Velocity Profile of a Boundary Jet in a Rotating Fluid

Yamashita, Iwao
Ariake National College of Technology : Associate Professor

Takematsu, Masaki
Research Institute for Applied Mechanics, Kyushu University : Professor

https://doi.org/10.5109/6783298

出版情報：Reports of Research Institute for Applied Mechanics. 38 (109), pp.85-91, 1992-02. 九州大学応用力学研究所

Reports of Research Institute for Applied Mechanics
Vol. XXXVIII, No. 109, 1992

Velocity Profile of a Boundary Jet in a Rotating Fluid

By Iwao YAMASHITA* and Masaki TAKEMATSU†

A laminar boundary jet adjacent to a vertical sidewall of a rotating circular cylinder was studied experimentally. The boundary jet was produced by differential rotation of a ring-shaped plate along the periphery of the top surface of the working fluid. The velocity field of the resulting boundary jet was observed by a pH-indiator method with special interest in the case when the with of the driving ring was O $(E^{1/4})$ (E is the Ekman number).

It was demonstrated that the azimuthal velocity has an appreciable vertical shear and that the vertical velocity in the boundary jet is of order $E^{1/4}$. The vertically averaged azimuthal velocity profile was compared with an empirical formula which is a modified version of the theoretical solution given by Kimura (1976)

Key words: Rotating fluid, Boundary jet, Velocity profile, Experiment

1. Introduction

Consider a steady motion in a cylindrical container of fluid which is rotating with a constant angular velocity Ω about its vertical axis of symmetry. The motion is driven by differential rotation of a ring-shaped plate in contact with the upper free surface as pictured in Fig. 1. The theory of rotating fluid (Greenspan[1]) tells us that when the Ekman number (E) is sufficiently small there appear two distinct vertical shear layers of thickness O $(E^{1/4})$ in order for the major azimuthal flow in the inviscid interior to satisfy the non-slip condition at the sidewall and to smooth out the stress discontinuity along the inner edge of

* Associate Professor, Ariake National College of Technology, Ohmuta 836, Japan.
† Professor, Research Institute for Applied Mechanics, Kyushu University, Kasuga 816, Japan.

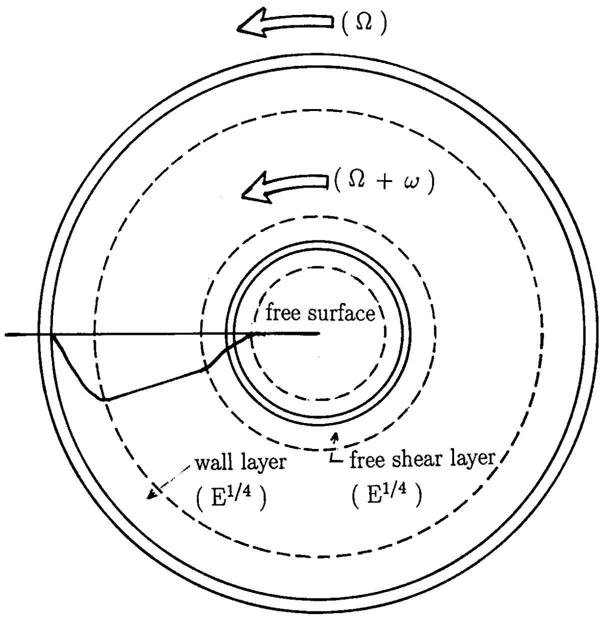

Fig. 1 Configuration of vertical shear layers in a rotating fluid.

the driving ring : One is a wall layer and the other is a free shear layer. It is also known that each shear layer is accompanied with an additional ageostrophic layer of thickness O $(E^{1/3})$ to satisfy the boundary conditions for the vertical component of the motion. Analytical solutions for these typical wall layer and free shear layer are now available in the literature (*e.g.*, Stewartson[2]). The major interest here is in a special but important case when the width of the driving, ring-plate is O $(E^{1/4})$ or less. In this case, the wall layer and the free shear layer merge into a single vertical layer of boundary jet type. The resulting boundary jet may be useful as a simple laboratory analogue of oceanic boundary currents. Indeed, Kimura[3] and Yamashita & Takematsu[4),5] previcusly considered the boundary jet for instability. As far as is known, however, no satisfactory solution has ever been obtained for the merged vertical shear layer.

In this study we examine the velocity structure of boundary jet experimentally. The dominant azimuthal flow is observed at three different depths by a pH-method to determine the velocity profile of the jet and its depth dependence. The vertically averaged profile is found to be well represented by an empirical formula if it is normalized by the maximum value of the velocity. Somehow, however, the maximum velocity itself significantly varies case by case even at the same experimental conditions. The vertical velocity in the boundary jet is also estimated by the vertical deflection of dye patterns.

2. Experimental methods

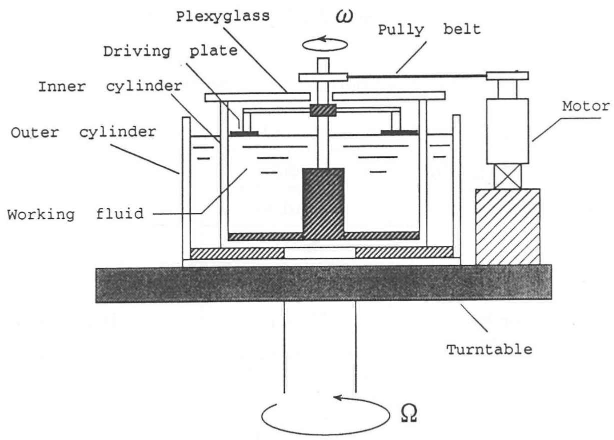

Fig. 2 Schematic of the experimental set-up

A sketch of the experimental set-up on a turntable is shown in Fig. 2. The working basin was a plexiglass cylinder with an inside diameter of 36 cm, which was placed in a larger cylindrical tank (80 cm in diameter) filled with tap water to minimize the effects of the variation in room temperature. In order to drive a boundary jet, a ring-shaped plate was suspended in the working basin by horizontal arms which was fixed at the center of the basin and rotated relative to the basin around the vertical axis. The top of the working basin was covered with a lucite plate to minimize the effect of air stream. The whole apparatus was mounted on a turntable and set rotating in the counter-clockwise direction.

In order to visualize the flow, a diluted solution of Thymol blue (a pH indicator) was used as the working fluid and several electrodes of 0.05 mm platinum wire were stretched radially across the basin at desired locations and depths. In most experiments, the dye wires were set in the upper layer (1.5 cm below the top surface), mid-layer and the lower layer (1.5 cm above the bottom surface). When a d.c. potential (15V) was applied across the electrodes, blue dye was swept off each electrode by the flow. Resulting dye patterns were recorded by a camera mounted on the turntable.

The controllable external parameters are the basic rotation rate Ω, the angular velocity ω of the driving ring-plate relative to the basic rotation, the height of the working fluid H and the width of the driving ing B. Important non-dimensional parameters of the flow are the Rossby number ε and the Ekman number E, which are respectively defined as

$$\varepsilon=\omega/\Omega \quad \text{and} \quad E=\nu/\Omega H^2,$$

where ν is the kinematic viscosity of the working fluid (water). A Reynolds number R of the flow is defined as

$$R=U_m b/\nu,$$

Table I. Experimental Conditions

fluid depth	H (cm)	12
width of the driving ring	B (cm)	3.0, 4.5
basic rotation	Ω(rad/s)	0.34~0.66
Rossby number ε	ω/Ω	0.06~0.35
Ekman number E	$\nu/\Omega H^2$	1.20×10^{-4}~3.47×10^{-4}
Reynolds number R	$U_m b/\nu$	105~222
ratio M	$B/(E/4)^{1/4}\cdot H$	2.5~5.0

where U_m and b are, respectively, the maximum velocity and a width of the azimuthal doundary jet. Another important quantity is the ratio $M=B/\delta$, where δ denotes a representative width of a vertical $E^{1/4}$-layer and is known to be given by $(E/4)^{1/4} H$. The ranges of the external parameters and the non-dimensional variables in the experiments are summarized in Table I.

3. Experimental results

Figure 3 shows horizontal flow patterns of a typical boundary jet as visualized by three radially-stretched dye wires at different depths. From these dye patterns azimuthal velocity profiles of each boundary jet in the upper-, mid- and lower layer were determined. Typical examples of the azimuthal velocity distributions thus determined are plotted in Figs. 4 and 5 , where the velocity and the radial distance from the sidewall are normalized by the maximum velocity of each profile and the width of the driving ring-plate, respectively, and the broken line represents an empirical formula which will be described later. The maximum azimuthal velocities of each boundary jet at the three depths are normalized by the speed of the ring-plate and are plotted in Fig. 6 against the Rossby number.

Somehow the intensity of the boundary jet was too dispersive to be determined uniquely for a prescribed experimental condition; the normalized maximum velocity, for example, varied in a random fashion between 0.24 and 0.44 as is illustrated in Fig. 6. Note also that the boundary jet generally has an appreciable vertical structure; the maximum velocity (Fig. 6) as well as the velocity profile (Figs. 4, 5) vary with depth. Such a vertical structure may be ascribed to the precense of an ageostrophic $E^{1/3}$-layer. Despite the apparent

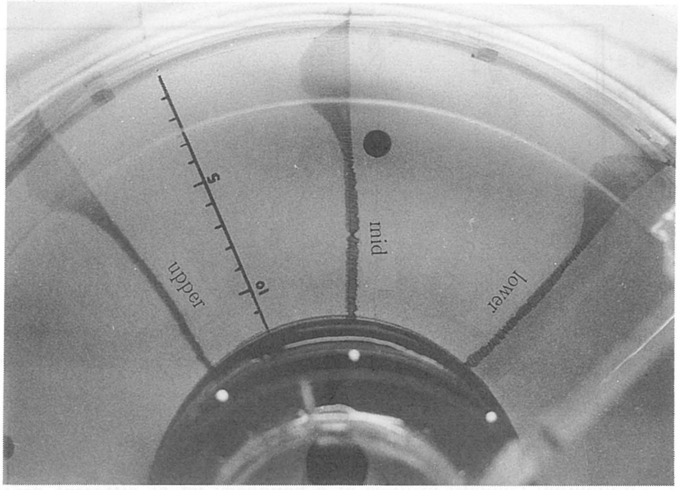

Fig. 3 Azimuthal velocity field of a boundary jet as visualized by three horizontal dye-lines: $\varepsilon=0.064$, $E=1.07\times10^{-4}$, $M=3.5$

vertical structure and the highly dispersive nature of the flow velocity, the normalized profile of vertically averaged azimuthal velocity can be well represented by an empirical formula of the form

$$U=\frac{1}{u_m}\cdot u(\frac{2}{3}Y\,;\,M), \tag{1}$$

where u_m is the maximum value of a theoretical velocity function u, which was originally proposed by Kimura[3] and is given in the form

$$u(Y\,;\,M)=\begin{cases}\frac{1}{2}(1-e^{-MY}-e^{-Y}\sinh MY), & 0\leqq Y\leqq 1\\ \frac{1}{2}(\cosh M-1)e^{-MY}, & 1<Y.\end{cases} \tag{2}$$

The empirical fornula (1) seems to be applicable to the boundary jet so long as the value of M is not too large (for $M<4.5$ crudely). Dye patterns swept off the radially stretched electrodes were deflected vertically in the boundary jet. It was observed that the maximum deflection angle of dye patterns was about 1/10 for all values of the Rossby number examined here. This implies that the ratio $|w/v|$ O $(E^{1/4})$, since the value of $E^{1/4}$ was about 0.1 in the present experiments. In this respect it should be noted that the theoretical solution was derived by assuming w of O $(E^{1/2})$, not of O $(E^{1/4})$ as suggested by the experiments.

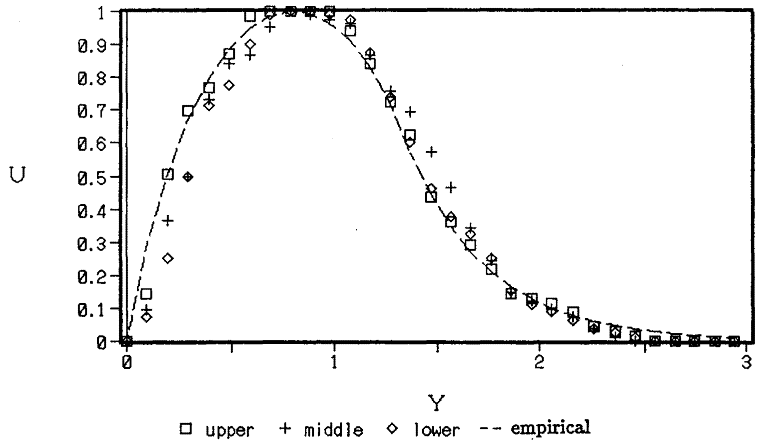

Fig. 4 Azimuthal velocity profiles of a boundary jet at different depths: $\varepsilon=0.14$, $E=1.2\times10^{-4}$, $M=3.4$

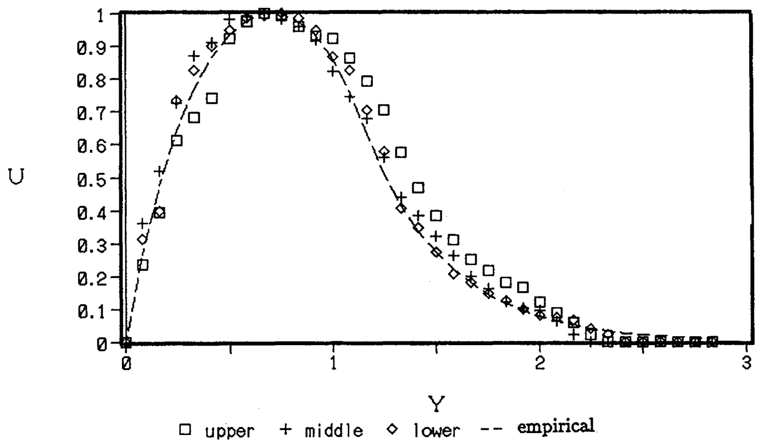

Fig. 5 Azimuthal velocity profiles of a boundary jet at different depths: $\varepsilon=0.24$, $E=2.3\times10^{-4}$, $M=2.9$

Acknowledgment

We would like to thank Professor T. Takemura for his continuous encouragement and interest in this work. Thanks are also due to Mr. T. Kita for his helpful suggestions in preparing the experiment.

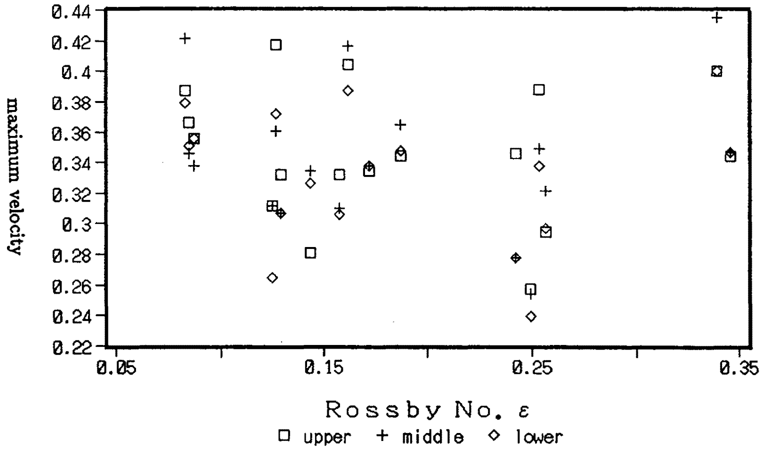

Fig. 6 Maximum value of each azimuthal velocity profile normalized by the speed of the driving ring-plate.

References

1) Greenspan, H.P.: *The theory of rotating fluids, Cambridge University Press,* 1969.
2) Stewartson, K.: *On almost rigid rotations*, J. Fluid Mech. *3* (1957) 17.
3) Kimura, R.: *Barotropic instability of a boundary jet on a sloping bottom,* Geophys. Fluid Dyn. *7* (1976) 205.
4) Yamashita, I. and Takematsu, M.: *Barotropic instability of a viscous boundary jet*, J. Oceanogr. Soc. Japan *44* (1988) 81.
5) Yamashita, I. and Takematsu, M.: *Barotropic instability of a viscous boundary jet (II)*, Bulletin of R.I.A.M. *63* (1987) 321 (in Japanese).

(Received January 8, 1992)