PARAMETRIC ESTIMATION FOR STOCHASTIC PARABOLIC EQUATIONS DRIVEN BY AN INFINITE DIMENSIONAL MFBM

B. L. S. Prakasa Rao
CR Rao Advanced Institute of Mathematics, Statistics and Computer Science

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PARAMETRIC ESTIMATION FOR STOCHASTIC PARABOLIC EQUATIONS DRIVEN BY AN INFINITE DIMENSIONAL MFBM

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B. L. S. Prakasa Rao*

Abstract

We study the problem of estimation of the parameter in a parabolic stochastic parabolic equation driven by an infinite dimensional mixed fractional Brownian motion.

Key Words and Phrases: Stochastic parabolic equation; Parametric estimation; Mixed fractional Brownian motion; Infinite dimensional mixed fractional Brownian motion.

1. Introduction

Our aim in this paper is to initiate the study of parametric inference for stochastic parabolic equations driven by an infinite dimensional mixed fractional Brownian motion (mfBm). As far as we are aware, this problem has not been investigated earlier. Results obtained in this paper and the proofs are analogous to those obtained earlier for stochastic partial differential equations (SPDEs) driven by an infinite-dimensional fractional Brownian motion but they are not consequences of those results. Following results obtained by Chigansky and Kleptsyna (2019) and Cai et al. (2016) for processes driven by a mfBm, we are able to generalize the results to stochastic parabolic equations driven by an infinite-dimensional mixed fractional Brownian motion. Parametric estimation for two cases of SPDEs driven by an infinite dimensional mfBm were discussed in a recent paper in Prakasa Rao (2022). The first case deals with a SPDE with linear drift (absolutely continuous case) given by

$$du_\epsilon(t,x) = (\Delta u_\epsilon(t,x) + \theta u_\epsilon(t,x))dt + \epsilon d\tilde{W}_Q^H(t,x)$$

where $\Delta = \frac{\partial^2}{\partial x^2}, \theta \in R$, and $\bar{W}_Q^H$ denotes the infinite dimensional mixed fractional Brownian motion on $L_2[0,1]$ with the Hurst index $H \in (1/2, 1)$ with a special nuclear covariance operator Q and the boundary conditions as specified in the equations (16) and (17) in Prakasa Rao (2022). The second case deals with a SPDE with linear drift (singular case) given by

$$du_\epsilon(t,x) = \theta \Delta u_\epsilon(t,x)dt + \epsilon(I-\Delta)^{-1/2} d\tilde{W}^H(t,x)$$

with $\theta > 0$, boundary conditions as specified in the equation (45) and $\tilde{W}^H(t,x)$ is a cylindrical infinite-dimensional mfBm with $H \in (1/2, 1)$. Our aim in this paper is to

* CR Rao Advanced Institute of Mathematics, Statistics and Computer Science, Hyderabad, India, blsprao@gmail.com

study problems of estimation of the parameter θ for the stochastic parabolic equations of the form

$$du(t) + (\mathcal{A}_0 + \theta\mathcal{A}_1)u(t)dt = \sum_{j\geq 1} g_j(t)d\tilde{W}_j^H(t), \quad 0 \leq t \leq T, \quad u(0) = u_0$$

where $\mathcal{A}_0, \mathcal{A}_1$ are linear operators, $g_j, j \geq 1$ are non-random functions, $\theta \in \Theta \subset R, \{\tilde{W}_J^H, j \geq 1\}$ is a family of *independent* mixed fractional Brownian motions with the same Hurst index H, and the equation is diagonalizable. The equation given above is said to be *diagonalizable* if the operators $\mathcal{A}_0, \mathcal{A}_1$ have the same system of eigenfunctions $\{h_j, j \geq 1\}$ such that $\{h_j, j \geq 1\}$ is an orthonormal basis in $\mathbf{H}$ and each h_j belongs to $\cap_{\gamma\in R}\mathbf{H}^\gamma$. Here $\mathbf{H}$ is a separable Hilbert space and $\mathbf{H}^\gamma$ is as defined in the Section 2.

For some basic properties of a mfBm and properties of processes driven by a mfBm, see Section 2 in Prakasa Rao (2022). We will use here the notation introduced in the Section 2 of Prakasa Rao (2022). We refer the reader to Section 2 there in for more details.

2. Parametric estimation for stochastic parabolic equations driven by an infinite dimensional mfBm

We now extend the work of Cialenco et al. (2009) (cf. Prakasa Rao (2010)) dealing with problems of estimation for SPDEs driven by an infinite dimensional fractional Brownian motion to SPDEs driven by an infinite dimensional mixed fractional Brownian motion. We introduce some notation and introduce the probability structure following the framework developed in Cialenko et al. (2009) (cf. Prakasa Rao (2010)). Even though the results obtained in this section are similar to those corresponding to a SPDE driven by an infinite-dimensional fBm, they are not consequences of those results. Proofs are analogous but the tools are different and depend on the recent results obtained by Chigansky and Kleptsyna (2019) and Cai et al. (2016).

Let $\mathbf{H}$ be a separable Hilbert space with the inner product $(.,.)_0$ and with the corresponding norm $\|.\|_0$. Let Λ be a densely defined linear operator on $\mathbf{H}$ with the property that there exists $c > 0$ such that

$$\|\Lambda u\|_0 \geq c\|u\|_0$$

for every u in the domain of the operator Λ. The operator powers $\Lambda^\gamma, \gamma \in R$ are well defined and generate the spaces $\mathbf{H}^\gamma$ with the properties (i) for $\gamma > 0, \mathbf{H}^\gamma$ is the domain of Λ^γ, (ii) $\mathbf{H}^0 = \mathbf{H}$, and (iii) for $\gamma < 0, \mathbf{H}^\gamma$ is the completion of $\mathbf{H}$ with respect to the norm $\|.\|_\gamma \equiv \|\Lambda^\gamma.\|_0$ (cf. Krein et al. (1982)). The family of spaces $\{\mathbf{H}^\gamma, \gamma \in R\}$ has the following properties:

(i) $\Lambda^\gamma(\mathbf{H}^r) = \mathbf{H}^{r-\gamma}, \gamma, r \in R$;

(ii) For $\gamma_1 < \gamma_2$, the space $\mathbf{H}^{\gamma_2}$ is densely and continuously embedded into $\mathbf{H}^{\gamma_1}$, that is, $\mathbf{H}^{\gamma_2} \subset \mathbf{H}^{\gamma_1}$ and there exists a constant $c_{12} > 0$ such that $\|u\|_{\gamma_1} \leq c_{12}\|u\|_{\gamma_2}$;

(iii) for every $\gamma \in R$ and $m > 0$, the space $\mathbf{H}^{\gamma-m}$ is the dual of the space $\mathbf{H}^{\gamma+m}$ with respect to the inner product in $\mathbf{H}^\gamma$, with duality $\langle .,. \rangle_{\gamma,m}$ given by

$$\langle u_1, u_2\rangle_{\gamma,m} = (\Lambda^{\gamma-m}u_1, \Lambda^{\gamma+m}u_2)_0, \;\; u_1 \in \mathbf{H}^{\gamma-m}, u_2 \in \mathbf{H}^{\gamma+m}.$$

Let $(\Omega, \mathcal{F}, P)$ be a probability space and let $\{\tilde{W}_j^H, j \geq 1\}$ be a family of independent mixed fractional Brownian motions on this space with the same Hurst index H in $(0,1)$.

Consider the SDE

$$du(t) + (\mathcal{A}_0 + \theta \mathcal{A}_1)u(t)dt = \sum_{j\geq 1} g_j(t) d\tilde{W}_j^H(t), \quad 0 \leq t \leq T, \quad u(0) = u_0 \tag{2.1}$$

where $\mathcal{A}_0, \mathcal{A}_1$ are linear operators, $g_j, j \geq 1$ are non-random and $\theta \in \Theta \subset R$. The equation (2.1) is said to be *diagonalizable* if the operators $\mathcal{A}_0, \mathcal{A}_1$ have the same system of eigenfunctions $\{h_j, j \geq 1\}$ such that $\{h_j, j \geq 1\}$ is an orthonormal basis in $\mathbf{H}$ and each h_j belongs to $\cap_{\gamma \in R} \mathbf{H}^\gamma$. It is called (m, γ)-parabolic for some $m \geq 0, \gamma \in R$, if (i) the operator $\mathcal{A}_0 + \theta \mathcal{A}_1$ is uniformly bounded from $\mathbf{H}^{\gamma+m}$ to $\mathbf{H}^{\gamma-m}$ for every $\theta \in \Theta$, that is, there exists $C_1 > 0$ such that

$$\|(\mathcal{A}_0 + \theta \mathcal{A}_1)v\|_{\gamma-m} \leq C_1 \|v\|_{\gamma+m}, \quad \theta \in \Theta, \quad v \in H^{\gamma+m}; \tag{2.2}$$

and (ii) there exists a $\delta > 0$ and $C \in R$ such that,

$$-2\langle(\mathcal{A}_0 + \theta \mathcal{A}_1)v, v\rangle_{\gamma,m} + \delta \|v\|_{\gamma+m}^2 \leq C\|v\|_\gamma^2, \quad v \in \mathbf{H}^{\gamma+m}, \quad \theta \in \Theta. \tag{2.3}$$

If the equation (2.1) is (m, γ)-parabolic, then the condition (ii) implies that

$$\langle(2\mathcal{A}_0 + 2\theta \mathcal{A}_1 + CI)v, v)\rangle_{\gamma,m} \geq \delta \|v\|_{\gamma+m}^2$$

where I is the identity operator. The Cauchy-Schwartz inequality and the continuous embedding of $\mathbf{H}^{\gamma+m}$ into $\mathbf{H}^\gamma$ will imply that

$$\|(2\mathcal{A}_0 + 2\theta \mathcal{A}_1 + CI)v\|_\gamma \geq \delta_1 \|v\|_\gamma$$

for some $\delta_1 > 0$ uniformly in $\theta \in \Theta$.

Let us choose $\Lambda = [2\mathcal{A}_0 + 2\theta_0 \mathcal{A}_1 + CI]^{1/(2m)}$ for some fixed $\theta_0 \in \Theta$. If the operator $\mathcal{A}_0 + \theta \mathcal{A}_1$ is unbounded, we say that $\mathcal{A}_0 + \theta \mathcal{A}_1$ has order $2m$ and Λ has order 1. If the equation (2.1) is (m, γ)-parabolic and diagonalizable, we will assume that the operator Λ has the same eigenfunctions as the operators $\mathcal{A}_0$ and $\mathcal{A}_1$. This is justified by the comments made above.

Suppose the equation (2.1) is diagonalizable.Then there exist eigenvalues $\{\rho_j, j \geq 1\}, \{\nu_j, j \geq 1\}$ such that

$$\mathcal{A}_0 h_j = \rho_j h_j \text{ and } \mathcal{A}_1 h_j = \nu_j h_j.$$

Without loss of generality, we can also assume that there exists $\{\lambda_j, j \geq 1\}$ such that

$$\Lambda h_j = \lambda_j h_j.$$

Following the arguments in Cialenco et al. (2009), it can be shown that the equation (2.1) is (m, γ)-parabolic if and only if there exists $\delta > 0, C_1 > 0$ and $C_2 \in R$ such that, for all $j \geq 1, \theta, \in \Theta$,

$$|\rho_j + \theta \nu_j| \leq C_1 \lambda_j^{2m} \tag{2.4}$$

and

$$-2(\rho_j + \theta \nu_j) + \delta \lambda_j^{2m} \leq C_2. \tag{2.5}$$

As the conditions in (2.4) and (2.5) do not depend on γ, we conclude that a diagonalizable equation (2.1) is (m,γ)-parabolic for some γ if and only if it is (m,γ)-parabolic for every γ. We will say that the equation (2.1) is m-parabolic. We will assume that the equation (2.1) is diagonalizable and fix the basis $\{h_j, j\geq 1\}$ in $\mathbf{H}$ consisting of the eigenfunctions of $\mathcal{A}_0$, $\mathcal{A}_1$ and λ. Observe that the set of eigenfunctions is the same for all the three operators. Since h_j belongs to every $\mathbf{H}^\gamma$, and since $\cap_\gamma \mathbf{H}^\gamma$ is dense in $\cup_\gamma \mathbf{H}^\gamma$, every element f of $\cup_\gamma \mathbf{H}^\gamma$, has a unique expansion $\sum_{j\geq 1} f_j h_j$ where $f_j =< f, h_j >_{0,m}$ for suitable m.

The functional structure described above follows the work in Cialenco et al. (2009) and Prakasa Rao (2010).

Definition 2.1: The *infinite dimensional mixed fractional Brownian motion* $\tilde{W}^H$ is an element of $\cup_{\gamma\in R}\mathbf{H}^\gamma$ with the expansion

$$\tilde{W}^H(t) = \sum_{j\geq 1} \tilde{h}_j \tilde{W}^H_j(t). \tag{2.6}$$

The solution of the diagonalizable equation

$$du(t) + (\mathcal{A}_0 + \theta\mathcal{A}_1)u(t)dt = d\tilde{W}^H(t), \quad 0\leq t\leq T, \quad u(0)=u_0, \tag{2.7}$$

with $u_0\in\mathbf{H}$, is defined to be a random process $u(t), 0<t\leq T$, with values in $\cup_\gamma\mathbf{H}^\gamma$ and has an expansion

$$u(t) = \sum_{j\geq 1} h_j u_j(t) \tag{2.8}$$

where

$$u_j(t) = (u_0,h_j)_0 e^{-(\theta\nu_j+\rho_j)t} + \int_0^t e^{-(\theta\nu_j+\rho_j)(t-s)}\ d\tilde{W}^H_j(s). \tag{2.9}$$

Let

$$\mu_j(\theta) = \theta\nu_j + \rho_j, \quad j\geq 1. \tag{2.10}$$

In view of the equation (2.5), we get that there exists a positive integer J such that

$$\mu_j(\theta) > 0 \ \text{ for } \ j\geq J \tag{2.11}$$

if the equation (2.1) is m-parabolic and diagonalizable.

Theorem 2.1 : *Suppose that $H\geq\frac{1}{2}$ and the equation (2.1) is m-parabolic and diagonalizable. Further suppose that there exists a positive real number γ such that*

$$\sum_{j\geq 1}(1+|\mu_j(\theta)|)^{-\gamma} < \infty. \tag{2.12}$$

Then, for every $t>0, \tilde{W}^H(t)\in L_2(\Omega,\mathbf{H}^{-m\gamma})$ and $u(t)\in L_2(\Omega,\mathbf{H}^{-m\gamma+m\min(2H,1)})$.

Proof : The condition (2.12) implies that $\lim_{j\to\infty}|\mu_j| = \infty$, and hence the operators $\mathcal{A}_0+\theta\mathcal{A}_1$ and Λ are unbounded. The parabolicity assumption and the equations (2.4) and (2.5) imply that, for sufficiently large j,

$$1+|\mu_j(\theta)|^m \leq C_2\lambda_j^{2m}$$

uniformly in $\theta \in \Theta$. Furthermore

$$E\|\tilde{W}^H(t)\|^2_{-m\gamma} = (t^{2H}+t)\sum_{j\geq 1}\lambda_j^{-2m\gamma} \leq C_2(t^{2H}+t)\sum_{j\geq 1}(1+\mu_j(\theta)|^{-\gamma} < \infty.$$

From the definition of the mixed fractional Brownian motion and the fact that the component fractional Brownian motion and the Brownian motion are independent and centered, it follows that

$$\begin{aligned} E[u_j^2(t)] &= H(2H-1)e^{-2\mu_j(\theta)t}\int_0^t\int_0^t e^{\mu_j(\theta)(s_1+s_2)}|s_1-s_2|^{2H-2}ds_1ds_2 \\ &\quad +e^{-2\mu_j(\theta)t}\int_0^t e^{2\mu_j(s)}ds \\ &= J_1(t)+J_2(t) \text{ (say)} \end{aligned} \tag{2.13}$$

by the properties of a fractional Brownian motion (cf. Prakasa Rao (2010)) and the Brownian motion. It can be checked that

$$\lim_{j\to\infty}|\mu_j(\theta)|^{2H}J_1(t) = H(2H-1)\int_0^\infty x^{2H-2}e^{-x}dx = H(2H-1)\Gamma(2H-1)$$

and

$$\lim_{j\to\infty}|\mu_j(\theta)|J_2(t) = \frac{1}{2}.$$

Hence

$$\lim_{j\to\infty}|\mu_j(\theta)|^{\min(2H,1)}E(u_j^2(t)) < \infty$$

and

$$\sum_{j=1}^\infty(1+|\mu_j(\theta)|)^{-\gamma+\min(2H,1)}E(u_j^2(t)) < \infty.$$

Maximum likelihood estimation :

Consider the diagonalizable equation

$$du(t)+(\mathcal{A}_0+\theta\mathcal{A}_1)u(t)dt = d\tilde{W}^H(t), \quad u(0)=0, \quad 0\leq t\leq T \tag{2.14}$$

with

$$u(t) = \sum_{j\geq 1}h_ju_j(t) \tag{2.15}$$

as given by (2.9). Suppose the processes $\{u_i(t), 0\leq t\leq T\}, i=1,\ldots,N$ can be observed continuously over the interval $[0,T]$. The problem is to estimate the parameter θ using these observed paths over the interval $[0,T]$.

Recall that $\mu_j(\theta) = \rho_j+\nu_j\theta$, where ρ_j and ν_j are the eigenvalues of the operators $\mathcal{A}_0$ and $\mathcal{A}_1$ respectively. Furthermore each process u_j is a mixed fractional Ornstein-Uhlenbeck process satisfying the stochastic differential equation

$$du_j(t) = -\mu_j(\theta)u_j(t)dt+d\tilde{W}_j^H(t), \quad u_j(0)=0, \quad 0\leq t\leq T. \tag{2.16}$$

Since the processes $\{\tilde{W}_j^H, j \geq 1\}$ are independent, it follows that the processes $\{u_j, 1 \leq j \leq N\}$ are independent. Following the notation introduced, let

$$M_j^H(t) = \int_0^t g_H(s,t) d\tilde{W}_j^H(s), Q_j(t) = \frac{d}{dw_j^H(t)} \int_0^t g_H(s,t) u_j(s) ds \tag{2.17}$$

and

$$Z_j(t) = \int_0^t g_H(s,t) du_j(s) \tag{2.18}$$

for $j = 1, \ldots, N$ where the function $g_H(s,t)$ is as defined by the equation (5) in Prakasa Rao (2022). Applying the Girsanov-type formula, it can be shown that the measure generated by the process $(u_1, \ldots, u_N)$ is absolutely continuous with respect to the measure generated by the process $(\tilde{W}_1^H, \ldots, \tilde{W}_N^H)$ and their Radon-Nikodym derivative is given by

$$\exp(-\sum_{j=1}^N \mu_j(\theta) \int_0^T Q_j(s) dZ_j(s) - \sum_{j=1}^N \frac{[\mu_j(\theta)]^2}{2} \int_0^T Q_j^2(s) dw_H(s)). \tag{2.19}$$

Maximizing this function with respect to the parameter θ, we get the maximum likelihood estimator

$$\hat{\theta}_N = -\frac{\sum_{j=1}^N \int_0^T \nu_j Q_j(s)(dZ_j(s) + \rho_j Q_j(s) dw_H(s))}{\sum_{j=1}^N \int_0^T \nu_j^2 Q_j^2(s) dw_H(s)}.$$

Theorem 2.2 : *Suppose the Hurst index $H > \frac{1}{2}$. Further suppose that*

$$\sum_{j=1}^\infty \frac{\nu_j^2}{\mu_j(\theta)} = \infty. \tag{2.20}$$

Then

$$\lim_{N\to\infty} \hat{\theta}_N = \theta \text{ a.s.}$$

where θ is the true parameter.

Proof : Observe that

$$\hat{\theta}_N - \theta = -\frac{\sum_{j=1}^N \int_0^T \nu_j Q_j(s) dM_j^H(s)}{\sum_{j=1}^N \int_0^T \nu_j^2 Q_j^2(s) dw^H(s)}. \tag{2.21}$$

The numerator of the expression on the right side (2.21) is a local martingale and the denominator is its quadratic variation. It is known that

$$\lim_{j\to\infty} \mu_j(\theta) \;\; E[\int_0^T Q_j^2(s) dw_H(s)] = \frac{T}{2} > 0 \tag{2.22}$$

from the equation (2.1) in Chigansky and Kleptsyna (2019) (cf. Maruskeevych (2016)) which implies that

$$\sum_{j=1}^{N}\int_0^T \nu_j^2 Q_j^2(s)dw^H(s)\to\infty \quad \text{a.s. as } \ N\to\infty$$

under the condition (2.20). Hence, an application of the Strong law of large numbers for local martingales (cf. Prakasa Rao (1999)) will imply that the expression on the right side of the equation (21) converges to zero almost surely and hence

$$\hat{\theta}_N-\theta\to 0 \ \text{ almost surely}$$

as $N\to\infty$.

Theorem 2.3: *Suppose there exists a norming function I_N such that*

$$I_N^2\sum_{j=1}^{N}\int_0^T \nu_j^2 Q_j^2(s)dw^H(s)\to\eta^2 \ \textit{ in probability as } \ N\to\infty \tag{2.23}$$

where η is a random variable such that $P(\eta>0)=1$. Let

$$R_N=\sum_{j=1}^{N}\int_0^T \nu_j Q_j(s)dM_j^H(s). \tag{2.24}$$

Then

$$(I_N R_N, I_N^2\langle R_N\rangle)\to(\eta Z,\eta^2) \ \textit{ in law as } \ N\to\infty \tag{2.25}$$

where the random variable Z has the standard normal distribution and the random variables Z and η are independent. Here $\langle R_N\rangle, N\geq 1$ is the quadratic variation of the process $\{R_N, N\geq 1\}$. Furthermore

$$I_N^{-1}(\hat{\theta}_N-\theta)\to\frac{Z}{\eta} \ \text{ in law as } \ N\to\infty \tag{2.26}$$

Proof: This theorem follows as a consequence of the central limit theorem for local martingales (cf. Theorem 1.49 and Remark 1.47, Prakasa Rao (1999)) and observing that

$$I_N^{-1}(\hat{\theta}_N-\theta)=\frac{I_N R_N}{I_N^2\langle R\rangle_N}, \quad N\geq 1. \tag{2.27}$$

Remarks : If the random variable η in Theorem 2.3 is a constant almost surely, then the limiting distribution of the maximum likelihood estimator $\hat{\theta}_N$ is Gaussian with mean zero and variance η^{-2}. Otherwise it is a mixture of the normal distributions with mean zero and variance η^{-2} with the mixing distribution as that of the random variable η. Applying Berry-Esseen bound for sums of independent random variables, it is possible to obtain the limiting distribution and the rate of convergence of the distribution of the MLE if a bound on the variance of the random variable

$$\int_0^T Q_j^2(s)\, dw^H(s)$$

can be obtained as $j\to\infty$ after suitable norming as in Lemma A.2 in Cialenco et al. (2009) in the fractional Brownian case. It has not been possible to obtain a similar result in the case of mixed fractional Brownian motion as the function $g_H(s,t)$ defining the martingale M_j^H defined by (2.17) is not explicitly known. The problem of obtaining a closed form for the function $g_H(s,t)$ remains open.

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References

Cai, C., Chigansky, P., and Kleptsyna, M. (2016). Mixed Gaussian processes: A filtering approach, *Ann. Probab.*, 44: 3032–3075.

Chigansky, P., and Kleptsyna, M. (2019). Statistical analysis of the mixed fractional Ornstein-Uhlenbeck process. *Theory Probab. Appl.*, 63: 408–425.

Cialenco, I., Lototsky, S. G., and Pospisil, J. (2009). Asymptotic properties of the maximum likelihood estimator for stochastic parabolic equations with additive fractional Brownian motion, *Stoch. and Dynam.*, 9: 169–185.

Krein, S. G., Petunin, Y. I., and Semenov, E. M. (1982). *Interpolation of linear operators*, Volume 54 of *Translations of Mathematical Monographs*, American Mathematical Society, Providence, Rhode Island.

Marushkevych, D. (2016). Large deviations for drift parameter estimator of mixed fractional Ornstein-Uhlenbeck process, *Mod. Stoch.: Theory Appl.*, 3: 107–117.

Prakasa Rao, B. L. S. (1999). *Semimartingales and Their Statistical Inference*, Boca Raton: CRC Press and London: Chapman and Hall.

Prakasa Rao, B. L. S. (2010). *Statistical Inference for Fractional Diffusion Processes*, London: Wiley.

Prakasa Rao, B. L. S. (2022). Parametric estimation for SPDEs driven by an infinite dimensional mixed fractional Brownian motion, *Bull. Inf. Cyb.*, 54(2): 1–14.

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