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NONLINEAR INSTABILITY AND EVOLUTION OF STEEP WATER WAVES UNDER WIND ACTION

By Tadao KUSABA* and Hisashi MITSUYASU**

Results of experimental studies of the evolution of nonlinear wave train are reported. In the first experiment, the evolution of mechanically generated monochromatic waves with various steepness is studied without wind action. Many of the results is quite similar to the results of previous studies (Lake et al 1977[1], Melville 1982[2], Su et al 1982[3], Mitsuyasu & Honda 1982[4]) ; Waves of small steepness show a simple attenuation due to viscous energy dissipation. Waves of moderate or large steepness show such an evolution of the spectra that initially generated discrete spectrum composed of fundamental, higher harmonics, and their side band frequency components tend to be a continuous spectrum, which is similar to that of wind waves, though energy level is generally smaller than that of wind waves and its level depends on the initial steepness of the waves. A recurrence phenomenon is not clear due to a length limitation of the wave tank.

In the second experiment the evolution of the same waves is studied by applying a favorable wind ($U_r=10$ m/s, $u_*\approx 60$ cm/s). The evolution of the wave spectra shows the following properties ; Sharp spectral lines in the wave spectrum disappear gradually with evolution of the wave spectrum in the order of side bands of higher harmonics, higher harmonics and higher side band of the fundamental frequency component. High frequency region of the wave spectrum approaches to the continuous wave spectrum which is almost the same to that of pure wind wave spectrum. In a later stage of the evolution even the fundamental frequency component is buried in the wind wave spectrum yielding a continuous spectrum with a peak near the lower side band of the fundamental frequency of the original wave. After this stage the waves are considered to develop as the wind wave, though they are not studied in the present experiment due to the limitation of the length of wind-wave tank.

key words : Wave evolution, Nonlinear instability, Wave breaking and Wave development.

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1. Introduction

The present study has two aspects ; One is a sequel of our previous study on the wind-induced growth of water waves, and another one is a preliminary study of the nonlinear instability of wave train under wind action.

Previously we measured the wind-induced growth of mechanically generated monochromatic waves in a wind-wave tank and derived an empirical relation of the growth rate of the monochromatic waves (Mitsuyasu & Honda 1982[4]). In the previous study, however, we used only stable waves of relatively small steepness to simplify the phenomena. When the waves developed to become steep and unstable and began to break, the spectral energy of the fundamental frequency component of the monochromatic waves began to decrease with an increase of the fetch even under the wind action. Due to a limited length of the wind-wave tank, we could not study further phenomena after the wave breaking. Therefore, this study is planned firstly to clarify the wind-induced growth of such steep unstable waves that they become gradually unstable with propagation and finally break even without wind action.

As for the instability of steep waves, the last two decades have seen much progress in the theoretical and experimental studies of the nonlinear instability of water waves of finite steepness ; Comprehensive review has been given by Yuen & Lake 1982[5]. However, many of the previous studies concerned with the evolution of steep waves without wind action and few studies were done on the instability and evolution of steep waves under wind action. When the wind blowes over water waves, energy and momentum are transferred from the wind to the waves. Therefore, it is interesting to clarify an energy balance in the unstable wave system under wind action. The present study aims to clarify experimentally the effects of wind action on the evolution of steep unstable waves.

In the first part of the experiment, we study the evolutions of stable and unstable waves without wind action to compair the results later with those obtained for the same waves under wind action. In the second part of the experiment, the evolutions of the waves under wind action are studies using the same waves with those in the first experiment. Due to a limitation of a length of the wind-wave tank the present study is mainly concerned with an initial stage of the evolution of the waves. Further studies are now in progress in a large wave tank which has been constructed recentry. More quantitative analysis of the data will be done after obtaining new data. Therefore, the present paper is a preliminaly one.

The experimental method will be described in §2, results of the experiment for waves without wind action in §3 and those for waves under wind action in §4.

2. Experiment

2.1. *Equipment and procedure*

The experiment was carried out in a wind-wave tank 0.8 m high, 0.6 m wide and with a usual test-section length of 15 m. Water depth in the tank was kept at 0.32 m throughout the experiment. Waves ware measured with registance-type wave gauges at eight stations;

x(m): 2.51, 3.51, 4.51, 5.51, 6.51, 7.82, 8.61 and 10.51;
$k_o x$: 40.4, 56.4, 72.5, 88.6, 105, 126, 139 and 169.
($k_o=16.1\ \mathrm{m}^{-1}$; initial wave number)

The arrangement of the equipments is almost the same to our previous study (Mitsuyasu & Honda 1982[4)]) except for wave properties and positions of wave gauges.

Wind speed was fixed to $U_r=10$ m/s, where U_r is the wind speed at 20 cm above a transition plate and it corresponds roughly to a cross-sectional mean wind speed. The following three waves were used for the measurements.

Waves No.	Period T(sec)	Height H(cm)	Steepness $(H/L)_o$	Steepness $(ak)_o$
1	0.5	1.7	0.044	0.139
2	0.5	3.2	0.082	0.257
3	0.5	4.1	0.105	0.331

(L: wave length, $a=H/2$ and $k=2\pi/L$)

In the first experiment for waves without wind action, monochromatic regular wave was generated by a flap-type wave generator at the up-wind side of the tank. The wave with initial wave steepness $(ak)_o=0.139$ is a stable one, though it shows a slight attenuation due to viscous energy dissipaion. The other waves with larger steepness, $(ak)_o=0.257$ and 0.331 become unstable as they propagate down in the tank and eventually break two-dimensionally in a large scale when the waves travel about 10 m ($k_o x=161$).

In the second experiment for waves under wind action, firstly wind waves were generated in the test-section by the wind, $U_r=10$ m/s. After the time when wind waves had attained a stationary state, the regular wave with given steepness was made to propagate into the wind-generated wave fields in the test section. Surface time history, $\eta(t)$, measured with wave gauges and wave profiles, $\eta(x)$ measured with a video-camera were recorded on a magnetic tape of TEAC PCU visual data recorder PU-400, which can record simultaneously the both signals. In addition to these measurements, photographs of wave trains were taken at various stages of evolution of waves with and without wind action. These photographs and images reproduced by the PU-400 visual recorder provide us useful informations about unstable wave trains with and without wind action.

2.2. *Analysis of the data*

Wave record of each run was dizitized at a sampling frequency 204.8 Hz, so that the length of one sample of the wave data containing $8192=2^{13}$ data point becomes an integral multiple of the given wave period. This procedure is used to reduce the leakage effect of line spectra of monochromatic waves. Power spectra of waves were computed through a FFT method using each data sample containing 8192 data points. Numbers of the samples of the spectra were three for waves without wind action and six for waves under wind action. Sample mean of these spectra was taken first, and moving average of three line spectra with a triangular filter was applied to the measured spectrum.

3. Results of experiment on the evolution of the waves without wind action

3.1. *General properties*

Figs. 1, 2 and 3 show wave profiles $\eta(x)$, surface time histories $\eta(t)$ and power spectra $\phi(f)$ of regular waves respectively in the absence of wind. Wave No. 1 which has a relatively small steepness $(ak)_0=0.139$ is stable but attenuate gradually with propagation distance due to viscous energy dissipation (Figs. 1(a), 2(a) and 3(a)). As shown clearly in Fig. 3(a), the measured spectra show many spikes of higher harmonics up to 8-th. Very tiny spectral peakes due to the side band instability appear only at long propagation distances $x=8.61$ m, 10.51 m ($k_o x=139$, 169).

Wave No. 2, which has a moderate steepness $(ak)_0=0.257$ becomes gradually unstable showing beat structure in $\eta(t)$ and finally breaks two-dimensionally at the location near $x=10$ m($k_o x=161$) (Figs. 1(b), 2(b) and 3(b)). The spectra of Fig. 3(b) show significant peaks at f_o, $f_o \pm \Delta f$, $2f_o$, $2f_o \pm \Delta f \ldots nf$, $nf_o \pm \Delta f$, with n an integer, where f_o is the fundamental frequency of the wave. These peaks correspond to the fundamental, its higher harmonics and the side bands both for the fundamental and for the higher harmonics. At shorter distance $x=4.51$ m the lower side band component is smaller than the higher one but it grows gradually and becomes larger than the higher side band component at near the breaking point $x=8.61$ m($k_o x=139$). Higher side band component also developes with propagation distance at an initial stage, but they reach a satulated level at about $x=8.61$ m($k_o x=139$).

Wave No. 3 which has the largest steepness, $(ak)_0=0.331$ in our present study shows clearly three-dimensional instability (Fig. 1(c)), and beat of the amplitude appears at the intial stage of generation (Fig. 3(c)). This wave propagates repeating small scale breakings at the crest and maintaining its three-dimensional properties up to $k_o x=139$. The beat of the amplitude becomes very large at $k_o x=139$ (Fig. 3(c)) and finally wave breaks two-dimensioally at $k_o x=161$. After the large scale breaking the waves show rather long crested

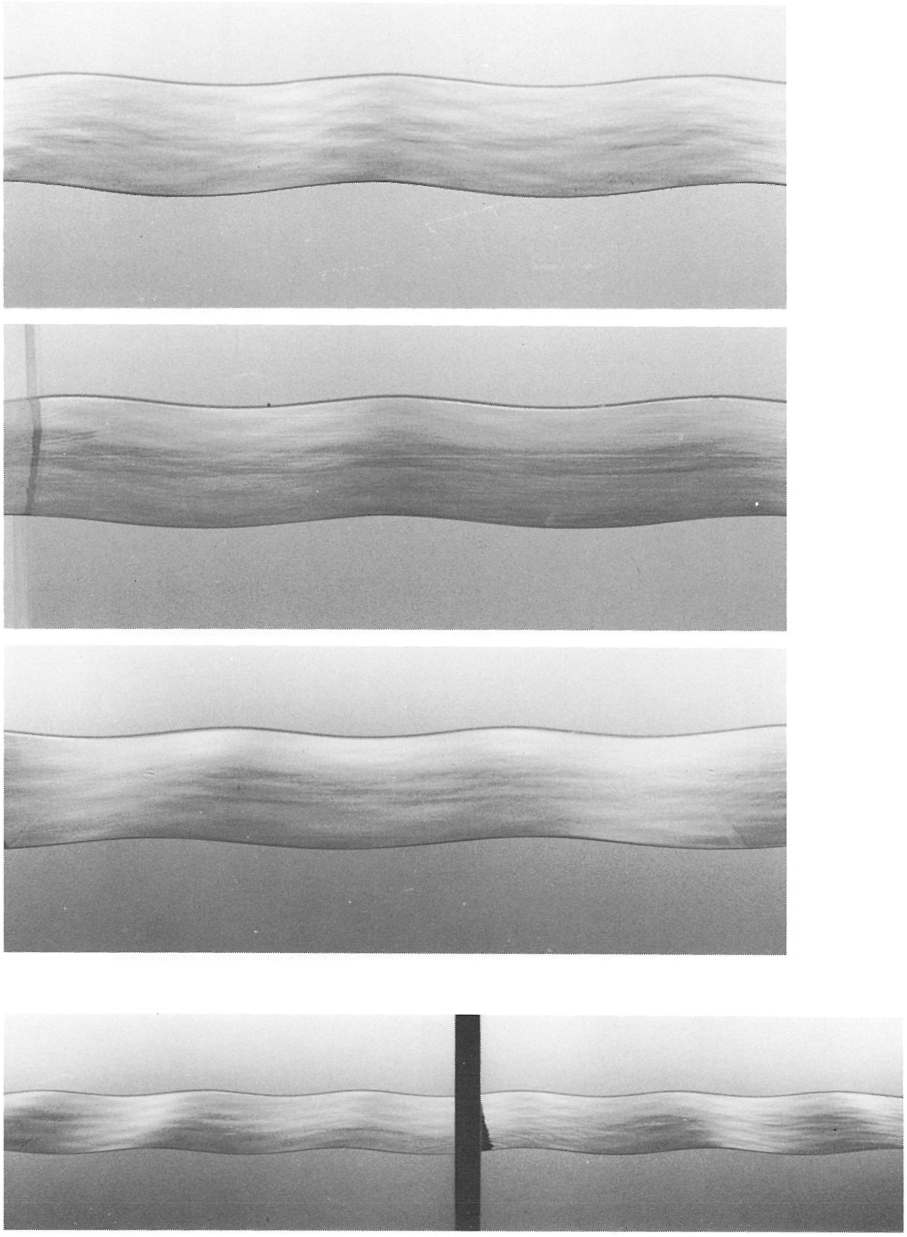

Fig. 1(a) Surface profile $\eta(x)$ without wind action for $(ak)_o=0.139$. From the top to the bottom, $x=4.5$ m, 7.2 m, 10.8 m. The lowest one is a wider view of waves at $x=10.2$ m. Wave Length is 0.39 m.

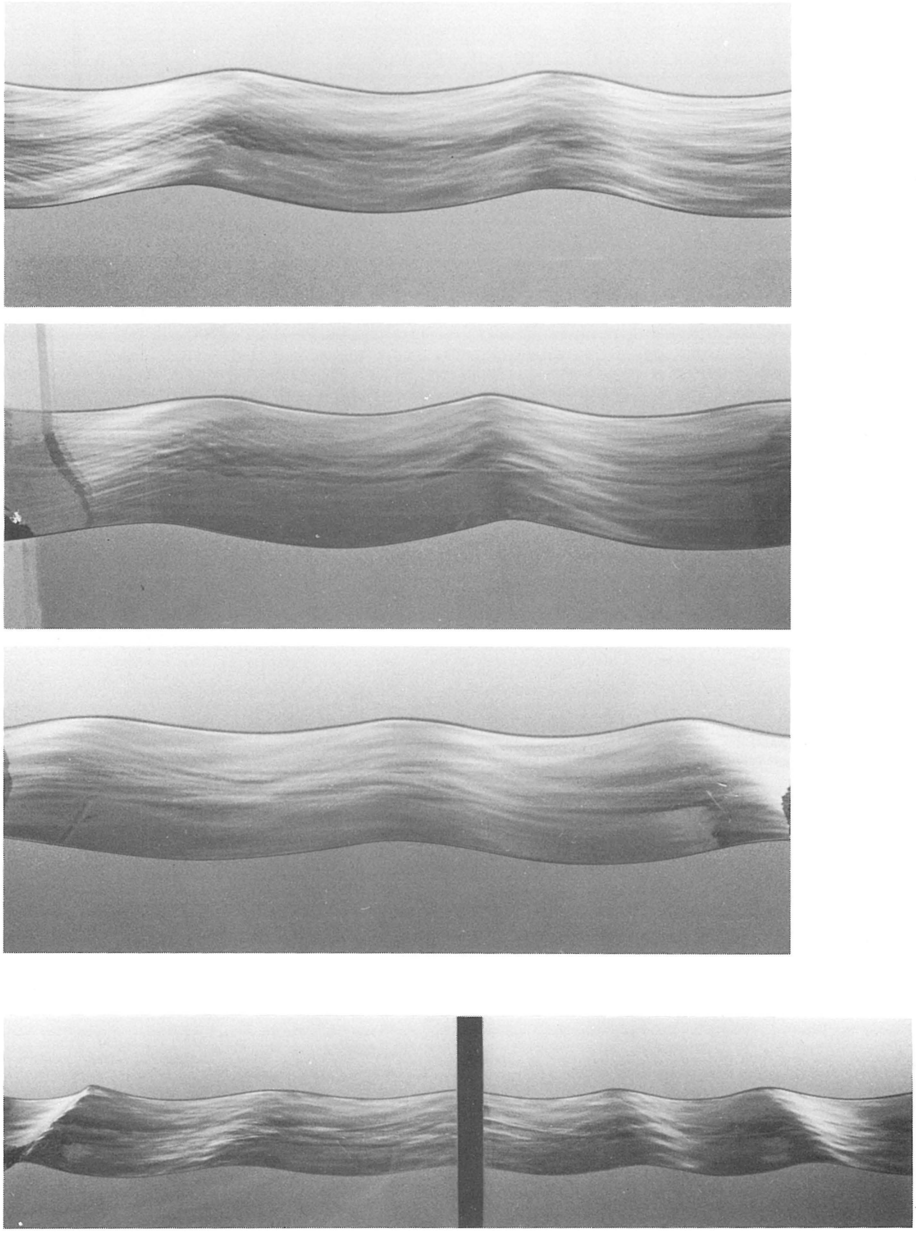

Fig. 1(b) The same as Fig. 1(a) except for $(ak)_o = 0.257$.

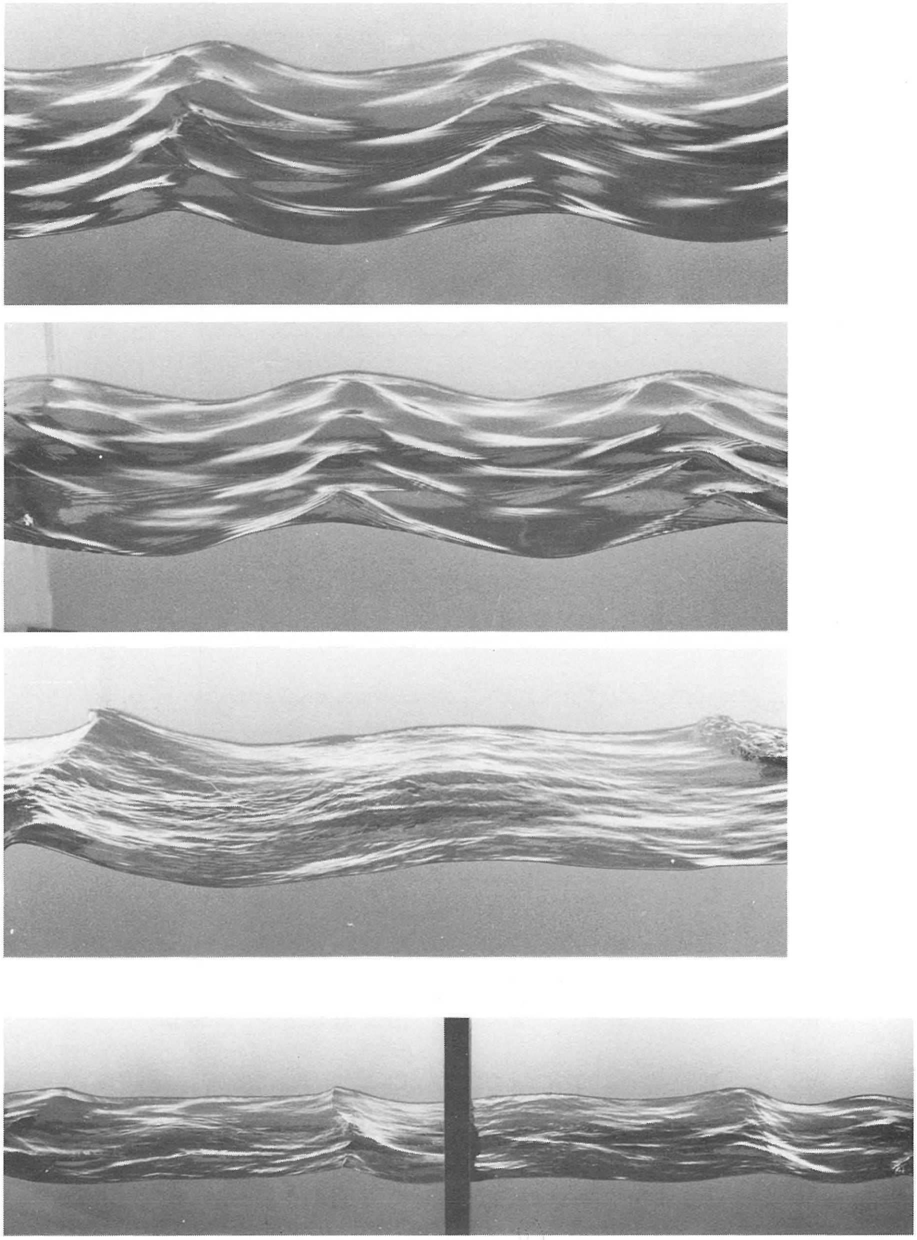

Fig. 1(c) The same as Fig. 1(a) except for $(ak)_0=0.331$.

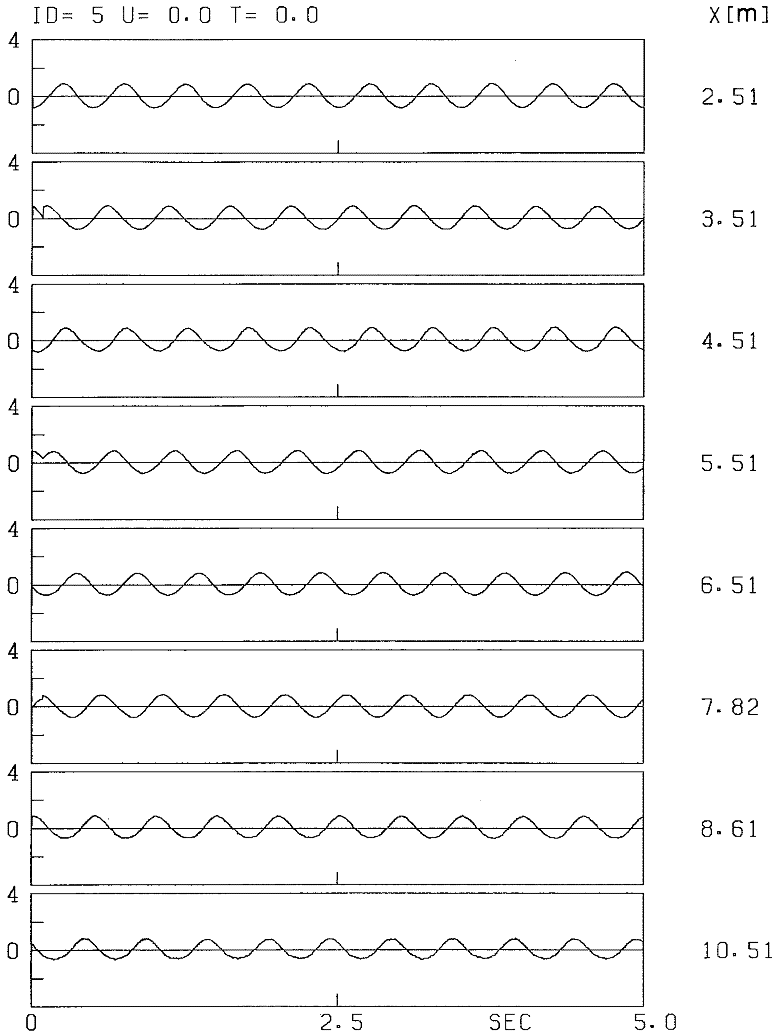

Fig. 2(a) Surface time history $\eta(t)$ without wind action for $(ak)_0=0.139$.

nature with steepness and frequency decreases (Fig. 2(c)). Random nature of the wave increases also after the wave breaking. The frequency spectra (Fig. 3(c)) show that the lower side band is larger than the higher side band at initial stage. Higher harmonics decrease gradually with wave propagation, so that the high frequency part of the spectra tends to be continuous spectra. In Figs. 3(a), 3(b) and 3(c), thin smooth curve superimposed on the spectra of regular waves without wind is the spectrum of pure wind waves measured at corresponding

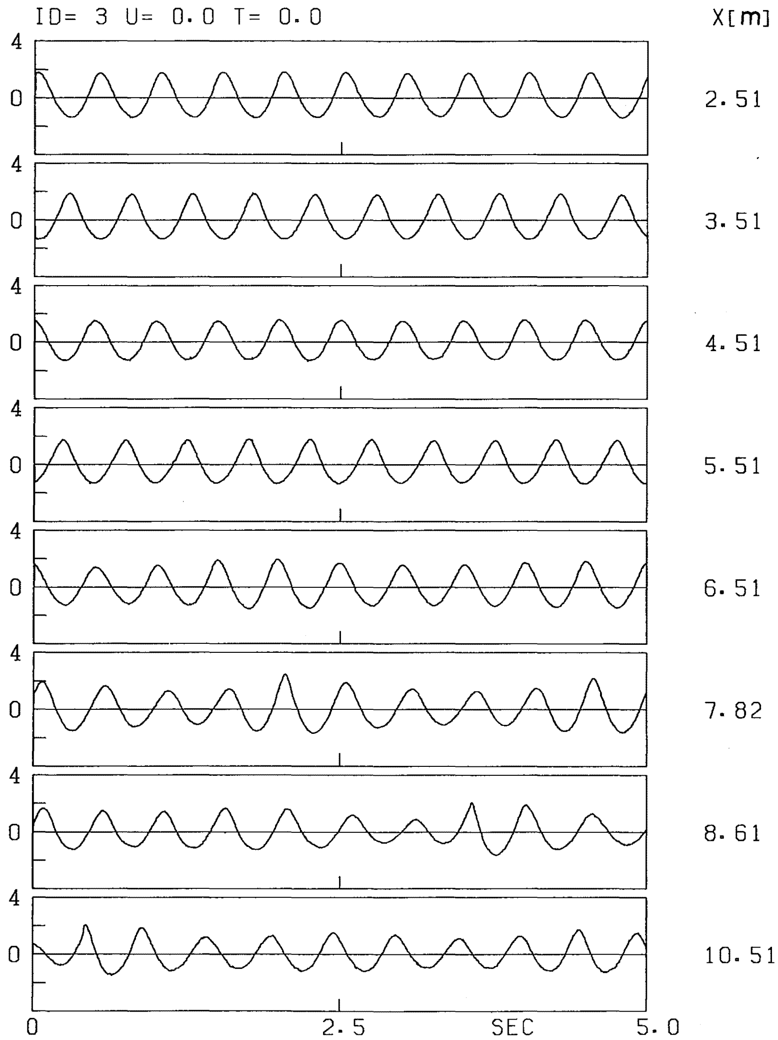

Fig. 2(b) The same as Fig. 2(a) except for $(ak)_0=0.257$.

fetch for wind speed $U_r=10$ m/s. A triangular filter with 21 spectral lines is used in the pure wind wave spectra to reduce the intrication of the figures. From these figures we can see that the high frequency region of power spectra of regular waves with small steepness is very lower in level than that of satulated wind wave spectra, except for the second harmonics. Energy level of the high frequency region of the spectra increases as wave steepness increases even for

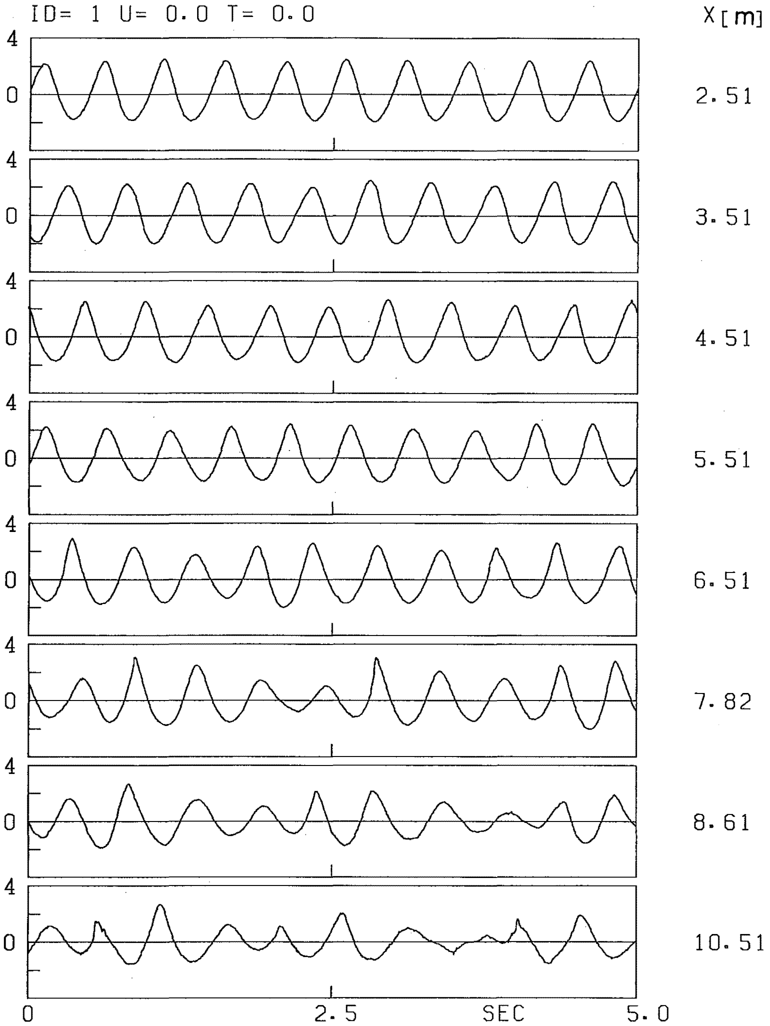

Fig. 2(c) The same as Fig. 2(a) except for $(ak)_o = 0.331$.

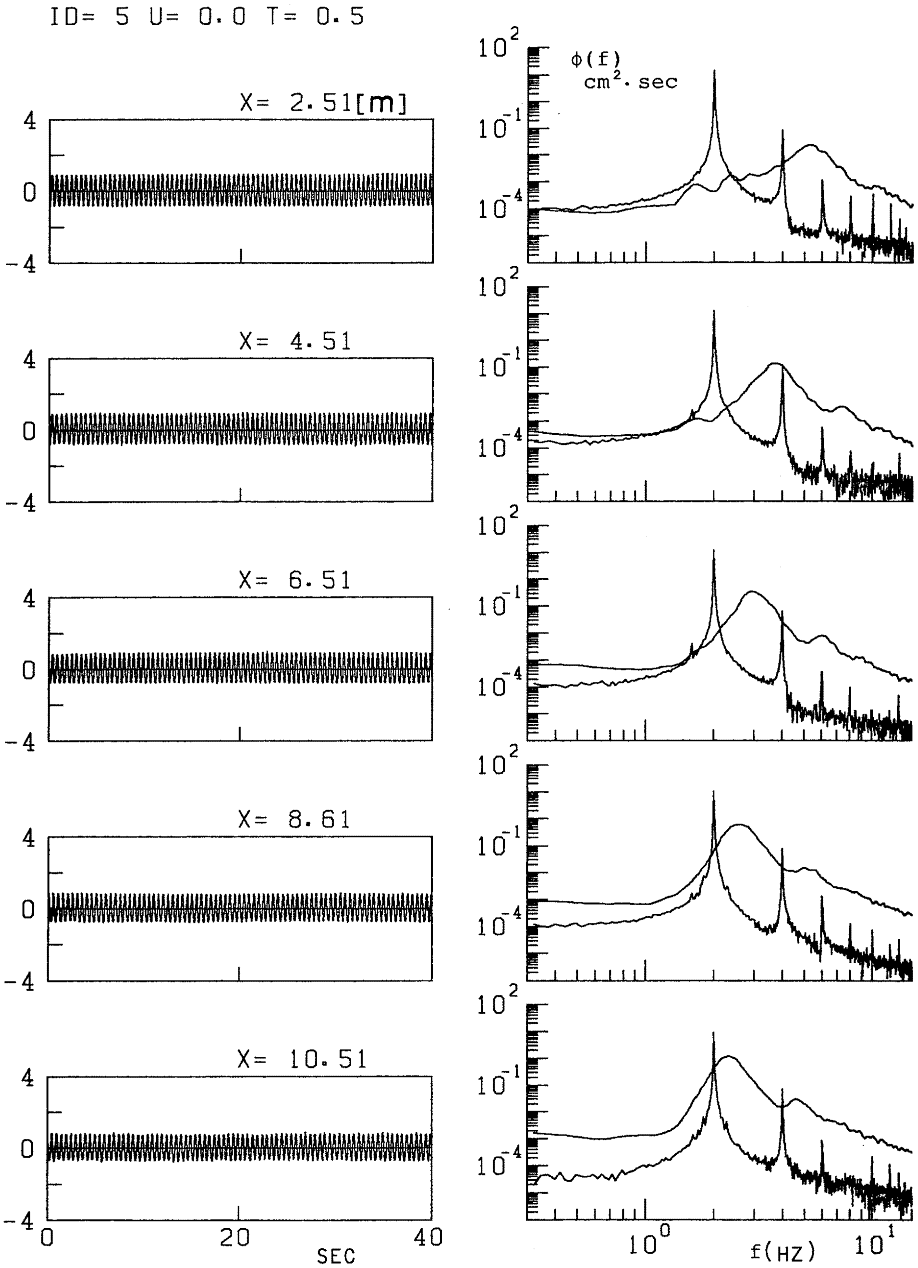

Fig. 3(a) Evolution of power spectra without wind action (right-hand column) and a part of corresponding wave records (left-hand column) for $(ak)_0=0.139$.

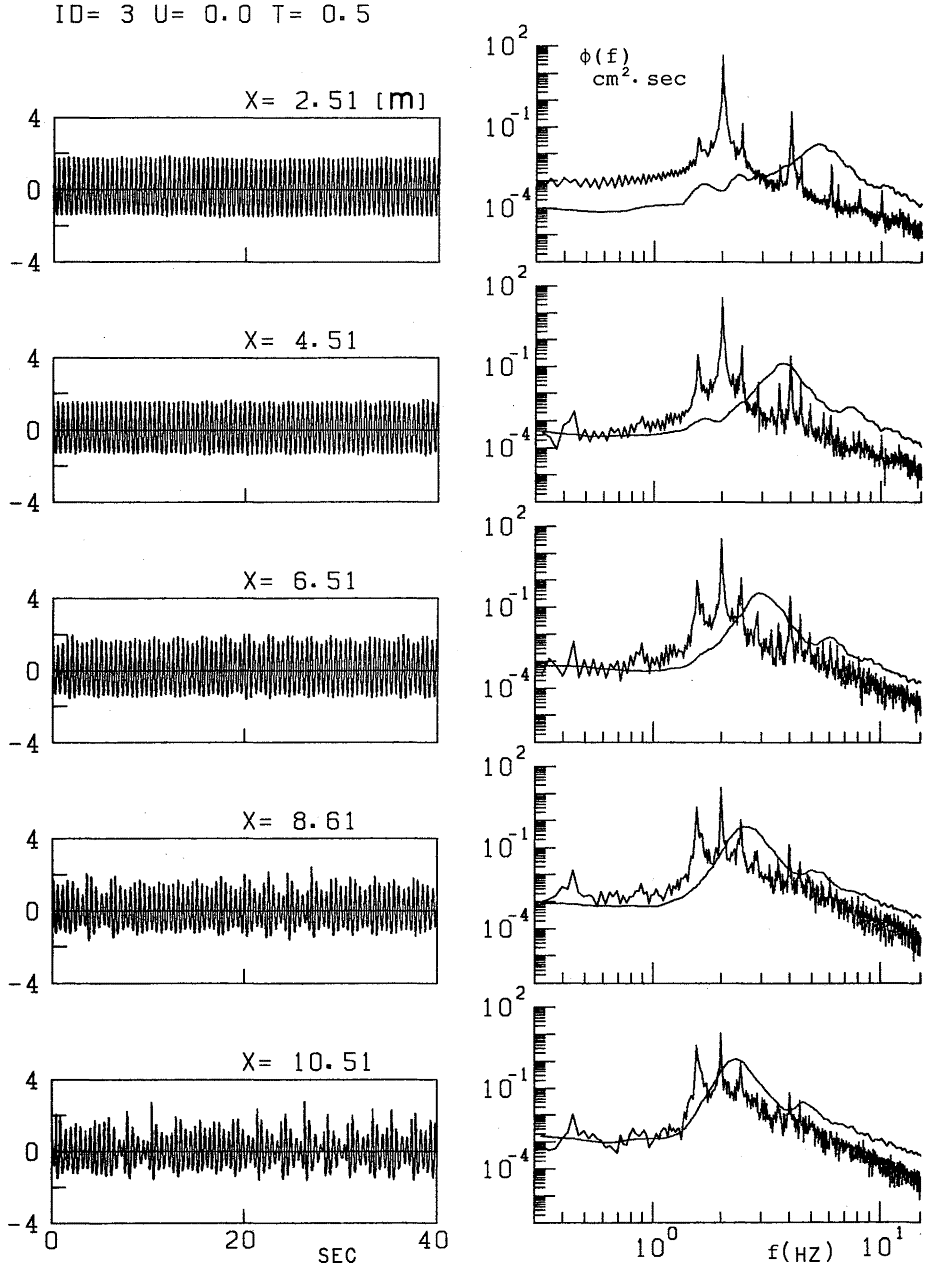

Fig. 3(b) The same as Fig. 3(a) except for $(ak)_o = 0.257$.

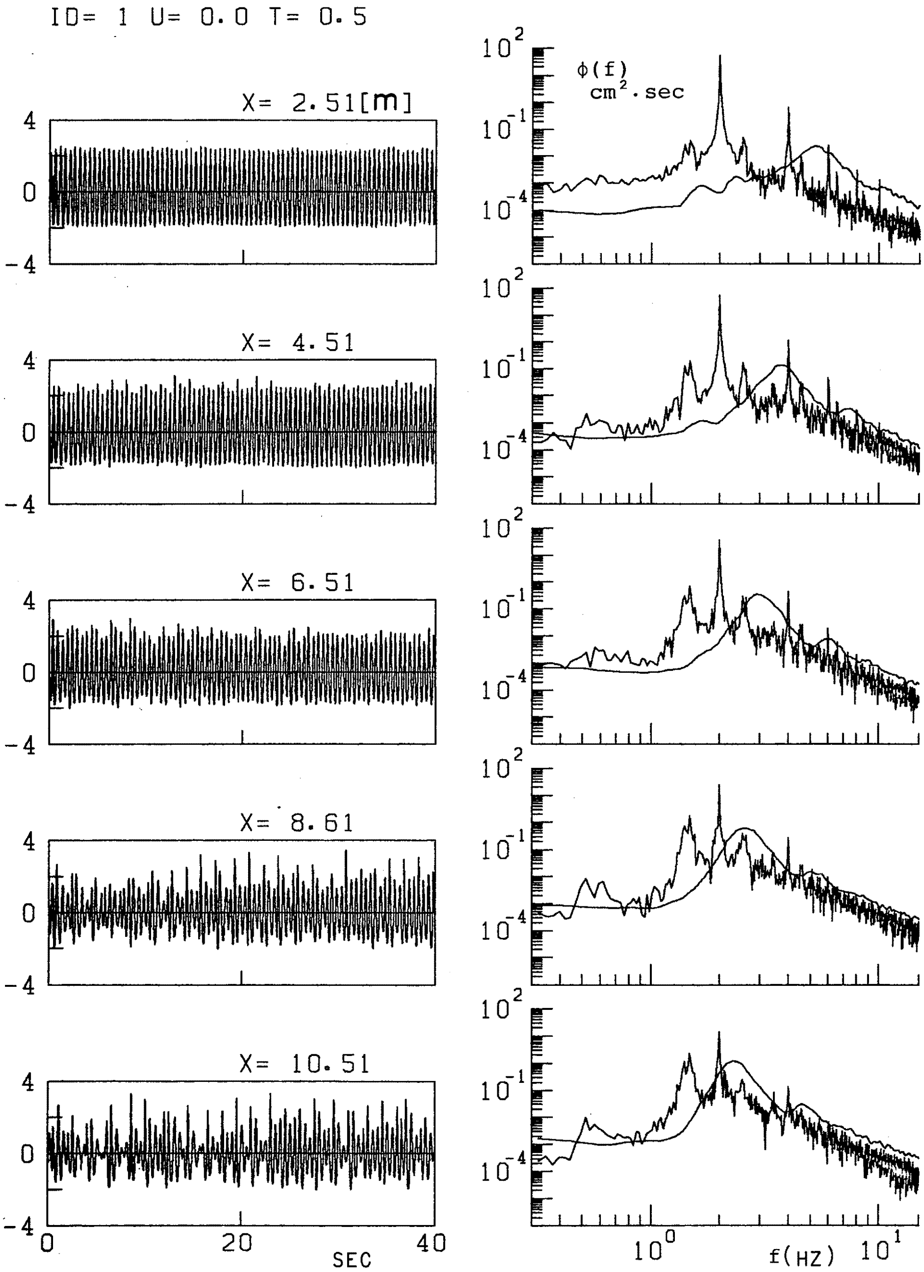

Fig. 3(c) The same as Fig. 3(a) except for $(ak)_o=0.331$.

the waves without wind. This may be caused by breaking of waves, which diffuses spectral energies of higher harmonics and their side band.

3.2. *Energy balance in the waves without wind action*

In order to clarify the energy exchange among the fundamental frequency component, its side bands and the other frequency components, the frequency spectrum of wave was divided into five frequency regions and the change of the spectral energy contained in each frequency region was investigated. Fig. 4 shows the five frequency regions schematically, where E_1, E_2, E_3, E_4 and E_5 are wave energies contained respectively in the low, the lower side band, the fundamental, the higher side band and the high frequency regions.

The change of wave energy in each frequency region for the stable wave $(ak)_0=0.139$ is shown in Fig. 5(a) together with total wave energy $E(=\sum_{i=1}^{5}E_i)$. In Fig. 5(a), since the difference between E and E_3 is negligibly small, only the data for E are plotted. The straight line in Fig. 5(a) shows a theoretical change of the wave energy due to viscous energy dissipations at water surface boundary and boundaries of side walls and bottom of the tank (Mitsuyasu & Honda 1982[4)]). The measured decay rates for E or E_3 are in good agreements with the theoretical one. Fig. 5(a) also shows that the energy levels of E_1, E_2, E_4 and E_5 are negligibly small and their changes with propagation distance are negligibly

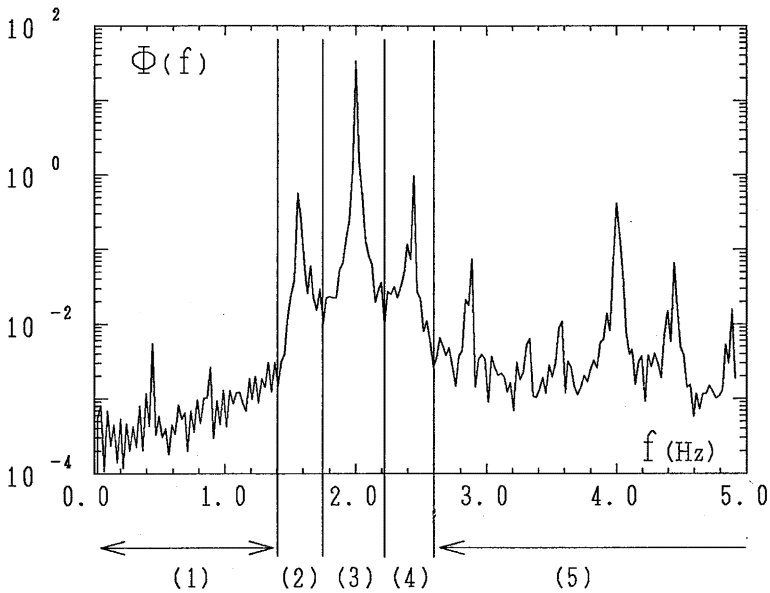

Fig. 4 Definition of characteristic frequency regions of the power spectrum. (1), low frequency region; (2), lower side band frequency region; (3), fundamental frequency region; (4), higher side band frequency region; (5), high frequency region.

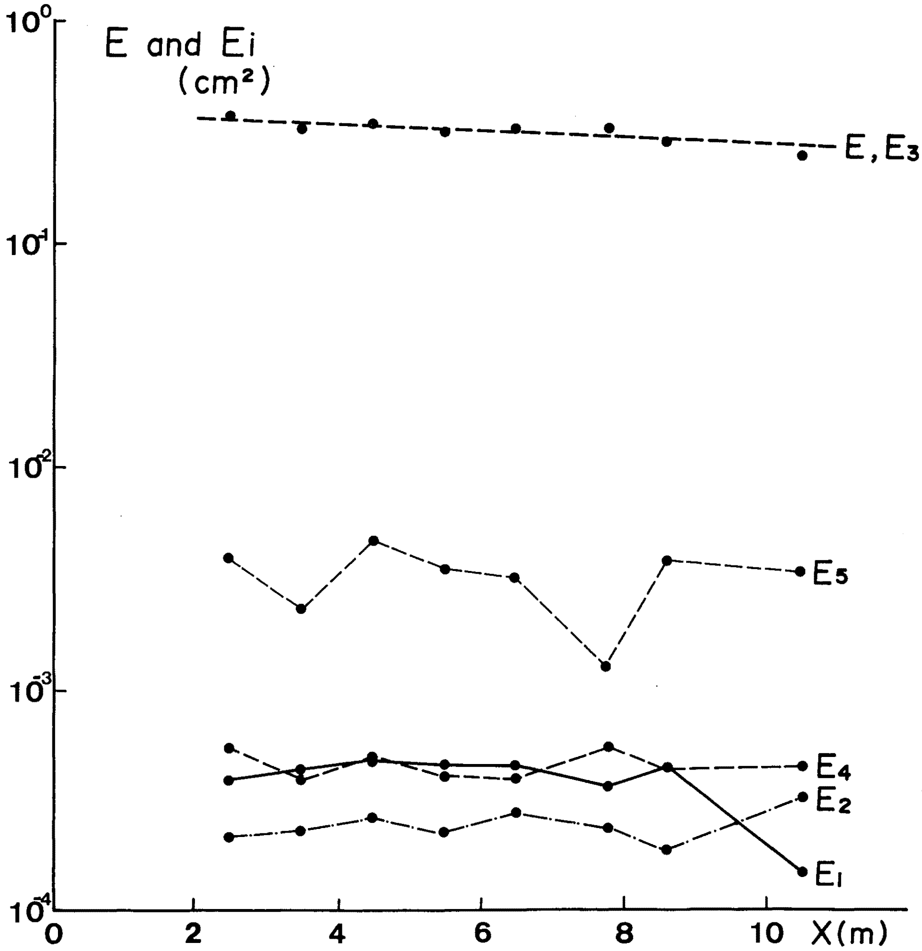

Fig. 5(a) The change of wave energy without wind action in the characteristic frequency regions for $(ak)_0=0.139$.

small, too. In summary, the wave energy of the stable wave with small steepness $(ak)_0=0.139$ is contained mostly in the fundamental frequency component and it attenuates gradually with propagation distance due to the viscous energy dissipation.

Figs. 5(b) and 5(c) show the growth and decay of spectral components for unstable waves $(ak)_0=0.257$ and 0.331. At an intial stage of evolution, the attenuation of E or E_3 for $(ak)_0=0.257$ shows fairly good agreement with the theoretical prediction of viscous energy dissipation, but the attenuation rate of the energy increases abruptly near $x=8$ m ($k_o x=129$). This abrupt departure from the theoretical curve is attributted to the onset of wave breaking. The

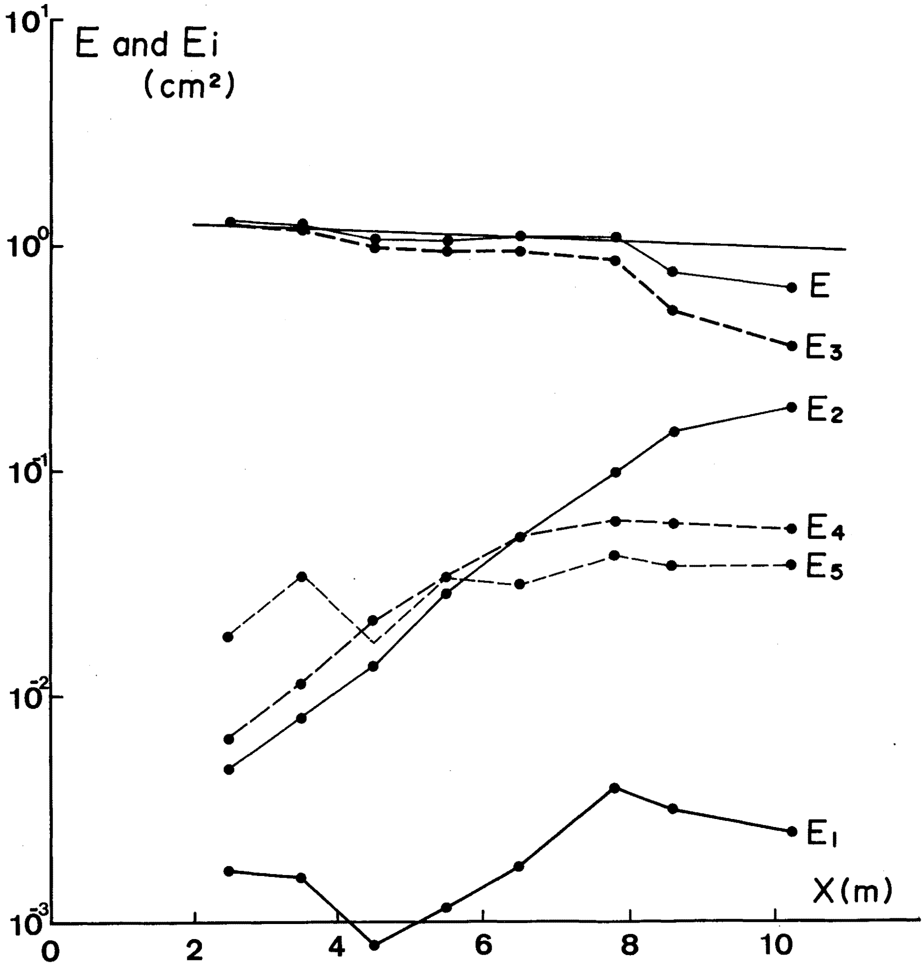

Fig. 5(b) The same as Fig. 5(a) except for $(ak)_o=0.257$.

energy dissipation due to the wave breaking, ΔE, is an order of 30 % of the total wave energy E. For the wave with large steepness $(ak)_o=0.331$, E and E_3 begin to decrease rapidly at shorter distance $x=4.5$ m ($k_o x=73$). The measured decay rate is larger roughly by a factor of 2.5 than the theoretical one estimated from the viscous energy dissipations. This is probably due to the energy dissipation by wave breaking of spilling type. The difference between E and E_3 is attributed to the evolution of the side bands E_2 and E_4. The higher and lower side bands of fundamental frequency component, E_2 and E_4 have an almost equal magnitude and they developes with an almost equal growth rate at an initial stage of evolution $x \lesssim 5$ m ($k_o x \lesssim 80$). In later stage, however, the growth rate of

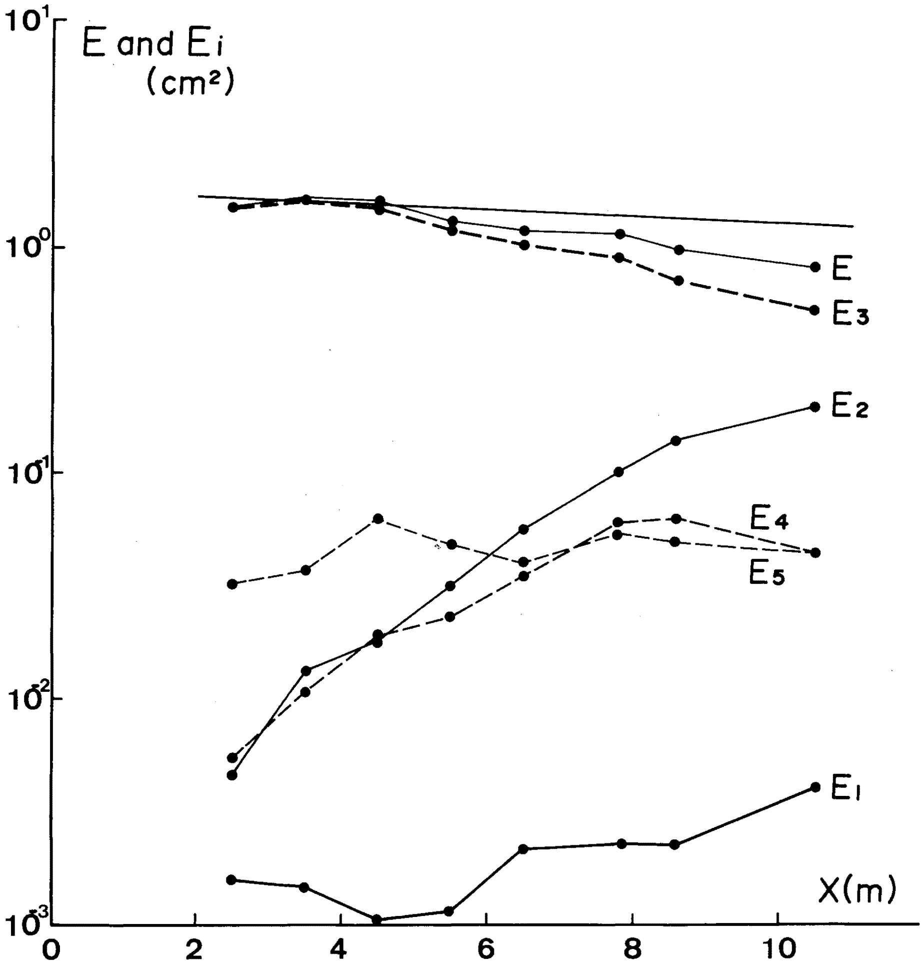

Fig. 5(c) The same as Fig. 5(a) except for $(ak)_0=0.331$.

E_4 decreases and becomes smaller than that of E_2. The energies of the both side bands, E_2 and E_4 show a trend to reach satulated leveles respectively, though the satulated level of E_2 is always larger than that of E_4. Figs. 5(b) and 5(c) also show that the wave energy at the low frequency region, E_1 is negligibly small, the wave energy at the high frequency region, E_5 is roughly the same order with the energy of the higher side band, E_4, and the changes of E_1 and E_5 are very small.

It is concluded that the evolution of the monochromatic waves without wind action shows many features which were commonly observed in previous studies (Lake et al 1977[1], Melville 1982[2], Su et al 1982[3], Mitsuyasu & Honda 1982[4]).

4. Results of experiments on the evolution of the waves under wind action

4.1. *General properties*

Wave profiles $\eta(x)$, surface time histories $\eta(t)$ and its energy spectra $\phi(f)$ under wind action are shown respectively in Figs. 6, 7 and 8. As shown in the left side of Fig. 8(a), the stable wave No. 1, $(ak)_0=0.137$, under wind action shows very rapid growth of its amplitude and increase of random fluctuations of the amplitude. The wave spectra in the right hand side of Fig. 8(a) show that when regular waves develope and become steep enough, wave components due to side band instability begin to grow at $x=6.5$ m ($k_o x=105$). The spectral form in a high frequency region ($f \gtrsim 2f_m$) is almost the same to that of pure wind waves. At short fetch $x \lesssim 8.6$ m ($k_o x \lesssim 139$), the energies of higher harmonics are much higher than those in the high frequency region of wind wave spectra. However, at larger fetch $x \gtrsim 8.6$ m ($k_o x \gtrsim 139$), the energies of the higher harmonics diffuse into the wind wave spectrum and spikes due to higher harmonics disappear. This does not mean that wind wave spectra covered them completely, because there are no great changes in the energy level of satulated wind wave spectra. Such a change is attributed to randomization of the regular waves. As shown in the left side of Figs. 8(b) and 8(c), the steep unstable waves, $(ak)_0=0.257$ and $(ak)_0=0.331$, do not grow much with fetch even under wind action, but random modulation of their amplitude increases appreciably. The corresponding energy spectra, right hand side of Figs. 8(b) and 8(c) show that spectral energy near the lower side band increases rapidly with fetch, but the change of the spectral energy near the higher side band is not appreciable, which is in contrast to the waves without wind action. In the case of the steepest wave, $(ak)_0=0.331$, spectral peaks of the lower and higher side bands are not clear. This is probably due to the fact that fundamental frequency components and its lower and higher side bands have relatively wide frequency ranges due to random fluctuations of the dominant waves, and spectral energies have diffused into a continuous spectrum.

More detailed analysis of the spectral data showed that the frequencies of the lower and higher side bands shift a little toward the fundamental frequency of the wave as compared with those of the waves without wind action, though the reason is not clear.

If we look at the main point of the phenomenon, the evolution of the wave spectra of monochromatic waves under wind action shows common features irrespective of their initial wave steepness; (i) the growth of fundamental frequency component accompanied by the generation of wind waves at a higi frequency region, (ii) the generation of side bands due to the instability of the fundamental wave, (iii) broadning of the spectral lines of the fundamental and side bands frequency components, and (iv) the transformation from the discrete line spectrum to the continuous spectrum. The difference in the phenomena due to the different initial steepness of the waves is reflected in a difference of

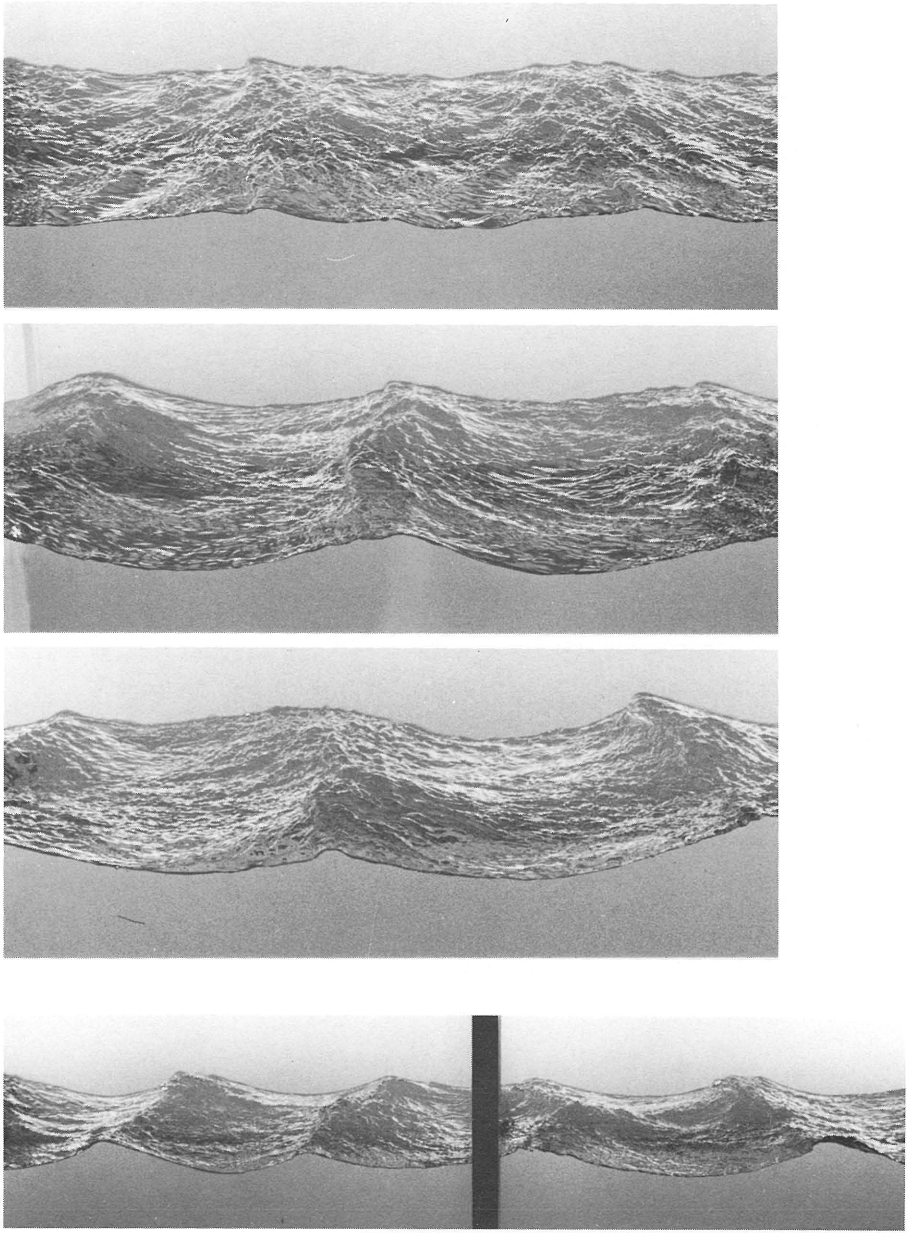

Fig. 6(a) Surface profile $\eta(x)$ under wind action for $(ak)_0=0.139$. Other conditions are the same as Fig. 1(a).

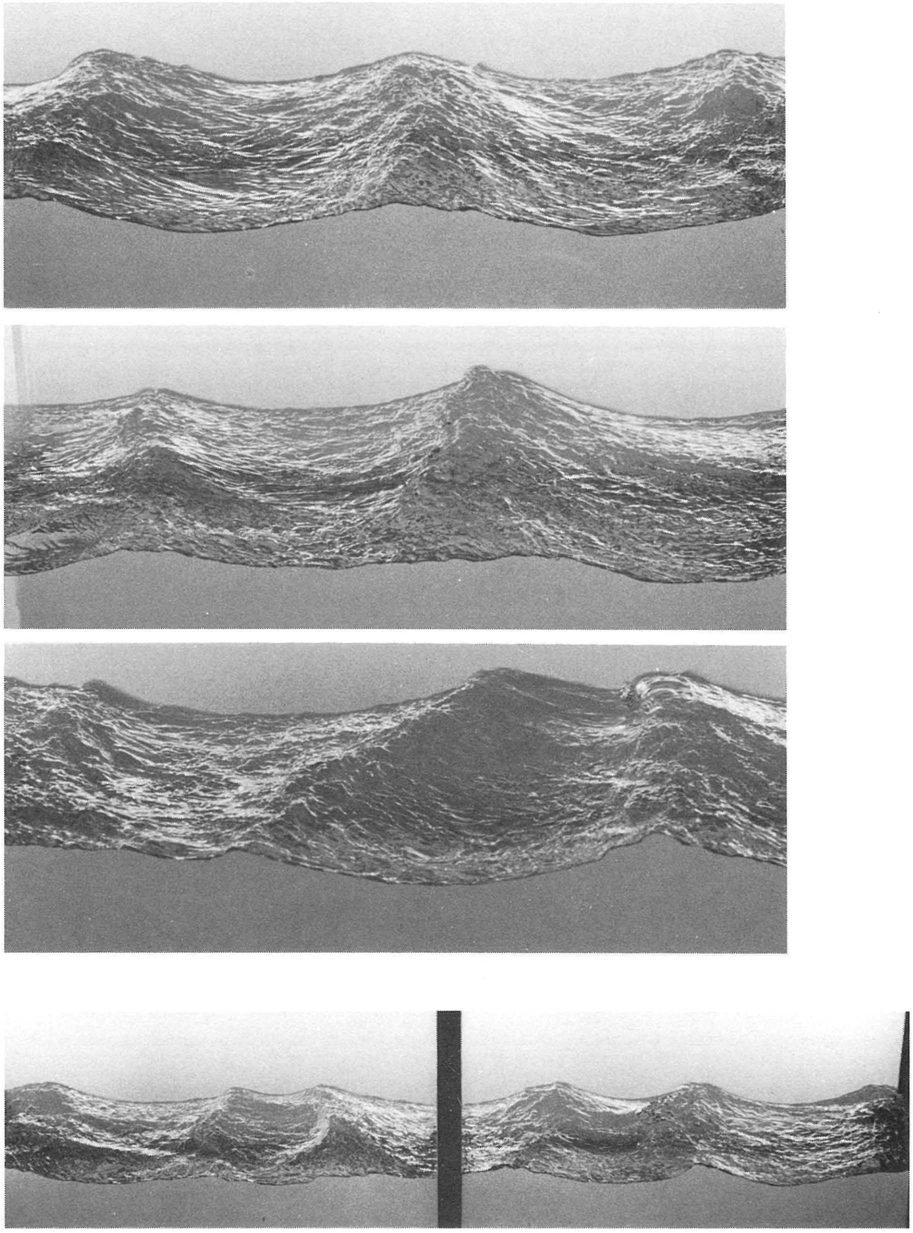

Fig. 6(b) The same as Fig. 6(a) except for $(ak)_o = 0.257$.

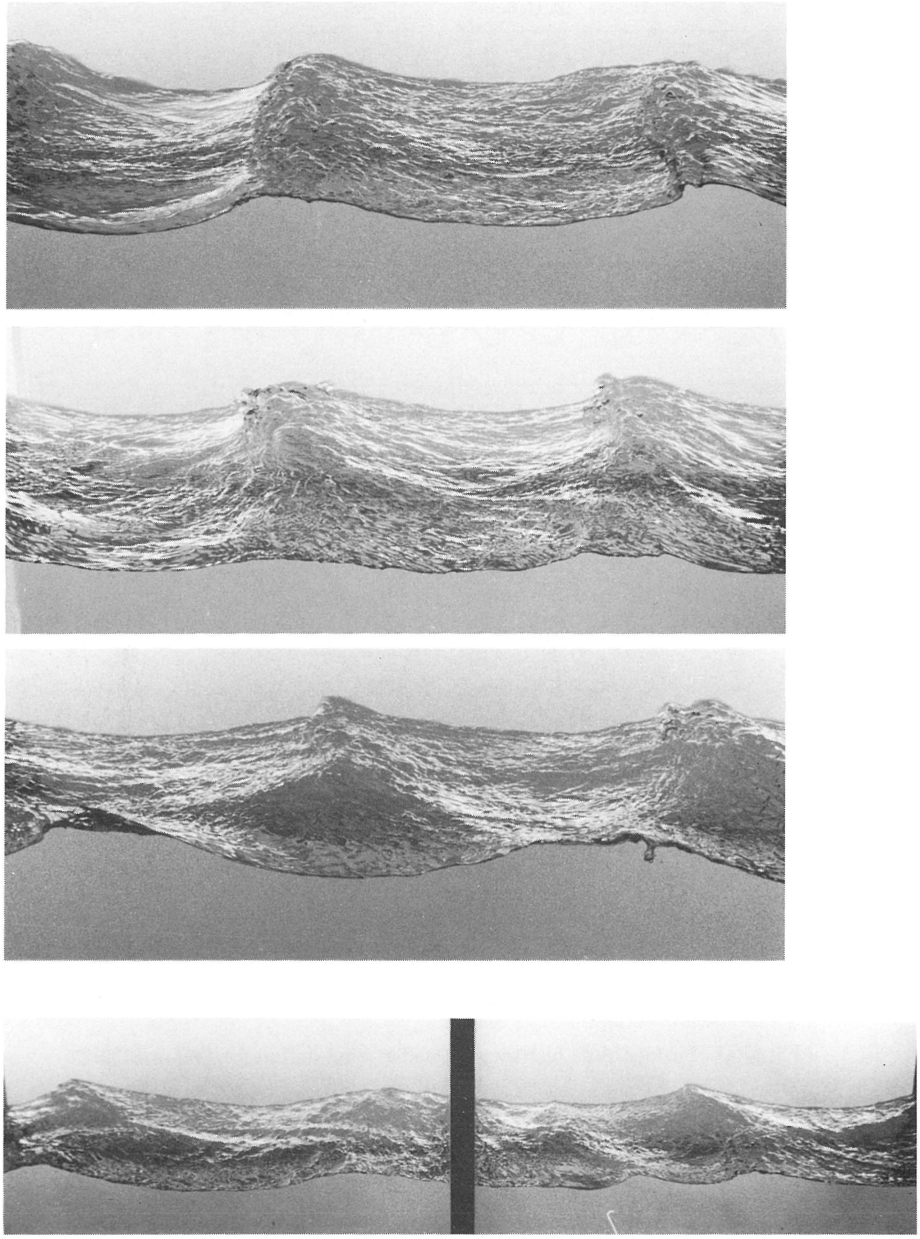

Fig. 6(c) The same as Fig. 6(a) except for $(ak)_o = 0.331$.

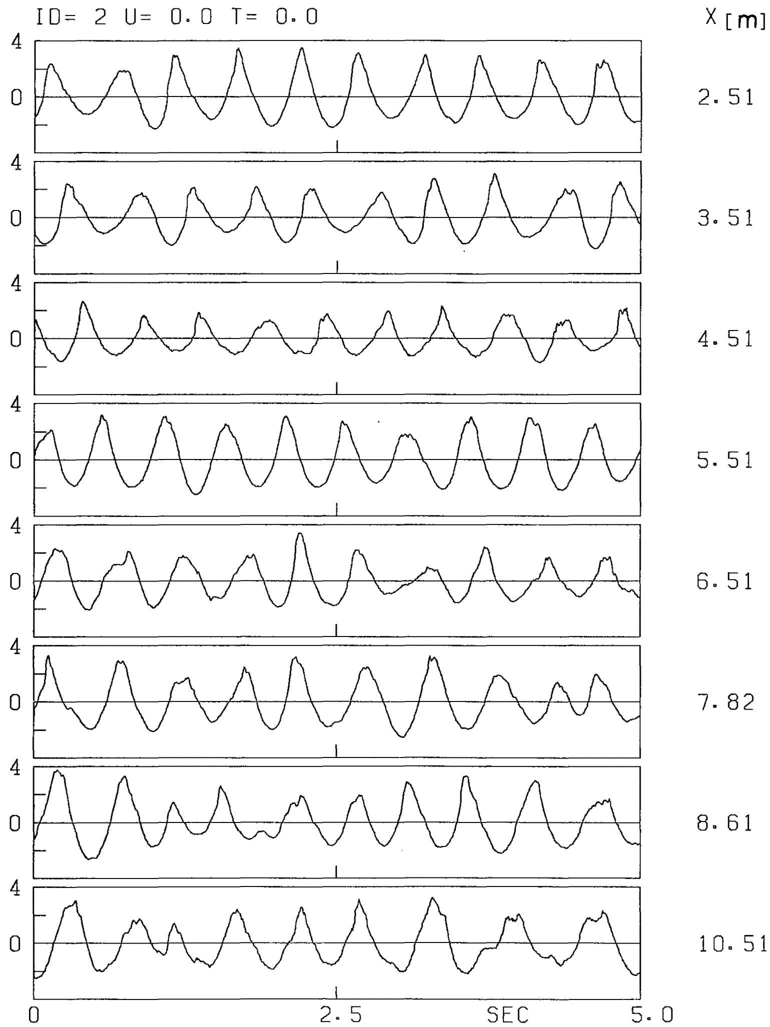

Fig. 7(a) Surface time history $\eta(t)$ under wind action for $(ak)_0=0.139$.

fetches where such a transformation of the spectrum has been accomplished. The peak frequency of the continuous spectrum evolved from the monochromatic wave spectrum lies near the lower side band of the fundamental frequency component of the monochromatic wave, and spectral energy of such a continuous spectrum is always higher than that of pure wind waves at corresponding fetches. For futher fetches, the waves will develope in the same way as pure wind waves. Such phenomena provide a useful experimental technique for the simulation of ocean waves in a experimental tank. Because,

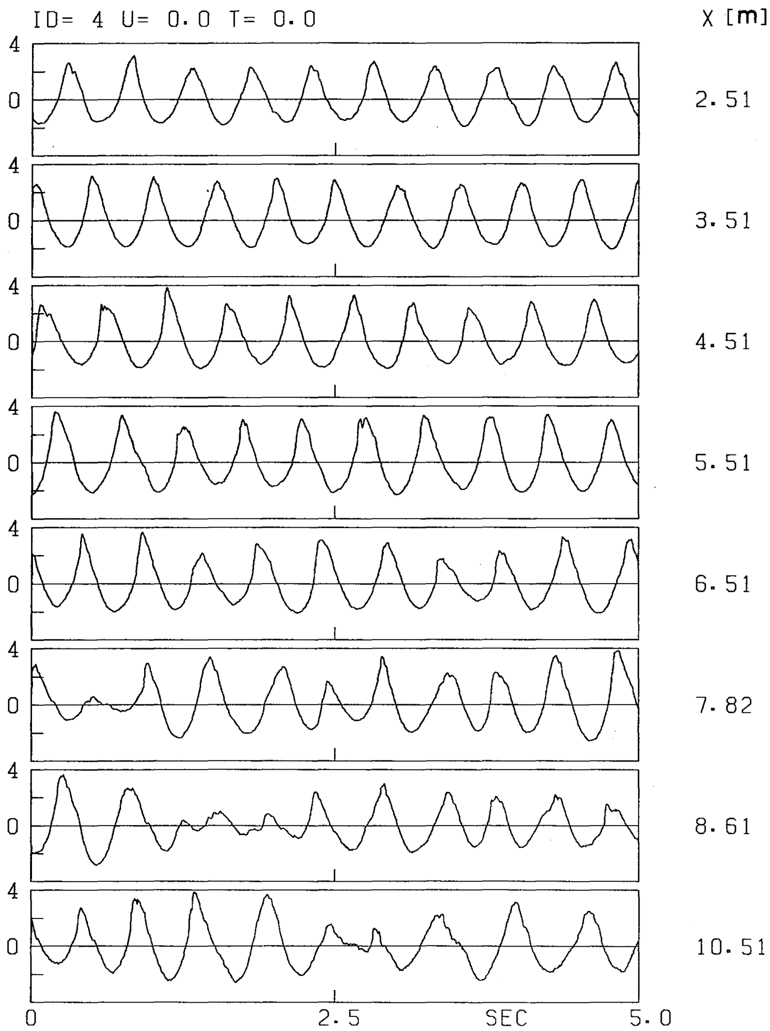

Fig. 7(b) The same as Fig. 7(a) except for $(ak)_0=0.225$.

we can generate, by adding wind over steep monochromatic waves, random waves which show almost the same properties with wind waves but have much larger scale than that of pure wind-generated waves limited by a fetch.

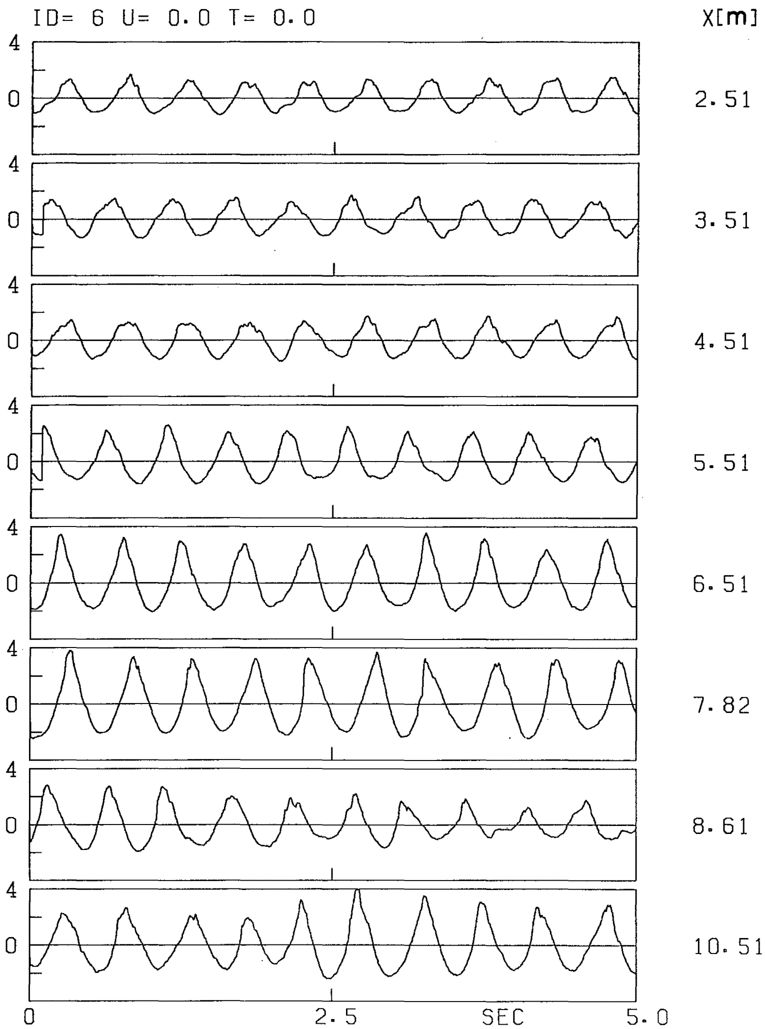

Fig. 7(c) The same as Fig. 7(a) except for $(ak)_o=0.331$.

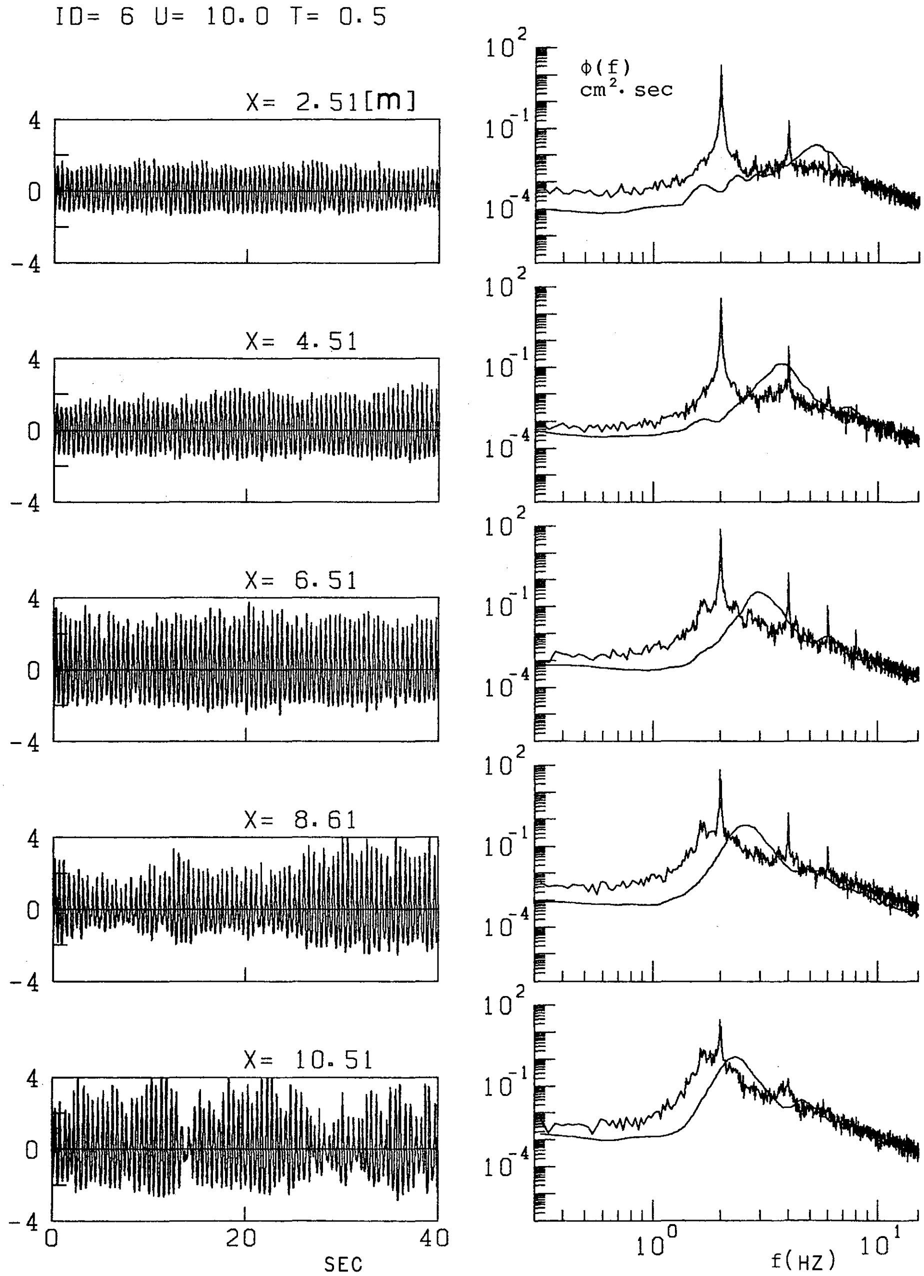

Fig. 8(a) Evolution of power spectra under wind action (right-hand column) and a part of corresponding wave records (left-hand column) for $(ak)_0=0.139$.

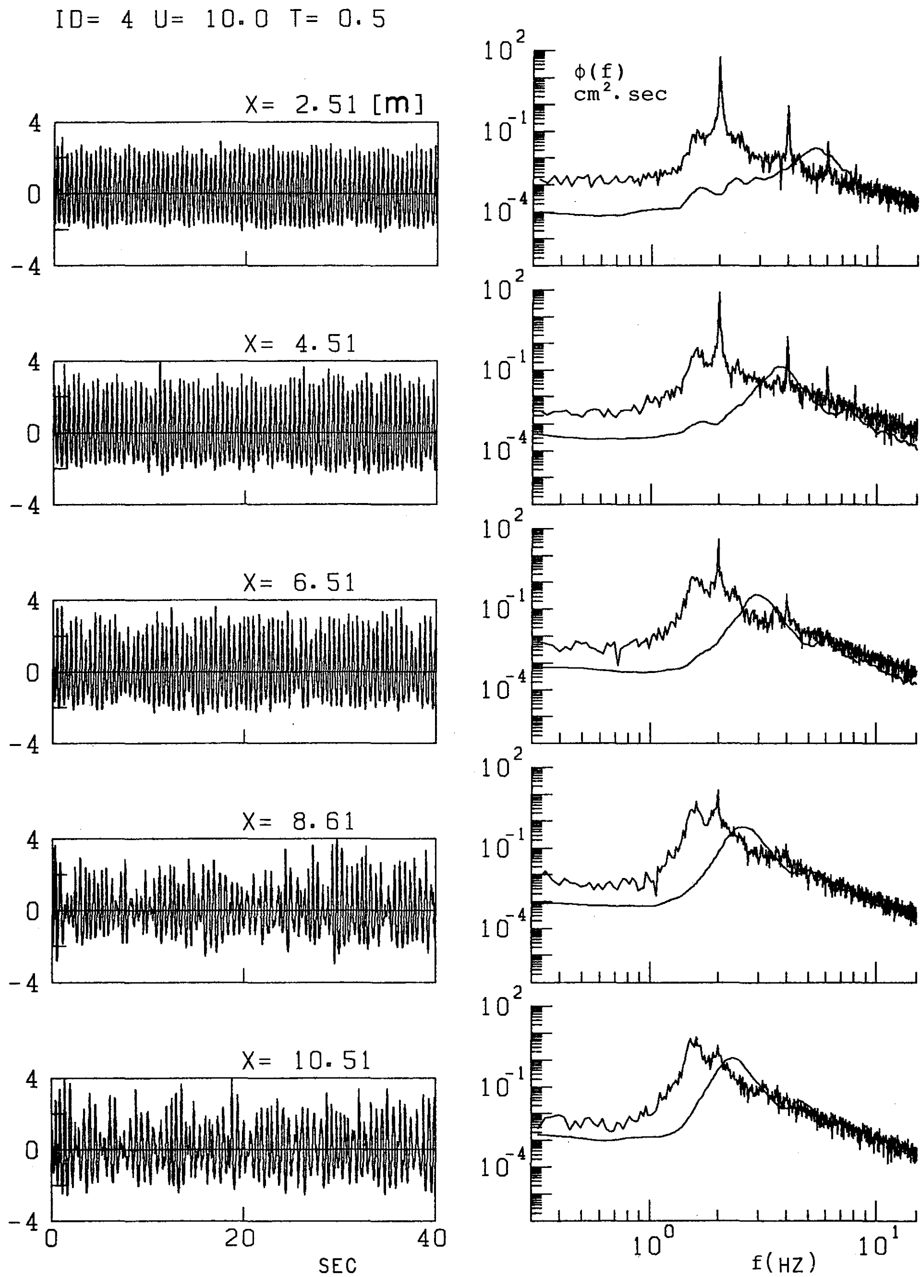

Fig. 8(b) The same as Fig. 8(a) except for $(ak)_o=0.257$.

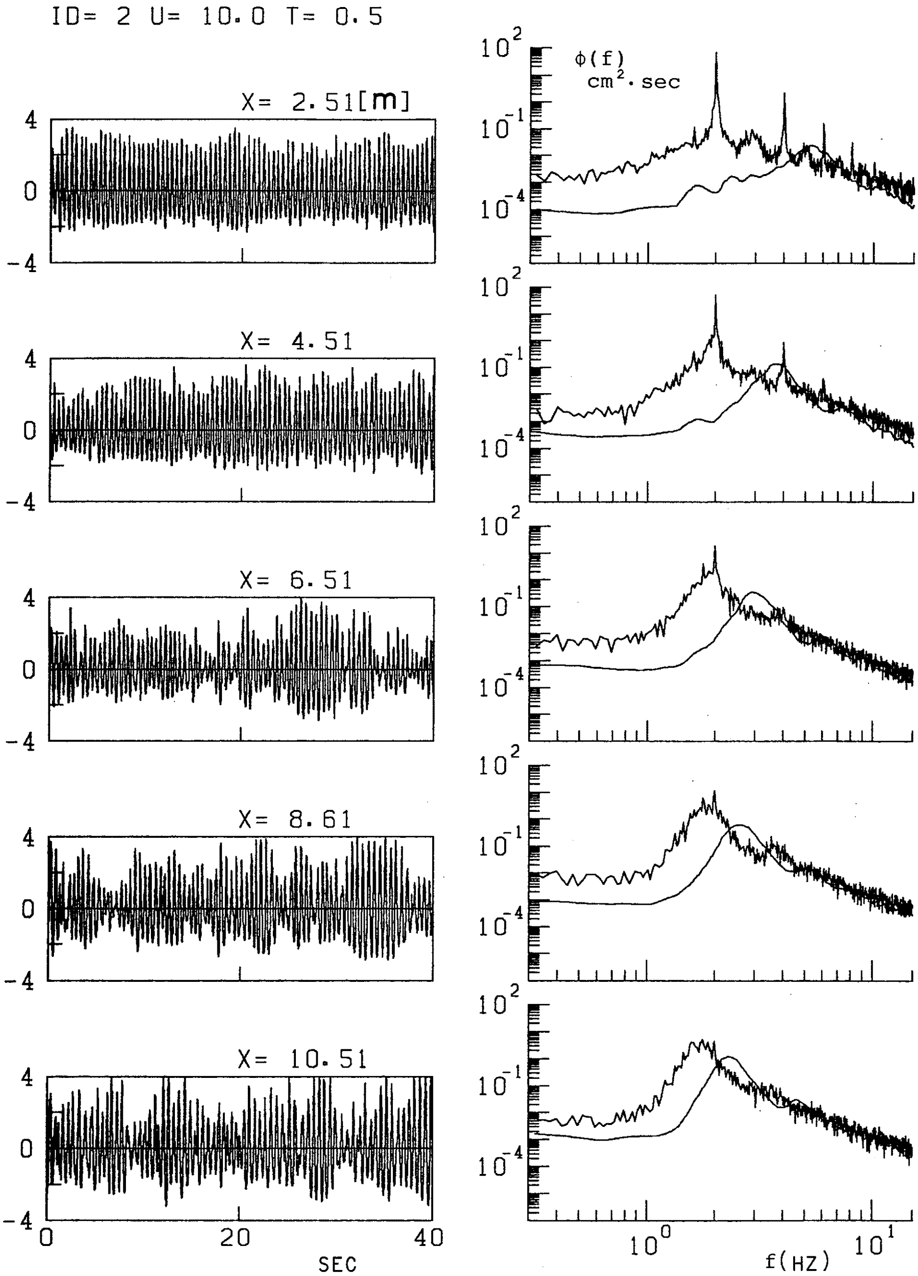

Fig. 8(c) The same as Fig. 8(a) except for $(ak)_o = 0.331$.

5.2. *Energy balance in the waves under wind action*

Figs. 9(a), 9(b) and 9(c) show the growth and decay of spectral components of the waves under wind action. For the stable wave with small steepness $(ak)_o=0.139$, E or E_3 increases exponentially in a range $k_o x \lesssim 105$. The measured growth rate is in good agreement with our previous result (Mitsuyasu & Honda 1982[4]). After reaching the maximum levels, E and E_3 begin to decrease gradually and E approaches to a satulated value E_s which satisfy the relation, $E_s f_m^3/g u_* \approx 2.5\times 10^{-4}$. Other energies, E_1, E_2, E_4 and E_5 also increase with fetch, but they are much smaller than E or E_3 within a fetch of our present experiment.

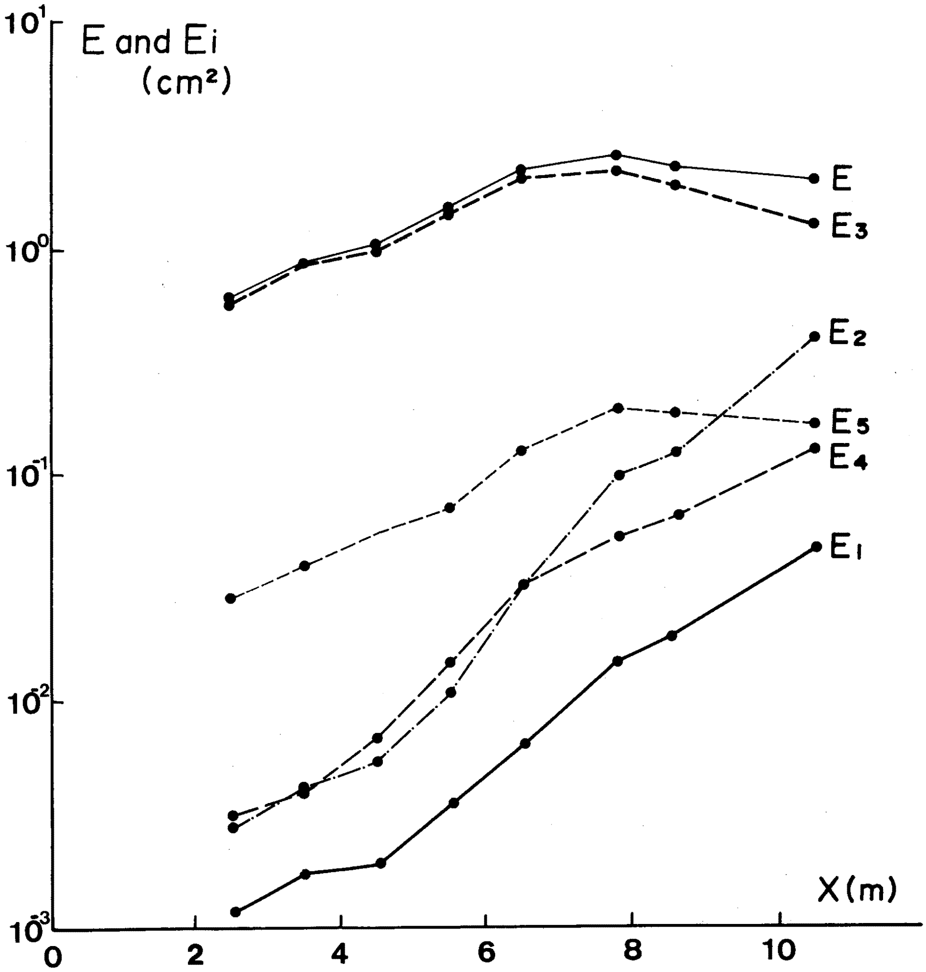

Fig. 9(a) The change of wave energy under wind action in the characteristic frequency regions for $(ak)_o=0.139$.

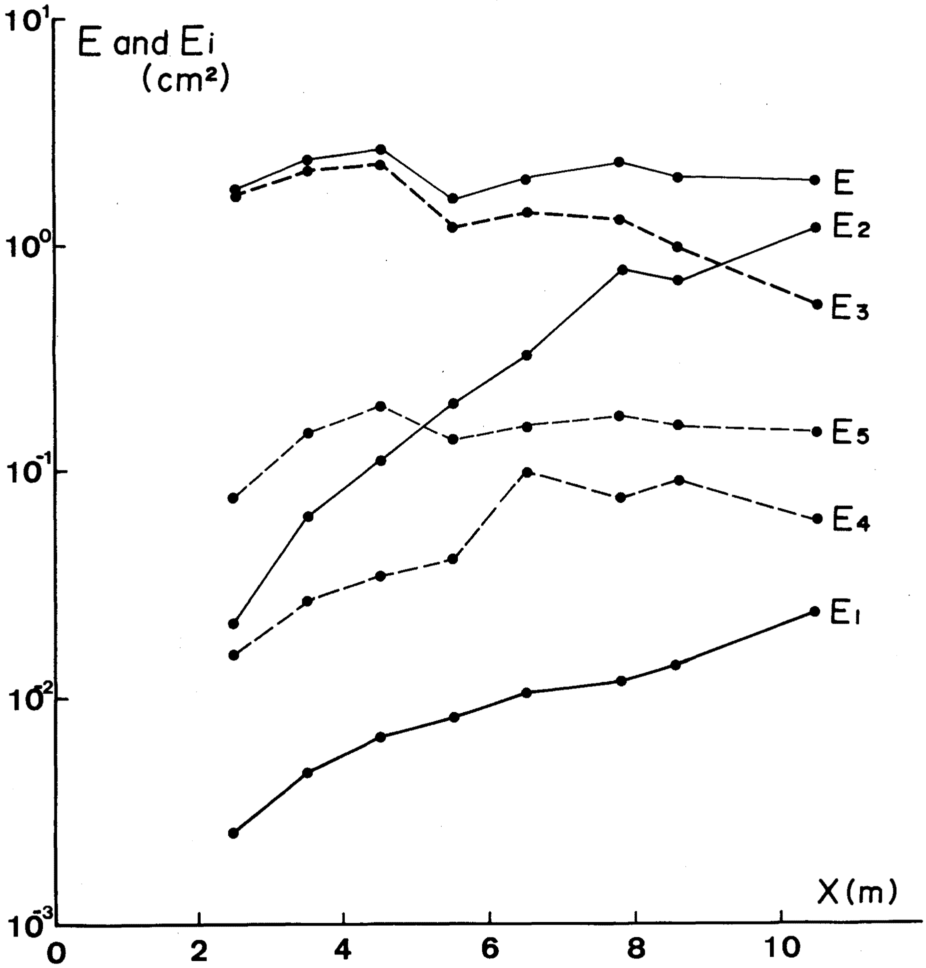

Fig. 9(b) The same as Fig. 9(a) except for $(ak)_0=0.257$.

The total wave energy E for the wave with steepness $(ak)_0=0.257$ shows a satulation at an initial stage and it does not change much with a fetch (Fig. 9(c)). The saturated value of E satisfy again $Ef_m^3/gu_*=2.5\times10^{-4}$. The energy of the lower side band, E_2 increases very rapidly and it becomes larger than the energy of the fundamental frequency component E_3, while the energy of the higher side band, E_4 increases roughly in the same rate with that for waves without wind and attains to a saturated level which is an order of magnitude smaller than E.

The energy for the wave with large steepness $(ak)_0=0.331$ shows properties similar to those of the wave with a moderate steepness $(ak)_0=0.257$, except that the energies of the lower and higher side bands, E_2 and E_4, are of equal

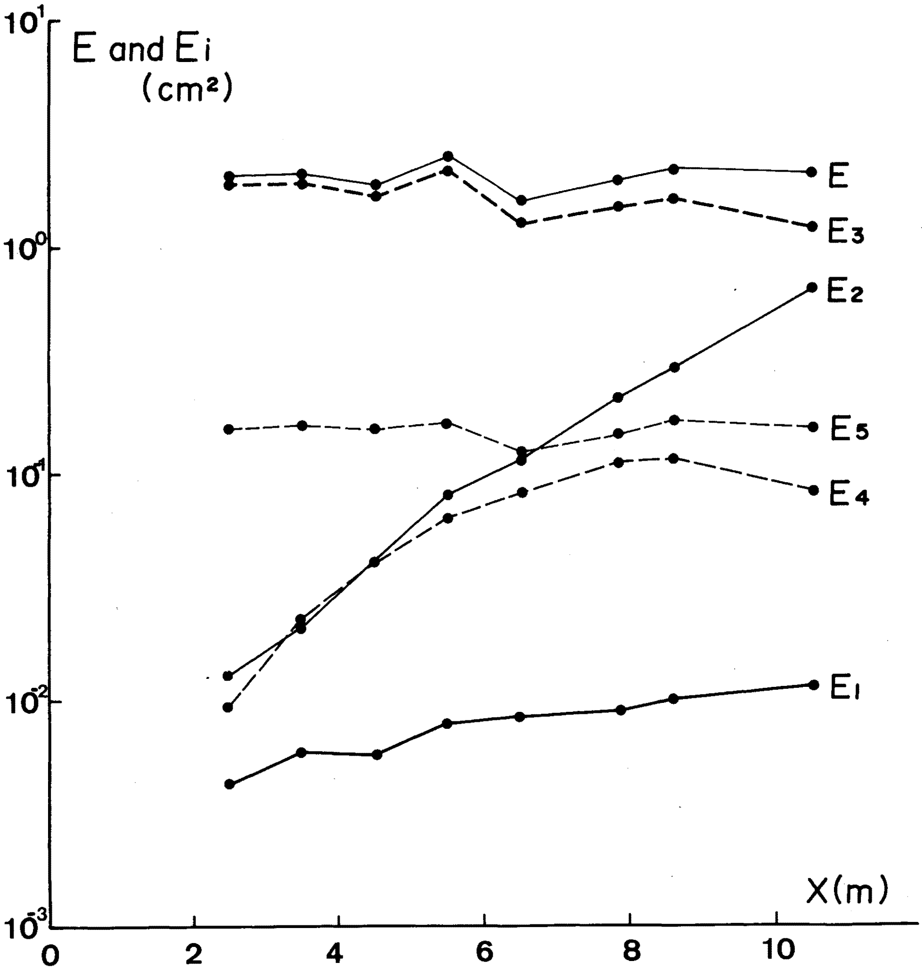

Fig. 9(c) The same as Fig. 9(a) except for $(ak)_o=0.331$.

magnitude and grow at almost the same exponential growth rate for $x \lesssim 4.5$ m($k_o x \lesssim 73$). The energy of the higher side band E_4 takes a satulated value at $x \approx 8.61$ m($k_o x \approx 139$), but that of the lower side band E_2 increases with fetch and becomes comparable to E_3. For unstable steep waves $(ak)_o=0.257$ and 0.331, wave energy at the low frequency region E_1 shows a gradual increase with fetch, but its level is negligibly small as compared to E. The wave energy at the high frequency region, E_5 shows an constant value independent of fetch reflecting a saturation of wave spectrum at the high frequency region.

It is interesting that the total wave energy E shows roughly constant values within a fetch of the present experiment, while the energy of the fundamental

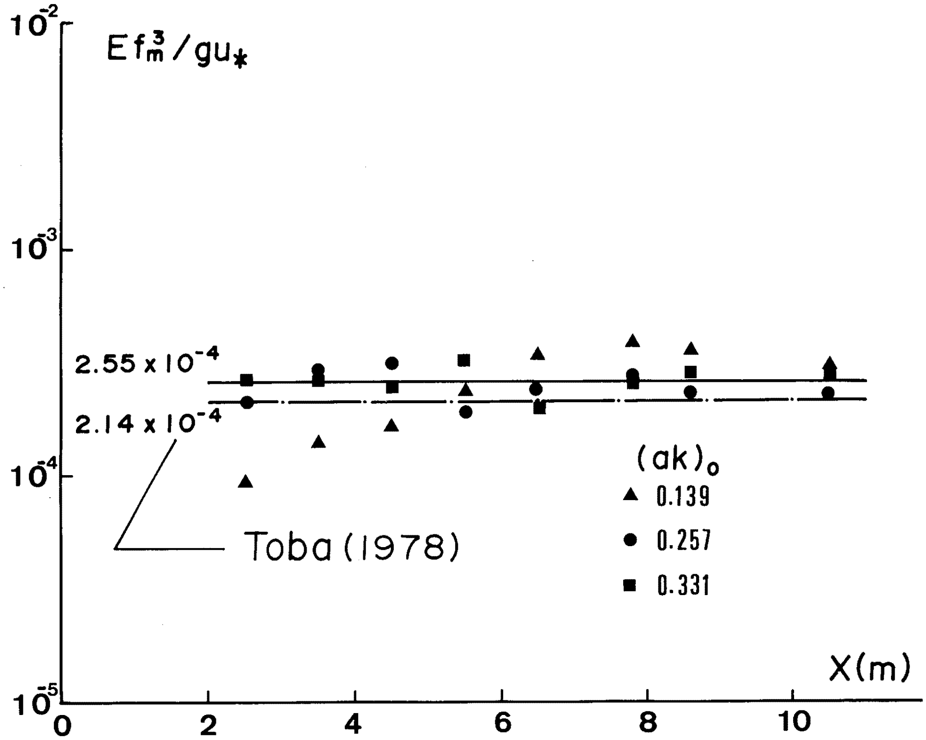

Fig. 10 The change of the normalized wave energy, Ef_m^3/gu_*, under wind action.

frequency component decreases gradually with fetch. This is largely due to the growth of wave energy of the lower side band conpensating the attenuation of the fundamental frequency component. Of course, energy supply from the wind to various frequency components of the wave is very important, because we could not find such a property in the absence of wind action.

It should be noted here that the present study is concerned with transitional phenomena from monochromatic waves to random waves which are caused by the wave instability and the wind action. After the transition to the random waves, the waves are considered to develop in the same way as wind waves. Thus, the total energy of the wave, E increases with fetch, while the dimensionless wave energy Ef_m^3/gu_* is considered to be still constant.

The present value $Ef_m^3/gu_*=2.5\times10^{-4}$, is very close to the value 2.1×10^{-4} presented by Toba (1978)[6] as the three-seconds power low for the growing wind waves. The dependece of the dimensionless wave energy Ef_m^3/gu_* on a fetch is shown in Fig. 10. For the wave with a small steepness, $(ak)_0=0.139$, Ef_m^3/gu_* takes a small value at short fetches, but it increases gradually with fetch and approaches to the satulated value as fetch increases, while the steep unstable waves, $(ak)_0=0.257$ and $(ak)_0=0.331$ show the constant value of Ef_m^3/gu_* from an initial stage of the wave developments.

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