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NUMERICAL ANALYSIS OF CURRENT-DRIVEN INSTABILITIES FOR LARGE DRIFT VELOCITIES IN A MAGNETIC FIELD

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The current-driven instabilities for the electron drift velocity larger than the electron thermal velocity in a parallel magnetic field is studied using the Vlasov equation. It is found that the instability due to the anomalous Doppler effect appears in addition to the Buneman instability when the magnetic field is applied and the neutralized ion Bernstein wave instability appears for the strong magnetic fields.

Key words: Buneman instability, Vlasov equation, Magnetic field, Neutralized ion Bernstein wave instability

§ 1. Introduction

The Buneman instability is considered to be one of important instabilities in the initial stage of the turbulent heating experiments.[1)~4)] Although the turbulent heating experiments have been carried out in a magnetic field, the results have been interpreted by the theory of the Buneman instability in the absence of the magnetic field. In the previous paper,[5)~6)] we studied the Buneman instability in a parallel magnetic field using fluid equation and found that the Buneman instability is modified by the application of the magnetic field. However, the dispersion equation derived using the fluid equation does not include the terms of Landau damping or cyclotron damping which reduces the growth rate of the instabilities. In this paper, we analyze numerically the dispersion relation of current-driven instabilities in the magnetic field derived using the Vlasov equation. It is found that the Buneman instability has qualitatively the same tendency as obtained using the fluid equation and when the magnetic field is strong, the neutralized ion Bernstein wave instability as well as the Buneman instability appears.

In section 2, we present the dispersion equation of current-driven instabilities. The results obtained by numerical calculations and some dis-

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cussions are given in the section 3.

§ 2. Dispersion Equation

We consider a homogeneous infinite collisionless plasma, in which magnetized electrons with a Maxwellian distribution drift toward the magnetic field through stationary and magnetized ions with a Maxwellian one. In this paper, we examine the case where $T_e = T_i$ and the drift velocity of electrons, v_d, is greater than the thermal velocity of the electrons, v_e $(=\sqrt{T_e/m})$.

The dispersion equation of electrostatic waves in such a system can be written as follows;[7]

$$\varepsilon = 1 + \frac{1}{k^2\lambda_D^2}\left\{1 + \frac{\omega - k_\parallel v_d}{\sqrt{2}\, k_\parallel \sqrt{T_e/m}} \sum_{n=-\infty}^{\infty} Z\left(\frac{\omega - k_\parallel v_d - n\omega_{ce}}{\sqrt{2}\, k_\parallel \sqrt{T_e/m}}\right) I_n(k_\perp^2 \rho_e^2) e^{-k_\perp^2 \rho_e^2}\right\}$$
$$+ \frac{T_e/T_i}{k^2\lambda_D^2}\left\{1 + \frac{\omega}{\sqrt{2}\, k_\parallel \sqrt{T_i/M}} \sum_{n=-\infty}^{\infty} Z\left(\frac{\omega - n\omega_{ci}}{\sqrt{2}\, k_\parallel \sqrt{T_i/M}}\right) I_n(k_\perp^2 \rho_i^2) e^{-k_\perp^2 \rho_i^2}\right\}$$
$$= 0 \qquad (1)$$

where $k^2 = k_\parallel^2 + k_\perp^2$, $\theta = \tan^{-1}(k_\perp/k_\parallel)$ and I_n is the n-th order modified Bessel function. The following definition is used as a plasma dispersion function Z:

$$Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-x^2}}{x - \zeta} d\zeta \qquad \text{for Im } \zeta > 0 \qquad (2)$$

and its analytic continuation for Im $\zeta \leqq 0$.
We solve numerically the dispersion equation for the complex frequency ω $(\equiv \omega_r + i\gamma)$ as a function of $k_\parallel$ for different parameters $(\omega_{ce}/\omega_{pe})^2$ and $\theta = \tan^{-1}(k_\perp/k_\parallel)$. In the present calculations, the drift velocity of $v_d/v_e = 4$ and 1.5, which is greater than the critical velocity of the Buneman instability $v_d/v_e = 1.3$, is selected. The hydrogen plasma is assumed.

§ 3. Results and Discussions

In this section, we show the results of numerical calculations. In the calculations, the plasma dispersion function $Z(\zeta)$ is calculated using the scheme introduced by Watanabe[8], who uses a modification of a trapesoidal rule for numerical integration of Eq. (2) for small $|\zeta|$ and uses a continued fraction expantion of $Z(\zeta)$ for large $|\zeta|$.

Figure 1 shows the dispersion relation for $(\omega_{ce}/\omega_{pe})^2 = 1$ and $v_d/v_e = 4$. The x-axis indicates the parallel wave number normalized to the Debye length $(k_\parallel \lambda_D)$ and y-axis the real and imaginary parts normalized to ω_{pi}. The dotted and solid lines show the growth rate and real part, respectively. It is found that the Buneman instability is not affected by the presence of magnetic field at $\theta=0°$. This is readily understood from the fact that if we take $\theta=0°$, namely $k_\perp=0$, all the $n\neq0$ terms in the summations in Eq. (1) vanish and only the $n=0$ term remains.

At $\theta=30°$, however, the growth rate expands towards the larger $k_\parallel$ side. When θ reaches 40°, the instability has two peaks. The instability on the large $k_\parallel$ side disappears at about 60°. The other instability on the smaller $k_\parallel$ side continues to exist up to about 90°. For the case of the unmagnetized ions, the same results are also obtained. Even if we take the terms $n=0$ and -1 in the summation associated with the electrons, two peaks also appear. Thus, the other instability due to the anomalous Doppler effect[9] ($\omega=k_\parallel v_d+n\omega_{ce}$, $n=-1$) is excited in addition to the Buneman instability.

Figure 2 shows the dispersion relation for $v_d/v_e=4$ and $(\omega_{ce}/\omega_{pe})^2=0.1$ which corresponds to the case of weak magnetic fields. The instability at $\theta=0°$ is the same as shown in Fig. 1(A). At $\theta=40°$, the growth rate of the instability on the larger $k_\parallel$ side is greater than that of the smaller $k_\parallel$ side. When $\theta=60°$, the growth rate of larger $k_\parallel$ side becomes smaller than that of smaller $k_\parallel$ side. The instability on the larger $k_\parallel$ side disappears at $\theta=70°$. and on the smaller $k_\parallel$ side at $\theta=90°$.

When $(\omega_{ce}/\omega_{pe})^2=10$, which corresponds to the case of strong magnetic fields, only one instability appears for all θ (see Fig. 3). This is different

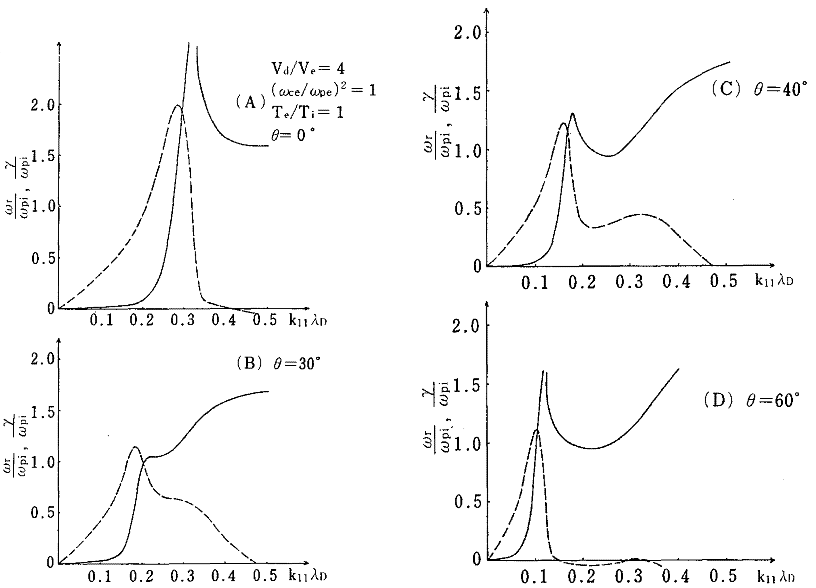

Fig. 1 The dispersion relation in the case of $(\omega_{ce}/\omega_{pe})^2=1$ for different angles.

from the results obtained using the fluid model, which is due to the effect of Landau damping.

Above discussed features of the Buneman instability are also seen from

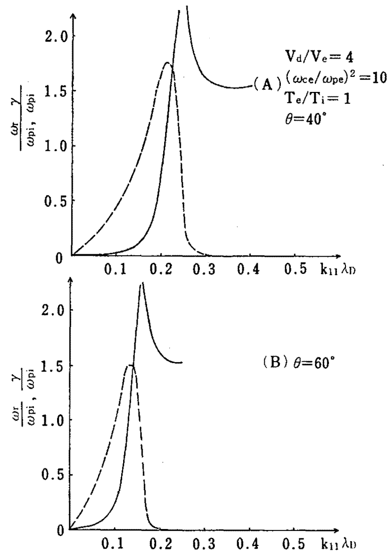

Fig. 2 The dispersion relation in the case of $(\omega_{ce}/\omega_{pe})^2=0.1$,

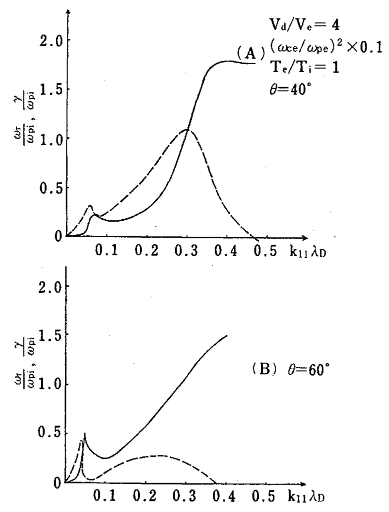

Fig. 3 The dispersion relation in the case of $(\omega_{ce}/\omega_{pe})^2=10$.

Fig. 4, where we have plotted the contour lines of the growth rate for $v_d/v_e=4$ and various magnitude of magnetic fields.

The slower the drift velocity of the electrons becomes, the smaller the growth rate of the instability does. Figure 5 shows the growth rate contour lines for $v_d/v_e=1.5$. In this case, neutralized ion Bernstein wave instabilities are excited in addition to the Buneman instability when the magetic field becomes strong (Fig. 5(D)). In this figure, the dotted circles correspond to the neutralized ion Bernstein wave instabilities. The modes

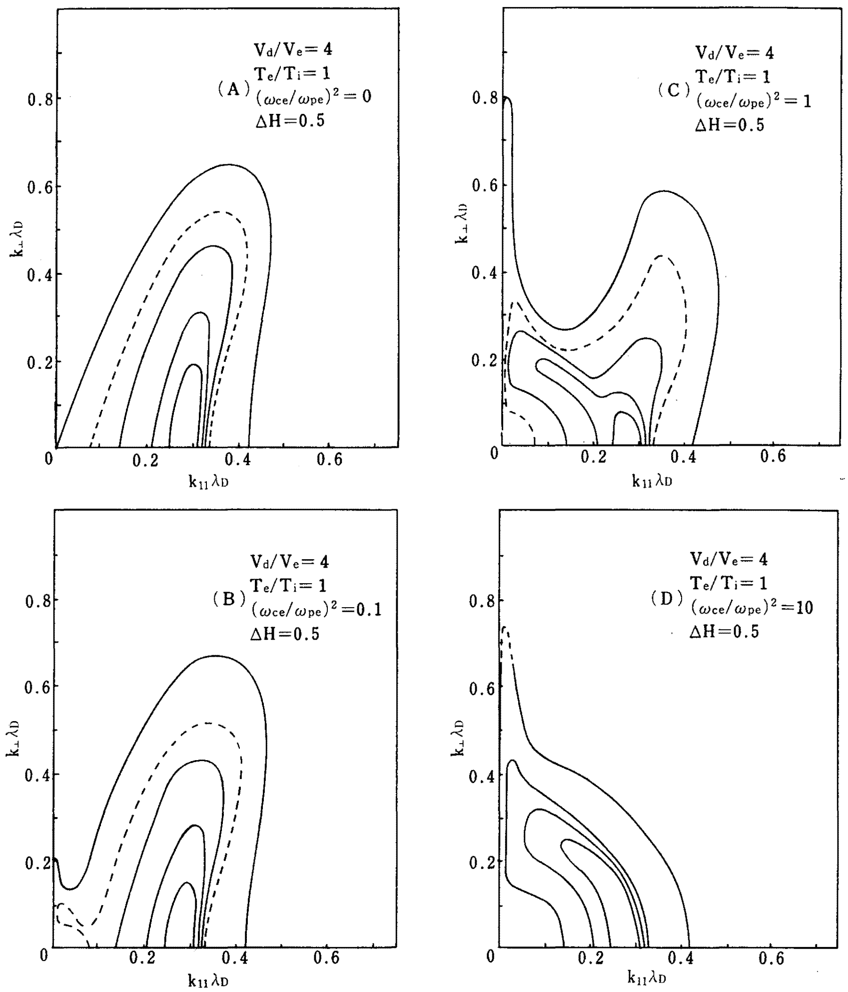

Fig. 4 Growth rate contour lines for various magnitude of the magnetic field in the case of $v_d/a_e=4$.

from the fundamental to 6-th are excited.

The detailed coupling structure between the neutralized ion Bernstein wave instabilities and the Buneman instability is shown in Fig. 6. The Buneman instability is on the large $k_{\parallel}$ side. The neutralized ion Bernstein wave instabilities up to the third harmonics are plotted in bold circles. The heights of the growth rates normalized to ω_{pi} is indicated by the number '0', '1',..... The fundamental and higher harmonics are decoupled each other

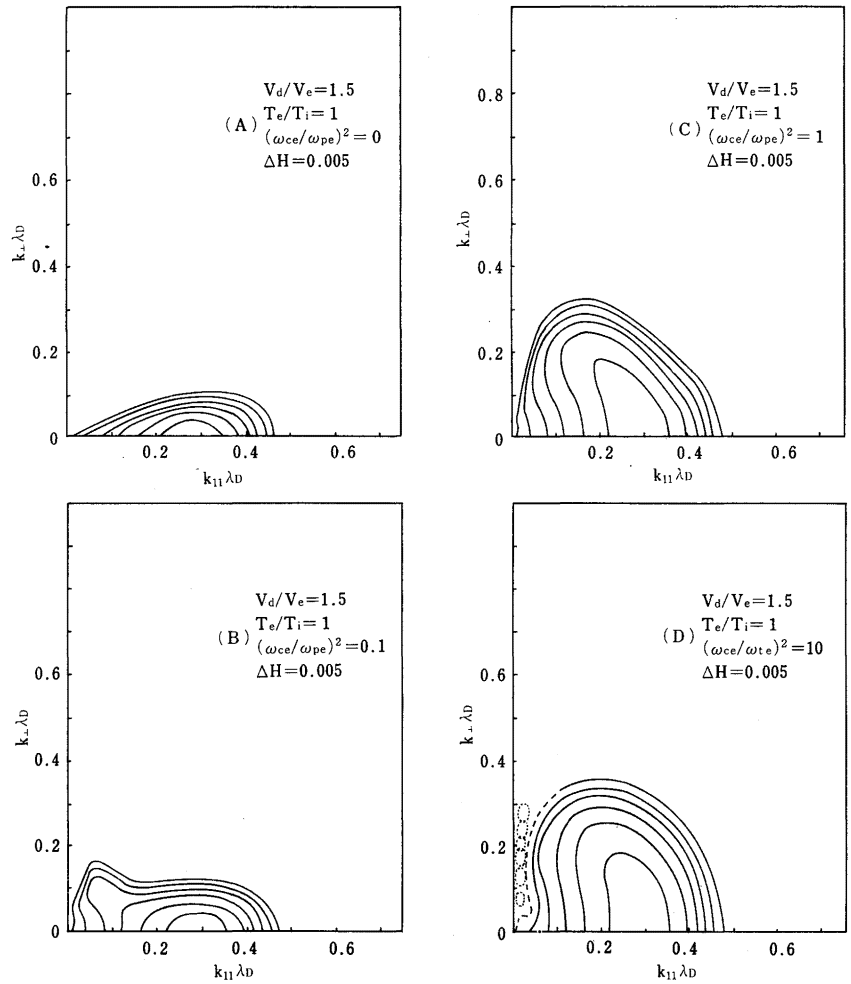

Fig. 5 Growth rate contour lines for various magnitude of the magnetic field in the case of $v_d/v_e=1.5$.

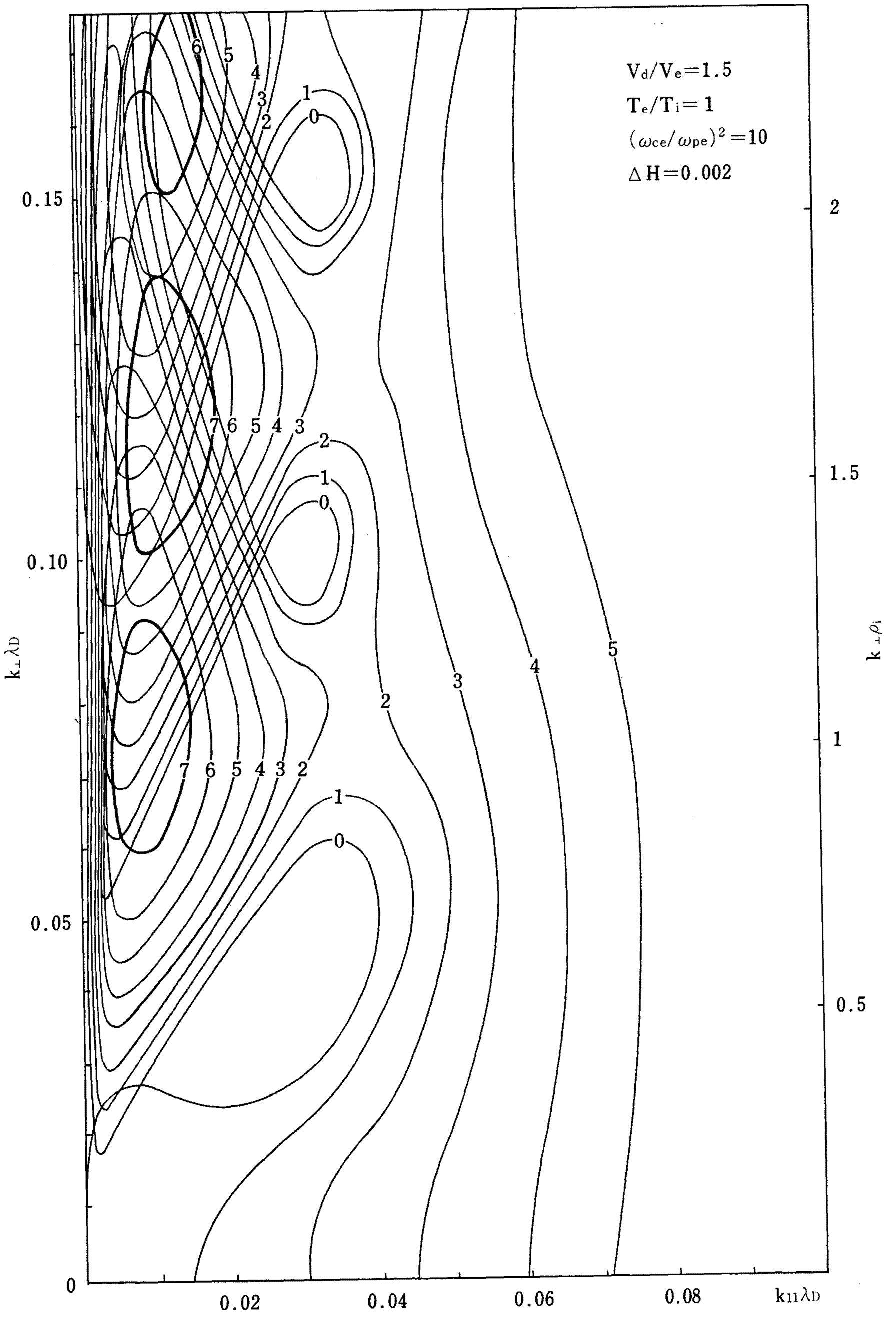

Fig. 6 Coupling structures between the neutralized ion Bernstein wave instability and the Buneman instability.

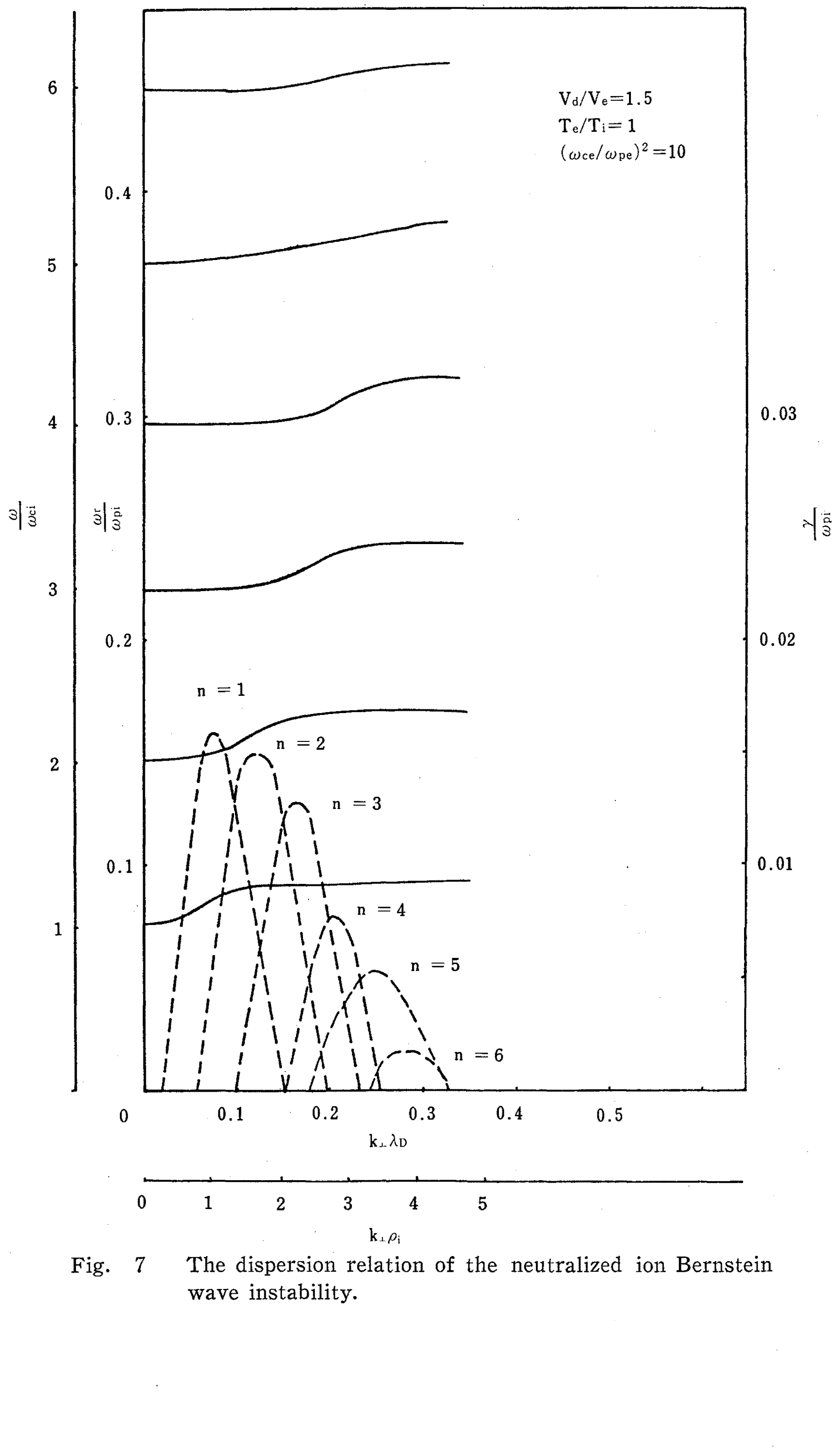

Fig. 7 The dispersion relation of the neutralized ion Bernstein wave instability.

at about $\theta=86°$ (also see Fig. 7). These instabilities couple as θ decreases. It is found easily that the growth rates of these instabilities become small gradually with the increasing 'harmonic' number. As v_d/v_e increases, these modes couple with each other and become the Buneman instability.

Finally, we show the parameter region where these instabilities exsist. In Fig. 8, we plot the boundary between two regions where there appears only one instability and there appear two instabilities. The number of the instabilities is determined from number of peaks of the growth rates. The solid circles describe the region where the growth rates have one peak and the open circles two peaks.

It is easily found from this figure that as v_d/v_e increases the instability due to the anomalous Doppler effect can easily be excited in weak magnetic fields. In the case of strong magnetic fields, namely $(\omega_{ce}/\omega_{pe})^2=10$, as v_d/v_e reaches 10, the instability due to the anomalous Doppler effect appears on the larger $k_{\parallel}$ side. This tendency is understood as follows: The resonance condition of the anomalous Doppler effect is given by

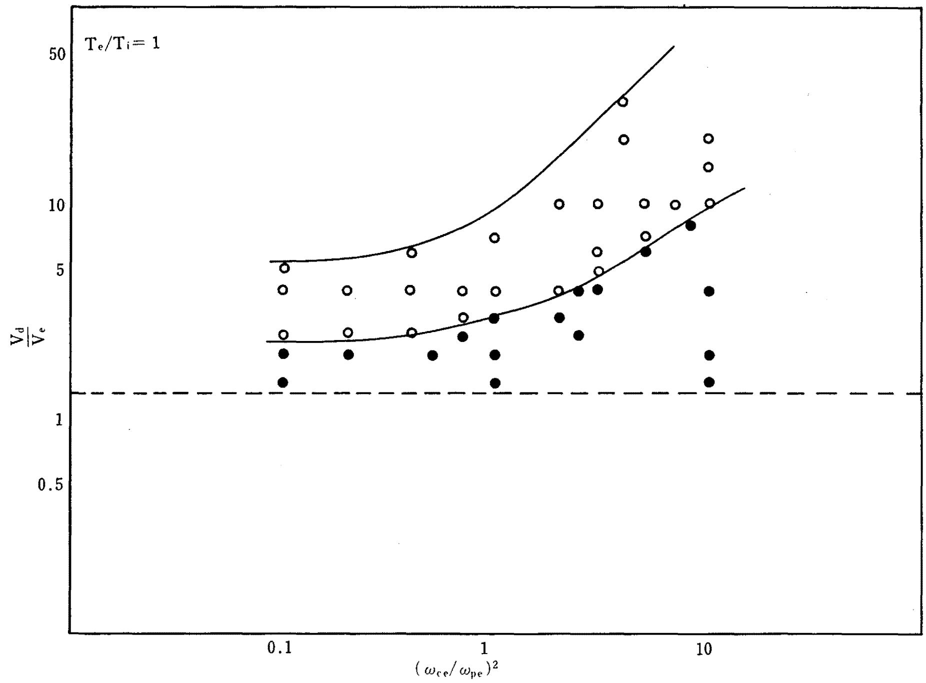

Fig. 8 Instability regions of the anomalous Doppler effect.

$$\omega - k_{\parallel} v_d = n\omega_{ce}, \quad n = -1, \ -2, \ldots \tag{3}$$

Since we consider the case of $\omega \ll \omega_{ce}$, this resonance condition gives an equality $v_d = |n| \omega_{ce}/k_{\parallel}$, which is rewritten as

$$v_d/v_e = |n| \, (\omega_{ce}/\omega_{pe}) \, \frac{1}{k_{\parallel}\lambda_D}. \tag{4}$$

Now we take $k_{\parallel}\lambda_D < 1$, so that $v_d/v_e > |n| \, (\omega_{ce}/\omega_{pe})$. We expect from this inequality, that when the value $(\omega_{ce}/\omega_{pe})$ is small, the resonance condition of the anomalous Doppler effect is easily satisfied even if v_d/v_e is small. Thus the instability due to the anomalous Doppler effect is excited in the weak magnetic fields. Our numerical calculations support these expectations. Furthermore, we expect that the higher order anomalous Doppler effect may appear at high v_d/v_e values, which will be reported elsewhere.

In summary, we calculated numerically the dispersion equation of current-driven instabilities in a parallel magnetic field using the Vlasov equation and the following results are found:

(1) The instability due to the anomalous Doppler effect appears in addition to the Buneman instability which is qualitatively the same tendency as obtained using the fluid equation.

(2) The neutralized ion Bernstein wave instabilities in addition to the Buneman instability are excited in the range of the angle 85°～87° when the drift velocity ratio v_d/v_e is comparatively low and the magnetic field is strong.

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