COMPUTER SIMULATION OF PARTICLE WAVES

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NOTE

COMPUTER SIMULATION OF PARTICLE WAVES

By Arata KANEKO* and Hiroyuki HONJI**

The formation process of particle waves in oscillatory flow has been simulated numerically by assuming fluid forces acting between two-nearby particles. It has been found that the stream-wise force attrac tive at short distances around a particle must be distantly repulsive for the regular waves to form.

Key words: Oscillatory flow, Sand particles, Rythmic pattern, Numerical experiment

1. Introduction

It has been observed that sand particles scattered sparsely on a plane floor grow into particle waves under oscillatory flow, and suggested that the initiation of oscillation ripples on a sand bed is due to these waves[1]. In this paper, the inter-particle fluid forces which have a key role for the formation of particle waves are investigated by means of numerical experiments.

2. Simulation model

Two circular cylinders or spheres placed closely in unidirectional flow experience the viscous forces as indicated by solid arrows in Fig. 1[2)3]. The pressure forces acting in unidirectional inviscid flow have the directions opposite to those of the viscous forces as indicated by dotted arrows in the same figure[4]. An oscillatory flow may be regarded as consisting of starting flows which have more or less the characteristics of inviscid flows. Therefore, one has to consider that the both types of fluid forces, viscous and inviscid, may be concerned with the particle wave formation under oscillatory flow.

A system composed of N-particles with unit mass scattered at ran-

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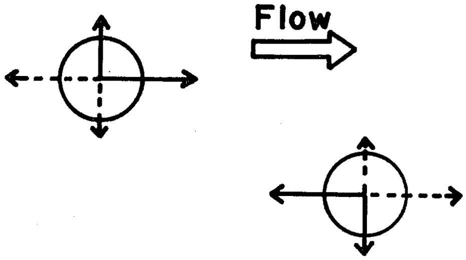

Fig. 1 Interactive fluid forces acting on two spheres. Solid arrows indicate viscous forces and dotted arrows pressure forces.

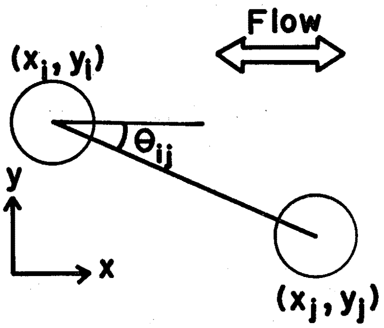

Fig. 2 Sketch of coordinate system.

dom in a plane is examined in the computer simulation. Let the positions of i'th and j'th particles denote (x_i, y_i) and (x_j, y_j), respectively. The coordinate system is illustrated in Fig. 2. The quantities (u_i, v_i) are the (x, y) components of the velocity of the i'th particle, and $(F_{x_{ij}}, F_{y_{ij}})$ are those of the interactive fluid forces between the i'th and j'th particles. The fluid forces induced in an oscillatory flow may be divided into the component independent of time and the component periodic with respect to time[5]. Attention is here focused on the steady component responsible for the relative displacement of the particles. We assume that the particle system obeys the following time evolution equations

$$
\begin{aligned}
\frac{dx_i}{dt} &= u_i , \\
\frac{dy_i}{dt} &= v_i , \\
\frac{du_i}{dt} &= \sum_{\substack{j=1 \\ j\neq i}}^{N} F_{x_{ij}} \cos^2 \theta_{ij} \frac{x_j - x_i}{|x_i - x_j|} - f u_i , \\
\frac{dv_i}{dt} &= \sum_{\substack{j=1 \\ j\neq i}}^{N} F_{y_{ij}} \sin^2 \theta_{ij} \frac{y_j - y_i}{|y_i - y_j|} - f v_i ,
\end{aligned}
\qquad (1)
$$

for $i=1, 2, \cdots\cdots, N$, where

$$\cos\theta_{ij} = \frac{|x_i - x_j|}{\sqrt{(x_i-x_j)^2 + (y_i-y_j)^2}},$$

$$\sin\theta_{ij} = \frac{|y_i - y_j|}{\sqrt{(x_i-x_j)^2 + (y_i-y_j)^2}}.$$

In the right-hand sides of eq. 1, $\cos^2\theta_{ij}$ and $\sin^2\theta_{ij}$ give the θ_{ij} dependence of the interactive forces, and $(x_j-x_i)/|x_i-x_j|$ and $(y_j-y_i)/|y_i-y_j|$ give their directions. A constant f is a frictional coefficient and the velocity of particles slows down appropriately by introducing the frictional force necessary for the computational stability. In the present computation f is taken as 0.2. For $F_{x_{ij}}$ and $F_{y_{ij}}$, we use the following simple functions

$$\begin{aligned} F_{x_{ij}} &= A + B\cos\frac{\pi}{10}|x_i-x_j| && \text{for } |x_i-x_j| \leqslant X_m, \\ F_{x_{ij}} &= 0 && \text{for } |x_i-x_j| > X_m, \\ F_{y_{ij}} &= C + D\cos\frac{5}{6}\pi|y_i-y_j| && \text{for } |y_i-y_j| \leqslant Y_m, \\ F_{y_{ij}} &= 0 && \text{for } |y_i-y_j| > Y_m, \end{aligned} \tag{2}$$

Where A, B, C and D are adjustable coefficients. X_m and Y_m are the maximum distances beyond which $F_{x_{ij}}$ and $F_{y_{ij}}$ do not work. The kinematic energy (E) of the particle system may be defined as

$$E = \frac{1}{2}\sum_{i=1}^{N}(u_i^2+v_i^2). \tag{3}$$

Equation 1 is composed of the $4N$ first-order ordinary differential equations. Let us seek numerically a solution of these equations by means of the Runge-Kutta method with the fourth-order accuracy. Two kinds of interactive forces are examined in the numerical experiments. The values of the coefficients, X_m, and Y_m for each case are presented in Table 1. The profiles of $F_{x_{ij}}$ and $F_{y_{ij}}$ for Cases 1 and 2 are illustrated in Fig. 3, where the forces are attractive when $F_{x_{ij}} > 0$ and $F_{y_{ij}} > 0$, and repulsive when $F_{x_{ij}} < 0$ and $F_{y_{ij}} < 0$. The force in Case 1 is repulsive around $|x_i - x_j| = 10$ distant from the origin. Case 2 gives an

Table 1 Numerical data for interactive forces

	A	B	C	D	X_m	Y_m
Case 1	0.08	0.12	0.1	0.1	12.7	1.2
Case 2	0.1	0.1	0.1	0.1	10.0	1.2

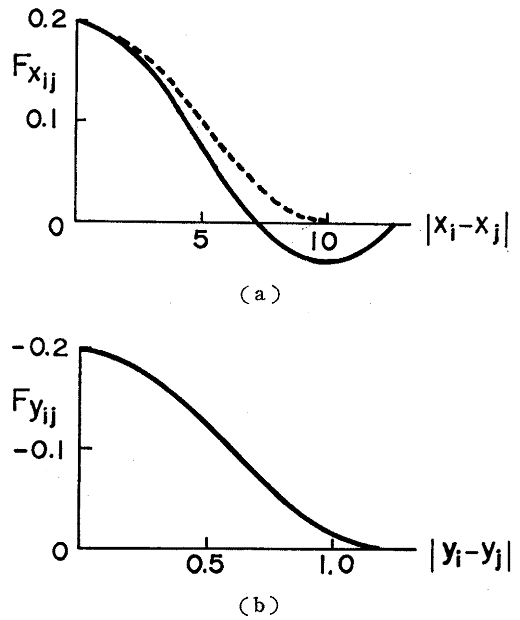

Fig. 3 Profiles of $F_{x_{ij}}$ and $F_{y_{ij}}$: (a) $F_{x_{ij}}$. A solid line indicates a profile for Case 1 and a dotted line that for Case 2, (b) $F_{y_{ij}}$. The same profile is used in Cases 1 and 2.

inter-particle force similar to the one acting in unidirectional flow; the force is attractive over the whole range of $|x_i-x_j|$.

3. Results and discussions

Figure 4 shows the development of particle waves computed for Case 1, in which $F_{x_{ij}}$ is attractive at $0 \leqslant |x_i-x_j| \leqslant 7.3$ and repulsive at $7.3 < |x_i-x_j| \leqslant 12.7$ $(=X_m)$, and $F_{y_{ij}}$ is repulsive at $0 \leqslant |y_i-y_j| \leqslant 1.2 (=Y_m)$. Initially 70 particles were scattered forming a row parallel to the flow direction, as shown in Fig. 4a. As time went on, the particles began to assemble with an approximately constant interval (Fig. 4b) and subsequently grew into the crest of regular particle waves (Fig. 4c). This formation process resembles remarkably that observed in an oscillatory-water tunnel (Fig. 4 in Kaneko and Honji, 1979)[1]. The wavelength of the waves in an equilibrium state was $1.1\ X_m$. The time variation of E during the wave pattern development is shown in Fig. 5. The particle system may be considered to reach an equilibrium state at $t=30$.

Figure 6 shows the final pattern of particles computed for Case 2, in which $F_{x_{ij}}$ is always attractive. Although a clear crest pattern formed, the intervals between two neighboring crests were not constant.

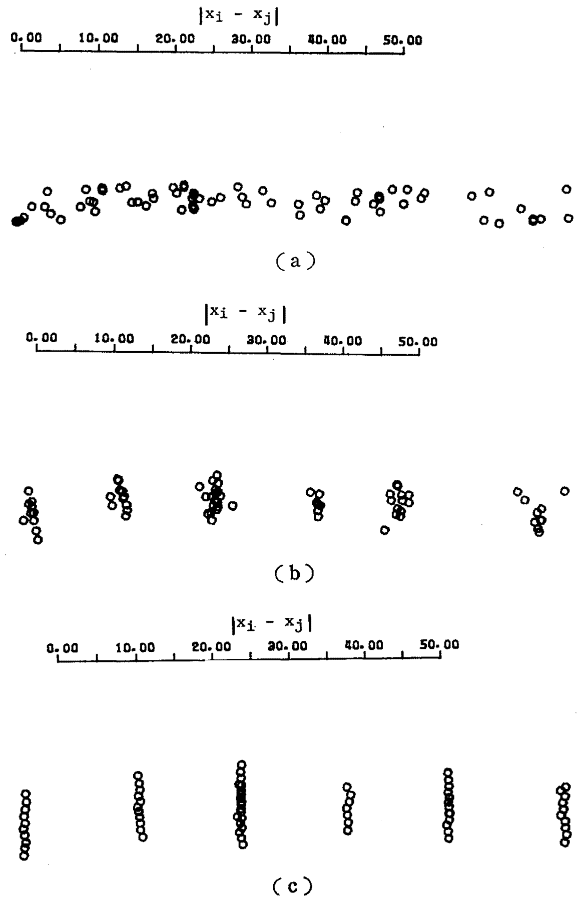

Fig. 4 Development of particle waves for Case 1. N=70 and S=375: (a) t=0 (b) t=10 (c) t=30.

This indicates that for the regular particle waves to form the force only attractive over the whole spatial range in the flow direction is not sufficient, but it must be at least distantly repulsive.

Figure 7 shows the development of particle waves computed under the same conditions as those for Fig. 4 except the number N of scattered particles and the area S over which they are initially distributed. As the result of the variation of N and S, the mean initial interval of particles defined as

$$d=\sqrt{\frac{S}{N}} \tag{4}$$

increased from 2.3 for Fig. 4 to 4.0 for Fig. 7. Nevertheless, the par-

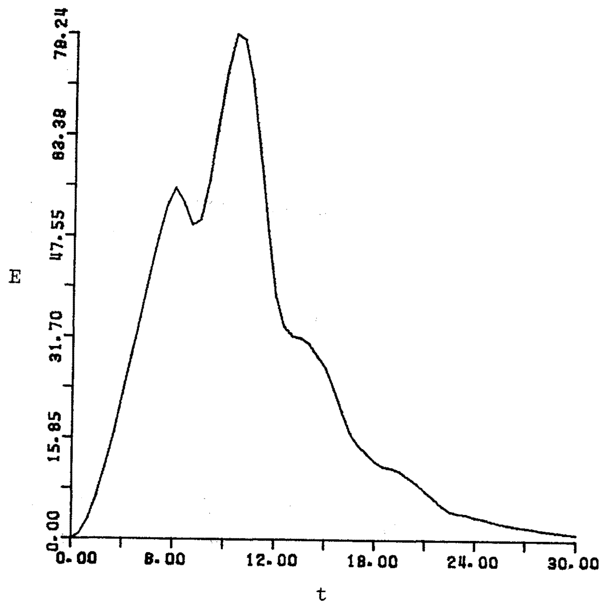

Fig. 5 Time variation of kinematic energy.

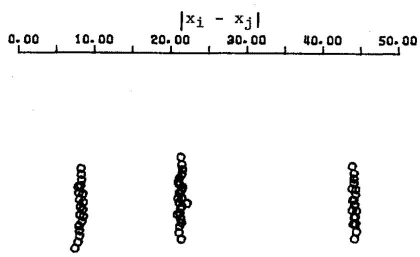

Fig. 6 Wave pattern formed at $t=30$ for Case 2. $N=70$ and $S=375$.

ticle waves formed similarly and the equilibrium wavelength was a the same as that in Fig. 4. This indicates that when $d \ll X_m$ the wavelength of particle waves is independent of d and roughly equal to X_m.

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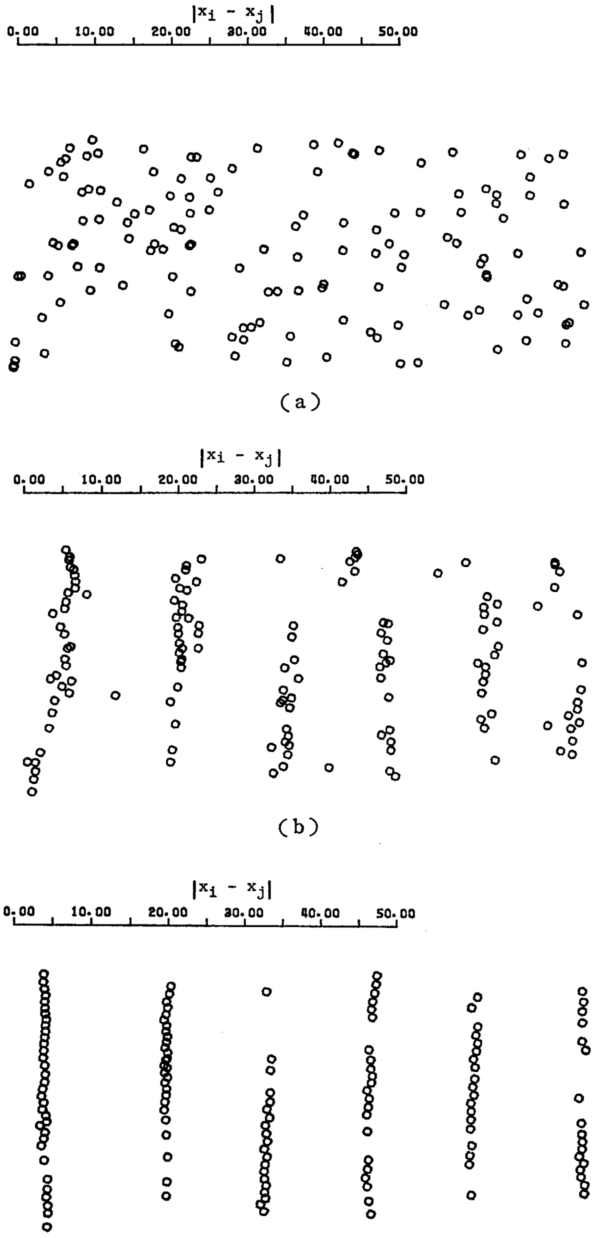

(c)

Fig. 7 Development of particle waves when particles are initially distributed over a wide area. $N=140$ and $S=2250$: (a) $t=0$ (b) $t=10$ (c) $t=30$

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