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ON THE TIME EVOLUTION OF A SOLITARY WAVE REFLECTED BY AN OBLIQUE WALL

By Mitsuaki FUNAKOSHI*

Initial-value problem of two-dimensional reflection of solitary waves in shallow water is studied with a finite-difference method. The development of reflected waves is examined in detail. If $\alpha < \sqrt{3\varepsilon}$ (α=the angle of incidence, $\varepsilon = a/h$, a=wave amplitude, h=undisturbed depth), reflected waves tend to those of the Mach reflection solution predicted by Miles. Calculated maximum run-up at the wall is considerably larger than that in Melville's experiment. The reason for this discrepancy is discussed in consideration of the effect of viscosity.

Key words : Mach reflection, Shallow water, Solitary wave

1. Introduction

We are concerned with the problem of oblique reflection of solitary waves at the wall in shallow water.

Let a be the amplitude of an incident solitary wave, h be an undisturbed depth, and $\varepsilon = a/h$, and α be the angle of incidence (see Fig. 1). Theoretical treatment of the reflection must be changed according to the ratio of α to $\sqrt{\varepsilon}$. The case of $\alpha \gg \sqrt{\varepsilon}$ is called as non-grazing reflection and the case of $\alpha = O(\sqrt{\varepsilon})$ as grazing reflection in this paper.

Non-grazing reflection corresponds to the "weak interaction" of two obliquely oriented solitary waves (The terms "weak interaction" and "strong interaction" were defined by Miles[1] for the interaction of two obliquely oriented solitary waves). The theoretical solution for this case describes a regular reflection pattern which is composed of an incident wave I and a reflected wave R (see Fig. 2 (a)).

Grazing reflection corresponds to the "strong interaction" of obliquely oriented solitary waves. Miles[2] obtained an explicit solution for this case which also describes a regular reflection pattern (see Fig. 2 (b)). He found that this solution describes singular behaviour when

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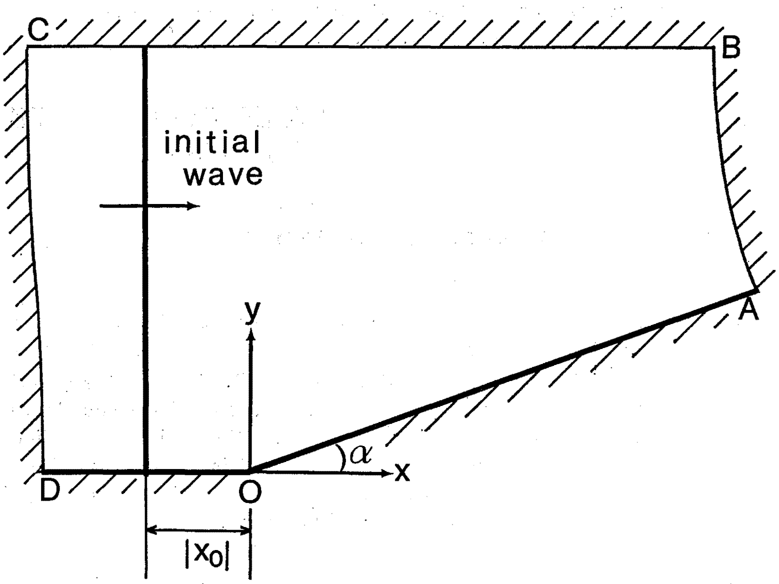

Fig. 1. Schematic diagram showing the geometry of calculations.

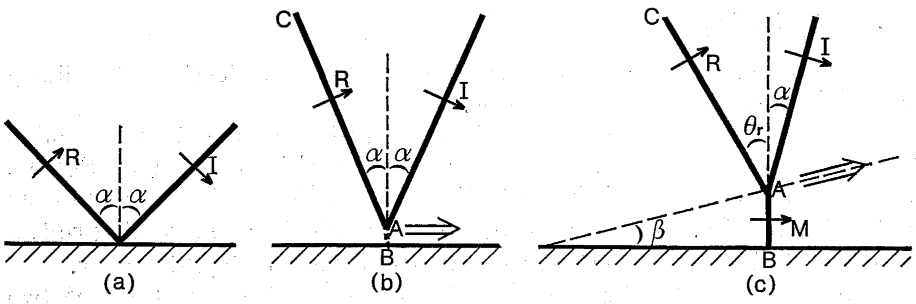

Fig. 2. Schematic representation of reflection patterns. (a), Regular reflection (non-grazing reflection): (b), regular reflection (grazing reflection): (c), Mach reflection.

$\alpha<\sqrt{3\varepsilon}$. For this case, he predicted that the singular solution does not appear really but that a resonantly interacting solution appears as an asymptotic solution for the initial-value problem of reflection. The resonantly interacting solution describes three solitary waves I, R and M (see Fig. 2 (c)) and is called a Mach reflection solution (and M is called a Mach stem). The intersection of these waves, A in Fig. 2 (c), moves away from the wall along a straight line inclined to the wall by the angle $\beta=\sqrt{\varepsilon/3}\,(1-\alpha/\sqrt{3\varepsilon})$ contrary to the case of regular reflection (Figs. 2(a) and (b)). This solution gives the angle of reflection $\theta_r=\sqrt{3\varepsilon}$

which is larger than α. The amplitude of the reflected wave of this solution is $a\zeta_r = a\alpha^2/3\varepsilon$ which decreases to zero as α decreases. The most interesting behaviour of the solution which Miles predicted for grazing reflection is that the maximum run-up at the wall $a\zeta_{max}$ increases to $4a$ (twice of the result in linearized theory) as α approaches to $\sqrt{3\varepsilon}$.

The Mach reflection solution predicted by Miles, however, satisfies the boundary condition of zero normal velocity at the wall only asymptotically. Furthermore, it is not clear whether a Mach stem is obtained as the solution of the initial-value problem and has the Boussinesq solitary wave profile.

Funakoshi[3] solved the initial-value problem of oblique reflection of solitary waves with the numerical method mentioned in § 2 and § 3. The results are as follows: (i) Mach reflection pattern appears for small α and regular reflection pattern does for large α, and the critical angle is consistent with the Miles' prediction $\alpha = \sqrt{3\varepsilon}$ for $\varepsilon \leqq 0.2$. (ii) Maximum run-up at the wall $a\zeta_{max}$ considerably larger than $2a$ is obtained when α is close to $\sqrt{3\varepsilon}$. The results for ζ_{max} agree well with the theoretical value $\zeta_{max} = (1+\alpha/\sqrt{3\varepsilon})^2$ for α considerably smaller than $\sqrt{3\varepsilon}$. (iii) The angle β in Fig. 2 (c), the angle of reflection θ_r and the amplitude of a reflected wave $a\zeta_r$ agree well with Miles' prediction. (iv) The wave profile of a Mach stem agrees well with the theoretical value. (v) It requires a long time for the reflection pattern to reach a stationary state for small α.

Perroud[4] made a laboratory experiment on the oblique reflection of solitary waves. He observed the Mach reflection pattern for small α. The obtained critical angle was about 45° independent of ε. Although some measured values of ζ_{max} seem to show the tendency to increase as α approaches to $\sqrt{3\varepsilon}$, the large values of ζ_{max} predicted by Miles were not observed.

Recently Melville[5] made a more accurate experiment on this problem with the apparatus of larger dimension. He obtained the critical angle which is close to $\sqrt{3\varepsilon}$ through a double extrapolation of the measured data. However, the measured ζ_{max} was much smaller than the theoretical value. The measured reflected wave was much narrower than a Boussinesq solitary wave. He tried to explain the discrepancy between the Miles' theory and the experiment by means of the conservation of mass, energy and momentum.

In this paper, the time evolution of reflected waves are examined numerically. The results about reflected waves and maximum run-up at the wall are compared with Miles' prediction or Melville's experimental results. Furthermore, the effect of viscosity is estimated in order to explain the reason for the discrepancy between the measurement and the numerical calculation.

2. Formulation

We deal with the problem of oblique reflection of solitary waves shown in Fig. 1.

We assume that the fluid is inviscid and incompressible and that the motion is irrotational. Let (lx, ly) and hz be horizontal and vertical co-ordinates, $lt/\sqrt{gh}$ the time, $a\zeta$ the free-surface displacement and $la\phi/\sqrt{g/h}$ the velocity potential, where l is a characteristic wavelength and g is the gravitational acceleration. Then the parameters relevant to the problem are $\varepsilon=a/h$ and $\delta=h^2/l^2$. As we are concerned with solitary waves, we assume that $\delta/\varepsilon=O(1)$ and $\varepsilon\ll 1$. Neglecting the terms of $O(\varepsilon^2)$, we get through the well known procedure,[1)6)7)]

$$\zeta_t+\Delta f+\varepsilon\nabla(\zeta\nabla f)-\frac{1}{6}\delta\Delta^2 f=0\ , \tag{1}$$

$$\left(f-\frac{1}{2}\delta\Delta f\right)_t+\zeta+\frac{1}{2}\varepsilon(\nabla f)^2=0\ , \tag{2}$$

where f is the non-dimensional velocity potential at the bottom, and ∇ and Δ are the gradient and Laplacian operators in a horizontal plane, and subscripts imply partial differentiation.

The boundary condition on the vertical wall is reduced to

$$f_{\mathrm{n}}=0\,,\quad f_{\mathrm{nnn}}=0\,,\quad \zeta_{\mathrm{n}}=0\,, \tag{3}$$

up to $O(\varepsilon)$, where subscript n implies the normal derivative to the vertical wall.

In one-dimensional case, i. e. $\partial/\partial y=0$, the following solution[8)] satisfies eqs. (1) and (2) with the error of $O(\varepsilon^2)$:

$$\zeta=\left(1-\frac{3}{4}\varepsilon\tanh^2\theta\right)\mathrm{sech}^2\theta\ , \tag{4a}$$

and

$$f=\frac{1}{k_0}\left[1-\frac{\varepsilon}{12}(5+4\,\mathrm{sech}^2\theta)\right]\tanh\theta\,, \tag{4b}$$

where

$$\theta=k_0(x-x_0-ct)\ ,$$

$$k_0=\sqrt{\frac{3\varepsilon}{4\delta}}\left(1-\frac{5}{8}\varepsilon\right)\quad\text{and}\quad c=1+\frac{1}{2}\varepsilon-\frac{3}{20}\varepsilon^2\,,$$

where x_0 is the initial position of the solitary wave. This solution is taken as the initial condition.

3. The Numerical Procedure

Evolution equations (1) and (2) for f and ζ are integrated with a finite-difference method under the boundary condition (3) and the initial condition (4).

In eqs. (1) and (2), spatial derivatives were approximated using centered differences, and the time evolution was calculated with the method which is similar to the leapfrog method. Calculations were made in a rectangular region which was transformed from the region shown in Fig. 1 by certain conformal mapping. The details of the numerical procedure are described in author's previous paper[3].

For most calculations, the position of an initial wave lx_0 was taken as $-5.6\sqrt{h^3/a}$. Since the variation of the ratio δ/ε changes only the scales of the normalization of the horizontal co-ordinates and of the time, the ratio can be chosen arbitrarily so long as it is O(1). The ratio was chosen as 0.75 or 0.2 in the present calculations.

4. Discussion of Results

Calculations were made for $\varepsilon=0.05$ in order to examine in detail the behaviours of reflected waves, troughs and maximum run-up at the wall. Furthermore, the calculations with $\varepsilon=0.1$, 0.15 were made for the comparison with Melville's experimental results.

4.1. The behaviour of reflected waves.

Figure 3 shows the examples of profiles of the surface elevation along the crest line of a reflected wave (and a Mach stem). At the first stage of interaction of an incident wave with an oblique wall, the profile such as the broken line is obtained. The amplitude of a reflected wave is not defined for this case. After the longer interaction time between an incident wave and an oblique wall, the profile such as the solid line appears. We define the surface elevation at the point P of locally minimum gradient as the amplitude of a reflected wave at that time and write it as $a\zeta_r^*(t)$.

The obtained values of $a\zeta_r^*(t)$ increase monotonically with the time and approach to certain values. We define this asymptotic value as the amplitude of a reflected wave in stationary state and write as $a\zeta_r$. For $\varepsilon=0.05$ ζ_r agrees well with the theoretical value as was shown in author's previous paper.

Time evolutions of profiles of the surface elevation along reflected waves (and Mach stems) are shown in Figs. 4 and 5. When $\alpha>\sqrt{3\varepsilon}$, the development of a Mach stem is not seen as is shown in Fig. 4. This corresponds to the regular reflection. On the other hand, if $\alpha<\sqrt{3\varepsilon}$ a Mach stem elongates with the time. This represents the Mach reflec-

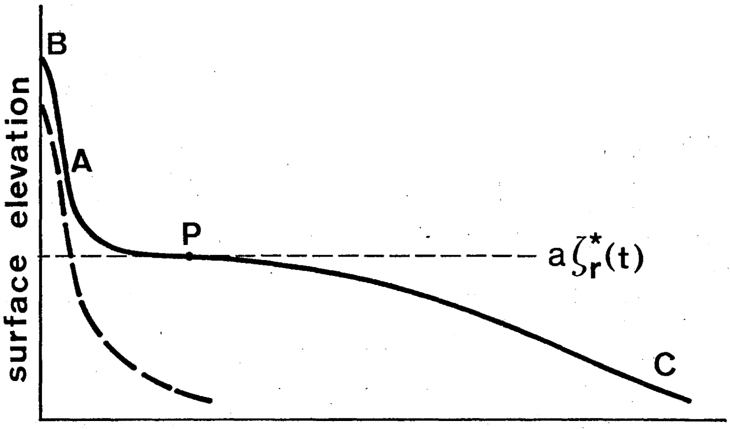

Fig. 3. Representative profiles of the surface elevation along the crest line of a reflected wave (and a Mach stem). The horizontal co-ordinate is the distance from the point of maximum run-up at the wall along the crest line of a reflected wave (and a Mach stem). ---, The profile at small interaction time: ——, the profile at the larger interaction time (B, A and C correspond to those in Figs. 2 (b) and (c)).

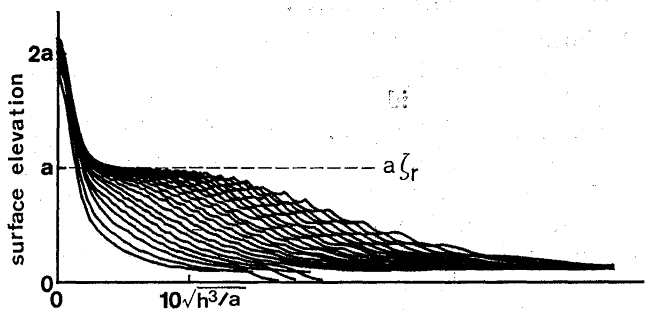

Fig. 4. Time evolution of the profile of the surface elevation along the crest line of a reflected wave. $\varepsilon=0.05$ and $\alpha=\pi/5$ $(\alpha/\sqrt{3\varepsilon}=1.62)$ corresponding to regular reflection. Horizontal co-ordinate is the same as that in Fig. 3. The lines are for the time $t_e'=5.08\ h/\sqrt{ag}$, $6.47\ h/\sqrt{ag}$, $\cdots$ of the same interval (t_e' is the interaction time of an incident wave with an oblique wall. When the crest line of an incident wave passes through the line $x=0$ in Fig. 1 far from the corner, t_e' is chosen as 0.).

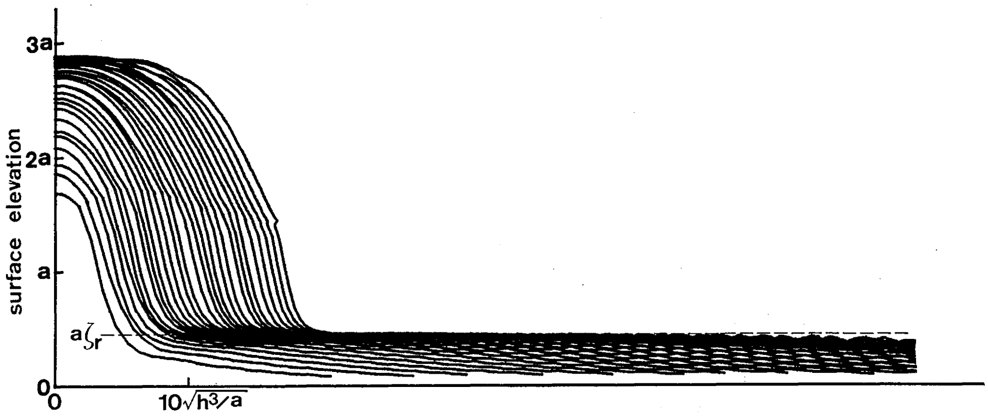

Fig. 5. Time evolution of the profile of the surface elevation along the crest line of a Mach stem and a reflected wave. $\varepsilon=0.05$ and $\alpha=\pi/13$ ($\alpha/\sqrt{3\varepsilon}=0.62$) corresponding to Mach reflection. Horizontal co-ordinate is the same as that in Fig. 3. The lines are for the interaction time $t'_e=27.95\ h/\sqrt{ag}$, $39.13\ h/\sqrt{ag}$, $\cdots$ of the same interval.

tion pattern (see Fig. 5).

The surface elevation along a reflected wave seems to have a constant value $a\zeta_r$ except for the region far from the wall after large interaction time as is shown in Figs. 4 and 5. The position of the tip of a reflected wave can not be determined definitely because the surface elevation decreases very slowly along the crest line of a reflected wave at the region far from the wall.

Melville considered the model with a reflected wave of finite length. In this model it is assumed that the surface elevation varies as a step function along the crest line of a reflected wave and that the incident and reflected waves and the Mach stem have the solitary wave profiles. He determined the length of the reflected wave for the values predicted by Miles of the amplitude and the length of the Mach stem and the amplitude of the reflected wave by means of the conservation of mass and energy. The result is inconsistent with the conservation of momentum in the neighbourhood of the tip of the reflected wave. For this reason, Melville suggested that Miles' solution does not appear asymptotically. However, the result of the present calculations differs from the assumption in this model, and the calculated amplitude of the reflected wave agrees with Miles' model. The above inconsistency seems due to the defect of the model itself. It seems that the discrepancy between the measurement and the theoretical solution is caused by the effect of viscosity.

Figure 6 shows the profile of the surface elevation across the crest line of a reflected wave. When $\alpha>\sqrt{3\varepsilon}$ the profile which agrees well

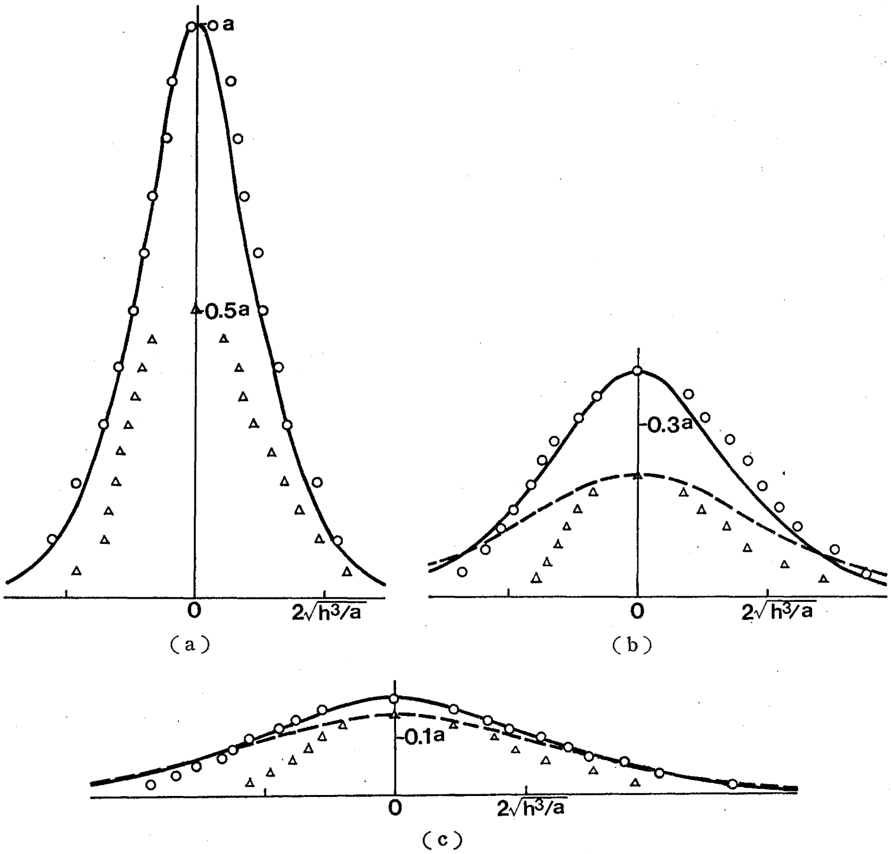

Fig. 6. Profiles of the surface elevation across the crest line of a reflected wave for $\varepsilon=0.05$. Circles denote the profiles of sufficiently developed reflected waves at the section where the maximum values of them agree with the theoretically predicted values. Triangles denote the profiles of insufficiently developed reflected waves. Solid lines denote the profiles of an incident wave (in (a)) and of the reflected wave in Miles' prediction (in (b) and (c)). Broken lines denote the profile of a Boussinesq solitary wave of the same amplitude as the profile of triangles. (a), $\alpha=\pi/6$, corresponding to regular reflection; $\triangle$, $t_e'=16.8\ h/\sqrt{ag}$; $\bigcirc$, $t_e'=72.7\ h/\sqrt{ag}$. (b), $\alpha=\pi/13$, corresponding to Mach reflection; $\triangle$, $t_e'=39.1\ h/\sqrt{ag}$; $\bigcirc$, $t_e'=274\ h/\sqrt{ag}$. (c), $\alpha=\pi/20$, corresponding to Mach reflection; $\triangle$, $t_e'=83.9\ h/\sqrt{ag}$; $\bigcirc$, $t_e'=330\ h/\sqrt{ag}$.

with that of an incident wave is obtained asymptotically (see the circles and a solid line in Fig. 6 (a)). Figures 6 (b) and (c) correspond to the case of Mach reflection. The profiles agree approximately with those of Miles' prediction

$$\zeta=\frac{\alpha^2}{3\varepsilon}\operatorname{sech}^2\left[\frac{\alpha}{\sqrt{3\varepsilon}}(\xi-\xi_0)\right]$$

after the sufficiently large interaction time, where $l\xi$ is the horizontal co-ordinate along the normal to the crest line of a reflected wave. The profiles for insufficient interaction time are not symmetric and represent the more rapid decrease at the backward side of waves as is shown by the triangles in Fig. 6.

Melville obtained "reflected wave amplitudes" which are considerably smaller than the theoretical values except for small α. The effect of viscosity seems negligible for his observation time as is discussed in §4.4. I infer that "the reflected wave amplitude" obtained by him is the surface elevation at some point on the crest line of the reflected wave at an insufficient interaction time from the following two reasons: The profiles such as the broken line in Fig. 3 are obtained in the present calculations with the same values of ε and α as experiments at the interaction time estimated crudely from the contents of his paper (see Fig. 7 for examples). Secondly, the much narrower profile than a Boussinesq solitary wave of a reflected wave measured by him is also obtained for the reflected wave at insufficient interaction time as is shown by the broken line and triangles in Figs. 6 (b) and (c). Therefore, the larger interaction time seems to be required for the measurement of fully developed reflected waves although the viscous effect may not be negligible then.

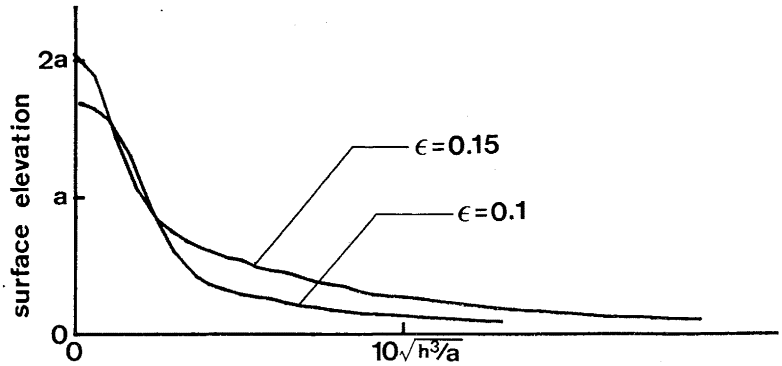

Fig. 7. The profiles of the surface elevation along the crest line of a reflected wave at the time which is inferred to be the largest observation time in Melville's experiment. Horizontal co-ordinate is the same as that in Fig. 3. One line is for $\varepsilon=0.1$, $\alpha=\pi/7$ and interaction time $t_e'=7.96\ h/\sqrt{ag}$. Another line is for $\varepsilon=0.15$, $\alpha=\pi/5$ and $t_e'=9.17\ h/\sqrt{ag}$.

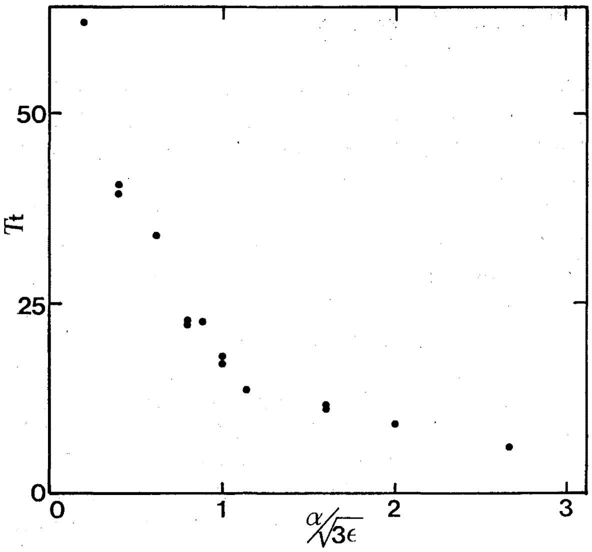

Fig. 8. The time required for the attainment of the largest depth of a trough at the wall. $\varepsilon=0.05$.

4.2. The behaviour of troughs.

In the present calculations a trough appears behind a reflected wave (and a Mach stem). The depth of it at the wall attains a maximum at certain time, $T_t h/\sqrt{ag}$. The time is measured from the instance when the incident wave passes through the line $x=0$ in Fig. 1. The values of T_t are shown in Fig. 8 for $\varepsilon=0.05$. They increases rapidly as α decreases to zero. The maximum depth decreases to zero as α decreases.

4.3. The maximum run-up at the wall.

The maximum run-up at the wall is the most interesting quantity partly because it is important in the investigation of the reflection of ocean waves and partly because Miles predicted that it is much larger than that in linearized theory when α is close to $\sqrt{3\varepsilon}$. The maximum run-up at the wall $a\zeta^*_{\max}(t)$ increases monotonically with the time and then tends to a constant approximately for most calculations (see Fig. 9 as examples). We define this asymptotic value as the maximum run-up at the wall for stationary state $a\zeta_{\max}$. Then $\zeta_{\max}$ is inferred to be close to that of Miles' prediction as was stated in author's previous paper.

The value of $\zeta^*_{\max}(t)$ at the largest observation time in Melville's experiment was much smaller than the theoretical value although mea-

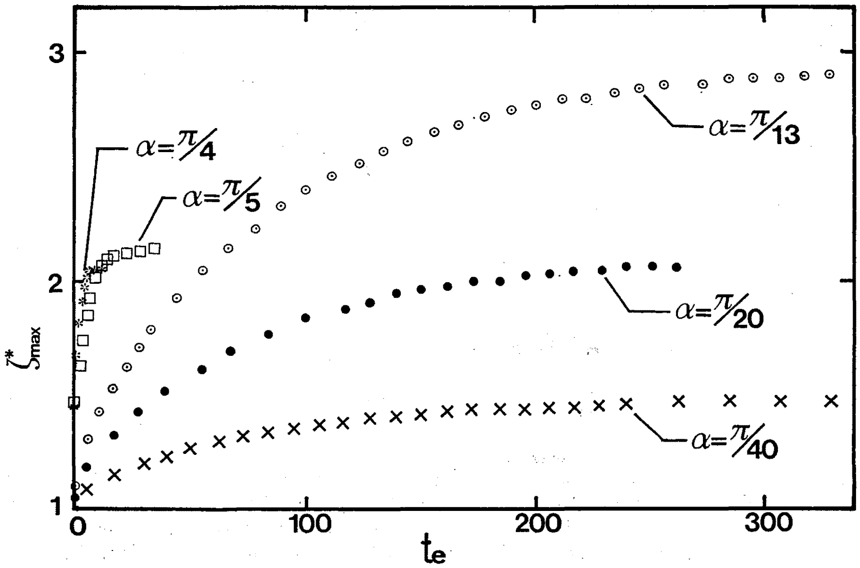

Fig. 9. Time evolution of maximum run-up at the wall for $\varepsilon=0.05$. Horizontal co-ordinate is the non-dimensional interaction time $t_e=t_e'\sqrt{ag}/h$.

sured $\zeta^*_{max}(t)$ increased slowly at that time for most cases. Furthermore, measured $\zeta^*_{max}(t)$ increased monotonically as α increased.

The values of $\zeta^*_{max}(t)$ in Melville's experiments and in the corresponding calculations are shown as a function of the co-ordinate along an oblique wall in Fig. 10. The measured values agree fairly well with the calculated values. However, the calculated values of $\zeta^*_{max}(t)$ continue to increase for the interaction time of 7 or 9 times as long as the largest interaction time in the experiments. Therefore, if the experiment with a little larger observation time is made, a little larger value of $\zeta^*_{max}(t)$ may be obtained (although much larger observation time does not neccessarily give the much larger value of it as is discussed in the following subsection).

4.4. The estimation of the effect of viscosity.

Although the accurate estimation of the viscous effect on the oblique reflection phenomena is difficult, the crude estimation may be possible if the theory for the viscous damping of a one-dimensional solitary wave is used.

Keulegan[9] examined the viscous attenuation of a one-dimensional solitary wave in a channel of constant depth and width. He derived a inverse-fourth-power decay law for a shallow water solitary wave of small amplitude on the assumptions that the flow is laminar and that the

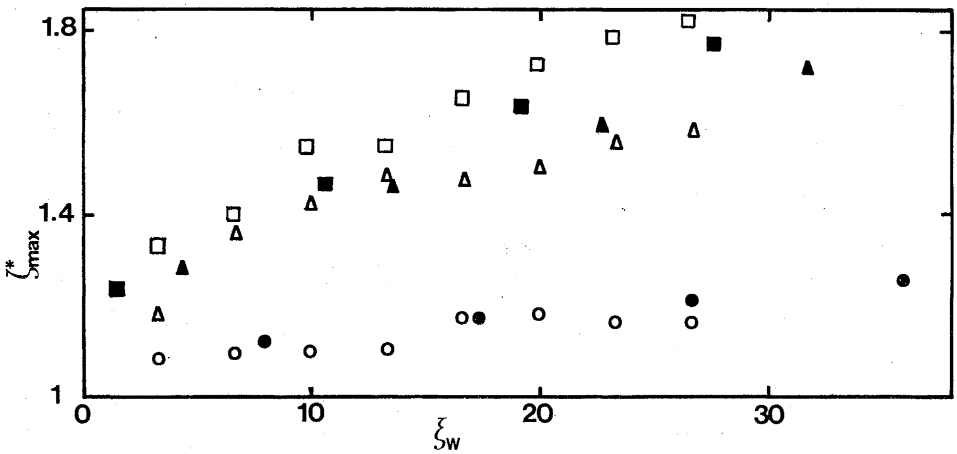

Fig. 10. Maximum run-up at the wall in the present calculations and in Melville's experiments for each value of the non-dimensional distance divided by h along an oblique wall from the corner. Open symbols denote experimental results. Solid symbols denote calculated results. ● and ○, $\varepsilon=0.1$, $\alpha=10°$ (experiment), 9° (calculation): ▲ and △, $\varepsilon=0.1$, $\alpha=25°$ (experiment), 25.7° (calculation): ■ and □, $\varepsilon=0.1$, $\alpha=30°$.

thickness of boundary layers is much smaller than the water depth and that the dissipative and dispersive effect of viscosity on wave form (except for the amplitude) can be neglected. This law is written as

$$\frac{p'(t')}{p'(0)}=\frac{1}{(1+\gamma t')^4}\ , \tag{5}$$

where $p'(t')$ is the amplitude of a solitary wave at the time t', and

$$\gamma=\frac{1}{12}\left(1+\frac{2h}{b}\right)\left(\frac{p'(0)g\nu^2}{h^6}\right)^{\frac{1}{4}}, \tag{6}$$

where h and b are the depth and the width of a channel and ν is the kinematic viscosity. This law was derived also by Ott and Sudan[10] and Kakutani and Matsuuchi.[11] Miles[12] also obtained a similar law. In his equation also the effect of surface contamination is taken into consideration, so eq. (6) is modified as

$$\gamma=0.0839\left(1+\frac{2h}{b}+\kappa\right)\left(\frac{p'(0)g\nu^2}{h^6}\right)^{\frac{1}{4}}, \tag{7}$$

where κ is a surface-contamination parameter and is chosen as 1 in the following.

We use the Miles' formula (5) and (7) for the estimation of viscous effect on the oblique reflection phenomena as a crude approximation. Melville's experiments were made with $h=20$, 30 cm and $\varepsilon=0.1$, 0.15.

The interaction time in his experiment is inferred to be about 4 seconds. If $t'=4$ sec., $h=30$ cm, $p'(0)=3$ cm, $\nu=0.01$ cm^2/s and $b=\infty$, Miles' formula give the damping of 1.2% of an initial wave amplitude. If $t'=4$ sec., $h=20$ cm, $p'(0)=3$ cm and $b=\infty$, the damping of 2.2% is given. Therefore, the viscous effect seems negligible in his experiment although maximum run-up at the wall may be affected also by the viscous dissipation in the boundary layer at a side wall.

In the present calculations for $\varepsilon=0.05$, the interaction time required for the attainment of the stationary state depends on α intensely as is shown in Fig. 9. If $\alpha/\sqrt{3\varepsilon}$ is larger than 1.5, then $\zeta^*_{max}(t)$ is saturated approximately in the interaction time less than $20h/\sqrt{ag}$, and the representative rate of increase of $\zeta^*_{max}(t)$ is fairly large. On the other hand, if $\alpha/\sqrt{3\varepsilon}$ is less than 0.81, the interaction time required for the saturation of $\zeta^*_{max}(t)$ is very large (more than $200h/\sqrt{ag}$), and the representative rate of increase of it is small.

Interaction times $20h/\sqrt{ag}$ and $200h/\sqrt{ag}$ correspond to 15 sec. and 150 sec. respectively if we use $h=30$ cm and $a=1.5$ cm. If $h=30$ cm, $p'(0)=1.5$ cm and $b=\infty$, Miles' formula with $t'=15$ sec. and $t'=150$ sec. give the damping of 4% and 30% of an initial wave amplitude. Therefore, if h is 30 cm, the viscous effect on the oblique reflection phenomena seems approximately negligible for $\alpha/\sqrt{3\varepsilon}>1.5$. In other words measured $\zeta^*_{max}(t)$ is expected to increase rapidly to the value close to that obtained from the inviscid theory before the viscous effect reduce the wave amplitude appreciably. However, for the Mach reflection case of $\alpha/\sqrt{3\varepsilon}<0.81$, the viscous effect is significant if h is 30 cm. That is, the viscous effect on $\zeta^*_{max}(t)$ becomes non-negligible before $\zeta^*_{max}(t)$ increases sufficiently, so $\zeta^*_{max}(t)$ tends to the considerably smaller value than that of inviscid theory and then decreases.

Figures 11 (a) and (b) show the viscous effect a little more definitely. The circles and stars represent the rate of increase of calculated $\zeta^*_{max}(t)$ per 1 second for $\varepsilon=0.05$ and $h=30$ cm. The solid line shows the rate of decrease of $p'(t')$ per 1 second based on eqs. (5) and (7) with $h=30$ cm, $p'(0)=1.5$ cm and $b=\infty$ (this seems to be underestimation of viscous effect in consideration of the existence of a side wall and the larger values of $\zeta^*_{max}(t)$ than 1 in the real problem). These rates are normalized by the values of ζ^*_{max} and p' at the time when they are calculated. When $\alpha=\pi/4$ ($\alpha/\sqrt{3\varepsilon}=2.0$), the rate of increase of $\zeta^*_{max}(t)$ is much larger than the viscous damping rate for the most time of evolution. On the contrary, if α is $\pi/13$ or $\pi/40$ ($\alpha/\sqrt{3\varepsilon}=0.62$ or 0.20), viscous damping overcome the increase of $\zeta^*_{max}(t)$ at the considerably smaller interaction time than that required for the attainment of stationary state in inviscid calculations.

Consequently, the large values of ζ_{max} which were obtained in the

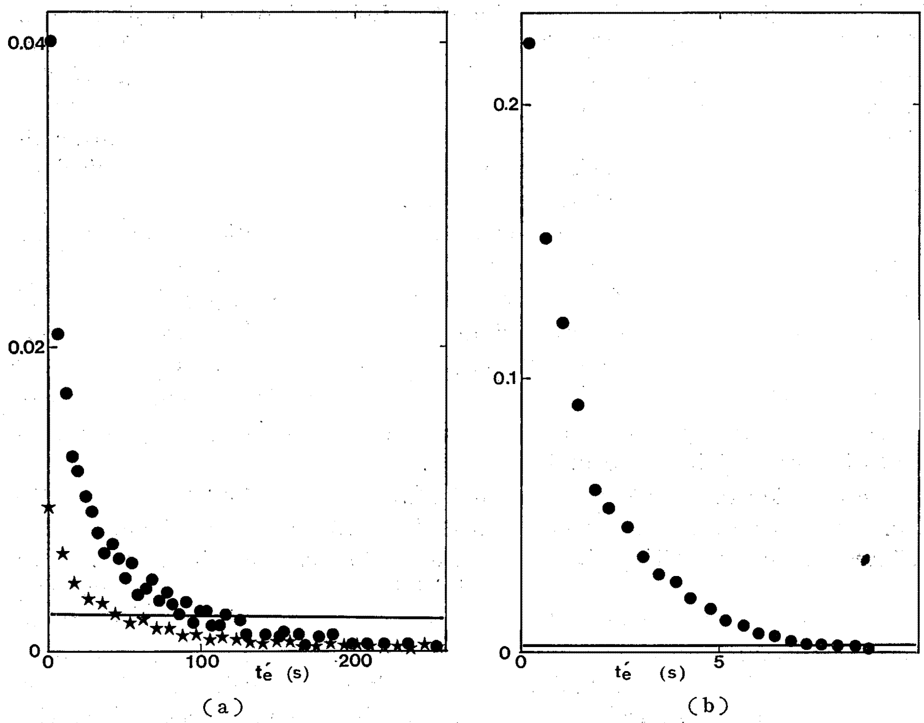

Fig. 11. The rate of increase of maximum run-up at the wall in inviscid calculations and the estimated rate of decrease of it due to viscosity for $h=30$ cm and $\varepsilon=0.05$. Circles and stars denote $(d\zeta^*_{\max}(t')/dt')/\zeta^*_{\max}(t')$. They are shown as a function of interaction time t_e'. Solid line denotes $-(dp'(t')/dt')/p'(t')$ based on eqs. (5) and (7). (a), Circles are for $\alpha=\pi/13$ and stars are for $\alpha=\pi/40$, both corresponding to Mach reflection. (b), Circles are for $\alpha=\pi/4$ corresponding to regular reflection.

inviscid calculations for $\alpha\sim\sqrt{3\varepsilon}$ seem unrealizable for small ε in the laboratory experiments with the apparatus of ordinary dimensions. The viscous effect seems relatively small for the large scale wave in the ocean. Therefore, the maximum run-up at the wall may become much larger than twice of the incident wave amplitude in the ocean.

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