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# **Quantitative explanation of the difference in AC magnetization curves between suspended and immobilized magnetic nanoparticles for biomedical application**

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## **Abstract**

We study the difference in AC magnetization ( $M$ - $H$ ) curves between suspended and immobilized magnetic nanoparticles (MNPs). We use three commercial MNP samples that are often used for biomedical application. First, we measure the hysteresis area of the AC  $M$ - $H$  curve,  $A$ , when the amplitude of the excitation field,  $H_{ac}$ , is changed. The  $A$  vs.  $H_{ac}$  curve is compared with previously obtained analytical results by taking account of the MNP size distribution in the sample. We show that they quantitatively agree for both suspended and immobilized samples. From the comparison, we clarify the mechanism that determines the AC  $M$ - $H$  curve of a suspended sample. For high  $H_{ac}$ , the Néel relaxation becomes dominant in the suspended sample, and alignment of easy axes caused by the AC field increases the hysteresis loss compared to the immobilized case. The effect of Brownian relaxation in the suspended sample increases with decreasing  $H_{ac}$  and gives additional loss at low field. The portion of Néel-relaxation- and Brownian-relaxation-dominant MNPs in the sample is quantitatively evaluated. We also clarify the difference in harmonic signals between suspended and immobilized samples. Finally, we discuss the condition for the MNP parameters and excitation field that determines the dominant relaxation mechanism in a suspended sample.

## **Key words**

Magnetic nanoparticle; Néel relaxation; Brownian relaxation; hysteresis loss; harmonic signals; easy axis alignment

## 1. Introduction

The AC magnetization ( $M-H$ ) curve of magnetic nanoparticles (MNPs) has been widely studied for biomedical application. For example, MNP hysteresis loss has been studied for magnetic hyperthermia, where the loss is used to raise the temperature of cancer cells to kill them [1-9]. Harmonic signals caused by nonlinear magnetization have been studied for magnetic particle imaging (MPI), where the three-dimensional position of MNPs is detected with high temporal resolution for medical diagnosis [10-13]. Harmonic signals are also used for magnetic biosensing, where disease-related biological targets are detected using bio-functionalized MNPs [14-18].

These magnetic properties are different between suspended and immobilized MNPs. The difference in these properties can also be used to magnetically distinguish the suspended and immobilized MNPs when they coexist in a sample. For immobilized MNPs, the dynamic behavior is determined by Néel relaxation. The easy axis of magnetization is fixed regardless of time, and only the magnetic moment vector rotates owing to Néel relaxation when an AC field is applied. However, for MNPs suspended in liquid, both the Brownian and Néel relaxations affect the AC  $M-H$  curve. Both the easy axis and magnetic moment vector rotate owing to Brownian and Néel relaxation, respectively.

The extent to which Brownian or Néel relaxation contributes to the AC  $M-H$  curve of suspended MNPs considerably depends on MNP parameters and excitation conditions. As a result, the AC  $M-H$  curve of suspended MNPs becomes very complicated. Thus far, theoretical studies have been done for two limiting cases. One is  $\tau_N(H) \gg \tau_B(H)$ , where  $\tau_N(H)$  and  $\tau_B(H)$  are the field-dependent Néel and Brownian relaxation times, respectively. In this case, the AC  $M-H$  curve of suspended MNPs is determined by linear or nonlinear Brownian relaxation [4, 19]. The other case is  $\tau_N(H) \ll \tau_B(H)$ . In this case, the dynamic behavior of suspended MNPs is primarily dominated by Néel relaxation. The magnetic moment rotation is caused by Néel relaxation. In addition, when the field  $H$  becomes strong, successive Brownian (or physical) rotation of suspended MNPs occurs, causing partial alignment of easy axes [20-25]. We note that the mechanism that gives the difference in AC  $M-H$  curves between suspended and immobilized MNPs is different for the two cases.

The condition under which these two cases can be applied is determined not only by the MNP parameters but also by the amplitude and frequency of the excitation field  $H$ . This is because  $\tau_B$  and  $\tau_N$  depend on  $H$ . As a result,  $H$  determines whether MNP behavior is Néel-relaxation-dominant or Brownian-relaxation-dominant even when MNP parameters are fixed. Therefore, it is necessary to quantitatively clarify the condition for each of these two cases in a suspended sample. When this condition is obtained, we can clarify the mechanism that results in the different AC  $M-H$  curves between suspended and immobilized MNPs.

In this paper, we study the difference in the AC  $M-H$  curves between suspended and immobilized MNPs. We use three commercial MNP samples that are often used for biomedical application. First, we evaluate the magnetic parameters such as the saturation magnetization,  $M_s$ , anisotropy constant,  $K$

and distribution of magnetic core size  $d_c$  in each sample. Next, the hysteresis area of the AC  $M$ - $H$  curve,  $A$ , is measured when the amplitude of the excitation field,  $H_{ac}$ , is changed. The measured  $A$  vs.  $H_{ac}$  curve is compared with analytical results. We show quantitative agreement between them for both suspended and immobilized samples. From the comparison, we clarify how the Brownian and Néel relaxations contribute to the hysteresis loss in a suspended sample. For our samples, easy axis alignment caused by the AC field increases the loss at high fields, while Brownian relaxation increases the loss at low fields. We also show the difference in harmonic signals between suspended and immobilized samples. Finally, we discuss the condition for the MNP parameters and excitation field that determines the dominant relaxation in a suspended sample.

## 2. Materials and methods

We used three commercial MNP samples: synomag (micromod GmbH, Germany), MS1 (Meito, Japan) and perimag (micromod GmbH, Germany), hereafter denoted as SY, MS and PE, respectively. The samples are multi-core based magnetic nanoparticles, and consist of elementally particles and crystallized agglomerates of elementary particles. We emphasize that the agglomerates are crystallized and can thus be regarded as an effective single core with size  $d_c$ , as discussed in previous papers [26, 27].

Synomag consists of maghemite ( $\gamma$ - $\text{Fe}_2\text{O}_3$ ) cores covered by a dextran shell, and their hydrodynamic diameter is nominally  $d_H = 50$  nm [28]. *Riahi et al* presented a transmission electron microscope (TEM) image of synomag, and obtained physical size of particles as  $d_{\text{TEM}} = 9$  and 24.3 nm for elementary particles and agglomerates, respectively [29]. They also obtained  $d_H = 66.5$  nm using the dynamic light scattering (DLS) measurement. MS1 are carboxydextran-coated nanoparticles of a mixture of magnetite ( $\text{Fe}_3\text{O}_4$ ) and maghemite, and are magnetically fractionated from Resovist (FUJIFILM RI Pharma, Japan) so as to have large core size [30]. The values of  $d_H = 55.2$  nm and  $d_{\text{TEM}} = 4.6$  nm for elementary particles were reported for Resovist in ref. [29], but the  $d_{\text{TEM}}$  value for agglomerates was unknown. Perimag consists of maghemite cores covered by a dextran shell with  $d_H = 130$  nm [28]. *Tay et al* obtained  $d_{\text{TEM}} = 5.5$  nm for elementary particles of perimag [31], but the  $d_{\text{TEM}}$  value for agglomerates was not reported.

We first measured the DC  $M$ - $H$  curve of each sample suspended in water up to  $\mu_0 H_{dc} = 1$  T using a homemade measurement system based on vibrating sample magnetometer (VSM) method. The distribution of  $d_c$  was estimated by analyzing the DC  $M$ - $H$  curve.

Then, the AC  $M$ - $H$  curves were measured for both the suspended and immobilized samples. For the suspended sample, 150  $\mu\text{g}$ -Fe of MNPs were dispersed in 150  $\mu\text{L}$  of water. For the immobilized sample, 150  $\mu\text{g}$ -Fe of MNPs were dispersed in 150  $\mu\text{L}$  of epoxy resin. The epoxy resin was initially liquid and began to solidify after 10 min, and almost completely solidified in about 1 h. When the epoxy resin is in liquid state, Brownian rotation of MNPs occurs, and the easy axes of MNPs become

randomly oriented. Then, solidification of the epoxy resin occurs. Therefore, MNPs are immobilized with randomly oriented easy axes when solidification of the epoxy resin completes [24].

The AC  $M$ – $H$  curves were measured using a homemade measurement system, whose details were mentioned in our previous paper [32]. The excitation field frequency was  $f = 20$  kHz, and the amplitude was changed from  $\mu_0 H_{ac} = 8$  to 20 mT.

### 3. Results

#### 3.1. Evaluation of MNP parameters

In the present paper, we assume that  $M_s$  and  $K$  are constant in sample, i.e., they are independent of core size  $d_c$ . Table I lists the parameters of the three MNP samples, obtained as follows. First, we measured the DC magnetization ( $M_{dc}$ – $H_{dc}$ ) curve. The  $M_{dc}$  value was almost saturated at  $\mu_0 H_{dc} = 1$  T. For simplicity, therefore,  $M_s$  was obtained from  $M_{dc}$  measured at  $\mu_0 H_{dc} = 1$  T. We obtained  $M_s = 345$ , 360 and 295 kA/m for synomag, MS1 and perimag, respectively. We note that the linear relationship between  $M_{dc}$  and  $1/H_{dc}$  was observed at high fields  $\mu_0 H_{dc} > 0.3$  T, as expected from the Langevin function (see Eq. (A3) in Appendix A). The  $M_s$  value can also be estimated by extrapolating the linear  $M_{dc}$  vs  $1/H_{dc}$  relation to  $1/H_{dc} = 0$  [33], and we obtained  $M_s = 348$ , 366 and 298 kA/m for synomag, MS1 and perimag, respectively. These values were almost the same as the  $M_s$  values obtained from  $M_{dc}$  measured at  $\mu_0 H_{dc} = 1$  T.

The distribution of the core size  $d_c$  in an MNP sample was obtained by analyzing the measured  $M_{dc}$ – $H_{dc}$  curve [34] (see Appendix A). The distribution was expressed as a  $d_c$  vs.  $n(d_c)V_c$  curve, where  $n(d_c)$  is the number density of MNPs with a core diameter  $d_c$  per unit of MNP volume, and  $V_c = (\pi/6)d_c^3$  is the volume of the magnetic core. Fig. 1 presents the results for the three samples. The  $d_c$  vs.  $n(d_c)V_c$  curve has two peaks for all three samples. The peak at small  $d_c$  (near  $d_c = 6$  nm) corresponds to elementary particles, while the peak at large  $d_c$  corresponds to crystallized agglomerates. The portion of agglomerates is much larger than that of elementary particles. The agglomerate  $d_c$  value is distributed from 12 to 30 nm for synomag. For MS1 and perimag,  $d_c$  for the agglomerates ranged from 10 to 45 nm. The typical agglomerate  $d_c$  value,  $d_{c,typ}$ , that gives the peak of  $n(d_c)V_c$  is  $d_{c,typ} = 17.8$ , 23.7 and 24.6 nm for synomag, MS1 and perimag, respectively.

Table I. Parameters of the three MNP samples

| sample  | $M_s$<br>(kA/m) | $K$<br>(kJ/m <sup>3</sup> ) | $\mu_0 H_k$<br>(mT) | $d_{c,typ}$<br>(nm) | $m_{typ}$<br>(10 <sup>-18</sup> Am <sup>2</sup> ) | $\sigma_{typ}$ |
|---------|-----------------|-----------------------------|---------------------|---------------------|---|----------------|
| synomag | 345             | 9.7                         | 56.2                | 17.8                | 1.02  | 6.9            |
| MS1     | 360             | 7.0                         | 38.9                | 23.7                | 2.51  | 11.8           |
| perimag | 295             | 5.8                         | 39.3                | 24.6                | 2.30  | 10.9           |

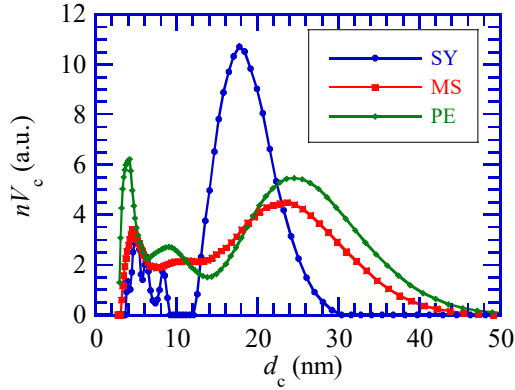


Fig. 1. Core size distribution for the three MNP samples: synomag (SY), MS1 (MS) and perimag (PE).

Examples of the AC  $M$ - $H$  curves are shown in Fig. 2(a). They were measured for immobilized (solid) and suspended (water) MS1 samples when  $f = 20$  kHz and  $\mu_0 H_{ac} = 20$  mT. The  $M$  value and hysteresis of the suspended sample were larger than those of the immobilized sample.

The anisotropy constant  $K$  was estimated by analyzing the coercive field  $\langle H_c \rangle$  of the AC  $M$ - $H$  curve for the immobilized sample. An analytical expression for  $\langle H_c \rangle$  accounting for the  $d_c$  distribution has been obtained in ref. [35] [Eq. (B4) in Appendix B]. This expression was compared with the measured  $\langle H_c \rangle$ - $H_{ac}$  curve taking  $K$  as an adjustable parameter. The value of  $K$  was determined to obtain the best fit between experiment and analysis. Fig. 2(b) presents the  $\langle H_c \rangle$ - $H_{ac}$  curve obtained for the three samples. Symbols represent the experimental results, while the solid lines are the analytical results. The value of  $K$  was chosen as 9.7, 7.0 and 5.8 kJ/m<sup>3</sup> for synomag, MS1 and perimag, respectively. We obtained good agreement between analysis and experiment for synomag. For MS1 and perimag, agreement was good at high fields, but deviation occurred at low fields. We note that the  $K$  values obtained for synomag and MS1 were a little larger than those reported in ref. [35]. This will be because magnetic parameters are slightly different for each product lot.

From  $M_s$  and  $K$ , the field  $H_k$  is defined as

$$H_k = \frac{2K}{\mu_0 M_s}. \quad (1)$$

Synomag has highest  $H_k$ , while MS1 and perimag have similar  $H_k$  values, as shown in Table I.

Using the  $d_{c,typ}$  values in Table I, we also estimated the typical values of  $m_{typ} = M_s V_{c,typ}$  and  $\sigma_{typ} = KV_{c,typ}/(k_B T)$ , where  $k_B$  is the Boltzmann constant and  $T$  is the absolute temperature. These values increase in the order synomag, perimag and MS1, as shown in Table I.

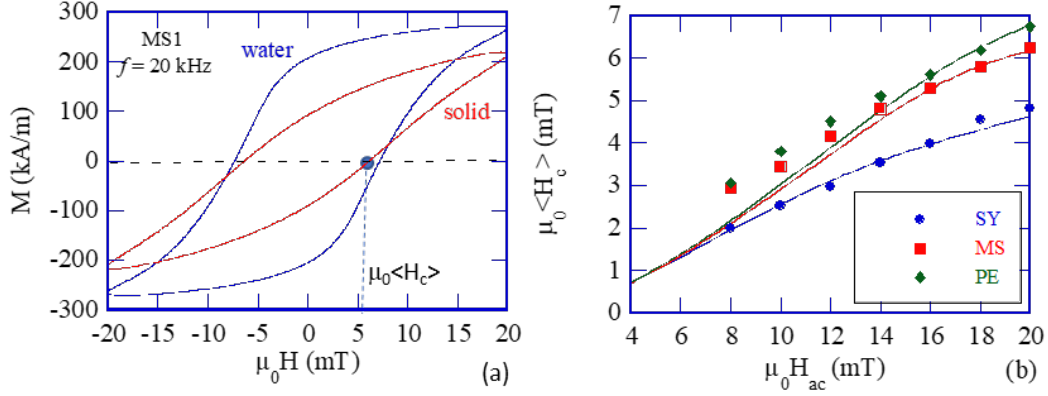


Fig. 2. (a) AC  $M$ - $H$  curve of the MS1 sample measured at  $f = 20$  kHz and  $\mu_0 H_{ac} = 20$  mT. (b)  $\langle H_c \rangle$ - $H_{ac}$  curves for the three immobilized MNP samples. Symbols are experimental results, while the solid lines are analytical results obtained from Eq. (B4). The calculation used the values  $K = 9.7, 7.0$  and  $5.8$  kJ/m<sup>3</sup> for synomag, MS1 and perimag, respectively.

### 3.2. Hysteresis loss

We consider the case when the AC excitation field

$$H(t) = H_{ac} \sin(2\pi ft) \quad (2)$$

is applied to an MNP sample. The hysteresis loss per unit cycle,  $\langle A \rangle$ , is given by

$$\langle A \rangle = -\oint \mu_0 \mathbf{M} \cdot d\mathbf{H} = \pi \mu_0 H_{ac} \text{Im}[M_1], \quad (3)$$

where  $\text{Im}[M_1]$  is the imaginary part of the fundamental component  $M_1$  when  $M(t)$  is expressed as a Fourier series. We obtained  $\text{Im}[M_1]$  from the measured  $M(t)$ , and then obtained  $\langle A \rangle$  using Eq. (3).

#### 3.2.1. Immobilized MNPs with randomly oriented easy axes

We first study  $\langle A \rangle$  for an immobilized MNP sample, where the AC  $M$ - $H$  curve is determined by Néel relaxation. Fig. 3 shows the dependence of  $\langle A \rangle$  on  $H_{ac}$ . Red circles represent the measured  $\langle A \rangle$  vs.  $H_{ac}$  curves for immobilized (solid) MNP samples. The results for synomag, MS1 and perimag are shown in Fig. 3(a), 3(b) and 3(c), respectively.

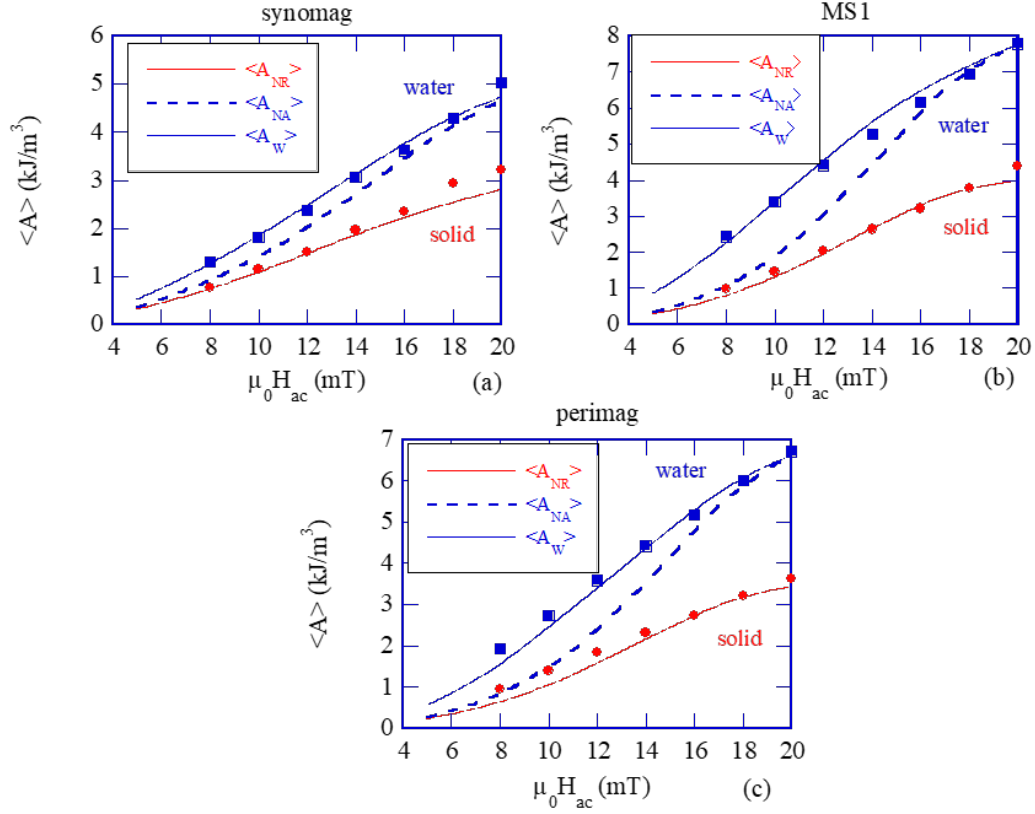


Fig. 3.  $\langle A \rangle$  vs.  $H_{ac}$  curve for (a) synomag, (b) MS1 and (c) perimag. Circles and rectangles represent the experimental results for the immobilized (solid) and suspended (water) samples, respectively. The lines are calculated from Eqs. (11) and (19).

We compare these experimental results with analysis. When MNPs have the same core size  $d_c$  and their easy axes are randomly oriented, an empirical expression for  $A$ , which we denote as  $A_{NR}$ , was obtained based on numerical simulation, and is given by [36]

$$A_{NR}(d_c) = \begin{cases} 1.92K(1 - 1.08\kappa) \times g_{NR} & \text{for } \kappa < 0.63 \\ 1.92K \exp\left\{-\left(\frac{\kappa}{0.6}\right)^{2.2}\right\} \times g_{NR} & \text{for } \kappa > 0.63, \end{cases} \quad (4)$$

with

$$\kappa = \frac{1}{\sigma} \ln\left(\frac{1}{4\xi_{ac}f\tau_0}\right), \quad (5)$$

where  $\sigma = KV_c/(k_B T)$ ,  $\xi_{ac} = \mu_0 m H_{ac}/(k_B T)$  and  $m = M_s V_c$  is the magnetic moment.  $\tau_0$  is the characteristic time and was set as  $\tau_0 = 10^{-9}$  s.

The value of  $g_{NR}$  in Eq. (4) is given by



$$g_{NR} = \frac{\exp\left(\frac{h_{ac} - h_1}{\Delta h}\right)}{1 + \exp\left(\frac{h_{ac} - h_1}{\Delta h}\right)}, \quad (6)$$

with

$$h_1 = 0.12(h_{ac,th})^2 + 0.72h_{ac,th} + 0.06, \quad (7)$$

$$h_{ac,th} = \begin{cases} 0 & \text{for } \sigma \leq \sigma_{th}, \\ 0.6\left(1 - \frac{\sigma_{th}}{\sigma}\right) & \text{for } \sigma > \sigma_{th} \end{cases} \quad (8)$$

$$\Delta h = 0.025, \quad (9)$$

where  $h_{ac} = H_{ac}/H_k$  is the normalized field amplitude. The value of  $\sigma_{th}$  in Eq. (8) is determined by the frequency  $f$  of the applied field that satisfies the condition

$$2\pi f\tau_N(0) = \frac{\pi^{\frac{3}{2}}}{\sqrt{\sigma_{th}}} f\tau_0 \exp(\sigma_{th}) = 1. \quad (10)$$

When  $d_c$  is distributed in the sample, the hysteresis loss of the sample,  $\langle A_{NR} \rangle$ , is given by

$$\langle A_{NR} \rangle = \frac{1}{\int_0^\infty nV_c dd_c} \int_0^\infty nV_c A_{NR}(d_c) dd_c. \quad (11)$$

The red lines in Fig. 3 represent  $\langle A_{NR} \rangle$  calculated with Eq. (11) for the three samples. In the calculation, the value of  $nV_c$  was obtained from the  $d_c$  vs.  $n(d_c)V_c$  curve in Fig. 1. The values of  $M_s$ ,  $K$  and  $H_k$  in Table I were used, and the value of  $\sigma_{th}$  was obtained from Eq. (10) as  $\sigma_{th}=10.3$  for  $f=20$  kHz. As shown in Fig. 3, we obtained good agreement between experiment and analysis for the three MNP samples, indicating the validity of Eq. (11). We emphasize that there is no adjustable parameter in the calculation.

### 3.2.2. Suspended MNPs

Next, we study  $\langle A \rangle$  for MNP samples suspended in water. In Fig. 3, blue rectangles represent the measured  $\langle A \rangle$  vs.  $H_{ac}$  curve for suspended (water) MNP samples. As shown,  $\langle A \rangle$  for the water sample was much larger than for the solid sample.

We compare these experimental results with analysis. For suspended MNPs, analytical expressions for hysteresis loss were obtained for two limiting cases, as shown below.

#### 3.2.2.1. Hysteresis Loss due to Brownian relaxation

An analytical expression for  $\text{Im}[M_1]$  in Eq. (3) has been obtained for the dynamics of suspended MNPs dominated by Brownian relaxation [19]. The hysteresis loss in this case,  $A_{BR}$ , is given by

$$A_{BR}(d_c, d_H) = \frac{\pi\mu_0 H_{ac} M_s}{3} \xi_{ac} k''(\xi_{ac}) g_{BR}(\xi_{ac}) \frac{2\pi f\tau_B(H)}{1 + [2\pi f\tau_B(H)]^2} \quad (12)$$

with

$$k''(\xi_{ac}) = 1 + \frac{0.024\xi_{ac}^2}{1 + 0.18\xi_{ac} + 0.033\xi_{ac}^2}, \quad (13)$$

$$g_{BR}(\xi_{ac}) = 1 - \frac{\xi_{ac}^3}{10 + 9\xi_{ac} + 3.81\xi_{ac}^2 + \xi_{ac}^3}. \quad (14)$$

The quantity  $\tau_B(H)$  in Eq. (12) is the field-dependent Brownian relaxation time given by [19, 37]

$$\tau_B(H) = \frac{\tau_{B,0}}{\sqrt{1 + 0.07\xi_{ac}^2}}, \quad (15)$$

where  $\tau_{B,0} = 3\eta V_H / (k_B T)$ ,  $\eta$  is the liquid viscosity (0.86 mPa s for water), and  $d_H$  and  $V_H = (\pi/6)d_H^3$  are the hydrodynamic diameter and hydrodynamic volume of an MNP, respectively.

### 3.2.2.2. Hysteresis loss due to Neel relaxation with easy axis alignment

We consider the case  $\tau_N(H) < 1/(2\pi f) \ll \tau_B(H)$ , where the MNP dynamics is primary dominated by Néel relaxation. In addition, the AC field causes successive Brownian (physical) rotation of suspended MNPs, and the easy axes are partially aligned along the applied field [20, 25]. The empirical expression for  $A$  in this case,  $A_{NA}$ , is [38]

$$A_{NA} = A_{NR} \times R_A, \quad (16)$$

where  $A_{NR}$  is the hysteresis loss when MNPs are immobilized with randomly oriented easy axes and is given in Eq. (4).  $R_A$  represents the increase in loss due to easy axis alignment and is given by

$$R_A = 1 + 1.2 \tanh(0.12\xi_{eff}), \xi_{eff} = \frac{k_{eff}}{\sqrt{2}} \xi_{ac}, \quad (17)$$

$$k_{eff} = \begin{cases} \frac{h_{ac}}{1.15(1 - \sqrt{\kappa})} [0.28 + 0.009(\sigma\kappa)^2] & \text{for } h_{ac} < 1.15(1 - \sqrt{\kappa}) \\ 0.28 + 0.009(\sigma\kappa)^2 & \text{for } h_{ac} \geq 1.15(1 - \sqrt{\kappa}) \end{cases}, \quad (18)$$

where  $k_{eff} = 0$  for  $\kappa > 1$ , and  $k_{eff} = 1$  when the right side of Eq. (18) is greater than 1.

### 3.2.2.3. Hysteresis loss for a suspended MNP sample

We combine  $A_{BR}$  in Eq. (12) and  $A_{NA}$  in Eq. (16) to obtain the hysteresis loss for the suspended sample,  $\langle A_w \rangle$ . As shown in Fig. 4,  $A_{NA}$  becomes dominant for smaller  $d_c$ , while  $A_{BR}$  becomes dominant for larger  $d_c$ . We obtain the following approximate expression for  $\langle A_w \rangle$  when  $d_c$  is distributed in the sample:

$$\langle A_w \rangle = \frac{1}{\int_0^\infty nV_c dd_c} \left\{ \int_0^{d_{c,B}} nV_c A_{NA}(d_c) dd_c + \int_{d_{c,B}}^\infty nV_c A_{BR}(d_c) dd_c \right\}, \quad (19)$$

where  $d_{c,B}$  is the value of  $d_c$  that distinguishes the two cases.

An example of the  $d_c$  vs.  $nV_c A$  curve is shown in Fig. 4. The blue and red lines show the  $d_c$  vs.

$nV_c A_{NA}$  and  $d_c$  vs.  $nV_c A_{BR}$  curves, respectively. These curves are calculated for the MS1 sample with  $d_H = 60$  nm when  $f = 20$  kHz and  $\mu_0 H_{ac} = 14$  mT. We obtained  $d_{c,B} = 31$  nm for this case. In Eq. (19), therefore, we use  $A_{NA}$  for  $d_c < d_{c,B}$  and  $A_{BR}$  for  $d_c > d_{c,B}$ .

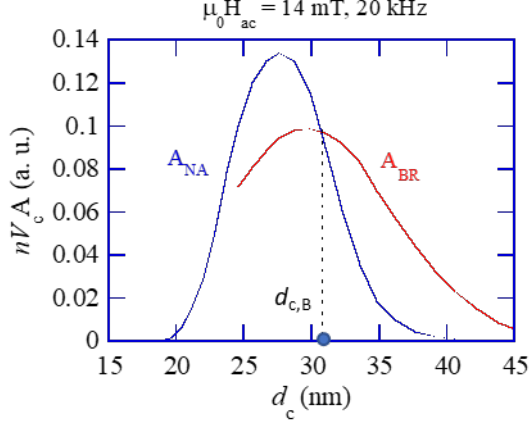


Fig. 4.  $d_c$  vs.  $nV_c A_{NA}$  and  $d_c$  vs.  $nV_c A_{BR}$  curves. The curves are calculated for the MS1 sample with  $d_H = 60$  nm when  $f = 20$  kHz and  $\mu_0 H_{ac} = 14$  mT.

In Fig. 3, the solid blue lines represent the  $\langle A_W \rangle$  vs.  $H_{ac}$  curves calculated from Eq. (19) for the suspended samples. In the calculation, we neglected the  $d_H$  distribution in the sample for simplicity. We took  $d_H$  as an adjustable parameter and set  $d_H = 50, 60$  and  $70$  nm for synomag, MS1 and perimag, respectively. The  $d_H$  values were consistent with the nominal ones for synomag and MS1, while  $d_H$  was smaller than the nominal value for perimag. We obtained good agreement between experiment and analysis for the three samples. These agreements indicate the validity of Eq. (19).

For comparison, the broken lines in Fig. 3 are the  $\langle A_{NA} \rangle$  vs.  $H_{ac}$  curves calculated by setting  $d_{c,B} = \infty$  in Eq. (19). We note that the difference between  $\langle A_W \rangle$  and  $\langle A_{NA} \rangle$  represents the effect of Brownian relaxation. In Fig. 3(a), the difference between  $\langle A_W \rangle$  and  $\langle A_{NA} \rangle$  is small for the synomag sample, indicating that the effect of Brownian relaxation is negligible. For the MS1 and perimag samples,  $\langle A_W \rangle$  was almost the same as  $\langle A_{NA} \rangle$  for  $\mu_0 H_{ac} > 16$  mT. However, the difference between them increased with decreasing  $H_{ac}$  for  $\mu_0 H_{ac} < 16$  mT. These results indicate that the effect of Brownian relaxation is small at high fields but becomes larger at low fields.

We now study the value of  $d_{c,B}$  in Eq. (19) that separates the Néel-relaxation-dominant and Brownian-relaxation-dominant MNPs in the sample;  $d_{c,B}$  can be calculated from Eqs. (12) and (16) with the condition  $A_{BR} = A_{NA}$ . We note that  $d_{c,B}$  becomes dependent on  $H_{ac}$ , indicating that the portion of Néel-relaxation- and Brownian-relaxation-dominant MNPs changes when  $H_{ac}$  changes. Fig. 5(a) shows the dependence of  $d_{c,B}$  on  $H_{ac}$ , whereby  $d_{c,B}$  increases with increasing  $H_{ac}$ . Because Néel relaxation becomes dominant for MNPs with  $d_c < d_{c,B}$ , this result indicates that the portion of Néel-

relaxation-dominant MNPs in the sample increases with  $H_{ac}$ . The volume fraction of these MNPs,  $VF_{Neel}$ , is estimated as

$$VF_{Neel} = \int_0^{d_{c,B}} nV_c dd_c / \int_0^\infty nV_c dd_c. \quad (20)$$

Fig. 5(b) presents the increase in volume fraction with  $H_{ac}$ . The volume fraction becomes larger than 90% at high fields. This result means that the AC  $M-H$  curve of the suspended sample is almost dominated by Néel relaxation at high fields. However, the volume fraction decreases with decreasing  $H_{ac}$ , corresponding to the increase in Brownian-relaxation-dominant MNPs at low fields. Therefore, the contribution of Brownian relaxation increases at lower fields. From Fig. 5(b), we see that the volume fraction of Brownian-relaxation-dominant MNPs at  $\mu_0 H_{ac} = 8$  mT was 30% for MS1 and 17% for synomag. Therefore, the effect of Brownian relaxation is largest for MS1 and smallest for synomag.

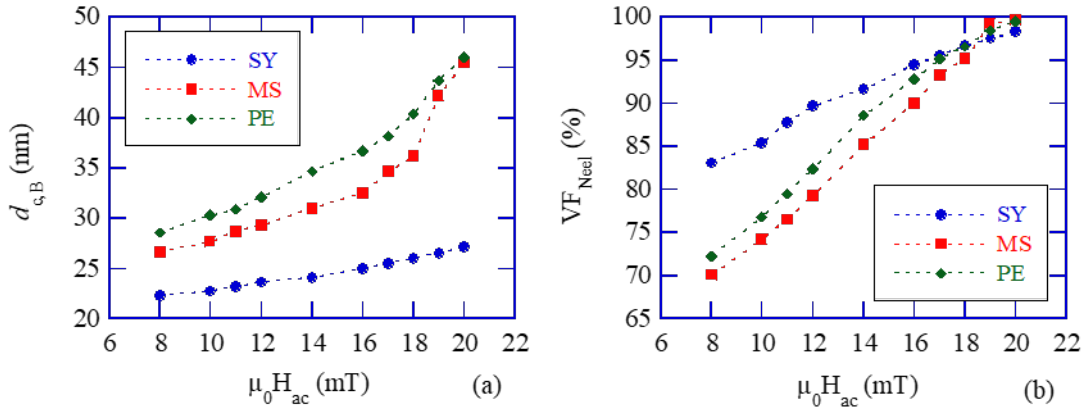


Fig. 5. (a)  $d_{c,B}$  vs.  $H_{ac}$  curves. (b) Change in volume fraction of Néel-relaxation-dominant MNPs in the three samples when  $H_{ac}$  is varied. The broken lines are for the eyes.

### 3.3. Comparison between immobilized and suspended samples

#### 3.3.1. Hysteresis loss and coercive field

We first compare  $\langle A \rangle$  between suspended (water) and immobilized (solid) samples. The ratio  $R_{\langle A \rangle}$  is defined as

$$R_{\langle A \rangle} = \frac{\langle A(\text{water}) \rangle}{\langle A(\text{solid}) \rangle}. \quad (21)$$

Fig. 6(a) presents the dependence of  $R_{\langle A \rangle}$  on  $H_{ac}$  for the three samples. For  $\mu_0 H_{ac} > 16$  mT, the  $R_{\langle A \rangle}$  values for MS1 and perimag were almost the same and were much larger than that for synomag. From Eq. (17),  $R_{\langle A \rangle}$  depends on  $\xi_{ac}$  (or  $m$ ). In Table I, the typical value of  $m$ ,  $m_{typ}$ , is estimated as 1.02, 2.51 and 2.30 aAm<sup>2</sup> for synomag, MS1 and perimag, respectively. Therefore, the small value of  $R_{\langle A \rangle}$

for synomag was caused by the low  $m_{\text{typ}}$ .

For  $\mu_0 H_{\text{ac}} < 16$  mT,  $R_{<A>}$  increased with decreasing  $H_{\text{ac}}$ . The increase was largest for MS1 and smallest for synomag. We note that Brownian relaxation is responsible for these properties because the fraction of Brownian-relaxation-dominated MNPs increases with decreasing  $H_{\text{ac}}$ , and the fraction is smallest for synomag, as shown in Fig. 5(b).

Next, we compare  $\langle H_c \rangle$  between suspended and immobilized samples. The ratio of  $\langle H_c \rangle$  for a suspended sample to the value for an immobilized sample is

$$R_{\langle H_c \rangle} = \frac{\langle H_c(\text{water}) \rangle}{\langle H_c(\text{solid}) \rangle}. \quad (22)$$

Fig. 6(b) presents the dependence of  $R_{\langle H_c \rangle}$  on  $H_{\text{ac}}$  for the three samples. The value of  $R_{\langle H_c \rangle}$  was much smaller than  $R_{<A>}$ . Furthermore,  $R_{\langle H_c \rangle}$  depended weakly on  $H_{\text{ac}}$ . Therefore, the difference in the coercive field was much smaller between the suspended and immobilized cases than the hysteresis loss.

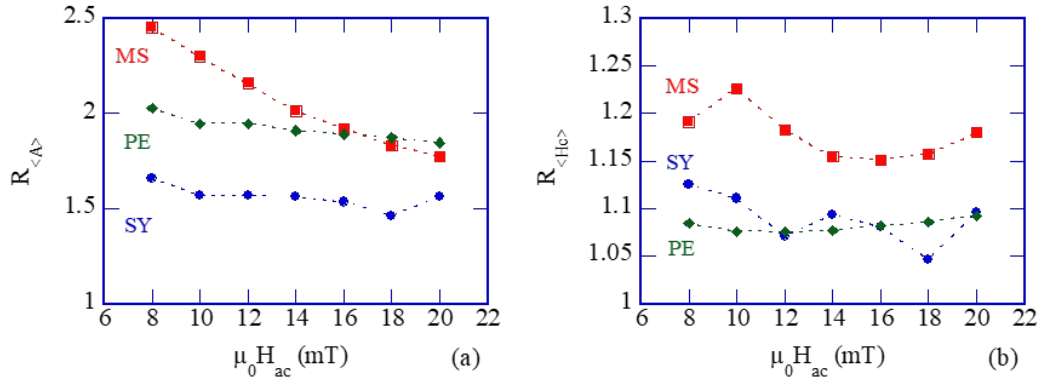


Fig. 6. (a)  $R_{<A>}$  vs.  $H_{\text{ac}}$  and (b)  $R_{\langle H_c \rangle}$  vs.  $H_{\text{ac}}$  curves for the three MNP samples.

We note that the parameter  $\kappa$  in Eq. (5) satisfies the condition  $\kappa > 0.4$  for the present sample parameters and excitation conditions. In our previous paper [38], we studied the dependence of  $H_c$  on the easy-axis angle  $\beta$ , and showed that the dependence becomes small for  $\kappa > 0.4$ . In this case, therefore,  $\langle H_c \rangle$  increases only slightly even when alignment of  $\beta$  occurs in suspended sample. As a result, the difference in  $\langle H_c \rangle$  is small between the suspended and immobilized cases in the present experiment. We note, however, that the dependence of  $H_c$  on  $\beta$  becomes stronger with decreasing  $\kappa$ . Therefore, the difference in  $\langle H_c \rangle$  will become large between the suspended and immobilized cases when  $\kappa$  becomes much smaller.

### 3.3.2. Harmonic signals

Finally, we compare the harmonic signals between suspended and immobilized samples. The amplitude of the harmonic signal,  $M_i$  ( $i = 1, 3, 5, \dots$ ), was obtained by expanding the measured waveform  $M(t)$  into a Fourier series. The  $M_i$  ratio between the two cases is

$$R_{Mi} = \frac{M_i(\text{water})}{M_i(\text{solid})}, i = 1, 3, 5, \dots \quad (23)$$

We studied the 3rd and 5th harmonics as examples. These harmonics are used for biosensing [14, 16, 18]. Fig. 7(a) presents the dependences of  $R_{M3}$  on  $H_{ac}$  for the three samples. These dependences are similar to the  $R_{<A>}$  vs.  $H_{ac}$  curves in Fig. 6(a). Specifically,  $R_{M3}$  increases in the order synomag, perimag and MS1. For MS1,  $R_{M3}$  considerably increases with decreasing  $H_{ac}$ . However,  $R_{M3}$  is almost independent of  $H_{ac}$  for synomag. This difference is due to the fact that the portion of Brownian-relaxation-dominated MNPs was small for the synomag sample.

Fig. 7(b) presents the relationship between  $R_{M5}$  and  $R_{M3}$ . The relation  $R_{M5}$  vs.  $R_{M3}$  is almost on the same curve for all three MNP samples. This indicates a strong correlation between  $R_{M3}$  and  $R_{M5}$  among the three samples, though the physical mechanism for this correlation is not clear at present. The solid line in Fig. 7(b) represents the correlation given by

$$R_{M5} = 0.8R_{M3} + 0.3R_{M3}^2. \quad (24)$$

We note that similar behavior was obtained for the relationship between  $R_{M7}$  and  $R_{M5}$ . However, the relation  $R_{M3}$  vs.  $R_{M1}$  was different among samples. It will be our future work to clarify the relationship between harmonics signals.

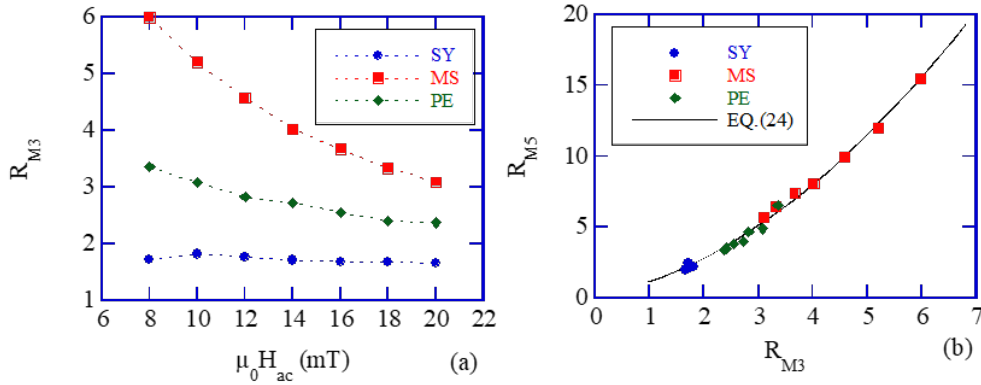


Fig. 7. Relationship (a) between  $R_{M3}$  and  $H_{ac}$  and (b) between  $R_{M5}$  and  $R_{M3}$  for the three samples. The broken lines in (a) are for the eyes. The solid line in (b) is calculated using Eq. (24).

## 4. Discussion

### 4.1 Condition for the Néel- and Brownian-dominant regions

The AC  $M$ - $H$  curve of suspended MNPs is affected by both Brownian and Néel relaxations. We will study the condition that determines the dominant relaxation. First, we study  $\tau_B(H)$  and  $\tau_N(H)$ . The

expression for  $\tau_B(H)$  is given in Eq. (15). The expression for  $\tau_N(H)$  is [39]

$$\tau_N(H) = \frac{\tau_0 \sqrt{\pi}}{\sqrt{\sigma}(1-h^2)} \frac{1}{(1+h)e^{-\sigma(1+h)^2} + (1-h)e^{-\sigma(1-h)^2}}, \quad (25)$$

where  $h = H/H_k$  is the normalized field. In this study, we use Eq. (25) for simplicity, though Eq. (25) is obtained when the easy axis is parallel to the applied field.

Fig. 8(a) shows examples of  $\tau_B(H)$  and  $\tau_N(H)$ . In the calculation, we used the parameters corresponding to the MS1 sample:  $M_s = 360$  kA/m,  $K = 7.0$  kJ/m<sup>3</sup>,  $\mu_0 H_k = 38.9$  mT and  $d_H = 60$  nm. The value  $d_c = 28$  nm was chosen because  $nV_c A$  has a peak value at approximately  $d_c = 28$  nm for MS1, as shown in Fig. 4. As shown in Fig. 8(a),  $\tau_N \gg \tau_B$  at low fields. Therefore, Brownian relaxation becomes dominant in the low-field region. However,  $\tau_N$  rapidly decreases with increasing  $H$ , and  $\tau_N = \tau_B$  at  $H = H_{ac,BR}$ ;  $\mu_0 H_{ac,BR} = 10.2$  mT for  $d_c = 28$  nm. As a result,  $\tau_N \ll \tau_B$  for  $H > H_{ac,BR}$ , and Néel relaxation dominates the dynamics of suspended MNPs at high fields.

For comparison, Fig. 8(a) also shows  $\tau_N(H)$  calculated for  $d_c = 25$  nm. The value of  $H_{ac,BR}$  becomes very small in this case, and Néel relaxation becomes dominant for almost all fields.

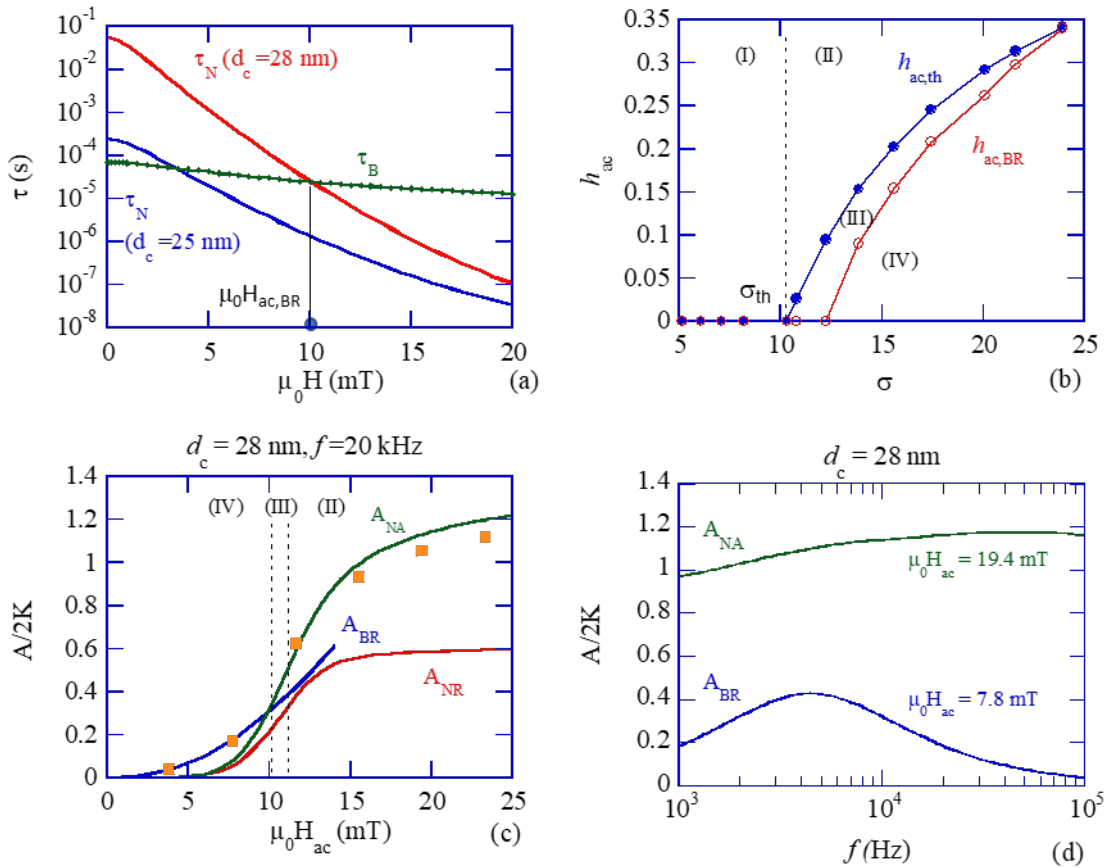


Fig. 8. (a) Field-dependent Brownian relaxation time  $\tau_B(H)$  and Néel relaxation time  $\tau_N(H)$ . (b) Four regions in the  $\sigma$ - $h_{ac}$  plane. The boundary between regions (II) and (III) is determined by  $h_{ac,th}$

calculated from Eq. (8). The boundary between regions (III) and (IV) is determined by  $h_{ac,BR}$ , which satisfies the condition  $\tau_N(h_{ac,BR}) = \tau_B(h_{ac,BR})$ . (c)  $A/(2K)$  vs.  $H_{ac}$  curves for  $d_c = 28$  nm. Lines show the results for  $A_{NR}$ ,  $A_{BR}$  and  $A_{NA}$  calculated using Eqs. (4), (12) and (16), respectively. Symbols represent the simulation results for suspended MNPs. (d)  $A/(2K)$  vs.  $f$  curves for  $d_c = 28$  nm and two values of  $H_{ac}$ . For  $\mu_0 H_{ac} = 7.8$  mT, line shows  $A_{BR}$  calculated using Eq. (12). For  $\mu_0 H_{ac} = 19.4$  mT, line shows  $A_{NA}$  calculated using Eq. (16).

We now study the condition that determines the dominant relaxation in suspended MNPs using the  $\sigma$ - $h_{ac}$  plane, as shown in Fig. 8(b). The  $\sigma$ - $h_{ac}$  plane is calculated for  $f = 20$  kHz and can be divided into four regions. Region (I) is determined by the condition  $\sigma < \sigma_{th}$ , where  $\sigma_{th}$  in Eq. (10) becomes 10.3 for  $f = 20$  kHz. In this region, the condition  $2\pi f \tau_N(0) < 1$  is satisfied, and Néel relaxation dominates the MNP dynamics. The hysteresis loss in this region is  $A_{NR}$  in Eq. (4). For sufficiently low  $\sigma$ , linear theory can be applied using the AC susceptibility of MNPs [4].

Region (II) is determined by the condition  $h_{ac} > h_{ac,th}$ , where  $h_{ac,th}$  is given in Eq. (8). We note that  $h_{ac,th}$  gives the threshold field for the magnetic moment reversal over the anisotropy energy barrier [25, 38]. In this region, the condition  $\tau_N(H) < 1/(2\pi f) \ll \tau_B(H)$  is satisfied, and Néel relaxation becomes dominant. When the AC field is applied, the magnetic moment changes its polarity by passing the anisotropy energy barrier. Furthermore, the AC field causes successive Brownian (physical) rotation in suspended MNPs, and easy axes are partially aligned along the AC field. The hysteresis loss in this region is  $A_{NA}$  in Eq. (16).

Region (IV) is determined by the condition  $h_{ac} < h_{ac,BR}$ , where  $h_{ac,BR}$  is the field satisfying  $\tau_N(h_{ac,BR}) = \tau_B(h_{ac,BR})$ . In this region,  $\tau_B(H) < \tau_N(H)$ , and therefore, Brownian relaxation becomes dominant. The hysteresis loss in this region is  $A_{BR}$  in Eq. (12). We note that  $A_{BR}$  strongly depends on  $2\pi f \tau_B(H)$ , as shown in Eq. (12). Namely, the effect of Brownian relaxation is largest for  $2\pi f \tau_B(H) \approx 1$ , but can be neglected at high frequencies satisfying  $2\pi f \tau_B(H) \gg 1$ .

Region (III) is determined by the condition  $h_{ac,BR} < h_{ac} < h_{ac,th}$ . In this region, both Brownian and Néel relaxations affect the AC  $M$ - $H$  curve of the suspended sample in a complicated way.

We note that the range of  $h_{ac}$  for regions (II) and (IV) changes with  $\sigma$ , as shown in Fig. 8(b). As  $\sigma$  decreases, the range of  $h_{ac}$  increases in region (II) and decreases in region (IV). This means that the effect of Brownian relaxation decreases for MNPs with lower  $\sigma$  (or  $d_c$ ). We also note that  $h_{ac,th}$  in Fig. 8(b) increases with increasing  $f$ , as can be seen from Eq. (8). This means that region (II) moves to a higher  $h_{ac}$  value for higher  $f$ . However,  $h_{ac,BR}$  is independent of  $f$ , and region (IV) does not change when  $f$  is changed.

We now show the  $A$  vs.  $H_{ac}$  curve calculated for each region. The lines in Fig. 8(c) represent the results calculated for parameters  $M_s = 360$  kA/m,  $K = 7.0$  kJ/m<sup>3</sup>,  $\mu_0 H_k = 38.9$  mT,  $d_c = 28$  nm and  $d_H$



= 60 nm. The results for  $A_{NR}$ ,  $A_{BR}$  and  $A_{NA}$  are calculated using Eqs. (4), (12), and (16), respectively. From Fig. 8(b), we obtained  $\mu_0 H_{ac, BR} = 10.2$  mT and  $\mu_0 H_{ac, th} = 11.4$  mT for  $d_c = 28$  nm ( $\sigma = 19.4$ ). Using these values, the field  $H_{ac}$  can be divided into regions (IV), (III) and (II), as shown in Fig. 8(c). In region (IV), Brownian relaxation is dominant and  $A$  is given by  $A_{BR}$ . In region (II), Néel relaxation is dominant and  $A$  is given by  $A_{NA}$ . In Region (III), we take the larger of  $A_{BR}$  and  $A_{NA}$  as the hysteresis loss. We note that region (I) does not exist because  $\sigma > \sigma_{th}$  in this case.

For comparison, the AC  $M$ - $H$  curve of suspended MNPs was numerically simulated for  $d_c = 28$  nm and  $f = 20$  kHz. The symbols in Fig. 8(c) represent the simulated  $A/(2K)$  vs.  $H_{ac}$  curves. In the simulation, both the dynamics of the magnetic moment  $\mathbf{m}$  and the unit vector along the easy axis,  $\mathbf{n}$ , were solved simultaneously via the stochastic Landau–Lifshitz–Gilbert equation and an equation from *Usov et al.* [22, 25]. We obtained good agreement between simulation and analysis, indicating the validity of the analysis.

Finally, we discuss the frequency dependence of  $A_{BR}$  in region (IV) and  $A_{NA}$  in region (II). As shown in Fig. 8(c), Brownian relaxation is dominant at  $\mu_0 H_{ac} = 7.8$  mT ( $h_{ac} = 0.2$ ), and  $A$  is given by  $A_{BR}$  in Eq. (12). The blue line in Fig. 8(d) shows the frequency dependence of  $A_{BR}$  calculated for  $\mu_0 H_{ac} = 7.8$  mT. As shown,  $A_{BR}$  strongly depends on  $f$ . Namely,  $A_{BR}$  becomes large for  $2\pi f \tau_B(H) \approx 1$  ( $f \approx 4.5$  kHz), but becomes very small at high frequencies satisfying  $2\pi f \tau_B(H) \gg 1$ . On the other hand, Néel relaxation is dominant at  $\mu_0 H_{ac} = 19.4$  mT ( $h_{ac} = 0.5$ ) as shown in Fig. 8(c), and  $A$  is given by  $A_{NA}$  in Eq. (16). The green line in Fig. 8(d) shows the frequency dependence of  $A_{NA}$  calculated for  $\mu_0 H_{ac} = 19.4$  mT. As shown, frequency dependence of  $A_{NA}$  is very weak. Therefore, frequency dependence is very different between  $A_{BR}$  and  $A_{NA}$ .

#### 4.2. Degree of easy axis alignment

As shown in Fig. 3, hysteresis loss increased owing to easy axis alignment in the suspended sample when  $H_{ac}$  was high. We will discuss the degree of easy axis alignment using the  $\langle A \rangle$  vs.  $H_{ac}$  curve. Fig. 9(a) presents the measured  $\langle A \rangle$  vs.  $H_{ac}$  curve, where the vertical axis is normalized by  $2K$  and the horizontal axis is normalized by  $H_k$ . The open and closed symbols are the results for the immobilized and suspended cases, respectively. In the immobilized case, the  $\langle A \rangle/(2K)$  vs.  $h_{ac}$  curves become similar among the three MNP samples. In the suspended case, MS1 and perimag have similar  $\langle A \rangle/(2K)$  vs.  $h_{ac}$  curves, while  $\langle A \rangle/(2K)$  is smaller for synomag.

The degree of easy axis alignment can be expressed using the mean orientation of the easy axes through  $\langle \cos \beta \rangle$ , where  $\beta$  is the easy axis angle, and  $\langle \rangle$  means the average over time and the MNP ensemble [20, 22, 38]. When easy axes are aligned,  $\beta$  becomes small and  $\langle \cos \beta \rangle$  becomes large. We expect the AC  $M$ - $H$  curve to depend on  $\langle \cos \beta \rangle H_{ac}$ . This is because the AC  $M$ - $H$  curve is determined by the distribution of  $\beta$ , and its distribution function,  $W_{EA}(\beta)$ , is determined by the value of  $H_{ac} \cos \beta$  [25, 38]. Therefore, we define the effective field

$$h_{ac,eff} = \begin{cases} h_{ac} & \text{for the immobilized case,} \\ ah_{ac} & \text{for the suspended case,} \end{cases} \quad (26)$$

where  $a$  represents the increase in  $\langle \cos\beta \rangle$  caused by the easy axis alignment in a suspended sample.

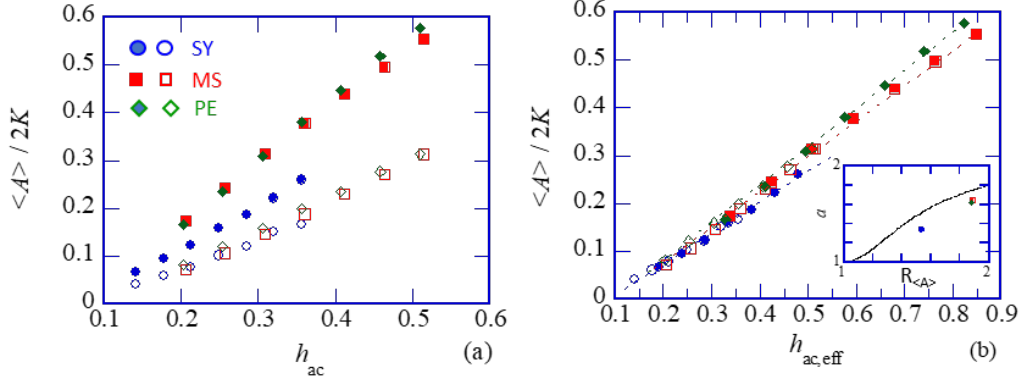


Fig. 9. (a)  $\langle A \rangle / (2K)$  vs.  $h_{ac}$  relations. Open and closed symbols represent the results for immobilized and suspended cases, respectively. (b)  $\langle A \rangle / (2K)$  vs.  $h_{ac,eff}$  relations. The value of  $h_{ac,eff}$  was calculated from Eq. (26). Broken lines are for eyes. The inset shows the relation between  $a = 2\langle \cos\beta \rangle$  and  $R_{\langle A \rangle}$ .

The  $\langle A \rangle / (2K)$  vs.  $h_{ac}$  relations in Fig. 9(a) were replotted as the  $\langle A \rangle / (2K)$  vs.  $h_{ac,eff}$  relations using Eq. (26). The results are shown in Fig. 9(b). To calculate Eq. (26), we used  $a = 1.35, 1.65$  and  $1.62$  for synomag, MS1 and perimag, respectively. The value of  $a$  for each MNP sample was determined so that the  $\langle A \rangle / (2K)$  vs.  $h_{ac,eff}$  relations for both immobilized and suspended cases would fall on the same curve, as shown by the broken lines in Fig. 9(b).

We note that  $a$  in Eq. (26) is the ratio of  $\langle \cos\beta \rangle$  between immobilized and suspended cases. It has been shown that  $\langle \cos\beta \rangle = 0.5$  (or effective angle  $\beta = 60$  deg) for the immobilized case with randomly oriented easy axes [22, 38]. Therefore,  $a$  is given by  $a = 2\langle \cos\beta \rangle$ . Using the obtained value of  $a$ , we can estimate  $\langle \cos\beta \rangle = 0.675, 0.825$  and  $0.810$  (or  $\beta = 48, 34$  and  $36$  deg) for synomag, MS1 and perimag, respectively. These results indicate that the degree of easy axis alignment is smallest for synomag. However, the degree of alignment is almost the same for MS1 and perimag.

We also study the relationship between  $\langle \cos\beta \rangle$  and  $R_{\langle A \rangle}$  in Fig. 6(a). The symbols in the inset of Fig. 9(b) show the relationship between  $R_{\langle A \rangle}$  at  $\mu_0 H_{ac} = 20$  mT and  $a = 2\langle \cos\beta \rangle$  for the three MNP samples. The solid line is the analytical result calculated as follows. The expression for  $R_A$  is given in Eq. (17) for MNPs with  $d_c$  (or  $\xi_{eff}$ ), and the following expression for  $\langle \cos\beta \rangle$  is obtained [38]:

$$\langle \cos\beta \rangle = 1 + \frac{1}{\xi_{eff}} \left( \frac{1 - \cosh \xi_{eff}}{\sinh \xi_{eff}} \right). \quad (27)$$

Combining Eqs. (17) and (27), we obtain the relationship between  $R_A$  and  $a = 2\langle \cos\beta \rangle$ , which is shown by the solid line in the inset of Fig. 9(b). The analytical result is consistent with the experimental

one, though the core size distribution is not taken into account in the analysis.

Finally, we note that the effect of the magnetic dipole–dipole interaction between MNPs is neglected in the present analysis. In our experiment, sample contained 150  $\mu\text{g-Fe}$  in 150  $\mu\text{L}$  sample volume. If we assume that MNPs consist of  $\text{Fe}_3\text{O}_4$ , volume concentration of MNPs becomes as low as  $c = 0.027\%$  for the present samples. From this concentration, we can estimate that the mean distance between particles is about 10 times larger than the diameter of particle. Therefore, dipole–dipole interaction will be small in the present experiment. However, because the dipole–dipole interaction increases with increasing MNP concentration, it is necessary to take account of this effect when the MNP density is much increased [40–42]. We also note that the hysteresis loss can be increased when linear chains of MNP are formed [42, 43].

## 5. Conclusion

We studied the effect of Néel and Brownian relaxation on the AC  $M$ – $H$  curve of suspended MNP samples. First, the  $\langle A \rangle$  vs.  $H_{\text{ac}}$  curves were measured for suspended and immobilized cases using three commercial MNP samples. The experimental results can be quantitatively explained with the previously obtained analytical results when the core size distribution in a sample is taken into account. We showed that Néel-relaxation- and Brownian-relaxation-dominant MNPs coexist in each sample, and that the portion of each changes when  $H_{\text{ac}}$  changes. At high fields, almost all MNPs became Néel-relaxation-dominant, and easy axis alignment caused by the AC field increased the hysteresis loss for a suspended sample. The portion of Brownian-relaxation-dominant MNPs increased with decreasing  $H_{\text{ac}}$ , and the Brownian relaxation gave additional loss at low fields. We can use the four regions in the  $\sigma$ – $h_{\text{ac}}$  plane in Fig. 8(b) to determine the dominant relaxation mechanism in a suspended sample. We also clarified the difference in harmonic signals between suspended and immobilized samples.

## CRedit authorship contribution statement

**Keiji Enpuku:** Data analysis, Writing - Original draft preparation. **Shuya Yamamura:** Experiment. **Takashi Yoshida:** Numerical simulation, Supervision.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## Appendix A: Estimation of core size distribution

The DC  $M$ - $H$  curve of a suspended sample can be expressed as

$$M_{dc} = M_s \int_0^\infty n(d_c) V_c L(\xi_{dc}) dd_c, \quad (A1)$$

where  $L$  is the Langevin function given by

$$L(\xi_{dc}) = \coth(\xi_{dc}) - \frac{1}{\xi_{dc}} \quad (A2)$$

with  $\xi_{dc} = \mu_0 M_s V_c H_{dc} / (k_B T)$ .

The  $d_c$  vs.  $n(d_c) V_c$  curve was obtained by numerically solving the inverse problem in Eq. (A1) using the nonlinear non-negative least squares (NNLS) method [34].

We also note that the Langevin function can be approximated as  $L(\xi_{dc}) = 1 - 1/\xi_{dc}$  for  $\xi_{dc} > 3$ . In this high field region, Eq. (A1) can be expressed as

$$M_{dc} = M_s - \frac{k_B T}{\mu_0 H_{dc}} \int_0^\infty n(d_c) dd_c \text{ for } \xi_{dc} > 3. \quad (A3)$$

Therefore, linear relationship can be obtained between  $M_{dc}$  and  $1/H_{dc}$  at high fields satisfying the condition  $\xi_{dc} > 3$ . We note that the  $M_s$  value can be estimated by extrapolating the linear  $M_{dc}$  vs  $1/H_{dc}$  relation to  $1/H_{dc} = 0$  [33].

## Appendix B: Expression for $H_c$ for the AC $M$ - $H$ curve

For immobilized MNPs with core diameter  $d_c$  and randomly oriented easy axes, the expression for coercive field  $H_c$  is empirically obtained as [35]

$$H_c(d_c) = \begin{cases} 0.48 H_k (1 - \kappa^{0.88}) \times g_c & \text{for } \kappa < 0.66, \\ 0.48 H_k \exp\left\{-\left(\frac{\kappa}{0.61}\right)^{2.1}\right\} \times g_c & \text{for } \kappa > 0.66, \end{cases} \quad (B1)$$

with

$$g_c = \frac{\exp\left(\frac{h_{ac} - h_{1c}}{\Delta h}\right)}{1 + \exp\left(\frac{h_{ac} - h_{1c}}{\Delta h}\right)}, \quad (B2)$$

$$h_{1c} = 1.19(h_{ac,th})^2 + 0.346h_{ac,th} + 0.037 \quad (B3)$$

where  $h_{ac,th}$  and  $\Delta h$  are given in Eqs. (8) and (9), respectively.

When  $d_c$  is distributed in a sample, the coercive field  $\langle H_c \rangle$  of the sample is approximately given by the volume-weighted average of  $H_c$  of each MNP:

$$\langle H_c \rangle = \frac{1}{\int_{d_{c,min}}^{\infty} nV_c dd_c} \int_{d_{c,min}}^{\infty} nV_c H_c(d_c) dd_c, \quad (B4)$$

where  $H_c(d_c)$  and  $nV_c$  are given in Eq. (B1) and Fig. 1, respectively. We note that the present sample consists of elementary particles and agglomerates, as shown in Fig. 1. In Eq. (B4), the integral is performed from  $d_c = d_{c,min}$  to neglect elementary particles because they do not contribute to  $H_c$  near room temperature owing to their very small  $\sigma$  values. We tentatively set  $d_{c,min} = 10$  nm from Fig. 1 for all samples.

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