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Gupta, Surbhi
Amity Institute of Applied Sciences, Amity University

Dixit, Shubhra
Amity School of Engineering and Technology, Amity University

Sharma, Ajay
Amity School of Engineering and Technology, Amity University

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Surbhi Gupta1*, Shubhra Dixit2, Ajay Sharma3
1Amity Institute of Applied Sciences, Amity University, Noida, India
2,3Amity School of Engineering and Technology, Amity University, Noida, India

*Author to whom correspondence should be addressed:
E-mail: sgupta11@amity.edu

Abstract: This article presents a study focused on the application of the algebra of logic approach to analyzing the stochastic behavior of an industrial ice cream plant. The goal is to investigate the plant's complex interactions and stochastic nature, which can impact its efficiency and performance. This research presents calculations for various reliability parameters aiming to analyze the performance of an Industrial Ice-Cream Plant. A comprehensive ten-component model is developed to demonstrate the plant's operational functionality. By adopting the algebra of logic framework, which combines logical reasoning and probabilistic modelling, the study aims to model and simulate the plant's operations, capturing the inherent uncertainties and randomness involved. The overall system reliability is evaluated considering the Weibull and Exponential distributions as failure rates models. Furthermore, numerical examples are employed to compute the Mean Time to Failure (MTTF), a crucial reliability metric. The results will provide valuable insights into the plant's stochastic behavior, enabling decision-makers to optimize processes, identify bottlenecks, and enhance overall productivity in the industrial ice cream production domain.

Keywords: Ice Cream plant, Reliability, Weibull distribution, Boolean Function Technique, Exponential Distribution.

1. Introduction

IoT devices can be integrated into ice cream production equipment to monitor and control various parameters, such as temperature, humidity, and energy consumption. These devices can communicate with each other and provide real-time data to ensure optimal production conditions, maintain quality, and reduce waste. By integrating advanced communication technologies into the ice cream industry, companies can streamline operations, improve product quality, enhance customer experiences, and gain a competitive edge in the market. These technologies enable real-time data analysis, effective communication, and better decision-making, leading to increased efficiency and customer satisfaction. Reliability has been a key aspect in the efficient operation of enterprises for decades. Demand for products is increasing every day as the world's population grows. It is critical for industries to adapt to new automated technology in order to meet such demands from the public. The complexity of the manufacturing line grows as automated technology improves. The availability of such complex production lines is greatly dependent on the components' dependability and the company's maintenance policies. The current financial condition, as well as the regular changes in the worldwide market, have an impact on how industries operate. Industrialists are now using reliability analysis in their production lines to improve efficiency in order to compete in the global market. There are many factors1 which depend upon the reliability of a production line, the company’s reputation being the one and the most important. It takes a lifetime to build a reputation and a fault in the system can destroy it in no time. Quality and availability are some other factors required to measure the performance of a production system. Reliability2 and maintainability3 are the two factors which assess the quality of a product. These factors provide optimal strategies after predicting the chances of failures in the production system. Unreliability can be caused by a variety of factors, but it most commonly occurs when the system's design is inefficient or when the machines are not adequately maintained. Inappropriate content selection can contribute to an inefficient design. Certain equipment performs differently depending on the surroundings. To minimise failure, it is critical to have a thorough understanding of a component's limitations, characteristics, and use. Ignoring small system problems can lead to the system's complete failure. As the system's complexity grows, reliability suffers, necessitating the
implementation of various preventative measures. Random changes in environmental conditions might potentially cause a product or component to fail. In a manufacturing process, there are two types of costs: Reliability cost and development cost. Testing, investigation, administration, and planning costs are all included in the reliability cost. Diagnostic costs, wastage costs, and other operational costs are all included in the internal failure cost. Maintenance and service charges are examples of external costs. Failure costs\(^1\), operating costs, purchase expenses, depreciation charges, and other costs are included in the development cost. If equipment is maintained on a regular basis, financial losses\(^2\) due to malfunctioning systems can be prevented. It is critical for an industry to have a planned dependability programme as a separate file to succeed. The dependability programme should be approved by a senior management team that can plan how to spend resources and improve maintainability and testing procedures. In order to make predictions regarding a product's overall performance, it is also necessary to have a high level of reliability. When a product fails before the warranty period expires, it can damage the company's reputation and result in financial loss in the form of repair costs. The quality of any product is largely determined by the industry's maintenance policy, which indicates that it should be properly maintained. Reliability is essential in all aspects of an industry's operations.

In recent decades, the ice cream industry has experienced a remarkable surge, establishing itself as a leading force within the rapidly expanding dairy sector of the Asia-Pacific region. The fact that India is the world's largest producer and manufacturer of dairy products is a major driving force behind this industry's expansion. Demand for frozen desserts is rising globally, not just in India, as the world's population grows, resulting in a growth in the global economy. For the Ice-Cream Industry, the Covid-19 issue was a catastrophic disaster. Industrialists have suffered losses of roughly 6000 crores INR from March 2020, which is the start of the peak season for this industry. Due to the nationwide shutdown, the market faced numerous logistical hurdles. The sales for the months of March to June were reduced by about 85% due to the closure of stores, malls, and street sellers.

2. Literature Review

Reliability, availability, and maintainability (RAM) analysis plays a crucial role in assessing the performance and efficiency of industrial systems. Several studies have been conducted to explore RAM analysis\(^6\) in different domains, such as food production lines power plants\(^7\), packaging plants\(^8\), and manufacturing plants\(^9\). This literature review\(^10\) aims to summarize and analyze the key findings and methodologies used in these studies.\(^11\) Conduct a performance analysis of Guava fruit using Artificial Neural Network.

22\(^2\) provides a comprehensive review of RAM analysis in food production lines. The author discusses various statistical methods for estimating RAM parameters\(^13\) and emphasizes the importance of these analyses in optimizing production processes.\(^14\) presents a case study on the evaluation of overall equipment effectiveness (OEE) in an automated ice cream production line, highlighting the significance of RAM analysis in improving productivity.

The authors\(^15\) analyze the failure data to assess the system's performance and propose strategies for enhancing reliability and maintainability,\(^16\) focus on reliability analysis based on bottle production plant. The study emphasizes the importance of considering micro downtime events for accurate reliability estimation. Similarly,\(^17\) Conducting a comprehensive analysis of reliability, maintainability, and availability in a solar photo voltaic power plant becomes crucial in optimizing its performance and reducing operational downtime. This study emphasizes the importance of such analyses in ensuring the efficient functioning of the power plant,\(^18\) present a modeling and analysis study on the reliability of an industrial bakery plant. The authors propose a stochastic model to estimate the reliability and availability parameters of the system.

Human reliability is another critical aspect to consider in RAM analysis.\(^19\) discusses human reliability and error in maintenance, emphasizing the need to account for human factors in RAM analysis.\(^20\) present a reliability modelling and analysis study on communication networks, where they consider the impact of human errors on network performance.

Boolean function technique (BFT)\(^21\) is a commonly used method in RAM analysis,\(^22\) evaluate the performance of an industrial plant using BFT.\(^23\) assess resilience factors and suggest improvements\(^24\) to enhance system performance.\(^25\) also employ BFT to analyze reliability in milk powder manufacturing plants and paper plants, respectively.

Other studies explore the application of different methodologies in RAM analysis,\(^26\) review reliability analysis of steel structures, highlighting the advantages of ANNs in predicting system behavior.\(^27\) presents a structural reliability analysis method using radial basis function networks, providing an alternative approach to estimating system reliability.

In conclusion, RAM analysis is crucial for evaluating the performance and efficiency of industrial systems. The reviewed studies demonstrate the application of various methodologies\(^28\) including statistical analysis, BFT, ANN models, and structural reliability analysis.\(^29\) These studies provide valuable insights and techniques to enhance the reliability, availability, and maintainability of different industrial systems\(^30\). Further research in this field can explore the integration of advanced technologies such as machine learning\(^31\) and artificial intelligence, to improve RAM analysis and optimize system performance.
3. Description of model:

3.1 Working Procedure of Ice Cream Plant

The Ice-Cream Plant under consideration consists of 10 components that are integral to the production line. The flowchart provided in Figure 1 illustrates the sequential process of ice cream production. These 10 components are defined as:

1. Raw Material Storage: This component involves the storage of raw materials required for ice cream production, such as milk, cream, sugar, flavourings, and additives. Proper storage conditions, including temperature and humidity control, are crucial to maintain ingredient quality.

2. Mixing and Blending: In this stage, the raw materials are measured and mixed together according to the desired recipe. The mixing process ensures uniform distribution of ingredients, creating a homogeneous ice cream base.

3. Pasteurisation is the process of heating the ice cream base to a specific temperature in order to eliminate harmful bacteria and ensure product safety. Pasteurisation extends the shelf life of ice cream while maintaining quality.

4. Homogenization: This process involves breaking down the fat globules in the ice cream base to achieve a smooth and creamy texture. Homogenization helps prevent fat separation and creates a stable emulsion.

5. Aging: After homogenization, the ice cream base is aged to allow flavors to develop and stabilize. Aging enhances the taste and texture of the ice cream.

6. Flavor Addition: Flavorings, such as fruits, nuts, chocolates, or syrups, are added to the ice cream base during this stage. It gives the ice cream its distinct taste and variety.

7. Freezing: The flavored ice cream base is rapidly frozen using continuous or batch freezers. The freezing process forms ice crystals and incorporates air to create the desired texture and consistency.

8. Packaging: Once the ice cream is frozen, it is packaged into containers or cartons. This step ensures hygienic storage, protects the ice cream from contamination, and provides convenience for consumers.

9. Storage and Distribution: Packaged ice cream containers are stored in temperature-controlled facilities to maintain their quality. They are then distributed to various retail outlets or directly to consumers.

10. Retail Display: Ice cream containers are displayed in retail stores or ice cream parlours to attract customers. Proper storage and display techniques are employed to maintain the quality of the product.

The process involving initial steps characterizes the operation involving unpasteurized, i.e., raw ingredients mix. Then the next stage involves the process involving pasteurized mix. The final step involves the operation involving freezing of Ice-Cream. Thus, the production can be categorized into three stages, the unpasteurized mix stage, the pasteurized mix stage, and the Frozen Ice-Cream Stage.

3.2 Methodology

The case study of an Ice-Cream Plant is explored in this research for reliability analysis. Because the life lifetime of these commodities is short, manufacturers are concerned about supplying fresh and high-quality food products. The Boolean function technique was used to examine several dependability variables in the automated Ice-Cream production line.

3.3 System Configuration

The Ice-Cream plant under consideration has 10 components. The components are denoted as $\psi_i$, where $i = 1, 2, \ldots, 10$. Fig. 2 shows the indicative diagram (systematic configuration) of the considered production line. The following assumptions for the model are taken into account before initializing the computations.

1) The system is functional at the initial stage.
2) No maintenance services for failed components.
3) For each component, reliability is in anticipation.
4) For each component, the state is statistically independent.
5) Failure time is random for each component.
6) The system is in either a good (operable) or failed state.
3.4 Mathematical Symbols Used:

\( \nu_1 \): Blending

\( \nu_2 \): Continuous Pasteurization

\( \nu_3 \): Batch Pasteurization

\( \nu_4 \): Ageing

\( \nu_5 \): Flavour Tank

\( \nu_6 \): Continuous Freezing

\( \nu_7 \): Batch Freezing/Whipping

\( \nu_8 \): Packaging

\( \nu_9 \): Hardening

\( \nu_{10} \): Storage/Distribution Stage

\( \ominus \) : Logical AND

\( \nu_i \) : The variable \( \nu_i \) takes a value of 1 when the component is in an operable state and 0 when the component is in a failed state.

\( \nu_i' \) : Negation for \( \nu_i \), \( i = 1, 2, \ldots, 10 \)

\( \mathcal{R}_i \) : \( i \)th component Reliability , where \( i = 1, 2, \ldots, 10 \).

\( \mathcal{Q}_i \) : \( i \)th component unreliability, where \( i = 1, 2, \ldots, 10 \).

\( \mathcal{R}_{\text{system}} \) : Reliability of the entire system

\( \mathcal{R}_{\text{SG}} \) : Reliability of the entire system when the failure rate follows the Weibull Distribution.

\( \mathcal{R}_{\text{SG}} \) : Reliability of the entire system when failure rate follows the Exponential Distribution.

3.5 Mathematical model formulation:

Using the Boolean Function Technique, the formulation of the model in terms of logical matrix is shown as Eq (1).

\[
F(\nu_1, \nu_2, \ldots, \nu_{10}) = \begin{vmatrix}
\nu_1 & \nu_2 & \nu_3 & \nu_4 & \nu_5 & \nu_6 & \nu_7 & \nu_8 & \nu_9 & \nu_{10} \\
\nu_1 & \nu_2 & \nu_4 & \nu_5 & \nu_6 & \nu_8 & \nu_9 & \nu_{10} \\
\nu_1 & \nu_3 & \nu_4 & \nu_5 & \nu_6 & \nu_8 & \nu_9 & \nu_{10} \\
\nu_1 & \nu_3 & \nu_4 & \nu_5 & \nu_7 & \nu_8 & \nu_9 & \nu_{10} \\
\end{vmatrix}
\]

\( (1) \)

\[
F(\nu_1, \nu_2, \ldots, \nu_{10}) = \nu_1 \wedge \nu_4 \wedge \nu_5 \wedge \nu_8 \wedge \nu_9 \wedge \nu_{10} \wedge
\delta(\nu_2, \nu_3, \nu_6, \nu_7)
\]

\( (2) \)

Where, \( \delta(\nu_2, \nu_3, \nu_6, \nu_7) = \begin{vmatrix}
\nu_2 & \nu_6 \\
\nu_2 & \nu_7 \\
\nu_3 & \nu_6 \\
\nu_3 & \nu_7 \\
\end{vmatrix} = \begin{vmatrix}
\nu_1 & \nu_2 \\
\nu_1 & \nu_3 \\
\nu_1 & \nu_4 \\
\nu_1 & \nu_5 \\
\end{vmatrix} = \begin{vmatrix}
K_1 \\
K_2 \\
K_3 \\
K_4 \\
\end{vmatrix}
\]

\( (3) \)

Such that \( K_1 = [\nu_2 \ \nu_6] \)

\( (4) \)

\( K_2 = [\nu_2 \ \nu_7] \)

\( (5) \)

\( K_3 = [\nu_3 \ \nu_6] \)

\( (6) \)

\( K_4 = [\nu_3 \ \nu_7] \)

\( (7) \)

Using Orthogonalization Algorithm, equation (1) becomes,

\[
F(\nu_1, \nu_2, \ldots, \nu_{10}) = \nu_1 \wedge \nu_2 \wedge \nu_4 \wedge \nu_5 \wedge \nu_6 \wedge \nu_8 \wedge \nu_9 \wedge \nu_{10} \\
\nu_1 \nu_2 \nu_4 \nu_5 \nu_6 \nu_8 \nu_9 \nu_{10} - \\
\nu_1 \nu_2 \nu_4 \nu_5 \nu_6 \nu_7 \nu_8 \nu_9 \nu_{10} + \\
\nu_1 \nu_3 \nu_4 \nu_5 \nu_6 \nu_8 \nu_9 \nu_{10} - \\
\nu_1 \nu_2 \nu_3 \nu_4 \nu_5 \nu_6 \nu_8 \nu_9 \nu_{10} \\
\nu_1 \nu_3 \nu_4 \nu_5 \nu_7 \nu_8 \nu_9 \nu_{10} - \\
\nu_1 \nu_2 \nu_3 \nu_4 \nu_5 \nu_7 \nu_8 \nu_9 \nu_{10} + \\
\nu_3 \nu_4 \nu_5 \nu_6 \nu_7 \nu_8 \nu_9 \nu_{10} + \\
\nu_1 \nu_2 \nu_3 \nu_4 \nu_5 \nu_6 \nu_7 \nu_8 \nu_9 \nu_{10}
\]

\( (8) \)

3.6 Some Particular Cases:

Case 1: When the reliability of each component is equal and is denoted as \( \mathcal{R} \). In this case, equation (10) becomes,

\[
\mathcal{R}_{\text{system}} = 4\mathcal{R}^8 - 4\mathcal{R}^9 + \mathcal{R}^{10}
\]

\( (10) \)

Case 2: When the failure rate follows the Weibull Distribution, the system's reliability at a given moment \( t \) is determined by the following expression:

\[
\mathcal{R}_{\text{SGW}}(t) = \sum_{j=1}^{5} \exp(-\gamma_j t^p) - \sum_{n=1}^{4} \exp(-\omega_n t^p)
\]

\( (12) \)

where, \( p > 0 \)

Case 3: When the failure rate follows Exponential Distribution.

In this case, by substituting \( p = 1 \) in equation (12), we get the system's reliability at the moment \( t \) by

\[
\mathcal{R}_{\text{EXP}}(t) = 1 - \exp(-\lambda t)
\]

\( (13) \)
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\[ R_{SE}(t) = \sum_{j=1}^{5} \exp(-\gamma_j t) - \sum_{n=1}^{4} \exp(-\omega_n t) \]  \hspace{1cm} (14)

Another crucial reliability parameter is the Mean Time to Failure, denoted as MTTF, which is expressed as:

\[ \text{MTTF} = \int_{0}^{\infty} R_{SE}(t) \, dt = \frac{1}{\gamma_1} - \frac{1}{\omega_4} \]  \hspace{1cm} (15)

3.7 Numerical Examples:

The following Table 1 shows the failure rate for each component in the production line. The following fig. 3 shows the calculated reliability of the system for different values of \( t \) when failure rate follows Weibull Distribution and Exponential Distribution.

Let the failure rate be \( \theta_i = \theta \) for all \( i = 1, 2, ..., 10 \) in equation (15), then,

\[ \text{MTTF} = \frac{0.155555555555}{\theta} \]  \hspace{1cm} (16)

Manufacturers strive for high system reliability to minimize losses, which requires increased investments. Consequently, it is crucial for them to evaluate system reliability while minimizing overall costs. To achieve this, the expected cost can be calculated using equation (16) based on \( S_1 \) (revenue cost per unit time) and \( S_2 \) (service cost per unit time).

\[ S(t) = S_1 \int_{0}^{t} R(t) \, dt - S_2 t \]  \hspace{1cm} (17)

The calculation of expected cost involves two cases: the useful life period and the wear out phase. In the useful life period, where the failure rate remains constant and follows an Exponential Distribution, the expected cost is determined. On the other hand, in the wear out phase, where failure is time-dependent and follows a Weibull Distribution, the expected cost is calculated. These two cases provide different perspectives on the system’s reliability and enable manufacturers to assess the associated costs in each phase.

Thus, for Case 1, for the constant failure rate, after substituting \( R(t) \) and solving the integral in equation 16, we have,

\[ S(t) = \left( S_1 \theta \right) \left[ 4 \left( 1 - e^{-8\theta t} \right) - 4 \left( 1 - e^{-9\theta t} \right) + \left( 1 - e^{-10\theta t} \right) \right] - S_2 t \]  \hspace{1cm} (18)

\[ s(t) = \left( \frac{S_1}{20} \right) \left[ 4 \left( 1 - e^{-8\theta t} \right) - 4 \left( 1 - e^{-9\theta t} \right) + \left( 1 - e^{-10\theta t} \right) \right] - S_2 t \]  \hspace{1cm} (19)

A numerical example is considered to calculate the expected cost in both cases. The revenue cost per unit time is taken as \( S_1 = $500 \) initially and time \( t \) is considered in months. The expected cost for different values of \( S_2 \), which is the service cost and different failure rates of the whole system with respect to time is shown in Fig. 5(a) and fig. 5(b).

**Fig. 3:** Variation of Reliability with respect to time

**Fig. 4:** MTTF with respect to failure rate

**Fig. 5(a):** Cost Analysis Graph for Useful Life Period
4. Conclusion:

This paper focuses on analyzing the reliability of an Ice-Cream plant by employing the Boolean Function Technique. The authors have determined various reliability parameters based on this analysis. Additionally, several examples were examined to gain insights into the behavioural patterns of reliability and its associated parameters in different circumstances. Some of the components in the line are at very crucial stage, if those components fail the whole line will fail and the production will be stopped as repair facility is not available. These stages include blending, ageing, flavour tank, hardening, packaging, and storage/distribution.

Based on the observations depicted in Fig. 3, it can be noted that the reliability of the components diminishes over time due to failures. The rate of decrease is relatively slower when failures adhere to the Exponential distribution. However, when failures are modelled by the Weibull distribution, the reliability experiences a significant decline. Fig. 4 illustrates that initially, the Mean Time to Failure (MTTF) shows a gradual decrease as the failure rate rises. However, as time progresses and failures accumulate, the MTTF exhibits a more indistinct decrease.

The initial revenue cost is taken as $500, and the initial service cost is taken as $200. Then different cases are considered for which service cost is increased at 2% of the current service cost. It was observed that for the useful life, initially, cost was increasing gradually till the 9th month and then it started decreasing exponentially. On the other hand, for the wear out phase, cost increased initially till the 6th month, but after the deterioration period started, it started decreasing vaguely. Weibull life distribution with shape parameter 2 which indicates wear-out is not acceptable as the plant starts incurring losses after 6 months. It also indicates necessity of repair facility for the plant. The result shows in Fig. 5(a) and Fig. 5(b) that there is a considerable increase in profit per unit time if revenue cost is increased. Failure rates were supposed to follow some probability distributions and this assumption can be used in other models of the similar nature. It was observed that using different probability distributions was helpful in making comparison in Boolean Function Technique.

Traditional statistical methods may not be suitable for conducting reliability analysis, as they may not effectively address the complexities involved. However, employing techniques such as mathematical models and optimization methods can help achieve reliability goals in a cost-effective manner. Unlike traditional methods, these approaches do not necessitate a large sample size, reducing both costs and time requirements.

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