temporarily the velocity at 2 cm as the typical velocity u_t for flow in the grass, we obtain from curves given in Fig. 4 and figures in (9) as the typical velocities for z_0's given by (8)

$$\left.\begin{array}{l}\text{(i) } u_t = 0.948\ \text{m/sec}, \\ \text{(ii) } 0.804\ \text{m/sec}, \\ \text{(iii) } 0.600\ \text{m/sec}.\end{array}\right\} \quad (10)$$

Although the relation between z_0 and u_t thus obtained shows a similar behaviour as that shown in Fig. 3 a), it is desirable to add another set which concerns with strong wind. For the case, where the temperature gradient lies between $0.0° \sim 0.9°$ and velocities at 1m lie between $4.0 \sim 8.0$ m/sec, a similar calculation gives

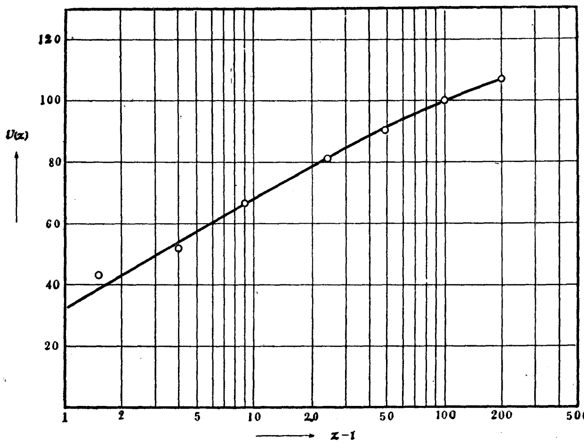

FIG. 4 a). Velocity Distribution (Best's Observation), Temperature Gradient $-3°$ F/m.

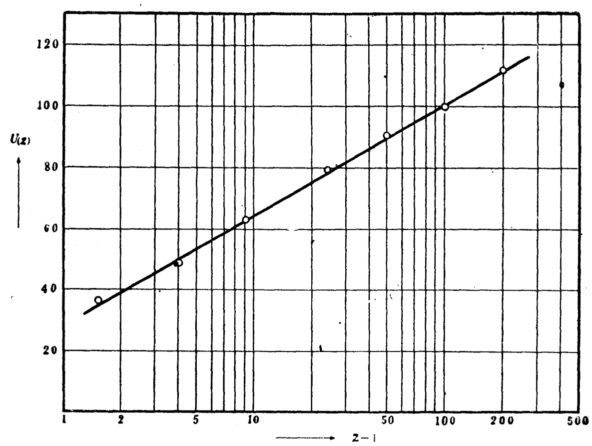

FIG. 4 b). Velocity Distribution (Best's Observation), Temperature Gradient Zero.

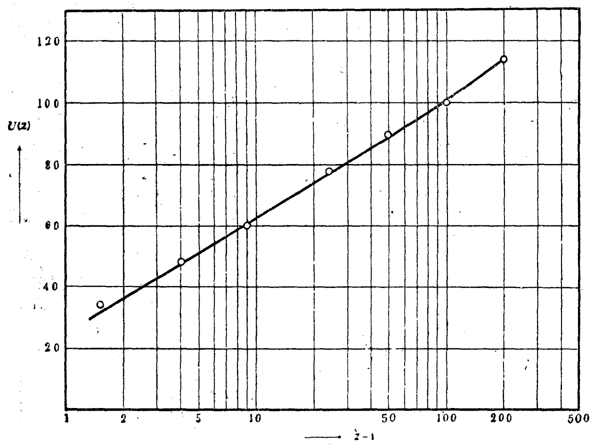

FIG. 4 c). Velocity Distribution (Best's Observation), Temperature Gradient 1° F/m.

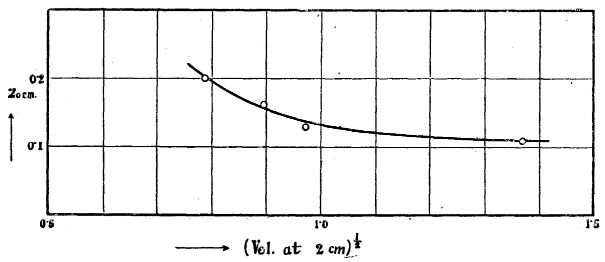

FIG. 5. Characteristic Roughness Curve, Best's Observations.

$$\left.\begin{aligned} &u(100) \\ &\quad = 5.70\,\text{m/sec}, \\ &u_t = 1.88\,\text{m/sec}, \\ &z_0 = 0.110\,\text{cm}. \end{aligned}\right\} \quad (11)$$

These results (8), (10), (11) are shown in Fig. 5, which is very similar to Fig. 3 a) and shows the existence of characteristic roughness curve.

Lastly it is of interest to mention that if instead of the typical wind velocity u_t and the hydrodynamical roughness length z_0 we introduce some Reynolds number concerning the mean lateral dimension of leaves or stems of the grass and u_t and some non-dimensional roughness length which is a function of shape, size and distribution of the grasses, their irregularities etc., the relation of z_0 and u_t may be rewritten in more general form having more physical meanings. In fact, if we put *Reynolds number for Best's observations* $= 0.6 \times$ *that for Takeda's* and make agree the case (ii) of Best with Takeda's curve, then we observe that two sets of results are in harmony as shown in Fig. 3 a) with the crosses. This fact seems to show that some similarity law prevails in the turbulent flows in and near grass-fields of various types.

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