Shear Correction Factor Effect on Functionally Graded Material (FGM) Beam with Timoshenko Hencky Beam (THB) Element

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Shear Correction Factor Effect on Functionally Graded Material (FGM) Beam with Timoshenko Hencky Beam (THB) Element

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Abstract: This study is about the effect of the Shear Correction Factor on static analysis of a Functionally Graded Material (FGM) beam using the Timoshenko Hencky Beam (THB) element. The material properties are evaluated according to the Power Law index. The convergence of the FGM beam by using the constant shear correction factor is compared with the non-constant shear correction factor (called k-FGM). This study shows that the shear correction factor did not affect the displacement in a thick FGM beam with a small Young modulus ratio (0.35). The shear correction factor significantly affects the displacement in an FGM beam with a high Young modulus ratio (20). The results yield that the THB element can be used to apply FGM beams structures.

Keywords: Functionally Graded Material; Timoshenko Hencky Beam; static evaluation; MATLAB

1. Introduction

Functionally Graded Material (FGM) are materials that change in composition, structure, and properties as they function in the spatial direction of the material. The FGM concept was first proposed by a materials researcher from the Sendai, Japan, in 1984. The material composition of FGM can vary, depending on the specifications required. FGM can be produced with continuous gradation variations so that the material properties of FGM are continuous, do not require an interface to reduce debonding or cracking within the materials, and can satisfy the needs of high performance in very high temperatures environments. FGM can be applied to various fields such as aerospace, aircraft, automotive, and civil structures (R. Menaa, 2012 [1]).

In structural problems, generally, the approach taken for analysis is using the Finite Element Method (FEM). The recent development of FEM in plate and shell structures can be found in (Katili et al. 2015, Katili et al. 2014, Katili et al. 2018a, Maknun et al. 2016, Irpanni et al. 2017, Katili et al. 2018b, Katili et al. 2017, Katili et al. 2018c, Katili et al. 2018d, Katili et al. 2019a, Katili et al. 2019b, Maknun et al. 2020, Maknun et al. 2015, Katili et al. 2021, Maknun et al 202, Sidara et al. 2021, Harahap et al 2021 and Maknun et al. 2020 [2-19]). In composite application can be found in (Kumar et al. 2017, Kumar et al. 2021a, Akash et al. 2020, Amr et al. 2017 [20-23]). FEM gives a good performance in beam analysis. Various beam analysis methods have been used, but due to the complexity of the theory, assumption theory is used for simplification. The most common theories are the Bernoulli-Euler Beam theory and the Timoshenko Beam Theory. Timoshenko's beam theory is a theory that considers the effect of shear deformation on the beam and requires a shear correction factor k. One of the elements based on Timoshenko's beam theory is the Timoshenko Hencky Beam (THB) element. There are 3 degrees of freedom of the THB elements in each node, namely the axial displacement (u), the vertical displacement (v), and rotation (θ).

In calculating the effect of shear forces, a correction factor k is needed to improve the ideal shear stiffness in the beam. These factors affect the level of accuracy of the theory used. The shear correction factor value commonly used for beam cross-sections in isotropic materials is 5/6. However, the constant shear correction factor value cannot be used on FGM beams. So it is necessary to have a shear correction factor value obtained from the energy equivalent equation.

In this study, an evaluation of the effect of the shear correction factor will be carried out in the case of a static FGM beam using the power-law index. Timoshenko Hencky Beam (THB) elements will be employed in this paper. This study will also evaluate the phenomenon of shear locking on the THB element.

2. Literature Review

2.1 Functionally Graded Material Theory

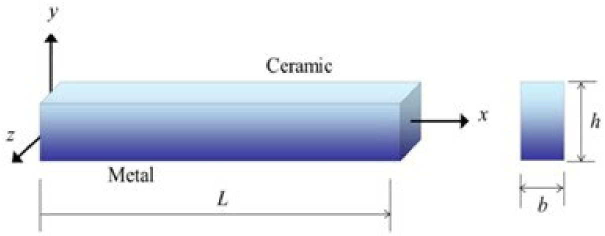

Fig. 1: Illustration of FGM Beam

Functionally Graded Material is the new concept of composite materials, consisting of two or more types but with a composition that changes continuously according to specific functions (Fig. 1). FGM can be used in various fields, such as for aerospace needs (rocket components), nuclear projects (plasma walls of fusion reactors), communications sector (optical fiber, lenses), the energy sector (thermoelectric generators, solar cells), medical sector (artificial bone) and other uses such as materials in buildings. In this study, the power-law function will be used as a volume variation of material properties. Power law function is expressed by:

$$V_{up}(y) = \left(\frac{y}{h} + \frac{1}{2}\right)^p \tag{1}$$

$$V_{lw}(y) = 1 - V_{up} \tag{2}$$

The Modulus of elasticity $E(y)$ function of the FGM beam based on the power law is defined:

$$E(y) = (E_{up} - E_{lw})\left(\frac{y}{h} + \frac{1}{2}\right)^p + E_{lw} \tag{3}$$

2.2 Timoshenko Beam Theory

Beams are an essential component in civil engineering, so they have become a popular topic for researchers. Various beam analysis methods have been used, but due to the complexity of the theory, the assumption is used for simplification. The theory used in this study is the Timoshenko beam theory. In the Timoshenko Beam Theory, shear deformation effects are considered in the formulation and require a shear correction factor (k). The kinematics of the Timoshenko beam theory is:

$$\theta = \frac{dv}{dx} - \gamma \tag{4}$$

$$\chi = -\theta_{,x} = -\frac{d\theta}{dx} \tag{5}$$

$$\gamma = v_{,x} - \theta = \frac{dv}{dx} - \theta \tag{6}$$

Because the beam consists of two materials with different characteristics, the center of gravity is no longer at the center of the beam, and it must be included in the calculation, therefore:

$$e = \frac{du}{dx} \tag{7}$$

$$\varepsilon = u_{,x} - y\theta_{,x} = e + y\chi \tag{8}$$

2.3 Timoshenko Hencky Beam Element

The Timoshenko Hencky Beam (THB) element is formulated based on the Timoshenko beam theory, with rotation (θ) being independent, not a derivative of the translation function (v). The energy equation obtains the stiffness equation for the THB element.

$$\Pi = \Pi_{int} - \Pi_{ext} \tag{9}$$

$$\Pi_{int} = \frac{1}{2}\iiint_v \varepsilon\sigma dV + \frac{1}{2}\iiint_v \gamma\tau dV \tag{10}$$

The internal energy can be grouped according to the kinematics of the beam as follows:

$$\Pi^a_{int} = \frac{1}{2} D_a \int_0^L e^2 dx \tag{11}$$

$$\Pi^{ab}_{int} = \frac{1}{2} D_{ab} \int_0^L (e\chi) dx \tag{12}$$

$$\Pi^b_{int} = \frac{1}{2} D_b \int_0^L \chi^2 dx \tag{13}$$

$$\Pi^s_{int} = \frac{1}{2}[bGh]\int_0^L \gamma^2 dx \tag{14}$$

With:

$$D_a = bh\left(\frac{E_{up} - E_{lw}}{p+1} + E_{lw}\right) \tag{15}$$

$$D_{ab} = bh^2\left(\frac{(E_{up} - E_{lw})p}{2(p+1)(p+2)}\right) \tag{16}$$

$$D_b = bh^3\left(\frac{(E_{up} - E_{lw})(p^2 + p + 2)}{4(p+1)(p+2)(p+3)} + \frac{E_{lw}}{12}\right) \tag{17}$$

The approximation of displacement (Fig 2.) function of THB element in FGM beam application are :

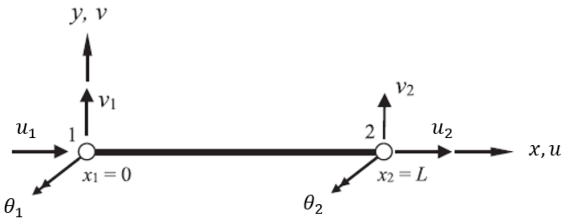

Fig. 2: Illustration of the beam with two nodes

$$u = \langle N_1 \quad 0 \quad 0 \quad N_2 \quad 0 \quad 0 \rangle \{U_n\} \tag{18}$$

$$v = \langle 0 \quad N_1 \quad 0 \quad 0 \quad N_2 \quad 0 \rangle \{U_n\} \tag{19}$$

$$\theta = \langle 0 \quad 0 \quad N_1 \quad 0 \quad 0 \quad N_1 \rangle \{U_n\} \tag{20}$$

With

$$N_1 = 1 - \frac{x}{L} \tag{21}$$

$$N_2 = \frac{x}{L} \tag{22}$$

$$L = x_2 - x_1 \tag{23}$$

$$\langle U_n \rangle = \langle u_1 \quad v_1 \quad \theta_1 \quad u_2 \quad v_2 \quad \theta_2 \rangle \tag{24}$$

The values of axial deformation, curvature, and shear deformation are:

$$e = u_{,x} = \left\langle -\frac{1}{L} \quad 0 \quad 0 \quad \frac{1}{L} \quad 0 \quad 0 \right\rangle \{U_n\} = \langle B_a \rangle \{U_n\} \tag{25}$$

$$\chi = -\theta_{,x} = \left\langle 0 \quad 0 \quad \frac{1}{L} \quad 0 \quad 0 \quad -\frac{1}{L} \right\rangle \{U_n\} = \langle B_b \rangle \{U_n\} \tag{26}$$

$$\gamma = v_{,x} - \theta = \left\langle 0 \quad -\frac{1}{L} \quad -1 + \frac{x}{L} \quad 0 \quad \frac{1}{L} \quad -\frac{x}{L} \right\rangle \{U_n\} = \langle B_s \rangle \{U_n\} \tag{27}$$

Therefore, the internal energy equations become:

$$\Pi^{a}_{\text{int}} = \frac{1}{2}[D_a]\langle U_n \rangle \int \{B_a\} \langle B_a \rangle \{U_n\}\, dx \tag{28}$$

$$\Pi^{ab}_{\text{int}} = \frac{1}{2}[D_{ab}]\langle U_n \rangle \int \{B_a\} \langle B_b \rangle \{U_n\}\, dx \tag{29}$$

$$\Pi^{b}_{\text{int}} = \frac{1}{2}[D_b]\langle U_n \rangle \int \{B_b\} \langle B_b \rangle \{U_n\}\, dx \tag{30}$$

$$\Pi^{s}_{\text{int}} = \frac{1}{2}[D_s]\langle U_n \rangle \int \{B_s\} \langle B_s \rangle \{U_n\}\, dx \tag{31}$$

With

$$D_s = k \frac{1}{2(1+\nu)} \left(\frac{E_{up} - E_{lw}}{n+1} + E_{lw} \right) bh \tag{32}$$

For the external energy caused by a constant external force:

$$\Pi^{e}_{ext} = \{f_n\} \langle U_n \rangle \tag{33}$$

$$\langle f_n \rangle = f_y \frac{L}{2} \langle 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \rangle \tag{34}$$

2.4 Shear Locking on Timoshenko Hencky Beam Element

Theoretically, the effect of shear forces can be ignored in the thin beam problem, and the value of shear stiffness is close to zero. In the THB element, the shear stiffness value increases and dominates the stiffness value due to bending for thin beams case. This phenomenon is called shear locking. The shear locking phenomenon causes the THB element to require a larger number of elements to achieve convergence results, so the time required for calculation will certainly be longer. To solve this problem, an approach is used by assuming that the value of the shear deformation γ is constant at mid-span so that [*Bs*] matrix becomes:

$$\langle B_s \rangle = \left\langle 0 \quad -\frac{1}{L} \quad -0.5 \quad 0 \quad \frac{1}{L} \quad -0.5 \right\rangle \tag{35}$$

2.5 Shear Correction Factor Theory

Most FGM beam analysis using the Timoshenko beam theory requires a shear correction factor. This coefficient assumes that the shear strain force is uniform at a depth of the cross-section. For objects with homogeneous cross-sections and isotropic materials, the shear correction factor depends on the geometry of the cross-section. For example, values of 5/6 are used for rectangular sections and 6/7 for circular sections (I. Katili, 2006 [24]). The formulation of the shear correction factor for the FGM (*k*-FGM) beam is as follows (Meena, 2012 [1]):

$$k = \frac{1}{\int_{-h/2}^{h/2} \frac{E(y)}{2(1+v)}} \left[\int_{-h/2}^{h/2} \frac{\left[\int_{-h/2}^{y} \left(\frac{E(y)}{1-v^2}\beta_{11} + \frac{vE(y)}{1-v^2}\beta_{12} + y\frac{E(y)}{1-v^2}\delta_{11} + y\frac{vE(y)}{1-v^2}\delta_{12} \right) dy \right]^2}{\frac{E(y)}{2(1+v)}} dy \right] \quad (36)$$

3. Numerical Result

The study was conducted by evaluating two cases of beams using 2, 4, 8, 16, 32, 64, and 128 elements with seven power-law factors (p = 0, 0.2, 0.5, 1, 2, 5, and 10). Six boundary conditions (simply supported, pin-pin supported, fixed-fixed supported, fixed-simple supported, fixed-roll supported, and cantilever beam) are evaluated. Two cases are evaluated, the first case is for a thick FGM beam with a small Young modulus ratio (0.35), and the second is a thick FGM beam with a high Young modulus ratio (20).

3.1 Case 1 (E_{up}/E_{lw}=0.35)

The simply supported beam with a uniform distributed load q in Fig. 3 (proposed by Simsek, 2009 [25]) is evaluated. The materials used are Aluminum (*Al*) with a modulus of elasticity E_{up} 70 GPa at the top of the beam, and Zirconia (ZrO_2) with a modulus of elasticity E_{lw} 200 GPa, both with a constant Poisson ratio v =0.3. This test employed two-length thickness ratios, *L/h* = 4 as a thick beam and *L/h* = 500 for a thin beam. The study also used *L/h* = 16, 100, and 500 to evaluate shear locking in thin beams with simply supported boundary conditions.

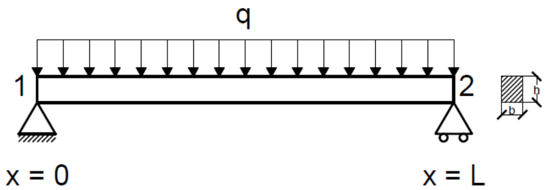

Fig. 3: Illustration of the simply supported beam

The displacement differences in case 1 show that the use of a constant shear correction factor value or shear correction factor according to the FGM theory (called *k* FGM) does not have much effect on the displacement. It found that the different values ranged from 0.185% to 1.395% for thick beams and 2.9% for thin beams. Details can be seen in table 1. This is due to the value of the shear correction factor *k* FGM being similar to 5/6 (constant shear correction factor).

Table 1. Numerical result for case 1

Boundary Condition	***L/h***	
	4	**500**
Simple supported beam	0.412%	2.934%
Pin-pin supported beam	0.419%	2.925%
Fixed-fixed supported beam	1.395%	2.934%
Fixed-simple supported beam	0.939%	2.933%
Fixed-roll supported beam	0.938%	2.934%
Cantilever beam	0.185%	2.934%

The following Figures 4 - 7 are a graphic illustration of case 1 for the value of power-law five as a value with the most displacement difference compared to other power-law factors:

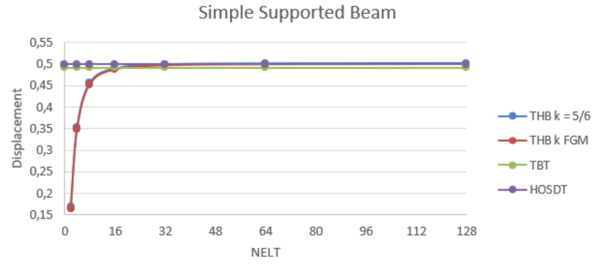

Fig. 4: simply supported boundary condition

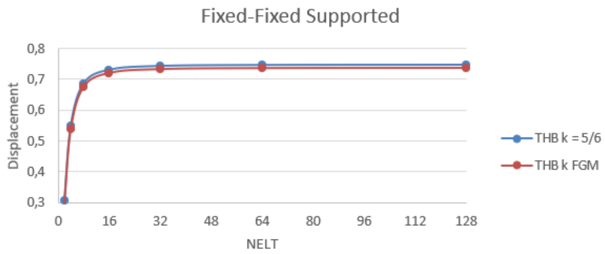

Fig. 5: fixed-fixed boundary condition

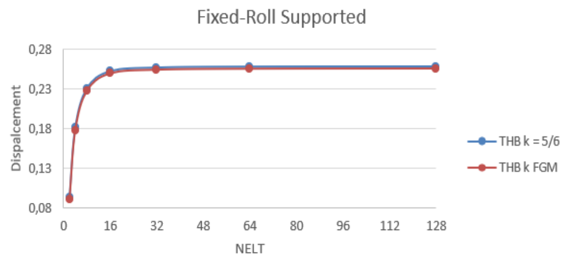

Fig. 6: fixed-roll boundary condition

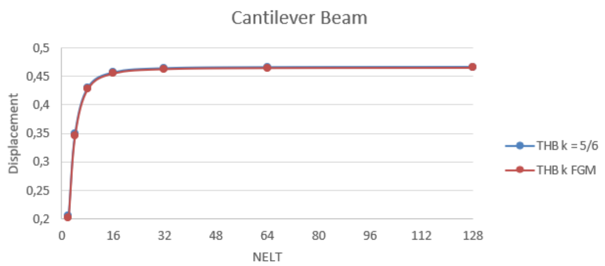

Fig. 7: cantilever boundary condition

Case 1 also evaluated shear locking on the THB element for thin beams with ratios L/h = 16, 100, and 500 (table 2). It was found that when the deformation is assumed in the middle of the span, it will reduce shear locking and is compatible with the theory that the shear energy has a negligible effect on thin beams.

Table 2. Numerical result for shear locking on simply supported beam on case 1

L/h	Displacement Gap
4	0.412%
16	0.048%
16 with constant γ	0.029%
100	0.625%
100 with constant γ	0.001%
500	2.934%
500 with constant γ	0.000%

Figures 8 - 10 show the result of the evaluation for the shear locking phenomenon in graphical form. It found that using a constant shear strain can reduce the effect of shear locking.

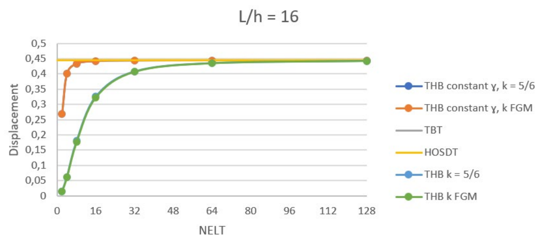

Fig. 8: Result of shear locking in case 1 simply supported beam with *L/h* 16

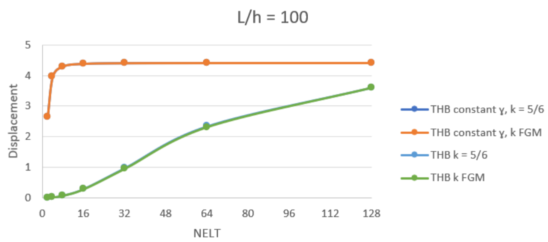

Fig. 9: Graphic result of shear locking in case 1 simply supported beam with *L/h* 100

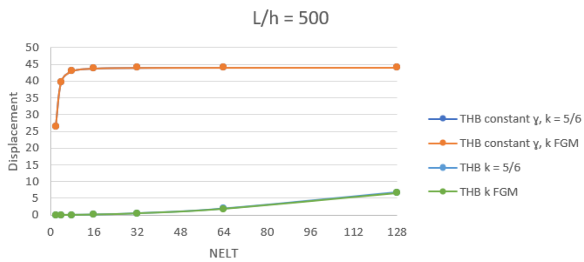

Fig. 10: Graphic result of shear locking in case 1 simply supported beam with *L/h* 500

3.1 Case 2 (E_{up}/E_{lw}=20)

This case is intended to evaluate the behavior of beams with a high ratio of Young's modulus ratio. The material used is a material with a Young modulus of E_{lw} 70 GPa at the bottom of the beam and material with a Young modulus of E_{up} 1400 GPa with a uniform Poisson ratio v= 0.3. Looking at the results of the previous case where there is no significant difference between thick and thin beams and THB elements that are not suitable for use on thin beams, in this case, only $L/h = 4$ (thick beams) is evaluated. The displacement results in case 2 using a constant correction factor or the FGM theory have a significant effect, with different values ranging from 4.447% to 25.76% for thick beams. Details can be seen in table 3:

Table 3. Numerical result for case 2

Boundary Condition	Displacement Gap
Simply supported beam	9.381%
Pin-pin supported beam	13.529%
Fixed-fixed supported beam	25.760%
Fixed-simple supported beam	19.593%
Fixed-roll supported beam	18.986%
Cantilever beam	4.447%

The following (figures 11 – 14) are a graphic illustration of case 2 for the value of power-law (p=10) as a value with the most displacement difference compared to another power-law factor. The results show that for the high Young's modulus ratio, the k FGM must be used in analysis to get the accurate results.

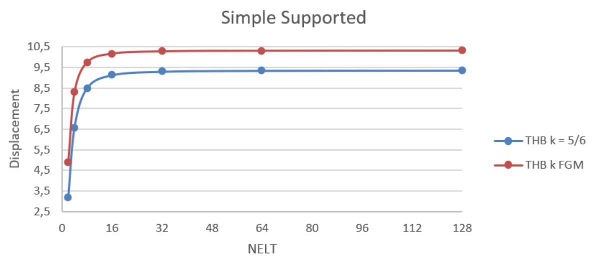

Fig. 11: Simply supported boundary condition

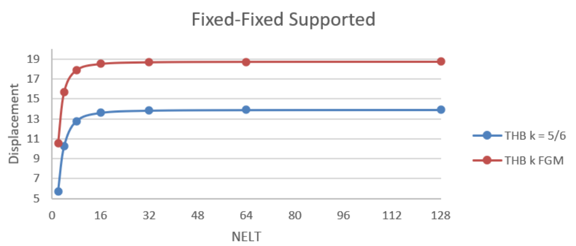

Fig. 12: fixed-fixed boundary condition

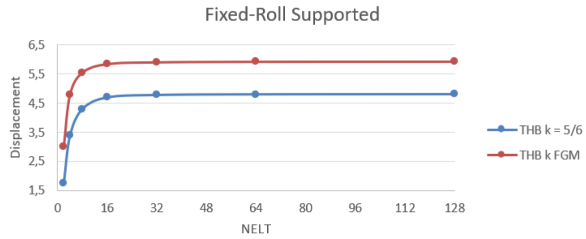

Fig. 13: fixed-roll boundary condition

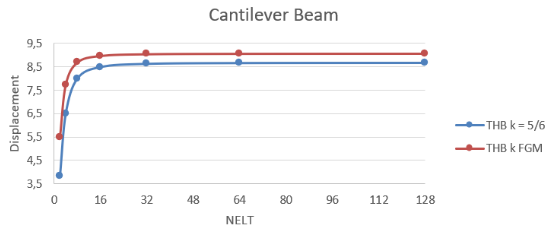

Fig. 14: cantilever boundary condition

3. Conclusions

The effect of the Shear Correction Factor on static analysis of a Functionally Graded Material (FGM) beam has been evaluated using the THB element. Different boundary conditions and slenderness ratio are performed in this study. Moreover, two modulus ratios are studied in this paper. This study shows that the Young's Modulus Ratio used affects the convergence results. At a low Young's modulus difference ratio, the shear correction factor used does not affect much regardless of the power-law value used. Though when the ratio of Young Modulus is high, the shear correction factor used has an effect, especially when the power-law value is greater than one. In addition, in found that the shear locking effect in THB element can be reduced by using constant shear deformation.

Acknowledgments

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Nomenclature

FGM	Functionally Graded Materials
THB	Timoshenko Hencky Beam
FEM	Finite Element Method
k	shear correction factor
u	degree of freedom on the axial direction
v	degree of freedom on the vertical direction
θ	degree of freedom on the rotation direction
V_{up}	volume of the upper material
V_{lw}	volume of the lower material
y	y-axis

h	beam thickness
p	power law index
E_{up}	the elasticity modulus of the upper material
E_{lw}	The elasticity modulus of the lower material
$E(y)$	elasticity modulus in y function
Π	energy equation
Π_{int}	internal energy
Π_{ext}	external energy
G	shear modulus

Greek symbols

γ	shear deformation
χ	curvature
e	axial deformation
ε	strain
σ	stress
τ	shear stress

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