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Kashiwagi, Masashi
Research Institute for Applied Mechanics, Kyushu University : Associate Professor

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Calculation Formulas for the Wave-Induced Steady Horizontal Force and Yaw Moment on a Ship with Forward Speed

By Masashi KASHIWAGI*

A new analysis method based on the theory of Fourier transform is provided for the added resistance, steady sway force, and yaw moment acting on an advancing ship in oblique waves. The principle of linear and angular momentum conservation is used to relate the steady force and moment to far-field disturbance waves generated by the ship. Maruo's added-resistance formula is derived easily with the present method in which Parseval's theorem is effectively used in place of the stationary-phase method. The new method is extended to the analysis of the steady sway force and yaw moment. Calculation formulas for these force and moment are obtained in a form analogous to that for the added resistance, involving only the Kochin function as unknown. In the limit of vanishing forward speed, the obtained formulas reduce to Maruo's for the drift force and Newman's for the drift moment.

Key words : Added resistance, Steady sway force, Steady yaw moment, Momentum principle, Kochin function, Forward-speed effects

1. Introduction

When a ship is floating on the surface of waves, the mean drifting force and yawing moment will be exerted on the ship as a result of wave actions. These drift force and moment are of second order in the wave amplitude, but of engineering importance in designing the control system to maintain the position or heading of ships in waves. A rational theoretical analysis of this subject, based on the principle of momentum conservation, was provided first by Maruo[1)]

* Associate Professor, Research Institute for Applied Mechanics, Kyushu University.

for the drift force in the horizontal plane and later by Newman[2] for the steady yawing moment. It has been common since these two papers to perform "exact" numerical computations of the drift force and moment when the ship's forward speed is zero.

When a ship is advancing at constant forward speed, the same kind of second-order steady force and moment will be also exerted on the ship. Maruo[3),4] applied the momentum-principle analysis to the case of forward speed present, and provided a formula for the ship's longitudinal component of the steady horizontal force. This component is known as the added resistance in waves and has interested many researchers in the field of naval architecture, because the prediction of wave resistence is crucial in considering economical operations of ships in actual seaways. With this engineering reason, many studies on the added resistance have been made so far ; references of these are included in the proceeding of symposium[5] held by the Society of Naval Architects of Japan.

In oblique waves, due to the steady sway force and yaw moment, the ship will advance with the drift angle and check helm to maintain a designated course and thus experience the increase of resistance arising from these secondary causes. Therefore in discussing the overall propulsive performance of a ship in waves, we need to focus more attention on the sway force and yaw moment besides the added resistance. However no calculation formulas exist for these steady force and moment, involving only the Kochin function as does the added-resistance formula. It may be true that Maruo's added-resistance analysis can be directly applied to the lateral force component, but it seems difficult to derive a compact formula for the yaw moment, as long as we follow Maruo's procedure of analyzing the momentum relation. His procedure is complicated, because the stationary-phase method is skillfully used to lead to a final experssion. Thus, to succeed in obtaining a compact formula for the steady yaw moment, we must first develop a new analysis method with which Maruo's added-resistance formula can be easily derived, and next apply it to the principle of angular momentum conservation which relates the moment on a ship to the far-field ship-generated waves.

The present paper reports the work performed along the above lines. In the new analysis method, Parseval's theorem in the Fourier-transform theory is effectively utilized, and thereby complicated calculi seen in Maruo's analysis are avoided. The obtained formulas permit the prediction of the steady sway force and yaw moment in terms only with the Kochin function equivalent to the far-field disturbance waves. Of course Newman's zero-speed results are recovered from the present formulas in the limit of vanishing forward speed.

2. Far-field asymptotic form of the velocity potential

For the sake of subsequent analyses on the principle of momentum and energy conservation, we need to obtain the asymptotic form of the disturbance velocity potential at large distances from a ship. Let us consider a ship advancing at constant forward velocity U into a plane progressive wave of amplitude a, circular frequency ω_0, and wavenumber k_0. The water depth is assumed infinite and thus $k_0 = \omega_0^2/g$, with g the acceleration of gravity. The angle of wave incidence is denoted by χ and measured as in Fig. 1, with $\chi = 0$ corresponding to the following wave. Due to the effect of this incident wave, the ship performs sinusoidal oscillations about its mean position with the circular frequency of encounter ω, which is related to ω_0 by $\omega = \omega_0 - k_0 U \cos\chi$.

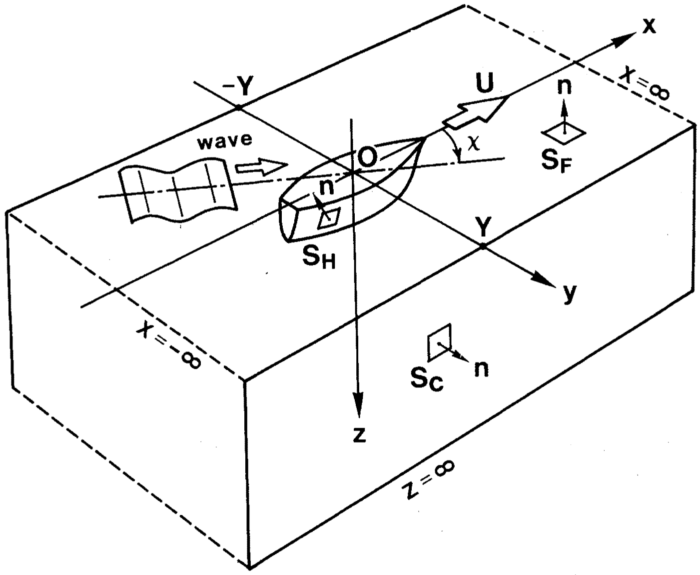

Fig. 1 Coordinate system and notations

As shown in Fig. 1, we take a right-hand Cartesian coordinate system O-xyz, translating with the same velocity as that of the ship. The x-axis is positive in the direction of ship's forward motion, the y-axis positive starboard, and the z-axis positive downward, with the origin placed on the undisturbed free surface.

To justify the linearity, we assume the amplitudes of incident wave and ship's oscillations to be small. Further we assume the flow inviscid with irrotational motion. Then the velocity potential can be introduced and written by linear assumption as

$$\Phi(x, y, z, t) = -Ux + \phi(x, y, z, t) \tag{1}$$

$$\phi(x, y, z, t) = \mathrm{Re}[\psi(x, y, z)e^{i\omega t}] \tag{2}$$

$$\psi(x, y, z) = \frac{ga}{i\omega_0}(\varphi_0 + \varphi) \tag{3}$$

$$\varphi_0 = e^{-k_0 z - ik_0(x\cos\chi + y\sin\chi)} \tag{4}$$

$$\varphi = \varphi_7 - \frac{\omega\omega_0}{g}\sum_{j=1}^{6}\frac{\xi_j}{a}\varphi_j \tag{5}$$

In the above, φ_0 is the potential of the incident wave and φ the disturbance potential due to the presence of a ship. The latter is divided into the scattered potential φ_7 and the radiation potential $\varphi_j\,(j=1,\,2,\,\cdots,\,6)$ due to forced motion of the ship in each mode of six degrees of freedom; ξ_j is the amplitude in the jth mode of motion. The symbol 'Re' in (2) means the real part to be taken.

The velocity potentials, φ_0 and φ, are governed by Laplace's equation and subject to the linearized free-surface boundary condition

$$\left(i\omega - U\frac{\partial}{\partial x}\right)^2\phi - g\frac{\partial\phi}{\partial z} = 0 \tag{6}$$

on $z=0$ and the condition of vanishing velocity as $z\to\infty$. In addition, the disturbance potential φ satisfies a suitable radiation condition.

From Green's theorem, the disturbance potential φ at any point $P=(x,\,y,\,z)$ in the fluid is given by

$$\varphi(P) = \int\!\!\int_{S_H}\left(\frac{\partial\varphi(Q)}{\partial n} - \varphi(Q)\frac{\partial}{\partial n}\right)G(P\,;Q)\,dS(Q) \tag{7}$$

where $Q=(\xi,\,\eta,\,\zeta)$ denotes the integration point on the wetted portion of ship hull S_H ; $\partial/\partial n$ is the normal differentiation with respect to the integration point, with the normal defined positive into the ship hull; and $G(P\,;Q)$ denotes the Green function or source potential which satisfies the same free-surface and radiation conditions as those to be satisfied by φ. With the Fourier-transform technique, this Green function can be written in the form[6)]

$$\begin{aligned} G(P\,;Q) = &-\frac{1}{4\pi}\left(\frac{1}{r}-\frac{1}{r'}\right) \\ &-\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-ik(x-\xi)}dk\cdot\mathrm{Re}\int_0^{\infty}\frac{e^{-in(z+\zeta)-|y-\eta|\sqrt{n^2+k^2}}}{(n+i\nu)\sqrt{n^2+k^2}}\,ndn \\ &-\frac{1}{2\pi}\left[\int_{k_1}^{k_2}+\int_{k_3}^{k_4}\right]\frac{\nu}{\sqrt{k^2-\nu^2}}e^{-\nu(z+\zeta)-|y-\eta|\sqrt{k^2-\nu^2}-ik(x-\xi)}dk \\ &+\frac{i}{2\pi}\left[-\int_{-\infty}^{k_1}+\int_{k_2}^{k_3}+\int_{k_4}^{\infty}\right]\frac{\nu}{\sqrt{\nu^2-k^2}} \\ &\qquad\times e^{-\nu(z+\zeta)-i\epsilon_k|y-\eta|\sqrt{\nu^2-k^2}-ik(x-\xi)}dk \end{aligned} \tag{8}$$

where

$$\left.\begin{matrix} r \\ r' \end{matrix}\right\} = \sqrt{(x-\xi)^2+(y-\eta)^2+(z\mp\zeta)^2} \tag{9}$$

$$\nu = \frac{1}{g}(\omega+kU)^2 = K+2k\tau+\frac{k^2}{K_0} \tag{10}$$

$$K=\frac{\omega^2}{g},\ \tau=\frac{U\omega}{g},\ K_0=\frac{g}{U^2} \tag{11}$$

$$\left.\begin{matrix}k_1\\k_2\end{matrix}\right\}=-\frac{K_0}{2}[1+2\tau\pm\sqrt{1+4\tau}] \tag{12}$$

$$\left.\begin{matrix}k_3\\k_4\end{matrix}\right\}=\ \frac{K_0}{2}[1-2\tau\mp\sqrt{1-4\tau}] \tag{13}$$

$$\epsilon_k=sgn(\omega+kU)=\begin{cases}-1 \text{ for } -\infty<k<k_1\\ \ \ 1 \text{ for } \ \ k_2<k<\infty\end{cases} \tag{14}$$

In the case of $\tau>1/4$, wavenumbers k_3 and k_4 given by (13) is not real, and thus the limits of integration in (8) should be interpreted such that $k_3=k_4$ for $\tau>1/4$. (Hereafter this convention will be understood.)

To obtain a far-field approximation to the disturbance potential φ when the transverse distance $|y|$ is large, let us first consider the asymptotic approximation of the Green function itself. It is obvious that all the terms except the last one in (8) vanish for large values of $|y|$. (These terms represent the local disturbance in the vicinity of the x-axis.) Therefore, substituting only the last term of (8) into (7), we obtain the desired approximation of the velocity potential valid at large distances from the x-axis:

$$\varphi(x,\ y,\ z)\sim\frac{i}{2\pi}\left[-\int_{-\infty}^{k_1}+\int_{k_2}^{k_3}+\int_{k_4}^{\infty}\right]H^{\pm}(k) \\ \times\frac{\nu}{\sqrt{\nu^2-k^2}}e^{-\nu z\mp i\epsilon_k y\sqrt{\nu^2-k^2}-ikx}dk \tag{15}$$

where

$$H^{\pm}(k)=\iint_{S_H}\left(\frac{\partial\varphi}{\partial n}-\varphi\frac{\partial}{\partial n}\right)e^{-\nu\zeta\pm i\epsilon_k\eta\sqrt{\nu^2-k^2}+ik\xi}dS \tag{16}$$

is the Kochin function equivalent to the complex amplitude of the far-field disturbance wave. The upper or lower of the complex signs in (15) and (16) is to be taken according as the sign of y is positive or negative, respectively. With the convention that the Kochin function is zero outside of the integration range explicitly shown in (15), we shall write (15) in the form

$$\varphi(x,\ y,\ z)\sim\frac{1}{2\pi}\int_{-\infty}^{\infty}i\epsilon_k H^{\pm}(k)\frac{\nu}{\sqrt{\nu^2-k^2}}e^{-\nu z\mp i\epsilon_k y\sqrt{\nu^2-k^2}}e^{-ikx}dk \tag{17}$$

Here the notation (14) has been used.

From this equation, we can readily obtain the Fourier transform of the disturbance potential in the far field:

$$F\{\varphi(x,\ y,\ z)\} \equiv \int_{-\infty}^{\infty} \varphi(x,\ y,\ z) e^{ikx} dx \tag{18}$$

$$= i\epsilon_k H^{\pm}(k) \frac{\nu}{\sqrt{\nu^2-k^2}} e^{-\nu z \mp i\epsilon_k y\sqrt{\nu^2-k^2}} \tag{19}$$

Note that neglected in (17) or (19) are only the local disturbances near the x-axis and that the momentum or the energy associated with these terms become infinitely small as the coordinate x tends to plus or minus infinity.

The Fourier transform of the incident-wave potential φ_0 will be derived by substituting (4) into the definition (18), with the result

$$F\{\varphi_0(x,\ y,\ z)\} = 2\pi\delta(k-k_0\cos\chi) e^{-k_0 z - ik_0 y \sin\chi} \tag{20}$$

where $\delta(k-k_0\cos\chi)$ is Dirac's delta function, thus contributing only for $k = k_0 \cos\chi$.

For convenience in subsequent derivations, we decompose the Kochin function in the form

$$H^{\pm}(k) = C(k) \pm i\epsilon_k S(k) \tag{21}$$

where

$$\left.\begin{aligned} C(k) &= \iint_{S_H} \left(\frac{\partial\varphi}{\partial n} - \varphi\frac{\partial}{\partial n}\right) e^{-\nu\zeta + ik\xi} \cos(\eta\sqrt{\nu^2-k^2}) dS \\ S(k) &= \iint_{S_H} \left(\frac{\partial\varphi}{\partial n} - \varphi\frac{\partial}{\partial n}\right) e^{-\nu\zeta + ik\xi} \sin(\eta\sqrt{\nu^2-k^2}) dS \end{aligned}\right\} \tag{22}$$

We note that $C(k)$ and $S(k)$ represent the symmetric and antisymmetric components, respectively, with respect to the center plane of a ship symmetrical about $y = 0$.

3. The added resistance

The principle of linear momentum conservation

In this section, we shall consider by use of the Fourier-transform technique the same problem as that analyzed by Maruo[4] and show that Maruo's added-resistance formula can be derived with considerable ease. Following Maruo, we begin by considering the rate of change of linear momentum within the fluid domain bounded by the ship's wetted surface S_H, the free surface S_F, and a control surface S_C at a large distance from the ship. Using Gauss' theorem and taking account of that there is no flux across S_H and S_F and that the pressure is zero on S_F, we get:

$$\frac{d\mathbf{M}}{dt} = -\iint_{S_H} p\,\mathbf{n}\,dS - \iint_{S_C} [p\,\mathbf{n} + \rho\nabla\Phi(\mathbf{n}\cdot\nabla\Phi)] dS \tag{23}$$

where p is the fluid pressure, ρ the fluid density, and $\mathbf{n}$ the normal vector.

As usual, we take time average of the above. Because of the periodicity of fluid motion, there can be no net increase of momentum in the control volume from one cycle to another. Therefore the steady force in the horizontal plane can be related to the far-field velocity potential, in the form

$$\begin{aligned}\overline{\mathbf{F}} &= \overline{\iint_{S_H} p\,\mathbf{n}\,dS} \\ &= -\overline{\iint_{S_C}\left[\,p\,\mathbf{n}+\rho\nabla\phi\left(\frac{\partial\phi}{\partial n}-Un_x\right)\right]dS}\end{aligned} \tag{24}$$

where, from Bernoulli's equation,

$$p = -\rho\left\{\frac{\partial\phi}{\partial t}-U\frac{\partial\phi}{\partial x}+\frac{1}{2}\nabla\phi\nabla\phi-gz\right\} \tag{25}$$

and n_x is the x-component of the normal vector. In (24) and (25), eq. (1) has been substituted and the overbar in (24) means taking time average.

Since a resistance is defined as the force in the negative x-direction, we obtain from (24) an expression for the added resistance:

$$\overline{R} = \overline{\iint_{S_C}\left[\,pn_x+\rho\frac{\partial\phi}{\partial x}\left(\frac{\partial\phi}{\partial n}-Un_x\right)\right]dS} \tag{26}$$

In the present analysis, instead of the usual control surface of a circular cylinder of large radius about the z-axis, we take two flat plates as the control surface, which are, as shown in Fig. 1, located at $y=\pm Y$ and extend from $x=-\infty$ to $x=+\infty$ and from the instantaneous free surface down to $z=+\infty$. (The value of Y is assumed large such that the local waves near the x-axis can be neglected.) Careful readers might be anxious about the momentum flux from the vertical planes parallel to the y-axis at $x=\pm\infty$. However the control surface considered here is of infinite length in the x-direction and all the disturbance waves radiating away from the x-axis are precisely taken into account. Thus, neglected are only the contributions from the local waves which exist only near the x-axis; these will become zero at $x=\pm\infty$ in the three-dimensional case.

Note that the x-component of the normal vector is zero on the present control surface. Then, neglecting terms higher than $O(\phi^3)$ as in the usual procedure, we readily obtain from (26)

$$\overline{R} = \rho\int_0^\infty dz\int_{-\infty}^{\infty}\left[\overline{\frac{\partial\phi}{\partial x}\frac{\partial\phi}{\partial y}}\right]_{-Y}^{Y}dx \tag{27}$$

Here $[\quad]_{-Y}^{Y}$ means the difference between the values of the quantity in brackets at $y=Y$ and at $y=-Y$. Substituting (2) into (27) and performing the time-average calculation, it follows that

$$\overline{R} = \frac{1}{2}\rho\,\mathrm{Re}\int_0^\infty dz\int_{-\infty}^\infty\left[\frac{\partial\phi}{\partial x}\frac{\partial\phi^*}{\partial y}\right]_{-Y}^{Y}dx \tag{28}$$

where the asterisk denotes the complex conjugate.

Next we substitute the velocity potential (3) for ϕ into the above. The result will involve terms which are quadratic in the disturbance potential φ and the incident-wave potential φ_0 separately, plus the cross terms of φ and φ_0. The contribution from φ_0 alone is zero, because there can be no force associated with the undisturbed incident wave system. Taking these into consideration, (28) can be written in the form

$$\overline{R} = \frac{\rho g a^2}{k_0}(\overline{R_1}+\overline{R_2}) \tag{29}$$

$$\overline{R_1} = \frac{1}{2}\mathrm{Re}\int_0^\infty dz\int_{-\infty}^\infty\left[\frac{\partial\varphi}{\partial x}\frac{\partial\varphi^*}{\partial y}\right]_{-Y}^{Y}dx \tag{30}$$

$$\overline{R_2} = \frac{1}{2}\mathrm{Re}\int_0^\infty dz\int_{-\infty}^\infty\left[\frac{\partial\varphi}{\partial x}\frac{\partial\varphi_0{}^*}{\partial y}+\frac{\partial\varphi_0{}^*}{\partial x}\frac{\partial\varphi}{\partial y}\right]_{-Y}^{Y}dx \tag{31}$$

We notice that the integrations with respect to x are of the form to which the following Fourier-transform theorem (Parseval's theorem) can be applied:

$$\int_{-\infty}^\infty f(x)g^*(x)dx = \frac{1}{2\pi}\int_{-\infty}^\infty F(k)G^*(k)dk \tag{32}$$

where $F(k)$ and $G(k)$ are Fourier transforms of $f(x)$ and $g(x)$, respectively, which may be calculated from the definition (18).

Let us consider first eq.(30). Since the potential φ has exponential dependence on the coordinate z as seen in (17), the z-integration in (30) can be carried out with the formula:

$$\int_0^\infty e^{-2\nu z}dz = \frac{1}{2\nu} \tag{33}$$

The x-integration in (30), on the other hand, can be performed by applying the Parseval's theorem (32) in terms of the Fourier transform of φ given by (19). After performing the x- and z-integrations in this manner, we get the following result with relative ease.

$$\begin{aligned}\overline{R_1} &= \frac{1}{8\pi}\int_{-\infty}^\infty \epsilon_k\{|H^+(k)|^2+|H^-(k)|^2\}\frac{\nu}{\sqrt{\nu^2-k^2}}kdk \\ &= \frac{1}{8\pi}\left[-\int_{-\infty}^{k_1}+\int_{k_2}^{k_3}+\int_{k_4}^{\infty}\right]\{|H^+(k)|^2+|H^-(k)|^2\}\frac{\nu}{\sqrt{\nu^2-k^2}}kdk\end{aligned} \tag{34}$$

Here we have used the convention concerning the integration range noted in deriving (17). In (34), it is understood that $k_3 = k_4$ in the case of $\tau > 1/4$.

We proceed to the second term $\overline{R_2}$ defined by (31). In the calculation of (31), it is sufficient to retain only terms which are independent of the coordinate y, because according to the theory of hyperfunction[7], sinusoidal terms will vanish when taking the limit of $Y \to \infty$ after performing the x- and z-integrations. Therefore only two cases should be considered here: $k_0 \sin \chi = \epsilon_k \sqrt{\nu^2 - k^2}$ and $k_0 \sin \chi = -\epsilon_k \sqrt{\nu^2 - k^2}$.

We begin with the first case, $k_0 \sin \chi = \epsilon_k \sqrt{\nu^2 - k^2}$. Since we are going to apply the Parseval's theorem (32) to the x-integration in (31), we must consider the product of the Fourier transforms of φ and φ_0, given by (19) and (20), respectively. Thus due to Dirac's delta function appearing in (20), we can put $k = k_0 \cos \chi$; from this and $k_0 \sin \chi = \epsilon_k \sqrt{\nu^2 - k^2}$, we have $\nu = k_0$. Therefore the z-integration in (31) takes the form

$$\int_0^\infty e^{-(\nu + k_0)z} dz = \frac{1}{(\nu + k_0)} = \frac{1}{2\nu} \tag{35}$$

Applying this result and Parseval's theorem, eq.(31) can be reduced to

$$\overline{R_2} = \frac{1}{2} k_0 \cos\chi \ \mathrm{Im}[H(k_0, \ \chi)] \tag{36}$$

where 'Im' denotes the imaginary part, and $H(k_0, \chi)$ is the function obtained after substituting $k = k_0 \cos \chi$ and $\epsilon_k \sqrt{\nu^2 - k^2} = k_0 \sin \chi$ into the Kochin function $H^+(k)$ and thus can be written as

$$H(k_0, \ \chi) = \int\int_{S_H} \left(\frac{\partial \varphi}{\partial n} - \varphi \frac{\partial}{\partial n} \right) e^{-k_0 \zeta + i k_0 (\xi \cos\chi + \eta \sin\chi)} dS \tag{37}$$

In the second case of $k_0 \sin \chi = -\epsilon_k \sqrt{\nu^2 - k^2}$, we can easily confirm that the reductions analogous to the first case lead to the same final result as (36) and (37). Therefore we have completed all of the necessary integrations.

Substituting (34) and (36) into (29) gives the formula for the added resistance in waves:

$$\frac{\overline{R}}{\rho g a^2} = \frac{1}{8\pi k_0} \left[-\int_{-\infty}^{k_1} + \int_{k_2}^{k_3} + \int_{k_4}^{\infty} \right] \{|H^+(k)|^2 + |H^-(k)|^2\} \times \frac{\nu}{\sqrt{\nu^2 - k^2}} k dk - \frac{1}{2} \cos\chi \ \mathrm{Im}[H(k_0, \ \chi)] \tag{38}$$

<u>Principle of energy conservation</u>

In Maruo's analysis, the last term of (38) is transformed further using the energy-conservation principle. Since no external force exists except the constant towing force and the gravitational force keeping the equilibrium position of the ship in space, there is no work done or no dissipation of energy. Thus, owing to the periodic nature of the fluid motion, we have the relation[4)]

$$\overline{\iint_{Sc} \frac{\partial \phi}{\partial t}\left(\frac{\partial \phi}{\partial n} - Un_x\right) dS} = 0 \tag{39}$$

Noting that $n_x = 0$ on the control surface shown in Fig. 1 and neglecting higher-order terms resulting from the free-surface elevation, the above equation can be transformed as

$$\int_0^\infty dz \int_{-\infty}^\infty \left[\overline{\frac{\partial \phi}{\partial t}\frac{\partial \phi}{\partial y}}\right]_{-Y}^{Y} dx = \frac{1}{2}\mathrm{Re}\int_0^\infty dz \int_{-\infty}^\infty \left[i\omega\phi\frac{\partial \phi^*}{\partial y}\right]_{-Y}^{Y} dx = 0 \tag{40}$$

Substituting (3) and decomposing the result into two parts like (29), we can write (40) in the form

$$\frac{1}{2}\mathrm{Im}\int_0^\infty dz \int_{-\infty}^\infty \left[\varphi\frac{\partial \varphi^*}{\partial y}\right]_{-Y}^{Y} dx$$
$$= -\frac{1}{2}\mathrm{Im}\int_0^\infty dz \int_{-\infty}^\infty \left[\varphi\frac{\partial \varphi_0^*}{\partial y} - \varphi_0^*\frac{\partial \varphi}{\partial y}\right]_{-Y}^{Y} dx \tag{41}$$

The procedure of performing these integrations with respect to x and z is the same as that for (30) and (31); that is, we apply Parseval's theorem (32) with the Fourier transforms of φ and φ_0. After straightforward reductions, we get the following result:

$$\frac{1}{8\pi}\left[-\int_{-\infty}^{k_1} + \int_{k_2}^{k_3} + \int_{k_4}^{\infty}\right]\{|H^+(k)|^2 + |H^-(k)|^2\}\frac{\nu}{\sqrt{\nu^2-k^2}}dk$$
$$= \frac{1}{2}\mathrm{Im}[H(k_0,\ \chi)] \tag{42}$$

Here $H(k_0,\ \chi)$ is the Kochin function defined by (37)

With this energy relation, the added-resistance formula (38) can be recast in the form

$$\frac{\overline{R}}{\rho g a^2} = \frac{1}{8\pi k_0}\left[-\int_{-\infty}^{k_1} + \int_{k_2}^{k_3} + \int_{k_4}^{\infty}\right]\{|H^+(k)|^2 + |H^-(k)|^2\}$$
$$\times \frac{\nu}{\sqrt{\nu^2-k^2}}(k - k_0\cos\chi)dk \tag{43}$$

If the relation (21) is substituted for $H^{\pm}(k)$, the above equation can be expressed as

$$\frac{\overline{R}}{\rho g a^2} = \frac{1}{4\pi k_0}\left[-\int_{-\infty}^{k_1} + \int_{k_2}^{k_3} + \int_{k_4}^{\infty}\right]\{|C(k)|^2 + |S(k)|^2\}$$
$$\times \frac{\nu}{\sqrt{\nu^2-k^2}}(k - k_0\cos\chi)dk \tag{44}$$

Introducing Hanaoka's variable transformation[8]

$$k=\frac{K_0}{2\cos\theta}\{1-2\tau\cos\theta\pm\sqrt{1-4\tau\cos\theta}\}, \tag{45}$$

we can confirm that (43) or (44) is identical to that derived by Maruo[4]. However, a point to be emphasized here is that the derivation in this paper ia quite simple in comparison to Maruo's, because the Fourie-transform technique is used in place of the stationary-phase method which was essential in Maruo's analysis. We can see from (44) that symmetric waves $C(k)$ and antisymmetric waves $S(k)$ contribute independently to the added resistance and no contribution exists from the interaction between them.

4. The steady sway force

The y-component of (24) gives the formula for the steady sway force:

$$\overline{F_y}=\overline{\int\int_{Sc}\left[pn_y+\rho\frac{\partial\phi}{\partial y}\left(\frac{\partial\phi}{\partial n}-Un_x\right)\right]dS} \tag{46}$$

Evaluating this on the control surface shown in Fig. 1, (46) can be reduced to

$$\begin{aligned}\overline{F_y}&=-\overline{\int_{\zeta_w}^{\infty}dz\int_{-\infty}^{\infty}\left[p+\rho\left(\frac{\partial\phi}{\partial y}\right)^2\right]_{-Y}^{Y}dx}\\&=\frac{\rho}{2}\int_0^{\infty}dz\int_{-\infty}^{\infty}\left[\overline{\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial z}\right)^2-\left(\frac{\partial\phi}{\partial y}\right)^2}\right]_{-Y}^{Y}dx\\&\quad+\frac{1}{2}\rho g\int_{-\infty}^{\infty}\left[\overline{\zeta_w^2}\right]_{-Y}^{Y}dx\quad+O(\phi^3)\end{aligned} \tag{47}$$

Here eq.(25) for the pressure p has been substituted and ζ_w is the unsteady elevation of the free surface, which is given by

$$\zeta_w=\frac{1}{g}\left(\frac{\partial\phi}{\partial t}-U\frac{\partial\phi}{\partial x}\right)_{z=0}+O(\phi^2) \tag{48}$$

Calculating time average in (47) and substituting (3) for the velocity potential, we can write (47) in the following decomposed form

$$\overline{F_y}=-\frac{\rho g a^2}{k_0}(\overline{Y_1}+\overline{Y_2}) \tag{49}$$

where

$$\begin{aligned}\overline{Y_1}&=-\frac{1}{4}\int_0^{\infty}dz\int_{-\infty}^{\infty}\left[\left|\frac{\partial\varphi}{\partial x}\right|^2+\left|\frac{\partial\varphi}{\partial z}\right|^2-\left|\frac{\partial\varphi}{\partial y}\right|^2\right]_{-Y}^{Y}dx\\&\quad+\frac{1}{4}\mathrm{Re}\int_{-\infty}^{\infty}\left[\left\{K|\varphi|^2+\frac{1}{K_0}\left|\frac{\partial\varphi}{\partial x}\right|^2+i2\tau\varphi^*\frac{\partial\varphi}{\partial x}\right\}_{z=0}\right]_{-Y}^{Y}dx\end{aligned} \tag{50}$$

$$\overline{Y_2} = -\frac{1}{2}\mathrm{Re}\int_0^\infty dz \int_{-\infty}^\infty \left[\frac{\partial\varphi}{\partial x}\frac{\partial\varphi_0^*}{\partial x} + \frac{\partial\varphi}{\partial z}\frac{\partial\varphi_0^*}{\partial z} - \frac{\partial\varphi}{\partial y}\frac{\partial\varphi_0^*}{\partial y} \right]_{-Y}^{Y} dx$$
$$+\frac{1}{2}\mathrm{Re}\int_{-\infty}^\infty \Big[\Big\{ K\varphi\varphi_0^* + \frac{1}{K_0}\frac{\partial\varphi}{\partial x}\frac{\partial\varphi_0^*}{\partial x}$$
$$+ i\tau\Big(\varphi_0^*\frac{\partial\varphi}{\partial x} - \varphi\frac{\partial\varphi_0^*}{\partial x} \Big) \Big\}_{z=0} \Big]_{-Y}^{Y} dx \tag{51}$$

Note that $\overline{Y_1}$ represents the contributions from ship-generated disturbance waves and $\overline{Y_2}$ the contributions from the interactions of incident wave and ship-generated waves.

Let us first consider $\overline{Y_1}$. In order to apply the Parseval's theorem (32) to the x-integrations in (50), we need to obtain the Fourier transform of the derivatives with respect to x, y, z of the disturbance potential φ; which can be done easily using (19). The z-integration, which is necessary in the first term in (50), can be performed by use of (33). Summarizing these, we obtain the result

$$\overline{Y_1} = -\frac{1}{8\pi}\int_{-\infty}^\infty (|H^+(k)|^2 - |H^-(k)|^2) \Big\{ \frac{k^2\nu}{2(\nu^2-k^2)} + \frac{\nu^3}{2(\nu^2-k^2)} - \frac{\nu}{2}$$
$$- \frac{\nu^2}{\nu^2-k^2}\Big(K + \frac{k^2}{K_0} + 2\tau k \Big) \Big\} dk$$
$$= \frac{1}{8\pi}\int_{-\infty}^\infty (|H^+(k)|^2 - |H^-(k)|^2)\nu dk \tag{52}$$

From (21), the following relation holds:

$$|H^+(k)|^2 - |H^-(k)|^2 = 2\epsilon_k \mathrm{Im}\{2C(k)S^*(k)\} \tag{53}$$

Thus, recalling the convention about the range of integration with respect to k, eq. (52) can be written in the form

$$\overline{Y_1} = \frac{1}{4\pi}\Big[-\int_{-\infty}^{k_1} + \int_{k_2}^{k_3} + \int_{k_4}^{\infty} \Big] \mathrm{Im}\{2C(k)S^*(k)\}\nu dk \tag{54}$$

Next we consider the second term, $\overline{Y_2}$, defined by (51). Also here, we apply the Parseval's theorem with the Fourier transforms of φ and φ_0; these are given by (19) and (20), respectively. With the reasons stated in transforming the interaction terms between φ and φ_0 in the added-resistance formula, we can concentrate on the case of $k = k_0 \cos\chi$, $\pm\epsilon_k\sqrt{\nu^2-k^2} = k_0 \sin\chi$, and thus $\nu = k_0$. Using these relations, eq. (51) can be transformed as

$$\overline{Y_2} = -\frac{1}{2}\mathrm{Re}\Big[\frac{i}{2}H(k_0, \chi) \Big\{ \frac{k_0\cos^2\chi}{\sin\chi} + \frac{k_0}{\sin\chi} - k_0\sin\chi$$
$$- \frac{2}{\sin\chi}\Big(K + \frac{(k_0\cos\chi)^2}{K_0} + 2\tau k_0\cos\chi \Big) \Big\} \Big]$$

$$= -\frac{1}{2}k_0 \sin\chi \,\mathrm{Im}[H(k_0, \chi)] \tag{55}$$

where $H(k_0, \chi)$ is given by (37).

As in the added-resistance formula, the above result can be put in a different form by applying the principle of energy conservation. Substituting the relation (42) in (55) and expressing the resulting equation in terms of $C(k)$ and $S(k)$ defined by (21), we get:

$$\overline{Y_2} = -\frac{1}{4\pi}k_0 \sin\chi\Big[-\int_{-\infty}^{k_1}+\int_{k_2}^{k_3}+\int_{k_4}^{\infty}\Big] \times \left\{|C(k)|^2+|S(k)|^2\right\}\frac{\nu}{\sqrt{\nu^2-k^2}}dk \tag{56}$$

Therefore, substitution of (54) and (56) into (49) gives the formula for the second-order steady sway force:

$$\frac{\overline{F_y}}{\rho g a^2} = -\frac{1}{4\pi k_0}\Big[-\int_{-\infty}^{k_1}+\int_{k_2}^{k_3}+\int_{k_4}^{\infty}\Big]\mathrm{Im}\{2C(k)S^*(k)\}\nu dk + \frac{\sin\chi}{4\pi}\Big[-\int_{-\infty}^{k_1}+\int_{k_2}^{k_3}+\int_{k_4}^{\infty}\Big]\{|C(k)|^2+|S(k)|^2\}\frac{\nu}{\sqrt{\nu^2-k^2}}dk \tag{57}$$

From this result, we can see that the first term comes from the interaction between symmetric and antisymmetric waves, whereas the second term comes from the independent contributions of symmetric and antisymmetric waves. Since the second term is multiplied by sin χ, both terms in (57) become zero in head and following waves for a ship with transverse symmetry.

5. The steady yaw moment

In order to relate the wave-induced steady yaw moment to the far-field velocity potential, we consider the principle of angular momentum about the z -axis. Newman[2] gave an expression for the rate of change of the vertical component of angular momentum, which is of general and thus applicable to the present problem. This can be expressed as

$$\frac{d\mathbf{K}_z}{dt} = -\iint_{S_H} p\,(\mathbf{r}\times\mathbf{n})_z\,dS - \iint_{S_C}[p\,(\mathbf{r}\times\mathbf{n})_z+\rho(\mathbf{r}\times\nabla\Phi)_z\,(\mathbf{n}\cdot\nabla\Phi)]dS \tag{58}$$

Here $\mathbf{r}$ is the position vector and the subscript z denotes the z-component of vector quantities. Note that the first term on the right-hand side of (58) is the minus yaw moment, because the unit normal is defined positive when pointing out of the fluid domain.

We take time average of (58). Since the fluid motion is periodic, there exists no net increase of angular momentum in the control volume. Therefore we get:

$$\overline{M_z} = \overline{\iint_{S_H} p(\mathbf{r}\times\mathbf{n})_z\, dS}$$
$$= -\overline{\iint_{S_C} [p(\mathbf{r}\times\mathbf{n})_z + \rho(\mathbf{r}\times\nabla\Phi)_z(\mathbf{n}\cdot\nabla\Phi)]dS} \tag{59}$$

Here the pressure of fluid p is given by (25), and it follows from (1) that

$$\left.\begin{aligned}
(\mathbf{r}\times\mathbf{n})_z &= xn_y - yn_x \\
(\mathbf{r}\times\nabla\Phi)_z &= x\frac{\partial\phi}{\partial y} - y\left(\frac{\partial\phi}{\partial x} - U\right) \\
\mathbf{n}\cdot\nabla\Phi &= n_x\left(\frac{\partial\phi}{\partial x} - U\right) + n_y\frac{\partial\phi}{\partial y}
\end{aligned}\right\} \tag{60}$$

Evaluating the above equations on the control surface shown in Fig. 1 and discarding terms higher than $O(\phi^3)$, eq. (59) can be reduced to

$$\begin{aligned}
\overline{M_z} = & \frac{\rho}{2}\int_0^{\infty} dz \int_{-\infty}^{\infty} \left[\overline{x\left\{\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right) - \left(\frac{\partial\phi}{\partial y}\right)^2\right\}} \right]_{-Y}^{Y} dx \\
& -\frac{1}{2}\rho g \int_{-\infty}^{\infty} \left[\overline{x\zeta_w^2} \right]_{-Y}^{Y} dx \\
& +\rho\int_0^{\infty} dz \int_{-\infty}^{\infty} \left[\overline{y\frac{\partial\phi}{\partial x}\frac{\partial\phi}{\partial y}} \right]_{-Y}^{Y} dx + \rho U \int_{-\infty}^{\infty} \left[\overline{y\zeta_w \frac{\partial\phi}{\partial y}} \right]_{-Y}^{Y} dx
\end{aligned} \tag{61}$$

where ζ_w is given by (48).

As before, calculating time average and substituting (3) gives the following expression:

$$\overline{M_z} = \frac{\rho g a^2}{k_0}(\overline{N_1} + \overline{N_2}) \tag{62}$$

where

$$\begin{aligned}
\overline{N_1} = & \frac{1}{4}\mathrm{Re}\int_0^{\infty} dz \int_{-\infty}^{\infty} \left[x\left(\left|\frac{\partial\varphi}{\partial x}\right|^2 + \left|\frac{\partial\varphi}{\partial z}\right|^2 - \left|\frac{\partial\varphi}{\partial y}\right|^2\right) + 2y\frac{\partial\varphi}{\partial x}\frac{\partial\varphi^*}{\partial y} \right]_{-Y}^{Y} dx \\
& -\frac{1}{4}\mathrm{Re}\int_{-\infty}^{\infty} \left[x\left\{K|\varphi|^2 + \frac{1}{K_0}\left|\frac{\partial\varphi}{\partial x}\right|^2 + i2\tau\varphi^*\frac{\partial\varphi}{\partial x}\right\}_{z=0} \right. \\
& \qquad \left. + 2y\left\{\left(i\tau\varphi^* + \frac{1}{K_0}\frac{\partial\varphi^*}{\partial x}\right)\frac{\partial\varphi}{\partial y}\right\}_{z=0} \right]_{-Y}^{Y} dx
\end{aligned} \tag{63}$$

$$\begin{aligned}
\overline{N_2} = & \frac{1}{2}\mathrm{Re}\int_0^{\infty} dz \int_{-\infty}^{\infty} \left[x\left(\frac{\partial\varphi}{\partial x}\frac{\partial\varphi_0^*}{\partial x} + \frac{\partial\varphi}{\partial z}\frac{\partial\varphi_0^*}{\partial z} - \frac{\partial\varphi}{\partial y}\frac{\partial\varphi_0^*}{\partial y}\right) \right. \\
& \qquad \left. + y\left(\frac{\partial\varphi}{\partial x}\frac{\partial\varphi_0^*}{\partial y} + \frac{\partial\varphi_0^*}{\partial x}\frac{\partial\varphi}{\partial y}\right) \right]_{-Y}^{Y} dx \\
& -\frac{1}{2}\mathrm{Re}\int_{-\infty}^{\infty} \left[x\left\{K\varphi\varphi_0^* + \frac{1}{K_0}\frac{\partial\varphi}{\partial x}\frac{\partial\varphi_0^*}{\partial x} + i\tau\left(\varphi_0^*\frac{\partial\varphi}{\partial x} - \varphi\frac{\partial\varphi_0^*}{\partial x}\right)\right\}_{z=0} \right.
\end{aligned}$$

$$+y\left\{i\tau\left(\varphi_0^*\frac{\partial\varphi}{\partial y}-\varphi\frac{\partial\varphi_0^*}{\partial y}\right)+\frac{1}{K_0}\left(\frac{\partial\varphi}{\partial x}\frac{\partial\varphi_0{}^*}{\partial y}+\frac{\partial\varphi_0{}^*}{\partial x}\frac{\partial\varphi}{\partial y}\right)\right\}_{z=0}\Big]_{-Y}^{Y}dx \quad (64)$$

In order to apply Parseval's theorem (32) to the x-integration in (63) and (64), the Fourier transform of the derivatives of φ times the coordinate x must be obtained. Considering $x(\partial\varphi/\partial x)$ as an example, it follows from (17) that

$$\begin{aligned} x\frac{\partial\varphi}{\partial x} &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\epsilon_k H^{\pm}(k)\frac{\nu k}{\sqrt{\nu^2-k^2}}e^{-\nu z\mp i\epsilon_k y\sqrt{\nu^2-k^2}}xe^{-ikx}dk \\ &= -\frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{d}{dk}\left[\ \epsilon_k H^{\pm}(k)\frac{\nu k}{\sqrt{\nu^2-k^2}}e^{-\nu z\mp i\epsilon_k y\sqrt{\nu^2-k^2}}\ \right]e^{-ikx}dk \quad (65)\end{aligned}$$

Therefore the Fourier transform of the above can be readily found:

$$\begin{aligned} F\left\{x\frac{\partial\varphi}{\partial x}\right\} &= -i\frac{d}{dk}\left[\ \epsilon_k H^{\pm}(k)\frac{\nu k}{\sqrt{\nu^2-k^2}}e^{-\nu z\mp i\epsilon_k y\sqrt{\nu^2-k^2}}\ \right] \\ &= -i\frac{d}{dk}\{H^{\pm}(k)\}\frac{\epsilon_k\nu k}{\sqrt{\nu^2-k^2}}e^{-\nu z\mp i\epsilon_k y\sqrt{\nu^2-k^2}} \\ &\quad -i\,H^{\pm}(k)\frac{d}{dk}\left\{\frac{\epsilon_k\nu k}{\sqrt{\nu^2-k^2}}e^{-\nu z}\right\}e^{\mp i\epsilon_k y\sqrt{\nu^2-k^2}} \\ &\quad \mp H^{\pm}(k)\frac{\nu k(\nu\nu'-k)}{\nu^2-k^2}e^{-\nu z}ye^{\mp i\epsilon_k y\sqrt{\nu^2-k^2}} \end{aligned} \quad (66)$$

where

$$\nu' = \frac{d\nu}{dk} = 2\left(\tau+\frac{k}{K_0}\right) \quad (67)$$

Regarding the Fourier transform of $\partial\varphi^*/\partial x$, we have from (17)

$$F\left\{\frac{\partial\varphi^*}{\partial x}\right\} = [H^{\pm}(k)]^*\frac{\epsilon_k\nu k}{\sqrt{\nu^2-k^2}}e^{-\nu z\pm i\epsilon_k y\sqrt{\nu^2-k^2}} \quad (68)$$

Similarly, we can obtain Fourier transforms which are necessary in carrying out the x-integration in (63). According to Parseval's theorem, we must consider the integration of the product of (66) and (68) with respect to k and similar integrations appearing in (63). In carrying out these integrations, we note that the integrand originating from the second term on the right-hand side of (66) is pure imaginary and thus does not contribute to the final result. Furthermore we can confirm that the summation of all the terms linearly proportional to y, including the contribution from the last term in (66), is precisely zero. Concerning the integration with respect to z in (63), eq. (33) can be used.

Summarizing these reductions, we shall get:

$$\overline{N_1} = -\frac{1}{8\pi}\mathrm{Im}\int_{-\infty}^{\infty}\Big[\ \frac{d}{dk}\{H^{+}(k)\}(H^{+}(k)\,)^*$$

$$-\frac{d}{dk}\{H^{-}(k)\}(H^{-}(k))^{*}\Big]\nu dk \tag{69}$$

Using (21), this equation can be rewritten in the form

$$\begin{aligned}\overline{N_1} &= \frac{1}{4\pi}\int_{-\infty}^{\infty}\epsilon_k \mathrm{Re}\{C'(k)S^{*}(k)-C^{*}(k)S'(k)\}\nu dk \\ &= \frac{1}{4\pi}\Big[-\int_{-\infty}^{k_1}+\int_{k_2}^{k_3}+\int_{k_4}^{\infty}\Big]\mathrm{Re}\{C'(k)S^{*}(k)-C^{*}(k)S'(k)\}\nu dk\end{aligned} \tag{70}$$

Here from (22), $C'(k)$ and $S'(k)$ are explicitly given as

$$\begin{aligned}\left.\begin{matrix}C'(k)\\ S'(k)\end{matrix}\right\} &= \int\int_{S_H}\Big(\frac{\partial\varphi}{\partial n}-\varphi\frac{\partial}{\partial n}\Big)e^{-\nu\zeta+ik\xi} \\ &\times\Big[(-\nu'\zeta+i\xi)\left\{\begin{matrix}\cos(\eta\sqrt{\nu^2-k^2})\\ \sin(\eta\sqrt{\nu^2-k^2})\end{matrix}\right\} \\ &\mp\frac{\nu\nu'-k}{\sqrt{\nu^2-k^2}}\eta\left\{\begin{matrix}\sin(\eta\sqrt{\nu^2-k^2})\\ \cos(\eta\sqrt{\nu^2-k^2})\end{matrix}\right\}\Big]dS\end{aligned} \tag{71}$$

It is clear from (70) that only the interactions between symmetric and antisymmetric waves contribute to the $\overline{N_1}$ term, which is the same as the steady sway force.

Next, we consider the second term, $\overline{N_2}$, defined by (64), which originates from the interaction of the incident wave and ship-generated waves. Following the foregoing procedure, the Parseval's theorem (32) will be used in conjunction with the Fourier transforms (20) and (66). Then we can put $k = k_0\cos\chi$ due to the property of Dirac's delta function in (20) and $k_0\sin\chi = \epsilon_k\sqrt{\nu^2-k^2}$ or $k_0 \sin\chi = -\epsilon_k\sqrt{\nu^2-k^2}$ depending on the value of χ due to the reasons stated in transforming (31); thus the relation $\nu = k_0$ holds.

After the x-integration using Parseval's theorem and the z-integration using

$$\int_0^{\infty} z e^{-2\nu z}dz = \Big(\frac{1}{2\nu}\Big)^2 \tag{72}$$

and (33), the interim result will consist of three parts, just like (66): the first (denoted by $\overline{N_{21}}$) includes the derivative of the Kochin function, the second ($\overline{N_{22}}$) incudes the terms linearly proportional to y, and the third ($\overline{N_{23}}$) is the remainder. After somewhat lengthy calculations, these three parts can be found to be :

$$\overline{N_{21}} = -\frac{1}{2}\sin\chi\,\mathrm{Re}\Big\{k_0\frac{d}{dk}[H^{\pm}(k)]\Big\} \tag{73}$$

$$\overline{N_{22}} = 0 \tag{74}$$

$$\overline{N_{23}} = -\frac{1}{2}\sin\chi\,\mathrm{Re}\left\{\left(\tau+\frac{k_0\cos\chi}{K_0}\right)H(k_0,\ \chi)\right\} \tag{75}$$

Here the quantity in braces in (73) should be evaluated at $k = k_0 \cos \chi$ and $\pm\epsilon_k\sqrt{\nu^2-k^2} = k_0\sin\chi$, with the complex sign taken according to $H^+(k)$ or $H^-(k)$, respectively. Therefore, using the relation $k_0 = \nu = (\omega+kU)^2/g$ and notation (21), the final result can be written as

$$\begin{aligned}\overline{N_2} &= \overline{N_{21}}+\overline{N_{22}}+\overline{N_{23}}\\ &= -\frac{1}{2g}\sin\chi\,\mathrm{Re}\left[(\omega+kU)\frac{d}{dk}\{(\omega+kU)H^{\pm}(k)\}\right]_{\substack{k=k_0\cos\chi\\ \pm\epsilon_k\sqrt{\nu^2-k^2}=k_0\sin\chi}} \qquad (76)\\ &= -\frac{1}{2}\sin\chi\,\mathrm{Re}\Big[k_0\{C'(k_0,\ \chi)+i\,S'(k_0,\ \chi)\}\\ &\qquad\qquad +\left(\tau+\frac{k_0\cos\chi}{K_0}\right)H(k_0,\ \chi)\Big] \qquad (77)\end{aligned}$$

where $C'(k_0,\ \chi)+i\,S'(k_0,\ \chi)$ is to be interpreted as

$$\left[\frac{d}{dk}\{C(k)+i\,S(k)\}\right]_{\substack{k=k_0\cos\chi\\ \sqrt{\nu^2-k^2}=k_0\sin\chi}}$$

Substituting (70) and (77) into (62), we obtain the formula for the steady yawing moment in waves:

$$\begin{aligned}\frac{\overline{M_z}}{\rho g a^2} &= \frac{1}{4\pi k_0}\left[-\int_{-\infty}^{k_1}+\int_{k_2}^{k_3}+\int_{k_4}^{\infty}\right]\mathrm{Re}\{C'(k)S)(k)-C^*(k)S'(k)\}\nu dk\\ &\quad -\frac{1}{2}\sin\chi\,\mathrm{Re}\Big[C'(k_0,\ \chi)+i\,S'(k_0,\ \chi)\\ &\qquad\qquad +\frac{1}{k_0}\left(\tau+\frac{k_0\cos\chi}{K_0}\right)H(k_0,\ \chi)\Big] \qquad (78)\end{aligned}$$

This is the result obtained for the first time by the present analysis. In the limit of vanishing forward speed, τ and $1/K_0$ are zero from (11), and $k_1 = -\infty$, $k_2 = -K$, $k_3 = K$, and $k_4 = \infty$ from (12) and (13). Thus we can confirm that Newman's result[2)] at zero forward speed is recovered from the present result.

6. Concluding remarks

The formulas obtained in this paper permit us to calculate the second-order sway force and yaw moment, provided that the Kochin function is determined from the velocity potential on the ship hull. Although there are still a number of problems to be resolved for a reliable solution by the three-dimensional panel method, some progress have been made recently in developing a fast algorithm of the Green function with forward speed and sinusoidal oscillation; for instance, Iwashita & Ohkusu[9)]. Therefore it will be possible in the near future to obtain the Kochin function from the "exact" solution of the entire boundary-value problem. However, from the viewpoint of economical computations with relatively good accuracy, the unified slender-ship theory developed by Newman[10] and Sclavounos[11] may be the first to be tested for the determination of the Kochin function. The computational work along this line is now in progress, and the results will be presented in the foreseeable future together with experiments to verify a part of them.

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