九州大学学術情報リポジトリ
Kyushu University Institutional Repository

The Surface Plate Calibration Comparison Based on The Grid Method

Hapiddin, Asep
Research Center for Testing Technology and Standards, BRIN

Novyanto, Okasatria
National Measurement Standards Laboratory, BSN

A. Praba Drijarkara
National Accreditation Body of Indonesia, KAN

Azzumar, Muhammad
Research Center for Testing Technology and Standards, BRIN

他

https://doi.org/10.5109/7151731

出版情報：Evergreen. 10 (3), pp.1819-1825, 2023-09. Interdisciplinary Graduate School of Engineering Sciences, Kyushu University, Japan
バージョン：
権利関係：Creative Commons Attribution-NonCommercial 4.0 International

KYUSHU UNIVERSITY

The Surface Plate Calibration Comparison Based on The Grid Method

Asep Hapiddin[1,*], Okasatria Novyanto[2], A. Praba Drijarkara[3], Muhammad Azzumar[1], Mohamad Syahadi[4], Miftahul Munir[1], Ninuk Ragil Prasasti[1], Nur Tjahyo Eka Darmayanti[1], Budhy Basuki[1], Hidayat Wiriadinata[1]

[1]Research Center for Testing Technology and Standards, BRIN, Indonesia

[2]National Measurement Standards Laboratory, BSN, Indonesia

[3]National Accreditation Body of Indonesia, KAN, Indonesia

[4]Research Center for Photonics, BRIN, Indonesia

*E-mail: asep056@brin.go.id

(Received April 27, 2023; Revised August 10, 2023; accepted August 18, 2023).

Abstract: The surface plate is a physical flatness reference that widely used in research and industrial applications, e.g., manufacturing, aerospace, automotive, and shipping engineering. Surface plate calibration is required; as a result to guarantee that it continues to fall within the parameters of what is considered to be an acceptable level of flatness. In most cases, Union Jack Moody's approach is relied on to gather data and determine the level of flatness and the Grid method is not widely recognized. The Grid technique can be broken down into two patterns: the Open Grid and the Full Grid. Spreadsheet application is utilized to processing flatness data with the Open Grid approach. A piece of computer software, e.g., Scilab or another program that can perform calculations with matrices, is required. Unlike the Open Grid technique, which has a greater degree of uncertainty, the Full Grid technique has a reduced degree of uncertainty. This study aims to report on assessing the surface plate flatness using two different types of Grid techniques. A surface plate measuring 1000 mm×1000 mm, as one of the measurement samples, was used to characterize both approaches. Open Grid and Full Grid measured the surface plate's flatness to be (6.93 ± 3.94) μm and (6.44 ± 0.68) μm, with Full Grid measuring it more accurately than Open Grid. It is consistently statistically shown that the E_n value is less than 1, which is 0.06 and 0.01, respectively. In conclusion, it can be said that statistically, the Full Grid method, as one of the proposed surface plate calibration methods, is considered an inlier result with smaller uncertainty.

Keywords: surface plate, flatness, calibration, full grid

1. Introduction

Various types of dimensional measurements are very important in determining the quality of products and facilities in various aspects of life, including flatness, roundness, strength, angular measurement, and long derivatives[1-5]. Various methods have been developed in dimensional measurements, e.g., based on the temperature parameter[6]. One of the applications of accurate dimensional measurements is to determine the accuracy of a design[7] and test the performance quality of machines and mechanics[8-9].

A surface plate is one of the devices used in calibration and dimensional measurement[10]. Its function is a datum or reference plane for measurements using dial and height gauges. Dial and height gauges are measuring instruments used widely in research and industrial applications, e.g., manufacturing, aerospace, automotive, and shipping engineering[11-14]. Therefore, the top quality that must be maintained is the surface's flatness, which needs to be measured periodically to ensure its suitability. The term "flatness" refers to the lowest gap between two parallel planes that border the plane being examined. There are a few different methods that can be used to evaluate the flatness of a surface plate. The most typical way to assess whether a surface is flat is to utilize an instrument capable of detecting small angles, e.g., an autocollimator, a laser interferometer, laser scanning, or a leveltronic[15-19].

The Grid system is one of the calibration methods for gathering data and evaluating the surface plate's flatness, and it is based on ISO 8512-2: 1990 Part 2: Granite. This method was developed in 1990[20-21].

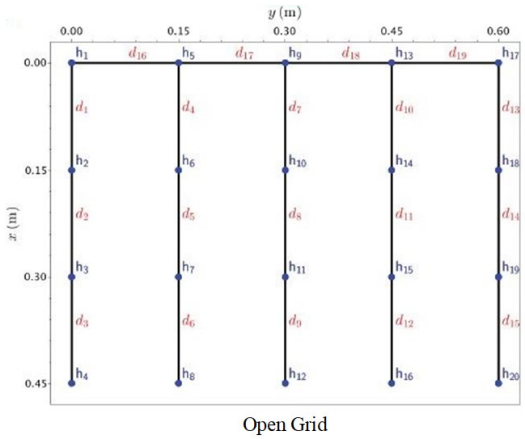
On the other hand, it is not as popular as Union Jack Moody's method. Moreover, only a few articles explore this method[22-26]. The calculation of the flatness of surfaces, the fundamental problems associated with the Grid method, was shown and solved in an NPL report by a least-squares approach and Haitjema by an iterative approach[27-28]. The commercially available software has already solved the mathematical problems related to the Grid method, although probably not in an optimal way.

Moody, a Sandia metrologist, developed a flatness for surface plate measurement method known as the Moody method. The measurement was carried out by measuring angular deviations along eight lines that consisted of four perimeter lines, two diagonal lines, and two center lines, adjusting the slope and height of the deviations on the diagonal lines to get zero displacements in the center of each line and equal deviation at either end, also adjusting slope and height of each perimeter line and center lines to maintain consistency at the intersections, and converting the angular deviations to linear displacements.

In this study, the calibration of a surface plate utilizing Grid methods, specifically Open and Full Grid, was discussed; Figure 1 and Figure 2 depict this discussion.

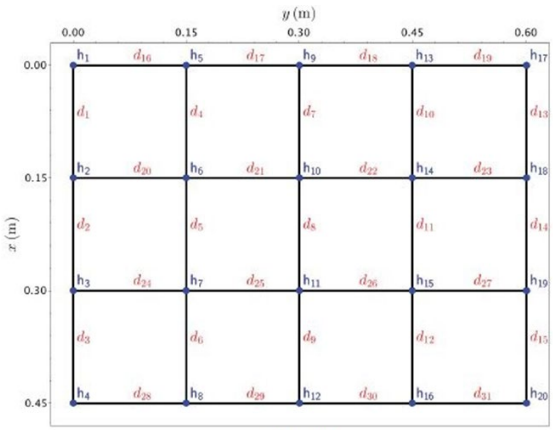
The surface plate flatness is determined by using inclination (angle) measuring tools with absolute or fixed tilt reference, i.e., an electronic level. The inclination data can calculate the height difference between any two points[29-30]. The height value of each point from the height difference data is then computed. The regression analysis approach is utilized to process the data[31]. The matrix approach is utilized to find a solution to the least squares calculation. Processing flatness data with the Open Grid approach necessitates using spreadsheet applications like Microsoft Excel. On the other hand, the Full Grid approach uses a piece of computer software called Scilab, which can calculate matrix operations. Guidelines for verifying software in measuring systems that NPL-UK released served as the basis for creating the software[32].

This project aims to enhance an Indonesian system for calibrating surface plates. The findings of this experiment serve as preliminary data that will be used to validate and improve the developing software based on the Full Grid approach. This software will calibrate surface plates using a method that has less uncertainty. The measurement is only carried out for this validation on one side of the surface plate.



Open Grid

Fig. 1: Open grid method pattern.



Full Grid

Fig. 2: Full grid method pattern.

2. Measurement and Research Methodology

The Open Grid pattern resembles a comb-like shape. This method simplifies the "standard" Grid method with more significant uncertainty measurement[29]. The evaluation of surface flatness is based on the theory of straightness evaluation, as shown in Figure 3.

The straightness of a line (relative to a datum line) can be calculated as the deviation z_i from the datum line. On the inclination measurement, the level indicates the gradient or inclination g_i, which shows the relative height h_i between the measured section ends. The deviation z_i of point i can be determined by accumulating the values h_1 through to h_i. If the level's indication G_i is in units of µm/m, the height h_i can be calculated from a level length L as

$$h_i = G_i \times L \quad (1)$$

While deviation z_i at point i from the datum line is calculated as

$$z_i = \sum_i h_i \quad (2)$$

Where h_i is the height of the second point relative to the first point, in µm. L is the length of the level, in m. G is

the inclination of the level, in µm/m. z_i at the point i from the datum line, in meter (m).

After calculating the deviation of each point relative to the datum line, the position of the line must be normalized to obtain the least total deviation. Normalization is carried out by "tilting" the line around its endpoint as a pivot point. Mathematically, this is carried out by correcting the value of z_i by k_i proportionally:

$$z'_i = z_i + k_i$$

$$k_i = \frac{i}{n} k_{\max} \qquad (3)$$

Where k_i is the correction at point i, n is several sections, and $k_{\max}$ is the correction at the furthest point. Figure 4 illustrates the magnitude of correction values, calculated proportionally concerning the importance of z_i at the furthest point. Dot lines represent the line after normalization.

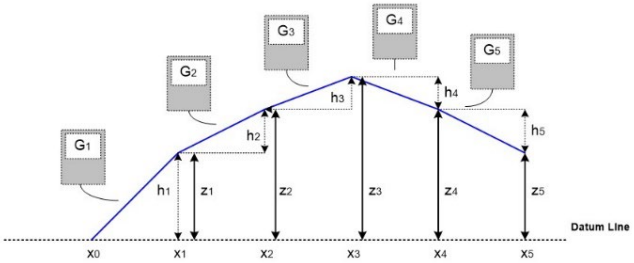


Fig. 3: Measurement of inclination and analysis of deviation.

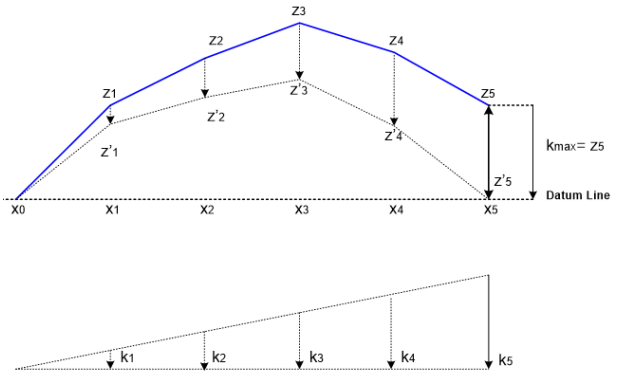


Fig. 4: Normalization of line concerning datum line. The value of $k_{\max}$ is taken from z_5.

After normalization, the total straightness deviation can be determined as the difference between the most positive and negative deviations. In a straightness evaluation, the value $k_{\max}$ must be defined in such a way as to obtain the slightest total straightness deviation. Flatness is evaluated by performing a straightness evaluation on all measuring lines[33].

Flatness can be determined using the Full Grid approach by measuring the height differences between adjacent points evenly distributed across the examined surface [34]. Errors can be reduced to a minimum by utilizing this strategy. An illustration of a surface in the shape of a box can be seen in Figure 5, which depicts points 1 through 9.

Following is a system of equations that can be used to figure out the connection between the height (related to a datum point or datum plane) of each point and the height differences d_i, through the following equation (4) system, as illustrated in Figure 5. The notation h_j indicates the height of the point in relation to the datum plane.

$$\begin{aligned} d_1 &= h_1 - h_2 & d_7 &= h_1 - h_4 \\ d_2 &= h_2 - h_3 & d_8 &= h_2 - h_5 \\ &\vdots & &\vdots \end{aligned} \qquad (4)$$

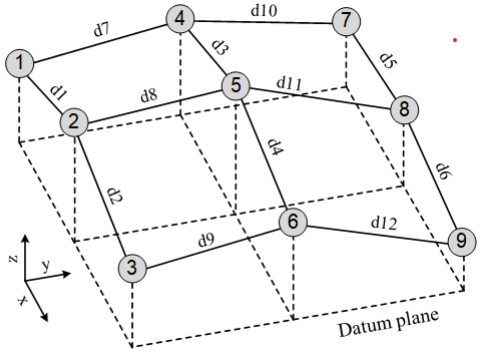


Fig. 5: An illustration of a surface that needs to be measured.

These computations could be written mathematically and expressed as an equation (5).

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \\ d_7 \\ d_8 \\ d_9 \\ d_{10} \\ d_{11} \\ d_{12} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} \qquad (5)$$

Alternatively, it can be expressed mathematically as an equation (6).

$$\underline{d_i} = A_0 \cdot \underline{h_j^*} \qquad (6)$$

Where $\underline{d_i}$ is a column matrix that contains the values that were measured, and $d_i (i = 1,...p)$ is the measured

values. h_j^* is a column matrix that contains the unidentified values $h_j (j = 1,...p)$. The link between d_i and h_j is defined by the design matrix A_0. Therefore, there is no solution to this system of equations because all heights are determined relative to one another. An arbitrary value may be assigned to one of the heights, as shown by the response. If it assigns no significance h_k, then equation (6) will change to become equation (7), as shown by the following expression:

$$\underline{d_i} = A \cdot \underline{h_j} \qquad (7)$$

Matrix A corresponds to the matrix A_0, except that the k-th column has been eliminated, and the variable h_j contains all of the h_j values where $j \neq k$. The solution of Equation (4) provides the observed heights h_j, which in turn offers the position for every sampled point as the product of $(x_j + y_j + z_j)$ where z_i equals h_i. Nevertheless, the table is at a tiny angle in relation to the measuring instrument, thus the surface represented by $(x_j + y_j + z_j)$ might be tilted. First, a new reference plane must be created, which should be taken from the plane that provides the greatest least-square fit. This will allow the flatness to be determined. The flatness deviation could then be calculated by measuring the difference in height between each point and the reference plane.

This study was conducted in a Length Laboratory with environment conditions (19.9±0.3) °C temperature and (48.5±2.4) % relative humidity. We utilized a thermo-hygrometer, a straight edge, and an electronic level. The base length of the electronic level was 150 mm. Through Indonesia's National Measurement Standards Laboratory (SNSU-BSN), both the electronic level and the other calibration support equipment may be traced back to the International System of Units (SI).

Microsoft Excel was utilized to develop flatness data processing with the Open Grid method. While the application for measuring the flatness of surface plates using the Full Grid approach has been constructed utilizing integrated software from Visual Studio and Scilab, as shown in Figure 6. A surface plate with a dimension of 1000 millimeters×1000 millimeters was utilized in characterizing both approaches.

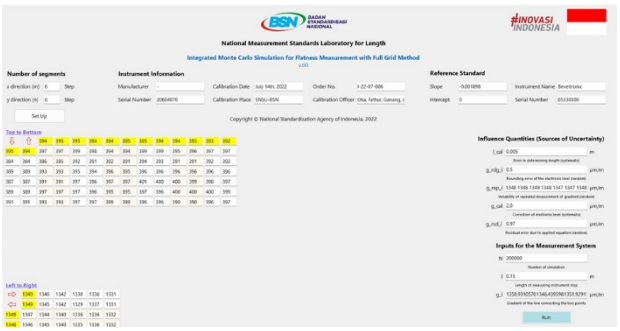


Fig. 6: A software application for measuring the levelness of a surface plate using the Full Grid method.

3. Result and Discussion

Figure 7 displays the findings of an analysis that compared the outcomes of flatness surface plate measurements obtained using various approaches. The flatness measurement data, both open and full grid methods, are random samples of 6 different types, both the size and grade of the surface plate.

The random samples came from the surface plate measuring 600 mm×450 mm, 1000 mm×750 mm, and 1000 mm×1000 mm. While the grade used is grade 0 and several surface plates whose grade is unknown. Using the open grid method, a spreadsheet method was utilized to analyze the uncertainty for flatness measurement. This method is one of the most common uncertainty estimation methods as a substitute for the Guide to the Expression of Uncertainty in Measurement (GUM method). Whereas in the full grid method, Monte Carlo Method (MCM) was utilized in the measurement uncertainty analysis due to the GUM method which is very difficult to be implemented for a certain surface plate measurement.

The weighted mean of comparing the Open Grid and Full Grid approaches determines the key comparison reference value (KCRV). This value is determined by calculating the weighted mean of the results. Equations (8), (9), and (10) were utilized in the process of data analysis for each measurement.

The measurement results were analyzed using statistical methods to check the correlation of the comparative test results between the two methods. The value of the measurement results is considered to have significantly deviated or an outlier if the E_n value obtained is greater than one[35]. A comparative test reference value is needed from all participant scores to calculate the E_n value. Assume that there are 2 participants in this comparison test, namely the Open Grid and Full Grid methods.

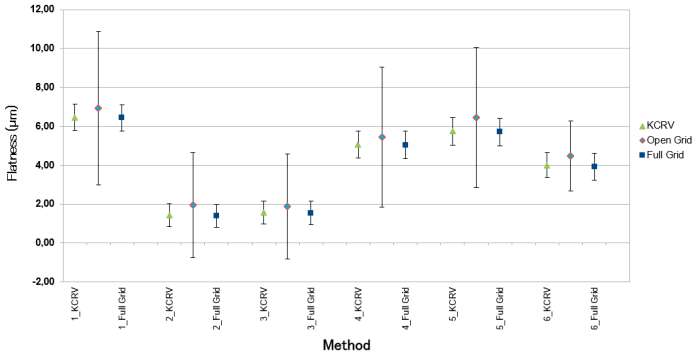


Fig. 7: Comparing the flatness surface plate measurements with different methods.

Each participant presents its measured value x_i, as well as its standard uncertainty $u(x_i)$. The normalized weighting value, w_i, can then be calculated using the following formula:

$$\underline{w_i} = C \cdot \frac{1}{\left[u(x_i)\right]^2} \quad (8)$$

the value of the normalization factor, C, is determined by:

$$C = \frac{1}{\sum_{i=1}^{I}\left[\frac{1}{u(x_i)}\right]^2} \quad (9)$$

When everything is considered, the weighted average value, x_w is:

$$\bar{x}_w = \sum_{i=1}^{I} w_i \cdot x_i \quad (10)$$

The uncertainty associated with this reference value is represented by the internal standard deviation, which is generated since the weighted average value will be used as the Key Comparison Reference Value (KCRV). The estimated standard uncertainty provided by every individual who took part in the comparison test is the input used to calculate the internal standard deviation[36], as represented by the equation (11).

$$u_{\text{int}}\left(\bar{x}_w\right) = \sqrt{\frac{1}{\sum_{i=1}^{I}\left[\frac{1}{u(x_i)}\right]^2}} = \sqrt{C} \quad (11)$$

Following the derivation of the value of the average weighting and the uncertainty associated with the weighted average, the equation $\left(x_i - \bar{x}_w\right)$ was used to determine the deviation value of each participant's measurement result relative to the weighted average deviation value. The uncertainty associated with this equation is determined by combining the participant's standard uncertainty $u\left(x_i\right)$ and the uncertainty associated with the weighted mean $u\left(\bar{x}_w\right)$. The uncertainty associated with the departure from the weighted mean is represented by equation (12).

$$u\left(x_i - \bar{x}_w\right) = \sqrt{\left[u\left(x_i\right)\right]^2 - \left[u_{\text{int}}\left(\bar{x}_w\right)\right]^2} \quad (12)$$

It is essential to ensure that the findings of the KCRV contribution are statistically consistent to calculate the reference value of the key comparative test (KCRV). The statistical consistency of measurement findings with the standard uncertainty can be determined by computing the E_n value for each participant.

E_n is defined as the departure ratio from the weighted mean to the uncertainty of this value's range. Multiplying the appropriate k value with a confidence level of 95% yields the spread uncertainty value.

$$E_n = \frac{x_i - \bar{x}_w}{\sqrt{\left[u\left(x_i\right)\right]^2 - \left[u_{\text{int}}\left(\bar{x}_w\right)\right]^2}} \quad (13)$$

Value $\bar{x}_w$ is the reference value in this comparison test with measurement uncertainty. In general, the results of calculating the reference uncertainty value $u(\bar{x}_w)$ are closer to the uncertainty value for measuring the flatness of a surface plate by utilizing the Full Grid system, as shown in Table 1.

Table 1. The outcome of a comparative analysis performed on a surface plate measuring 1000 mm × 1000 mm.

Measurand	Expected value (µm)			Uncertainty of measurement			E_n number	
	Open Grid	Full Grid	KCRV	Open Grid	Full Grid	KCRV	Open Grid	Full Grid
Flatness	6.93	6.44	6.45	3.90	0.68	0.67	0.06	0.01
Flatness	1.95	1.41	1.44	2.70	0.59	0.58	0.09	0.02
Flatness	1.89	1.55	1.57	2.70	0.60	0.58	0.06	0.01
Flatness	5.44	5.04	5.01	3.60	0.71	0.69	0.05	0.01
Flatness	6.45	5.71	5.74	3.60	0.71	0.70	0.10	0.01
Flatness	4.46	3.94	4.00	1.80	0.69	0.65	0.12	0.04

The E_n number of each participant is less than 1; therefore, both the Open Grid and Full Grid methods have a good agreement. On the open grid method, the measurement uncertainty analysis uses a spreadsheet method. Whereas in the full grid method, the measurement uncertainty analysis uses the Monte Carlo Method (MCM). Furthermore, this result shows that the developed software based on the Full Grid method has been successfully improved and validated.

4. Conclusion

Using the E_n evaluation, the developed software based on the Full Grid method has been validated by directly comparing it to the Open Grid method as a comparator

with KCRV as a reference. The comparison of the Full Grid and Open Grid methods is performed by measuring the flatness of the same surface plate 1000 mm×1000 mm.

Based on the results and discussion above, the results of the Open Grid and Full Grid methods have met the criteria of E_n value less than 1, which are 0.06 and 0.01, respectively. Thus, statistically, the Full Grid method is considered an inlier result as one of the proposed surface plate calibration methods.

However, this validation has only been performed on the one surface plate's side. In the future, this validation will be carried out on all sides of the surface plate to obtain the homogeneity of the flatness measurement. Furthermore, this software will be tested using "a software evaluation for surface plate measurement data set" provided by Haitjema.

Acknowledgements

The authors want to convey their gratitude to the administration and laboratory members of the National Measurement Standards Laboratory, a part of the National Standardization Agency of Indonesia-SNSU BSN, for their assistance in conducting this research.

References

1) J. Molleda, R. Usamentiaga, and D.F. García, “On-line flatness measurement in the steelmaking industry,” *Sensors*, **13** (*8*), 10245-10272 (2013). doi: 10.3390/s130810245
2) P. Hariharan, “Interferometric testing of optical surfaces: absolute measurements of flatness”, *Optical Engineering*, **36** (*9*) 2478-2481 (1997). doi: 10.1117/1.601474
3) J. Liu, C. Zhang, R. Cai, Y. Yao, Z. Zhao, and W. Liao, “Accuracy of 3-dimensional stereophotogrammetry: Comparison of the 3dMD and Bellus3D facial scanning systems with one another and with direct anthropometry," *American Journal of Orthodontics and Dentofacial Orthopedics*, 160 (6) 862-871 (2021). doi: 10.1016/j.ajodo.2021.04.020
4) Y. Kondo, A. Hirai, and B. Youichi, “Two-point diameter calibration of a sphere using a micro-coordinate measuring machine at NMIJ,” *Metrologia*, 59 (2) 024005 (2022). doi: 10.1088/1681-7575/ac579e
5) M.M. Takeyeldein, T.M. Lazim, I.S. Ishak, NAR Nik Mohd, and E.A. Ali. "Wind lens performance investigation at low wind speed," *EVERGREEN Joint Journal of Novel Carbon Resource Sciences & Green Asia Strategy*, **7**(*4*) 481-488 (2020). doi: 10.5109/4150467
6) A. S. Baskoro, M.A. Amat, and M.F. Arifardi. "Investigation Effect of ECR’s Thickness and Initial Value of Resistance Spot Welding Simulation using 2-Dimensional Thermo-Electric Coupled," *EVERGREEN Joint Journal of Novel Carbon Resource Sciences & Green Asia Strategy*, **8**(*4*) 821-828 (2021). doi: 10.5109/4742127
7) S.S. Chauhan, and A.K. Khare. "Kinematic analysis of the ABB IRB 1520 industrial robot using RoboAnalyzer software," *EVERGREEN Joint Journal of Novel Carbon Resource Sciences & Green Asia Strategy*, **7**(*4*) 510-518 (2020). doi: 10.5109/4150470
8) S.K. Lambha, V. Kumar, and R. Verma. "Performance Characteristics of a Deformed 120-Degree Partial Bearing with Couple Stress Lubrication," *EVERGREEN Joint Journal of Novel Carbon Resource Sciences & Green Asia Strategy*, **9**(*2*) 269-282 (2022). doi: 10.5109/4793662
9) M.M. Takeyeldein, T.M. Lazim, NAR Nik Mohd, I.S. Ishak, and E.A. Ali, “Wind turbine design using thin airfoil sd2030,” *EVERGREEN Joint Journal of Novel Carbon Resource Sciences & Green Asia Strategy*, **6**(*2*) 114-123 (2019). doi: 10.5109/2321003
10) O. Novyanto, and E. Pratiwi. “A Preliminary Study to Evaluate the Topography of Narrow Surface Plate,” *Instrumentasi,* **39** (*1*) 1–8 (2015).
11) L. Nowakowski, M. Skrzyniarz, S. Blasiak, and M. Bartoszuk, "Influence of the Cutting Strategy on the Temperature and Surface Flatness of the Workpiece in Face Milling", *Materials* **13** (20) 4542 (2020). doi: 10.3390/ma13204542
12) C. Shao, H. Wang, S. Suriano-Puchala, and S.J. Hu, “Engineering Fusion Spatial Modeling to Enable Areal Measurement System Analysis for Optical Surface Metrology”, *Measurement*, **13** 163-172 (2019). doi: 10.1016/j.measurement.2018.12.061.
13) H.S. Park, and U.M. Tuladhar, “Robust Inspection Technique for Detection of Flatness Defects of Oil Pans”, *International Journal of Automotive Technology*, **17** 119-126 (2019). doi: 10.1007/s12239-016-0011-3
14) H. Aanæs, E. Nielsen, and A.B. Dahl, "Autonomous Surface Inspection of Wind Turbine Blades for Quality Assurance in Production." 9th European Workshop on Structural Health Monitoring, EWSHM 2018, (2018). Manchester.
15) O. Novyanto, and N. Alfiyati. 2016. “An Approach Method to Calibrate the Autocollimator with Small Angle Measurement Range,” *MAPAN – J. of Metrology Soc. of India,* **31** (4) 9–15 (2011). doi: 10.1007/s12647-015-0153-1.
16) O. Novyanto, and N. Alfiyati. “Measurement Optimization of High Accuracy Check Master Based on Interferometry Method in National Standardization Agency of Indonesia: A Preliminary Study for Measuring Range of 10 Mm to 400 Mm,” J. Phys.: Conf. Ser., **1568** 012012 (2020). Bandung. doi: 10.1088/1742-6596/1568/1/012012.

17) A. Hapiddin, Y.I. Pawestri, A.M. Boynawan, Ratnaningsih, S. Agmal, and O. Novyanto. 2020. "Beat Frequency Measurement of the Stabilized He-Ne Laser 633 Nm Calibration in SNSU-BSN," J. Phys.: Conf. Ser., **1528** 012003 (2020). Padang. doi: 10.1088/1742-6596/1528/1/012003.
18) F. Li, H. Li, M.K. Kim, and K.C. Lo, "Laser Scanning Based Surface Flatness Measurement Using Flat Mirrors for Enhancing Scan Coverage Range" *Remote Sensing* 13 (4) 714 (2021). doi: 10.3390/rs13040714
19) A. Rahman, E. Pratiwi, O. Novyanto, N. Alfiyati, and O. Hedrony. "Optimization of Linear Transducer Calibration System Using Laser Interferometer Based on the Abbe Principle," IOP Conference Series: Materials Science and Engineering, 012049 (2020). doi: 10.1088/1757-899X/980/1/012049.
20) "International Standard ISO 8512-2:1990 Surface Plates-Part 2: Granite," n.d. https://www.iso.org/standard/15737.html (accessed February 13, 2023)
21) G.A. Skattum, "Estimating Thermal Effects for Granite Surface Plate Calibration," *NCSLI Measure*, **10** (*1*) 50–58 (2015). doi: 10.1080/19315775.2015.11721716.
22) J. Meijer, "From Straightness to Flatness", PhD thesis Twente University (The Netherlands), ISBN 90-9003117-0
23) J. Meijer, C. J. Heuvelman, "Accuracy of surface plate measurements - general purpose software for flatness measurement," CIRP Annals - Manufacturing Technology 39(1), 545-548 (1990) doi:10.1016/S0007-8506(07)61115-9
24) W. de Bruin, J. Meijer, R. van Hasselt, R. (1980), "Analysis of flatness measurement and form stability of a granite surface plate," CIRP Annals - Manufacturing Technology, 29(1), 385-390 (1980) doi:10.1016/S0007-8506(07)61357-2
25) J. Meijer, W. de Bruin, "Determination of flatness from straightness measurements and characterization of the surface by four parameters," Precision Engineering, 3(1), 17 (1991)
26) H. Haitjema H, J. Meijer, "Evaluation of surface plate flatness measurements," European Journal of Mechanical and Environmental Engineering 38 (4):165-172 (1993)
27) K. G. Birch, M. G. Cox, "Calculation of the Flatness of Surfaces: A Least-Squares Approach," National Physical Laboratory (1973)
28) H. Haitjema, "Iterative Solution of Least-Square Problems Applied to Flatness and Grid Measurement," Advanced tools in Metrology II, World Scientific Publishing Company (1996)
29) S.Z. Zahwi, M.A. Amer, M.A. Abdou, and A.M. Elmelegy. "On the Calibration of Surface Plates," *Measurement* **46** (*2*) 1019–1028 (2013). doi: 10.1016/j.measurement.2011.10.009.
30) "Operator's Manual Leveltronic 'Classic' and 'NT' Minilevel 'Classic' and 'NT' Levelmeter 25 / C25 Levelmeter 2000." n.d. https://www.wylerag.com/fileadmin/pdf/manuel/nt_eng.pdf. (accessed February 13, 2023)
31) J. Herrmann, "Least-Squares Wave Front Errors of Minimum Norm," *J. Opt. Soc. Am.*, 70 (1) 28 – 35 (1980). doi: 10.1364/JOSA.70.000028
32) B.A. Wichmann, G.I. Parkin, and R.M. Barker, "Software Support for Metrology Best Practice Guide No. 1 Validation of Software in Measurement Systems," NPL Report DEM-ES 014, 2007. https://www.nist.gov/system/files/documents/2017/05/09/NPL-Best-Practice-Guide-No-1-Validation-of-Software-in-Measurement-Systems-Jan2007.pdf
33) S. Rab, M.A. Sanjid, A. Zafer, S. Yadav, and A. Haleem. "Simulation of Kinematic Supports of Surfaces Plates for Optimum Flatness Tolerance." *MAPAN – J. of Metrology Soc. of India* **36** (*2*) 279–286 (2021). doi: 10.1007/s12647-021-00446-0.
34) H. Haitjema, "Uncertainty Propagation in Surface Plate Measurements," ISMQC 4th International Symposium on dimensional metrology in production and Quality control, 304 – 320 (1992). Tampere.
35) O. Novyanto, A.R. Nugraha, A. Rahman, N. Alfiyati, M. Azzumar, A. Hapiddin, E. Pratiwi, O. Hedrony, G. Ginanjar, R. Yuniarty, Istiqomah, Nurlathifah, R.A. Samodro, and A.P. Drijarkara. "Validation of An Interferometric Calibration Method for High Accuracy Check Master Using Applied Passive Compensation of Angular Motion," *Instrumentasi*, **45** (1) 77–93 (2021). doi: 10.31153/instrumentasi.v45i1.251
36) G. Hermann. "Uncertainty of Minimum-Zone Straightness and Surface Plate Flatness Measurement," 2012 7th IEEE International Symposium on Applied Computational Intelligence and Informatics (SACI), 93 – 96 (2012). Timisoara. doi: 10.1109/SACI.2012.6249982