

Development of theoretical model correlating elastic modulus and porosity of bone

王, 心

九州大学大学院総合理工学府総合理工学専攻機械・システム理工学メジャー

<https://hdl.handle.net/2324/6788193>

出版情報：九州大学, 2022, 修士, 修士
バージョン：
権利関係：



Development of theoretical model correlating elastic modulus and porosity of bone

Bio-Engineering Lab. (Supervisor: Prof. Mitsugu Todo) Wang Xin

1 Introduction

In Ct-based finite element analysis, Young's modulus of bone has been evaluated as a function of bone mineral density using empirical formulae such as Keyak's formulae. Those formulae were determined based on experiments and there was no theoretical background. Therefore, the principal objectives of this study were to develop the relationship between bone mineral density (porosity) and Young's modulus of bone theoretically and compare with Keyak's results.

2. Theory

In this study, bone is first considered as an ideal poroelastic medium. Through Cowin's research, the Young's modulus of cortical bone (E_c) and cancellous bone (trabecular bone) (E_t) can be expressed by E_s (the elastic moduli of trabecula bone tissue) such that

$$E_c = E_s \left(1 - \frac{3(1 - \nu_s)}{2(1 - 2\nu_s)}\right) \phi_c \quad (1)$$

$$E_t = E_s \beta^2 \quad \text{while} \quad \phi_t = 2\beta^3 - 3\beta^2 + 1 \quad (2)$$

where ϕ_c, ϕ_t : the porosity of cortical bones and trabecular bones ($BMD=1-\phi$).

Afterwards, a micromechanical model for the Young's modulus of bone proposed by Wanger was introduced. The model incorporates the platelet-like geometry of the basic reinforcing unit, the presence of alternating thin and thick lamellae, and the orientations of the crystal platelets in the lamellae. The thin and thick lamellae are modeled as orthotropic composite layers made up of thin rectangular apatite platelets within a collagen matrix, and classical orthotropic elasticity theory is used to calculate the Young's modulus of the lamellae. Bone is viewed as an assembly of such orthotropic lamellae bent into cylindrical structures, and having a constant, alternating angle between successive lamellae. The micromechanical model employs a modified rule-of-mixture to account for the two types of lamellae to derived a formula for E_s .

For the E_1 and E_2 (the longitudinal and transverse Young's moduli of bone), two important parameters required when calculating E_s , Halpin and Tsai (HT) derived their formulae:

$$E_1 = E_m \left(\frac{1+2\alpha_1\eta_1\phi_p}{1-\eta_1\phi_p} \right) \quad \text{with} \quad \eta_1 = \frac{E_p/E_m - 1}{E_p/E_m + 2\alpha_1} \quad (3)$$

While, Padawer and Beecher (PB) also derived:

$$E_1 = \varsigma E_p \phi_p + E_m (1 - \phi_p) \quad \text{and} \quad \varsigma = 1 - \frac{\tanh \gamma}{\gamma} \quad \text{with} \quad \gamma = \alpha_1 \left(\frac{\phi_p}{1 - \phi_p} \right)^{1/2} \left[\frac{G_m}{E_p} \right]^{1/2} \quad (4)$$

where E_p, E_m are the elastic moduli of the apatite platelets and the collagen matrix, G_p, G_m the shear moduli of the apatite platelets and the collagen matrix, ϕ_p the volume fraction of the platelets. It is noted that E_2 is given by the same expression as E_1 , but with α_2 (the ratio of the platelet length to thickness) instead of α_1 (the ratio of the platelet width to thickness)).

E_s can be calculated from:

$$E_s = \frac{2}{\pi} \int_0^{\pi/2} [V_{thin} E_{thin}(\theta) + V_{thick} E_{thick}(\theta)] d\theta \quad (5)$$

with

$$E_{thin}(\theta) = \left[\frac{\cos^4(\theta + \theta_{thin})}{E_1} + \frac{\sin^4(\theta + \theta_{thin})}{E_2} + \left(\frac{1}{G_{12}} - 2 \frac{\nu_{12}}{E_1} \right) \sin^2(\theta + \theta_{thin}) \cos^2(\theta + \theta_{thin}) \right]^{-1} \quad (6)$$

$$E_{thick}(\theta) = \left[\frac{\cos^4(\theta + \theta_{thick})}{E_1} + \frac{\sin^4(\theta + \theta_{thick})}{E_2} + \left(\frac{1}{G_{12}} - 2 \frac{\nu_{12}}{E_1} \right) \sin^2(\theta + \theta_{thick}) \cos^2(\theta + \theta_{thick}) \right]^{-1} \quad (7)$$

where θ_{thin} , θ_{thick} : the angles between the longitudinal direction of the thin and thick lamellae and the long axis of bone, V_{thin} , V_{thick} the volume fractions of thin and thick lamellae, G_{12} the shear modulus of a single lamella, and ν_{12} the Poisson's ratio of a single lamella.

By using different models, the final calculated E_s value will change. Then, we use the revised model to calculate the E value of normal bones, bones with osteopenia and osteoporosis, and compare with the experimental data of Keyak to get the final conclusion of this stage of the experiment.

3 Results and discussion

Comparing the results of this calculation with the previous studies reviewed, the variation trend of the PB model is the same as the previous research results, while the HT model is numerically similar to it as shown in Fig.1. Therefore, using these two models for the next calculation.



Fig.1 Variation of Young's modulus of bone as calculated from the (a)HT model, (b)PB model

For normal bones, bones with osteopenia and bones with osteoporosis, the variation trend of E with BMD is similar to Keyak's model, and the results calculated by using the HT model are closer to Keyak's experimental results than those using the PB model as shown in Fig.2.

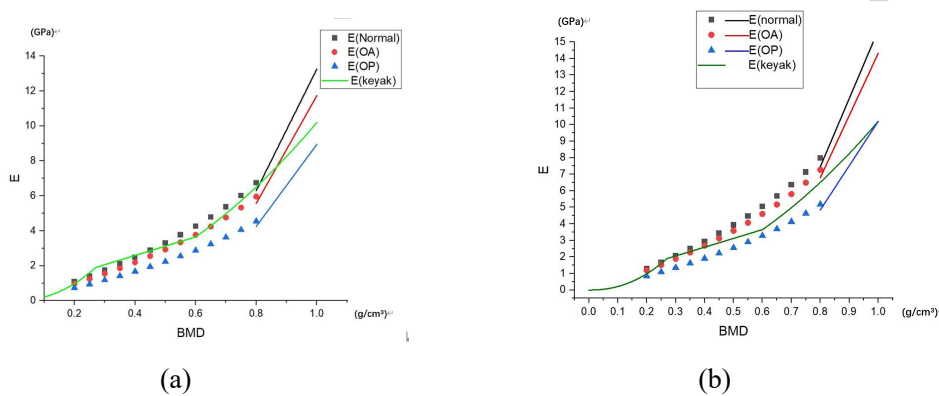


Fig.2 Variation of Young's modulus (E) with BMD from (a)HT model;(b)PB model

4 Conclusion

In this study, a theoretical model of the relationship between E and BMD was established based on the previous studies known from reading the paper. In the study, through theoretical calculations, it is proved that the calculation results of the theoretical model are basically consistent with Keyak's experimental results, and it is found that the results obtained by using the HT model in the research are closer to Keyak's experimental results than the PB model.