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OSCILLATION RIPPIE MARKS OF HIGHLY VISCOUS FLUIDS

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Ripple marks of highly viscous fluids similar to those of sands have been observed to form under oscillating water in a closed-type flume, when a parameter $R(=I\sqrt{gL}/\nu)$ is less than a certain critical value, g being the acceleration due to gravity, ν the kinematic viscosity of highly viscous fluids, and L the wavelength of ripple marks. On the basis of the damping theory of gravity waves, the region of R for the formation of ripple marks is determined.

Key words: Oscillation ripples, Viscous fluids, Damping wave theory

1. Introduction

Formation of oscillation ripple marks (ripples under oscillatory flow) is common to the nearshore sea-floors, where the influence of waves is pronounced, and well documented (Kommar [1)]). Quite naturally, most investigations have been concerned only with sand ripples; the granulation of bed materials has been considered important for the ripple mark formation. Based on the idea, however, that the ripple marks may be nonpropagating 'frozen' waves on the surface of a single fluid layer, we have considered a possibility that they may also form on the surface of nongranular highly viscous continuum materials under oscillating water. We report the results of experiments for testing out this possibility in § 2. In § 3, we examine a critical condition of nonpropagating surface waves using a dispersion relationship. In § 4, we compare the results of experiments with those of a simple theory of damping of gravity waves, and consider a limitation of the results.

2. Experimental methods and results

Experiments were performed using a closed-type flume with a 15×15 cm rectangular cross section; its middle horizontal portion 60cm long was used as a test section illustrated in Fig. 1. A uniform layer of a highly viscous

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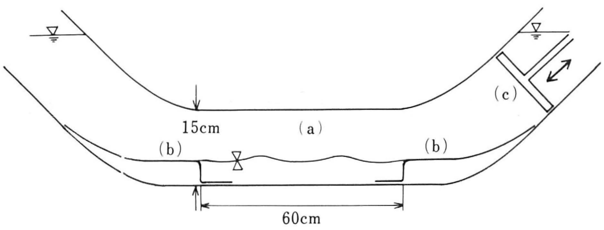

Fig. 1 Schematic diagram of experimental set-up.
(a) test section, (b) end plates, (c) piston.

(a)

(b)

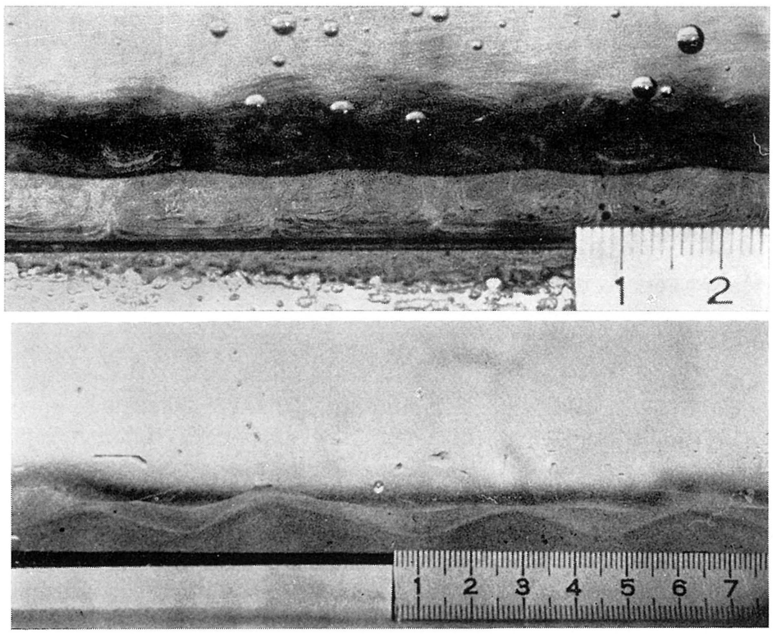

Fig. 2 (a) Ripple marks of glycerin (unit in cm). A glycerin layer is seen on the white bottom wall of the flume seen on the same level as the scale. $\nu=9.7\mathrm{cm}^2\mathrm{s}^{-1}$. $d=0.60$cm.
(b) Ripple marks of silicon-oil (unit in cm). $\nu=2.9\mathrm{cm}^2\mathrm{s}^{-1}$. $d=0.50$cm.

nongranular material (heavy silicon-oil or pure glycerin) was made over the bottom wall of the test section, which was filled fully with water. This overlying water was oscillated harmonically in the direction of the horizontal interface between two fluids by using a motor-driven piston. The kinematic viscosity of lower layer fluids ν ranged from 0.47 to 13 $\mathrm{cm}^2\mathrm{s}^{-1}$. The angular frequency of water oscillation ω ranged from 3.8 to 16 rads^{-1}, and the maximum velocity of water oscillation U from 1.8 to 21 cms^{-1}. The depth of the lower layer d was varied between 0.30 and 2.2 cm. A 35 mm camera was used to take photographs of the distortion of fluid layers, from which ripple wavelengths were measured.

It has been observed clearly that the oscillating ripple marks of highly viscous fluids form in a similar way as those of sands form. Two examples of those are shown in Fig. 2, where the direction of water oscillation is left and right. In Fig. 2(a), induced secondary streaming is seen in the lower layer of glycerin. Figure 2(b) shows similar ripple marks of silicon-oil.

3. Calculation of critical conditions of a single fluid layer

The aforementioned idea of regarding the ripple marks as nonpropagating waves leads to the consideration that such waves may form only when the waves would damp monotonically unless the underlying fluid layer were excited continuously. Therefore we consider the monotonically damping waves on the surface of a single fluid layer at rest as a whole, and assume that no surface tention works effectively at the surface. The condition of wave damping may be provided by using the dispersion relationship for surface gravity waves. However, previous calculations of the wave damping coefficients are valid only for small values of ν (Krauss [3]). Based on the Navier-Stokes equations, we derive the dispersion relationship for two-dimensional small amplitude surface gravity waves on the surface of highly viscous fluids.

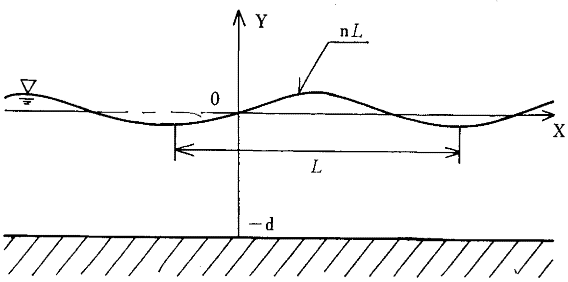

Fig. 3 Definition sketch.

Figure 3 shows a definition sketch of a horizontal layer of the fluid which is bounded by an infinite plane at the depth of d. The symbols ν, ρ, L, and ηL are the kinematic viscosity of the fluid, the fluid density, wavelength and an initial displacement of the free surface, respectively. The flow is considered two-dimensional. We consider the gravity waves, of which the amplitude is so small that we may linearize the Navier-Stokes equations.

We introduce the following dimensionless quantities

$$\left.\begin{aligned} &\tau=t\frac{\sqrt{gL}}{L}, \quad x=\frac{X}{L}, \quad y=\frac{Y}{L}, \\ &\hat{u}=\frac{u}{\sqrt{gL}}, \quad \hat{v}=\frac{v}{\sqrt{gL}}, \quad \hat{p}=\frac{p}{\rho gL}, \end{aligned}\right\} \tag{1}$$

where t is the time, u the horizontal velocity, v the vertical velocity, p the pressure in the fluid. Using these quantities and the stream function Ψ, the basic equations are

$$\frac{\partial^2\Psi}{\partial\tau\partial y}=-\frac{\partial\hat{p}}{\partial x}+\frac{1}{R}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\frac{\partial\Psi}{\partial y}, \tag{2}$$

$$-\frac{\partial^2\Psi}{\partial\tau\partial x}=-1-\frac{\partial\hat{p}}{\partial y}-\frac{1}{R}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\frac{\partial\Psi}{\partial x}, \tag{3}$$

where $R=\sqrt{gL}L/\nu$.

The boundary conditions for Ψ at the bottom $y=-H(=-d/L)$ are

$$\frac{\partial\Psi}{\partial x}=0, \tag{4}$$

$$\frac{\partial\Psi}{\partial y}=0. \tag{5}$$

The boundary conditions at the free surface $y=\eta$ are

$$\frac{\partial^2\Psi}{\partial y^2}-\frac{\partial^2\Psi}{\partial x^2}=0, \tag{6}$$

$$\hat{p}+\frac{2}{R}\ \frac{\partial^2\Psi}{\partial x\partial y}=0, \tag{7}$$

which mean that the tangential and normal stresses must vanish there. The kinematic condition is

$$\frac{\partial\eta}{\partial\tau}=-\frac{\partial\Psi}{\partial x}\ . \tag{8}$$

We assume that Ψ is a plane wave solution as

$$\Psi=f(y)e^{i(kx-\sigma\tau)}, \tag{9}$$

in which $f(y)$ is a function of y, $k=2\pi$, $\sigma=\sigma_d\dfrac{L}{\sqrt{gL}}=\sigma_r+i\sigma_i$ (σ_r, σ_i; real numbers), σ_r is the angular frequency, and σ_i is the decay or growth rate of a surface gravity wave and suffix d denotes a dimensional quantity. The possibilities of a wave propagating in the negative x-direction and a standing wave are not necessary for this calculation, because these types are the same as eq. (9) when a wave is the non-propagating one ($\sigma_r=0$).

We have the dispersion relationship

$$\begin{aligned} &\mathrm{A}+\mathrm{B}\sinh k\mathrm{H}\sinh m\mathrm{H}+\mathrm{C}\sinh k\mathrm{H}\cosh m\mathrm{H} \\ &\quad +\mathrm{D}\cosh k\mathrm{H}\sinh m\mathrm{H}+\mathrm{E}\cosh k\mathrm{H}\cosh m\mathrm{H}=0, \end{aligned} \tag{10}$$

in which

$$\begin{aligned} &\mathrm{A}=-\frac{4mk^2}{R}(m^2+k^2), \\ &\mathrm{B}=-\frac{k}{R}\{(m^2+k^2)^2+4k^2m^2\}, \\ &\mathrm{C}=mkR, \\ &\mathrm{D}=-k^2R, \\ &\mathrm{E}=\frac{m}{R}\{(m^2+k^2)^2+4k^4\}, \\ &m^2=k^2-i\sigma R\,. \end{aligned} \tag{11}$$

When $R\rightarrow+\infty$, (11) becomes the well-known dispersion relationship of an inviscid fluid.

4. Results and discussions

Considering the monotonic damping of the waves, we put $\sigma_r=0$ and obtain the region of wave damping corresponding to $\sigma_i<0$ ($\sigma_i\neq 0$) from eq. (10), as shown (shaded) in Fig. 4.

Before we discuss the meaning of this figure, we turn our attention to the two special cases of eq. (10). (i) When $\mathrm{H}\rightarrow 0$, eq. (10) has the real solution $\sigma_i(<0)$ for all the values of R, and hence the asymptote of the boundary line of the regions having the solution is $\mathrm{H}=0$ when $R\rightarrow+\infty$. The long wave (ripples) are formed in spite of the large value of R (i. e. the small value of ν), because the 'bottom friction' (Lighthill[4]) is intence when the depth of ths lower layer is small. (ii) When $\mathrm{H}\rightarrow+\infty$, we put $\sigma_r=0$ and the condition for having the real solution $\sigma_i(<0)$ is obtained approximately as $R\leq 12.0$. This region shows the 'internal dissipation' (Lighthill[4]). When $R\ll 1$ and $H\gg 1$, all waves are non-propagating ('frozen'), because $R\ll 1$ means that a relaxation time of viscous dissipation (L^2/ν) is smaller than that of the restoring force ($\sqrt{L/g}$). This result coincides with that of Landau and Lifshitz[3].

In Fig. 4, the marks for the ripple marks are included in the region obtained using the theory of wave damping, and the marks for three-dimensional ripple marks are approximately in the vicinity of the boundary of the region.

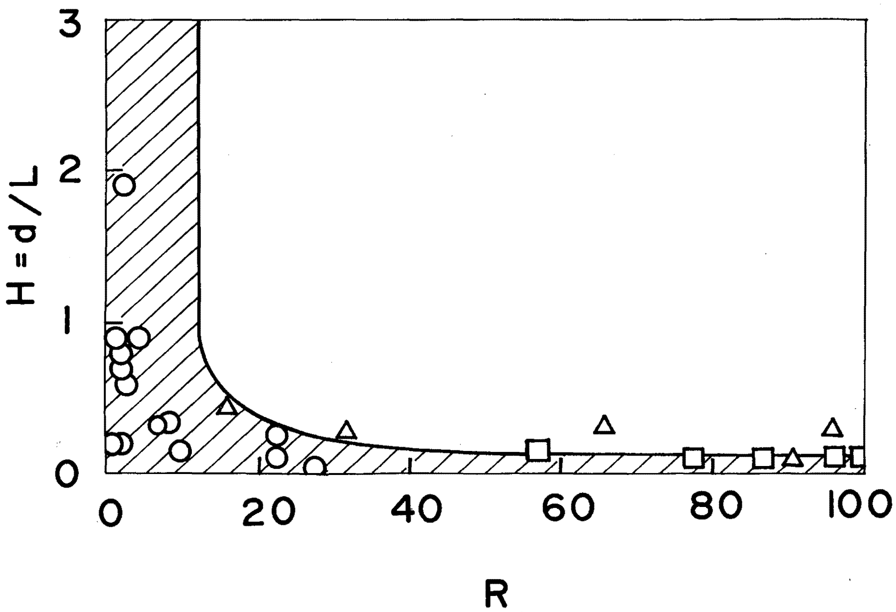

Fig. 4 Comparison between experiments and theory; the region for $\sigma_i<0$ and $\sigma_r=0$ is shaded.
○, glycerin ripples; □, silicon-oil ripples;
△, three-dimensional silicon-oil ripples.

In a two-layer fluid system, because the effect of gravity (g) reduces $\frac{\Delta\rho}{\rho}g(=g^*)$ where $\Delta\rho$ is the difference of densities between upper and lower layers, the situation of $L^2/\nu \ll \sqrt{L/g^*}$ is likely to occur. The shaded region in Fig. 4 may be extended.

In the experiments, wave field varies with time periodically (under oscillatory flows). When this time scale is larger than the damping time scale of waves, the theory may be useful.

In conclusion, the oscillation ripple marks form even on the surface of nongranular materials, and the theory of viscous wave damping seems valid for estimating approximately the region of R for the formation of highly viscous ripple marks.

Acknowledgements

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