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MOTIONS OF MOORED FLOATING BODY AND DYNAMIC TENSION OF MOORING LINES IN REGULAR WAVES*

By Wataru KOTERAYAMA**

In this paper, the theoretical and the experimental studies of moored floating cylinder and tension of mooring lines in waves are made.

The main conclusions obtained are as follows.

(1) Heaving and swaying motion of moored floating cylinder in regular waves can be estimated theoretically. As for roll, the calculated values are noticeably higher than those given by experimental results at resonance frequency.

(2) The experimental results of the average tension of mooring line in waves are interpreted theoretically.

(3) The experimental results of the tension amplitude of mooring chain due to oscillation of floating cylinder in waves are much larger than calculated results by using static catenary theory.

Here is proposed an approximate calculation method in which both an inertia force and hydrodynamic forces acting on the chain are taken into account. The experimental results agree with the calculated results.

Key word: Moored floating body, Dynamic tension, Approximate calculation method

1. Introduction

The present naval architecture technology has not devoted extensive efforts to the investigation of the tension of lines mooring a floating body in waves. So far ships and floating structures were moored in a harbor or calm sea; therefore demand for studying this problem from practical point of view were not serious. But the recent ocean development in the deep sea has created the needs to investigate the problem in detail. In this paper, the theoretical and experimental studies of motions of moored floating cylinder and tension of mooring lines in beam sea are made, and the validity

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of the linear theory are examined. An approximate method for calculating the dynamic tension of mooring chains are obtained.

2. Drifting force and motions of free floating cylinder in beam sea

Experiments and calculations on the drifting force and motions of a free floating cylinder in beam sea were carried out in order to compare with those of a moored cylinder. The section contour of the cylinder is expressed by a Lewis form ($\sigma=0.942$), and the principal dimensions are shown in Table 1. The experimental apparatus are shown in Fig. 1. Let Z, Y, and Φ represent the displacements of heaving, swaying and rolling motion, then the equations of the motions are expressed as[1]

Table 1 Particulars of floating cylinder and chain.

Model			
$\triangle$	282.6kg	σ	0.942
L	3.0m	KB	0.105m
B	0.5m	Ho(=B/2d)	1.25
d	0.2m	KG	0.177m
		GM	0.038m
Chain			
So	15.0m		
w	0.127kg/m		

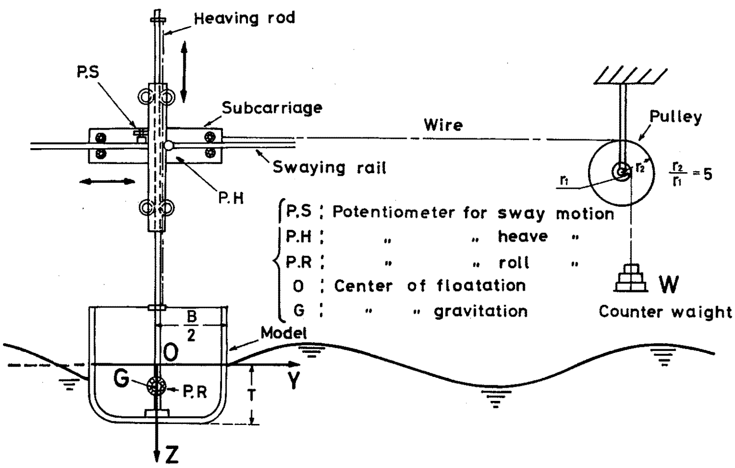

Fig. 1 Apparatus for mesuring motions and drifting force.

$$
\rho\nabla L(1+m_z)\ddot{Z}+N_zL\dot{Z}+(\rho gBL+C_{zz})Z=\frac{i\rho g}{K}\zeta_a\bar{A}_zLe^{i(\varepsilon_H+\omega t)}
$$

$$
\left.\begin{aligned}
&\rho\nabla L(1+m_y)\ddot{Y}+N_yL\dot{Y}+C_{yy}Y+a_{y\varphi}L\ddot{\phi}+b_{y\varphi}L\dot{\phi}+C_{y\varphi}\phi=\frac{i\rho g}{K}\zeta_a\bar{A}_yLe^{i(\varepsilon_S+\omega t)}\\
&(J_\varphi+I_{\varphi\varphi})L\ddot{\phi}+N_{\varphi G}L\dot{\phi}+(\rho g\nabla GML+C_{\varphi\varphi})\phi+a_{y\varphi}L\ddot{Y}+b_{y\varphi}L\dot{Y}+C_{\varphi y}Y\\
&\qquad=\frac{i\rho g}{K}\zeta_a\bar{A}_y(\overline{OG}-l_w)Le^{i(\varepsilon_S+\omega t)}
\end{aligned}\right\}\quad(1)
$$

where ρ=water density, L=length of the cylinder, B=breath of the cylinder, ∇=section area of the cylinder under the water plane, g=acceleration of gravity, m_z and m_y=added mass coefficients of heaving and swaying motion, J_ϕ=mass moment of inertia of rolling, $I_{\phi\phi}$=added mass moment of inertia of rolling, N_z, N_y, N_ϕ=damping coefficient of heaving, swaying and rolling motion, $a_{y\phi}, b_{y\phi}$=coupling coefficients of swaying and rolling motion, $OG-l_w$= lever of rolling moment, $K=\frac{\omega^2}{g}$=wave number, ω=circular frequency, ζ_a= amplitude of wave, $\bar{A}_z, \bar{A}_y$=ratio of radiation wave amplitude for heaving and swaying motion, $\varepsilon_H, \varepsilon_S$=the phase of radiation wave to heaving and swaying motion, $C_{zz}, C_{yy}, C_{y\phi}C_{\phi\phi}$=linear spring constants of mooring lines.

The equations of motions for the free floating body can be obtained by substituting the relations $C_{zz}=C_{yy}=C_{\phi\phi}=C_{y\phi}=C_{\phi y}=0$ into eq. (1). The coefficients $J_\phi+I_{\phi\phi}$ and $N_{\phi G}$ are obtained from the experiments on the free rolling and the other coefficients are calculated with Ursell-Tasai method[2]. The solutions of eq. (1) are written as

$$
\left.\begin{aligned}
Z&=Z_a\cos(\omega t+\varepsilon_z)\\
Y&=Y_a\cos(\omega t+\varepsilon_y)\\
\Phi&=\Phi_a\cos(\omega t+\varepsilon_\varphi)
\end{aligned}\right\}\quad(2)
$$

We can obtain the amplitudes Z_a, Y_a, Φ_a and the phases of the motions ε_z, ε_y, ε_ϕ by solving the above equations.

The calculated results are compared with experimental values in Fig. 2, 3 and 4. In those figures, H_w denotes a wave height and λ is a wave length. The theoretical and measured values of motions are generally in good agreement.

The theoretical results[3] of the drifting forces are compared with the experimental ones in Fig. 5. The figure shows the fact that the theoretical values are in good accord with the experimental ones when the wave frequency is small, and the experimental values are larger than theoretical ones when the wave frequency is large.

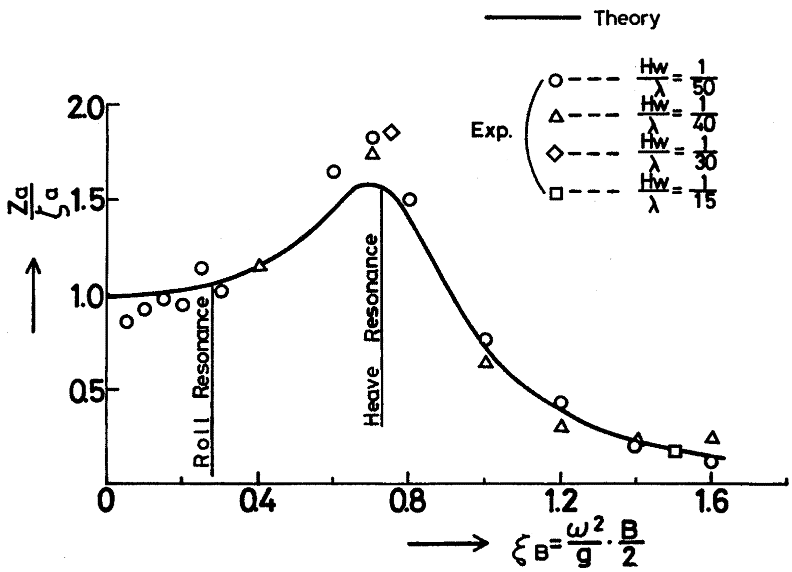

Fig. 2 Heave amplitude of freely floating cylinder.

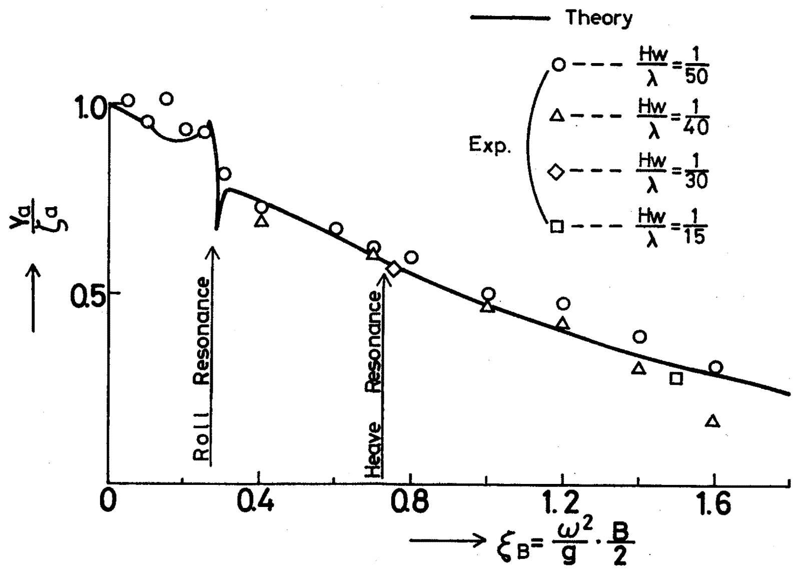

Fig. 3 Sway amplitude of freely floating cylinder.

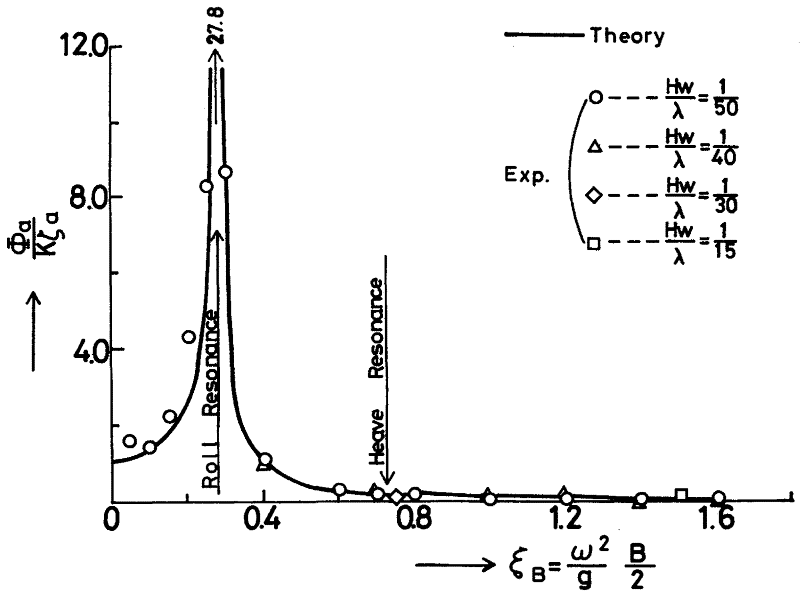

Fig. 4 Roll amplitude of freely floating cylinder.

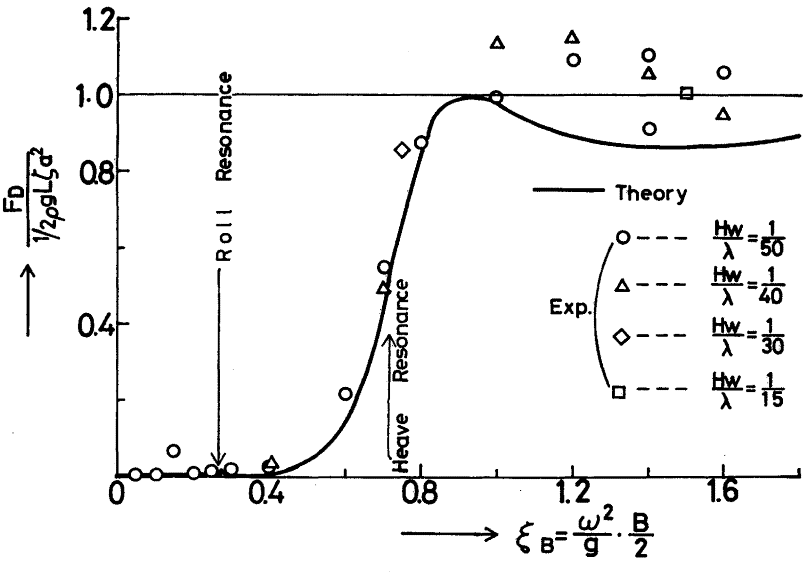

Fig. 5 Drifting force acting on freely floating cylinder.

3. Tension of mooring lines and motions of the moored cylinder in beam sea

The experiments were carried out on the same model that was used to study the free floating motions, as described in chapter 2. The principal dimensions of the mooring lines are shown in Table 1, where w is the weight of mooring line of unit length in water.

Two initial mooring conditions for horizontal component of chain tension were selected: $T_{h0}=1.84$kg and $T_{h0}=0.5$kg as shown in Fig. 6. Since the two conditions did not make any remarkable resulted motions of the model, a mention will be made here only to the case of $T_{h0}=1.84$kg.

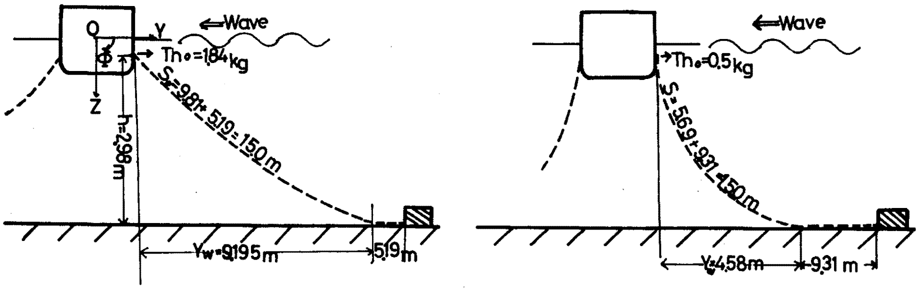

Fig. 6 Mooring conditions.

The comparison between the theoretical and experimental results of the heaving motion are shown in Fig. 7. The theoretical results are obtained by solving eq. (1) and the calculation method for the linear spring constant of the mooring chains in the equation is derived in Appendix I. The theoretical values show the good agreement with the experimental ones and the non-linear effect due to wave heights can be neglected.

The theoretical and experimental results of the swaying and rolling motions are shown in Fig. 8 and Fig. 9. A damping force in the swaying motion not only in the rolling motion is very impotant at its resonance frequency, so the experimental data of the free swaying oscillation should be introduced into the theoretical calculation. It is assumed that the damping coefficient N_y is composed of a viscous damping coefficient N_D and a potencial damping coefficient $N_p(\omega)$, where N_D is assumed to be indipendent of ω. For simplicity, the damping force obtained from the free swaying test will be assumed to be expressed as $N_y(\omega_y)LY$, where ω_y is the resonanse frequency.

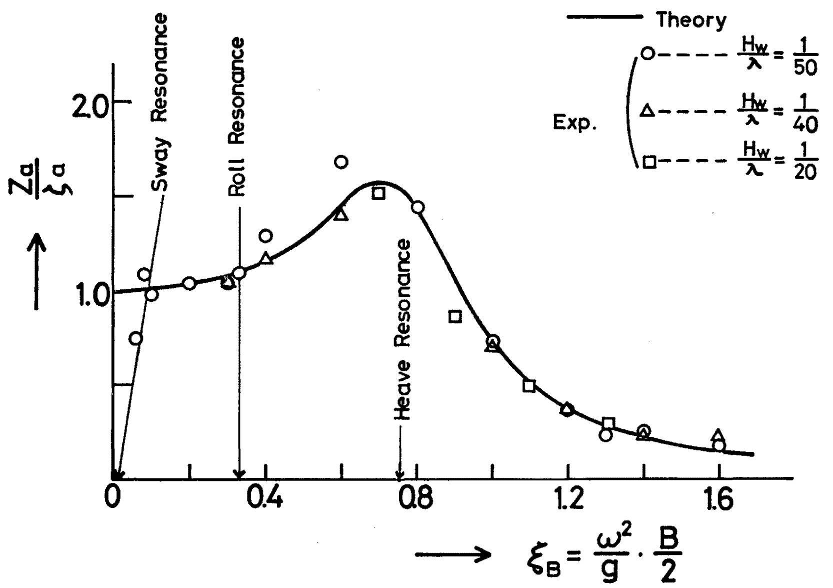

Fig. 7 Heave amplitude of moored cylinder.

Then, the viscous damping coefficient N_D is obtained from the following equation

$$N_y(\omega_y)=N_p(\omega_y)+N_D \tag{3}$$

where $N_p(\omega_y)$ is calculated by the linear theory. The damping coefficient at an arbitrary frequency is then given by the following equation.

$$N_y=N_D+N_p(\omega)=N_y(\omega_y)-N_p(\omega_y)+N_p(\omega) \tag{4}$$

The damping coefficient derived from the moored-free rolling test is much greater than that obtained from unmoored-free rolling one, which suggests that the viscous damping forces exerted on the mooring lines are very large. The difference between the calculation and the experiment on the rolling motion of the moored cylinder in the vicinity of its resonance frequency is mainly due to the non-linearity of the viscous damping force exerted on the mooring lines. As seen from Fig. 8 and Fig. 9, the calculated results of linear theory using the viscous damping coefficients obtained from the free oscillations agrees with the experiments except the case of the rolling motion in the vicinity of its resonance frequency.

The non-dimensional drifting displacements are shown in Fig. 10, the calculated results in the figure are obtained using the drifting force calculated with Maruo's theory and the linear spring constants of mooring lines

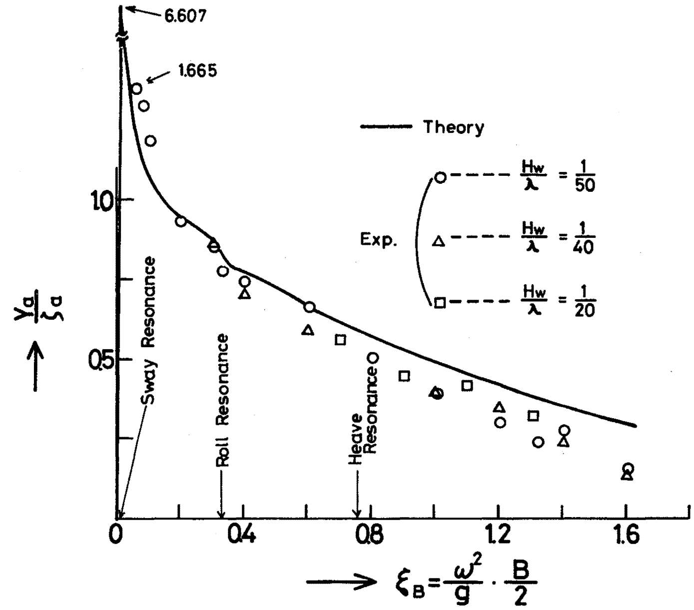

Fig. 8 Sway amplitude of moored cylinder.

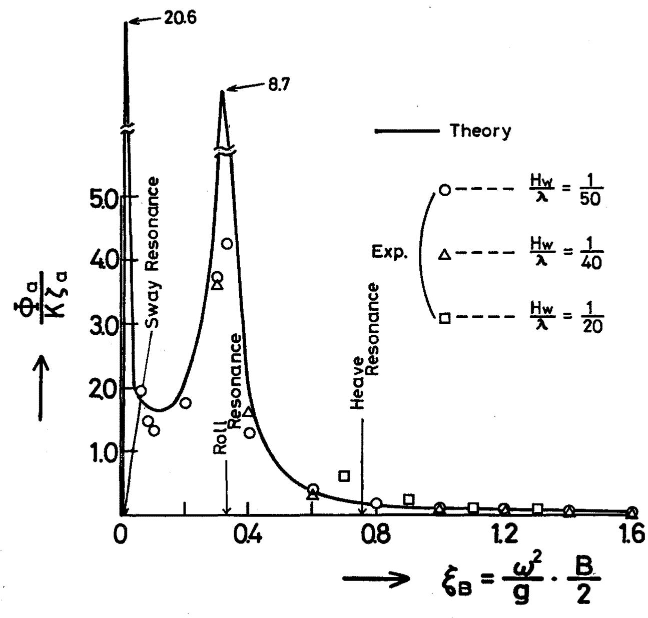

Fig. 9 Roll amplitude of moored cylinder.

as follows:

$$\frac{Y_0}{\zeta_a^2}=\frac{\frac{1}{2}\rho gL\zeta_a^2C_R^2}{C_{yy}\zeta_a^2}=\frac{\rho gLC_R^2}{2C_{yy}} \tag{5}$$

where C_R is the coefficient of a wave reflection. The calculated results show the good agreement with the experimental ones except the cases in the high wave frequency range. In case of high wave frequency, the tendency of the experiments agrees with those of the experiments on the drifting force and an average increase in the mooring force.

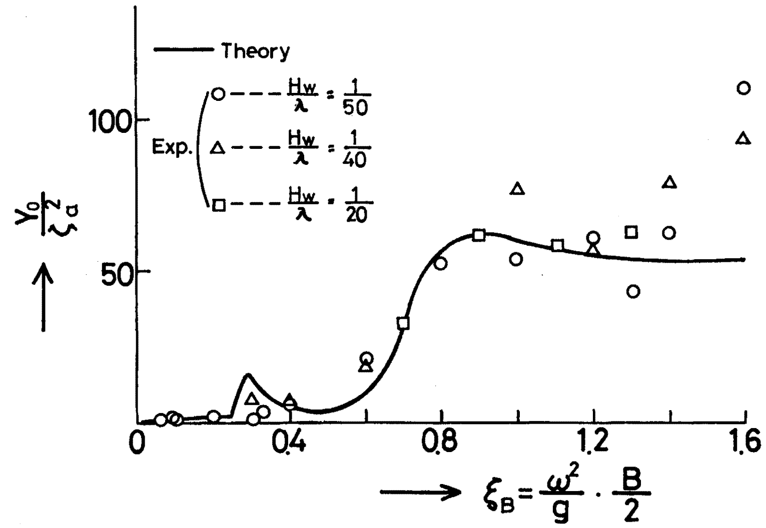

Fig. 10 Drifting displacement.

The difference between the horizontal component of an average increase in weather side line forces and that in lee side line forces should be identical with the drifting force acting on the cylinder. In Fig. 11, those values divided by $1/2\rho gL\zeta_a{}^2$ are shown, and the experimental results have same tendency as those of the drifting forces.

The tension amplitudes of mooring lines can be calculated with the catenary theory if the effects of the inertia and drag forces acting on the chains are taken into no account of.

Horizontal component of the average tension of a weather side chain (T_{hW}) and a lee side chain (T_{hL}) are

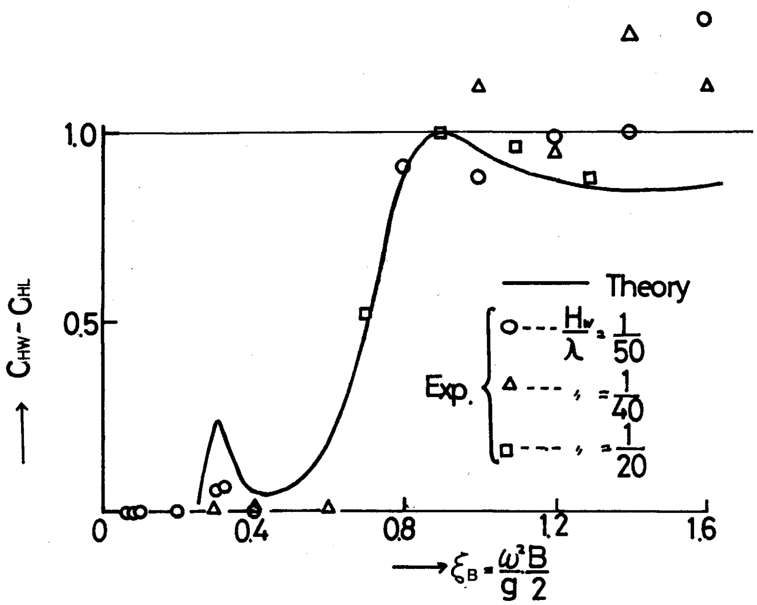

Fig. 11 Horizontal component of average increase in mooring force.

$$\left.\begin{aligned} T_{hW} &= T_{h0}+\frac{1}{4}F_D \\ T_{hL} &= T_{h0}-\frac{1}{4}F_D \end{aligned}\right\} \qquad (6)$$

where F_D is the drifting force acting on the cylinder. The linear spring constants of each two chains $C_{yy}^{(W)}$, $C_{yz}^{(W)}$, $C_{zz}^{(W)}$, $C_{yy}^{(L)}$, $C_{yz}^{(L)}$, $C_{zz}^{(L)}$, can be calculated using the value of T_{hw}, T_{hL}, and the computing method is shown in appendix I. Let the motions of the cylinder be Z, Y, Φ, as shown in Fig. 12, and the horizontal component of the weather side chain tension $F_{HW}^{(S)}$ can be calculated as

$$F_{HW}^{(S)}=[b(1-\cos\Phi)-Y]C_{yy}^{(W)}-(Z+b\sin\Phi)C_{yz}^{(W)} \qquad (7)$$

Usually, ϕ is assumed to be small, then

$$\left\{\begin{aligned} \cos\Phi &= 1-\frac{1}{2}\Phi^2=\left(1-\frac{1}{4}\Phi_a^2\right)-\frac{1}{4}\Phi_a^2\cos 2(\omega t+\varepsilon_\varphi) \\ \sin\Phi &= \Phi-\frac{1}{6}\Phi^3=\Phi_a\left(1-\frac{1}{8}\Phi_a^2\right)\cos(\omega t+\varepsilon_\varphi)-\frac{1}{24}\Phi_a^3\cos 3(\omega t+\varepsilon_\varphi) \end{aligned}\right. \qquad (8)$$

Substituting the relation into eq. (7) and neglecting the higher order forces, we obtain $F_{HW}^{(S)}$ as follows:

$$\begin{aligned} F_{HW}^{(S)} \doteqdot &-\left[Y_a C_{yy}^{(W)}\cos\varepsilon_y+Z_a C_{yz}^{(W)}\cos\varepsilon_z+\Phi_a\left(1-\frac{1}{8}\Phi_a^2\right)C_{yz}^{(W)}b\cos\varepsilon_\varphi\right]\cos\omega t \\ &+\left[Y_a C_{yy}^{(W)}\sin\varepsilon_y+Z_a C_{yz}^{(W)}\sin\varepsilon_z+\Phi_a\left(1-\frac{1}{8}\Phi_a^2\right)C_{yz}^{(W)}b\sin\varepsilon_\varphi\right]\sin\omega t \end{aligned} \qquad (9)$$

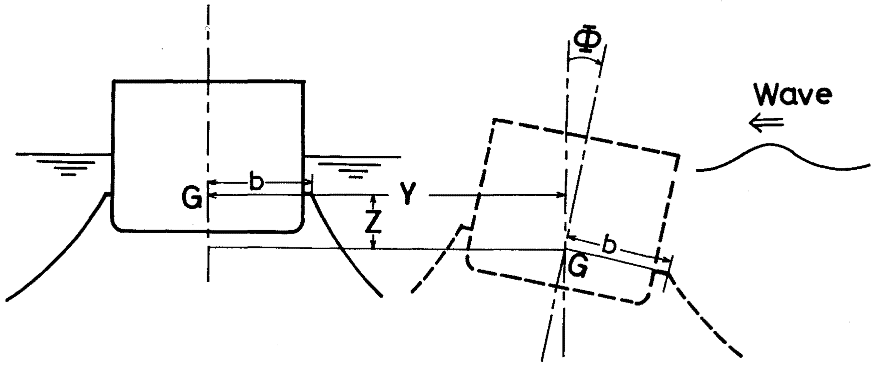

Fig. 12 Displacement of mooring points on cylinder.

Similarly on the vertical component of the weather side chain tension, we have

$$
\begin{aligned}
F_{VW}^{(S)} &= -(Z+b\sin\Phi)C_{zz}^{(W)}+[b(1-\cos\Phi)-Y]C_{yz}^{(W)} \\
&\doteqdot -\left[Y_a C_{yz}^{(W)}\cos\varepsilon_y+Z_a C_{zz}\cos\varepsilon_z+\Phi_a\left(1-\frac{1}{8}\Phi_a^2\right)bC_{zz}\cos\varepsilon_\varphi\right]\cos\omega t \qquad (10) \\
&\quad +\left[Y_a C_{yz}^{(W)}\sin\varepsilon_y+Z_a C_{zz}\sin\varepsilon_z+\Phi_a\left(1-\frac{1}{8}\Phi_a^2\right)bC_{zz}\sin\varepsilon_\varphi\right]\sin\omega t
\end{aligned}
$$

The horizontal component of lee side chain tension is

$$
\begin{aligned}
F_{HL}^{(S)} &= [b(1-\cos\Phi)+Y]C_{yy}^{(L)}-(Z-b\sin\Phi)C_{yz}^{(L)} \\
&\doteqdot -\left[-Y_a C_{yy}^{(L)}\cos\varepsilon_y+Z_a C_{yz}^{(L)}\cos\varepsilon_z-\Phi_a\left(1-\frac{1}{8}\Phi_a\right)bC_{yz}^{(L)}\cos\varepsilon_\varphi\right]\cos\omega t \qquad (11) \\
&\quad +\left[-Y_a C_{yy}^{(L)}\sin\varepsilon_y+Z_a C_{yz}^{(L)}\sin\varepsilon_z-\Phi_a\left(1-\frac{1}{8}\Phi_a\right)bC_{yz}^{(L)}\sin\varepsilon_\varphi\right]\sin\omega t
\end{aligned}
$$

and the vertical one is

$$
\begin{aligned}
F_{VL}^{(S)} &= [b(1-\cos\Phi)+Y]C_{yz}^{(L)}-(Z-b\sin\Phi)C_{zz}^{(L)} \\
&\doteqdot -\left[-Y_a C_{yz}^{(L)}\cos\varepsilon_y+Z_a C_{zz}^{(L)}\cos\varepsilon_z-\Phi_a\left(1-\frac{1}{8}\Phi_a^2\right)bC_{zz}^{(L)}\cos\varepsilon_\varphi\right]\cos\omega t \qquad (12) \\
&\quad +\left[-Y_a C_{yz}^{(L)}\sin\varepsilon_y+Z_a C_{zz}^{(L)}\sin\varepsilon_z-\Phi_a\left(1-\frac{1}{8}\Phi_a^2\right)bC_{zz}^{(L)}\sin\varepsilon_\varphi\right]\sin\omega t
\end{aligned}
$$

we call above mentioned method a statical one. The calculated and the experimental results of F_{HW} divided by $\rho g A_w \zeta_a$ are shown in Fig. 13, where A_w is water plane area (BxL). In this figure, a fine line indicate the results of the statical calculation, a thick line and a dotted line show the results of the dynamic calculations which is explained later. The results of the statical calculations show the good agreement with those of the experiments in the range of $\xi_B<0.3$, In the range of $\xi_B>0.3$, the results of the statical calculation are much smaller than those of the experiments.

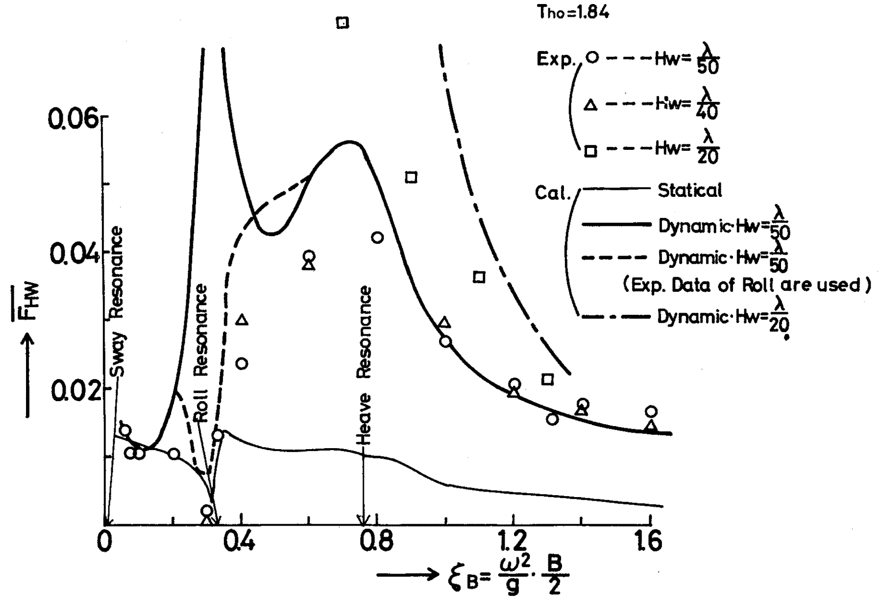

Fig. 13 Horizontal component of tension amplitude of weather side chains (T_{h0}=1.8 kg).

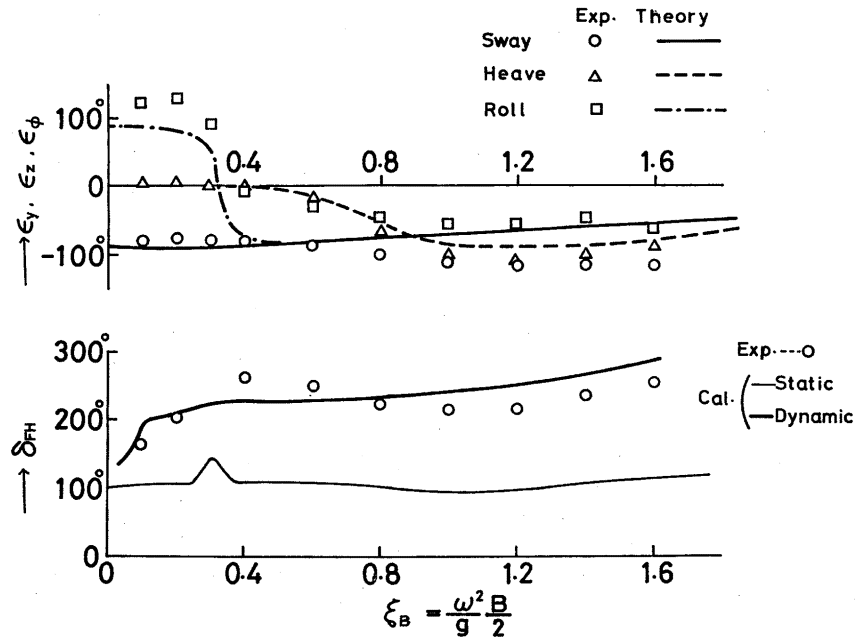

Fig. 14 Phase differences between wave and motions, wave and tension of weather side chains.

Fig. 14 shows the phase differences between the wave and the motions or the horizontal component δ_{FH} of the weather side chain tension. The theoretical results of the phase differences and the amplitudes of the motions show good agreement with the experimental ones except the cases of the rolling motion in the vicinity of its resonance frequency. However, values of δ_{FH} obtained by the statical calculations differ from the experimental results in almost all frequency range.

The differences between the statical calculations and experiments are caused by the ignoring an inertia and a drag force exerted on the mooring chains. We investigated further on this problem. The forced oscillation tests on a mooring chain were carried out in two initial conditions as shown in Fig. 6. The forced swaying and heaving motion at the upper end of the chain were carried out in various frequencies in still water. An example of record of experiment is shown in Fig. 15 and the tension amplitude of the chain at the upper end are in Fig. 16～Fig. 22. In any case of them, the results of the statical calculation which is indicated by a fine line and a fine chain line differ from the experimental ones. An exact equation for the dynamic tensions of the mooring lines is presented by Ried[4], but his theory is not convenient for a direct solution of a practical problem. A calculated result which is obtained with a finite element method introduct-

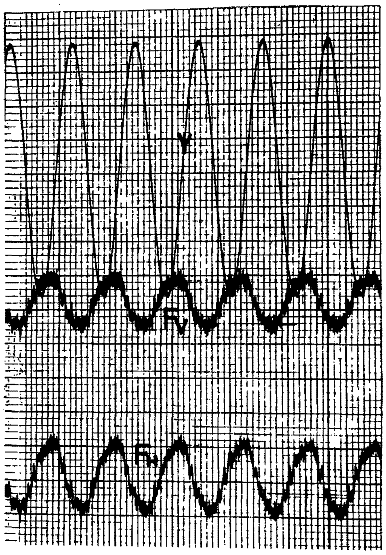

Fig. 15 Record of forced oscillation of mooring chain.

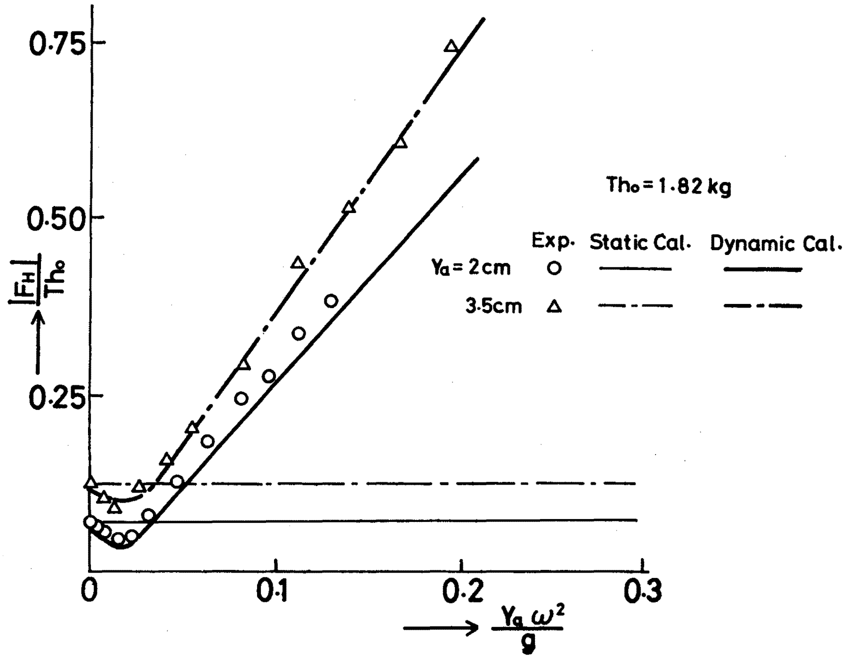

Fig. 16 Horizontal component of tension amplitude in forced sway (T_{h0}=1.8 kg).

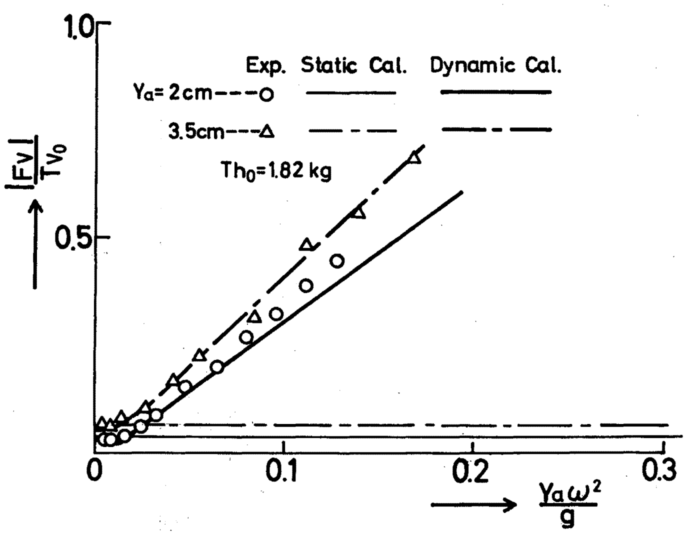

Fig. 17 Vertical component of tension amplitude in forced sway(T_{h0}=1.8 kg).

ing the inertia and the drag force has been shown in Ref. (5). In this paper, a simple and practical method for calculating the dynamic tensions of the mooring lines is offered.

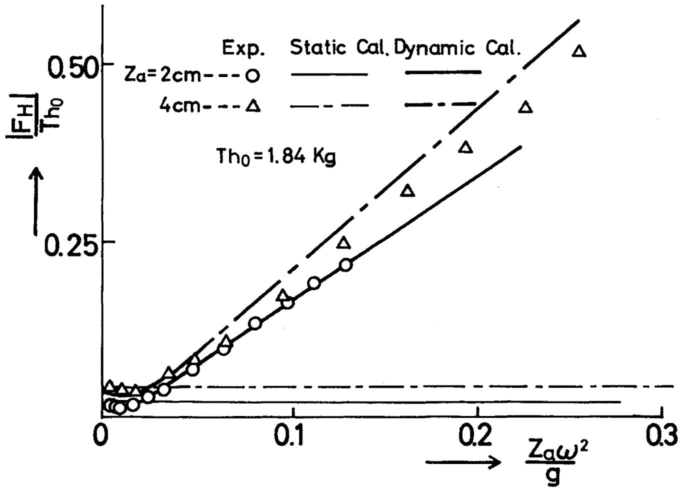

Fig. 18 Horizontal component of tension amplitude in forced heave (T_{h0}=1.8 kg).

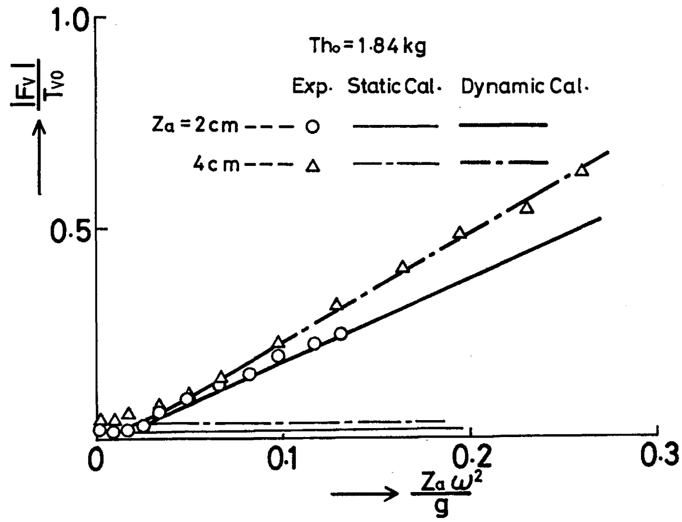

Fig. 19 Vertical component of tension amplitude in forced heave (T_{h0}=1.8 kg).

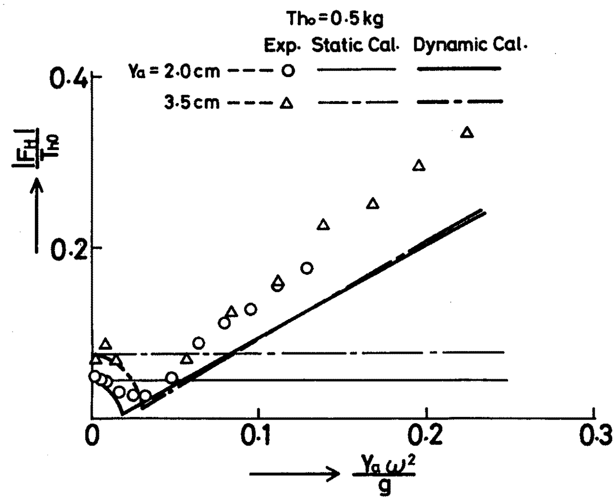

Fig. 20 Horizontal component of tension amplitude in forced sway ($T_{h0}=0.5$ kg).

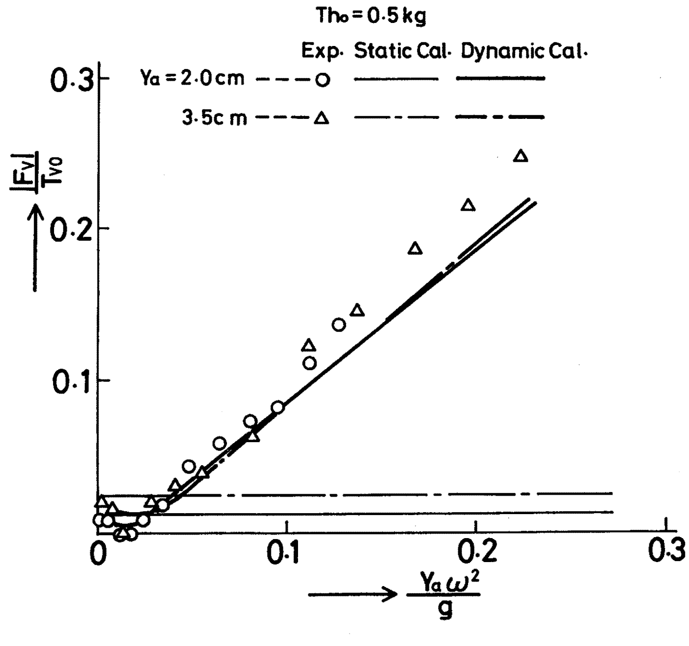

Fig. 21 Vertical component of tension amplitude in forced sway ($T_{h0}=0.5$ kg).

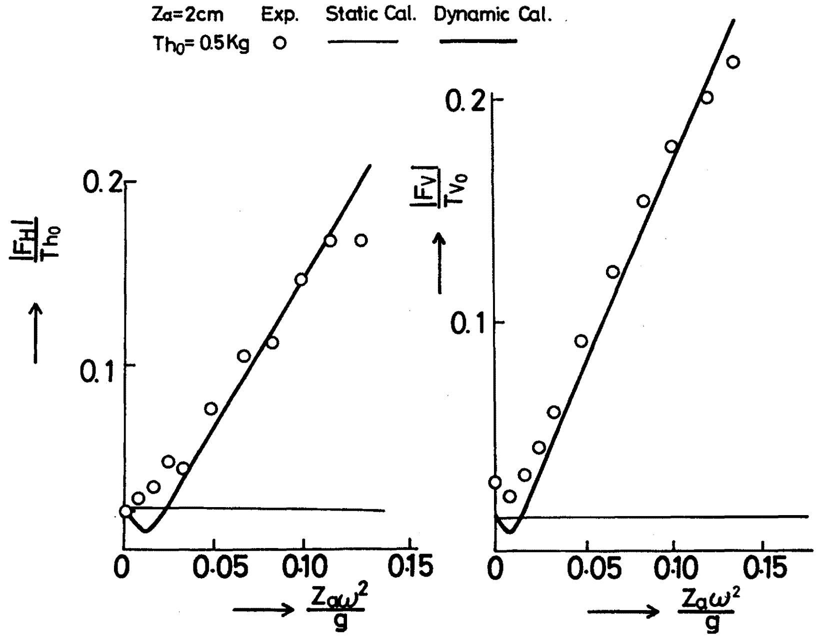

Fig. 22 Tension amplitude in forced heave ($T_{h0}=0.5$ kg).

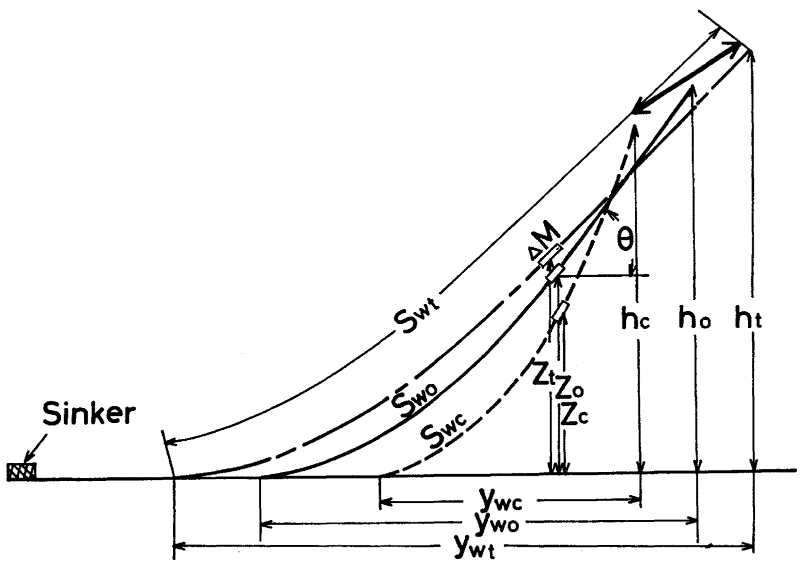

Fig. 23 Notation and assumption in dynamic calculation of chain tension.

Consider the two-dimensional case of a upper end of a mooring line swaying or heaving as shown in Fig. 23. The displacement Δz of a given point on the mooring line in z direction is expressed as

$$\Delta Z=\Delta Z_a \cos \omega t \tag{13}$$

Then, an inertia and a drag force exerted on an element of the mooring line is calculated as follows:

$$\Delta F_V^d=-\Delta\ddot{z}\Delta M(1+m)-\frac{1}{2}\rho C_D \Delta A \cos^3\theta|\Delta\dot{z}|\Delta\dot{z} \tag{14}$$

Linearizing $|\Delta\dot{z}|\Delta\dot{z}$ in eq. 14 leads to the following equation.

$$\Delta F_V^d=\omega^2\Delta Z_a \Delta M(1+m)\cos\omega t+\frac{1}{2}\rho C_D \Delta A\cos^3\theta\frac{8}{3\pi}\Delta z_a^2\omega^2\sin\omega t \tag{15}$$

where ΔM is a mass of the element, m is a added mass in z direction, C_D is the drag coefficient, ΔA is a projective area of the element of the mooring line, θ is an angle of an inclination of the element.

The vertical force F_{vd} exerted on the whole mooring line is calculated as

$$F_V^d=\int_0^S \Delta F_V^d ds=\int_0^{S_{wt}}\Delta F_V^d ds$$

$$=\Delta M\omega^2\cos\omega t\int_0^{S_{wt}}\Delta Z_a(1+m)ds+\frac{4\rho}{3\pi}\Delta A\omega^2\sin\omega t\int_0^{S_{wt}}C_D\cos^3\theta\Delta z_a^2 ds \tag{16}$$

wheer S_{wt} is the length of the mooring line catenary when the mooring line tension become largest in a period as shown in Fig. 23. It is widely known that m and C_D is dependent on Keulegan-Carpenter's Number[6] or $2\pi\Delta Z_a/D$ in this paper[7]. Therefore, they are not constant along the line. In this paper, for simplicity they are assumed to be constant along whole line.

Expression (16) can be written as

$$\begin{aligned}F_V^d&=\omega^2 M(1+m)\Delta z_m\cos\omega t+\frac{4\rho}{3\pi}C_D A\cos^3\theta_m\Delta z_m^2\omega^2\sin\omega t\\&=|F_{Vi}|\cos\omega t+|F_{VD}|\sin\omega t\end{aligned} \tag{17}$$

in which

$$\Delta z_m=\frac{1}{S_{wt}}\int_0^{S_{wt}}\Delta z_a ds$$

$$\cos^3\theta_m=\frac{1}{S_{wt}\Delta z_m^2}\int_0^{S_{wt}}\cos^3\theta\Delta z_a^2 ds$$

$M=\Delta M\times S_{wt}$, $A=\Delta A\times S_{wt}$,

D=the diameter of the chain.

The height Z of a point on the catenary is expressed as follows.

$$z=a\left(\cosh\frac{y}{a}-1\right) \tag{18}$$

where y is a horizontal distance from a point at which the mooring line contacts with the bottom, $a=\frac{T_h}{w}$, T_h: the horizontal component of the mooring line tension, W; a weight of the mooring line of a unit length in water.

An average height of the line catenary is given by

$$\begin{aligned}\bar{z}&=\frac{1}{s_w}\int_0^{S_w} z\,ds=\frac{1}{s_w}\int_0^{y_w} z\sqrt{1+\left(\frac{dz}{dy}\right)^2}dy\\ &=\frac{1}{s_w}\int_0^{y_w} a\left(\cosh\frac{y}{a}-1\right)\cosh\frac{y}{a}dy\\ &=\frac{a^2}{s_w}\int_0^{\frac{y_w}{a}}\cosh\frac{y}{a}\left(\cosh\frac{y}{a}-1\right)d\left(\frac{y}{a}\right)\\ &=\frac{a^2}{s_w}\left(\frac{1}{4}\sinh\frac{2y_w}{a}-\sinh\frac{y_w}{a}+\frac{y_w}{2a}\right)\end{aligned}\tag{19}$$

where y_w is the horizontal length of S_{wt}.

The displacement amplitude $\varDelta Z_m$ of the mean height of the line catenary is expressed as follows:

$$\begin{aligned}\varDelta z_m=\frac{1}{2}(\bar{z}_t-\bar{z}_c)=&\frac{a_t^2}{S_{wt}}\left(\frac{1}{4}\sinh\frac{2y_{wt}}{a_t}-\sinh\frac{y_{wt}}{a_t}+\frac{y_{wt}}{2a_t}\right)\\ &-\frac{a_c^2}{S_{wt}}\left(\frac{1}{4}\sinh\frac{2y_{wc}}{a_c}-\sinh\frac{y_{wc}}{a_c}+\frac{y_{wc}}{2a_c}\right)\end{aligned}\tag{20}$$

in which the suffics c means the slack condition and the suffics s shows the taut condition. In eq. (20), the quatities a_t, a_c, y_{wt}, y_{wc} and S_{wt} are calculated by following equations:

$$\begin{aligned}&a_t=\frac{T_{h0}+|F_H^{(S)}|}{w},\quad a_c=\frac{T_{h0}-|F_H^{(S)}|}{w}\\ &s_{wt}=\sqrt{h_t(h_t+2a_t)},\\ &y_{wt}=a_t\log\left[\frac{s_{wt}}{a_t}+\sqrt{\left(\frac{s_{wt}}{a_t}\right)^2+1}\right],\quad y_{wc}=a_c\log\left[\frac{s_{wc}}{a_c}+\sqrt{\left(\frac{s_{wc}}{a_c}\right)^2+1}\right]\end{aligned}\tag{21}$$

in which $|F_H^{(S)}|$ is the amplitude of the horizontal component of the line tension obtained by the statical calculation, and h is the height of the upper end of the mooring line.

The vertical component F_{vd} of the dynamic tension can be calculated by substituting $\varDelta Z_m$ in to eq. (17). Then the total vertical dynamic tension F_v is calculated as follows.

$$F_V=(|F_V^{(S)}|-|F_{Vi}|)\cos\omega t-|F_{VD}|\sin\omega t\tag{22}$$

in which $F_V^{(S)}$ is a value calculated from the statical method.

The horizontal component F_H of the total dynamic tension can be obtained through the following procedure. In the high frequency range for the forced oscillation test shown in Fig. 16~22, we can induce the following relation between $|F_H|$, $|F_V|$ and T_{h0}, T_{V0},

$$\frac{|F_H|}{T_{h0}}=\alpha\omega^2, \quad \frac{|F_V|}{T_{V0}}=\beta\omega^2 \tag{23}$$

The figures show the fact that α is neary equal to β, then we have

$$\frac{|F_H|}{T_{h0}}=\frac{|F_V|}{T_{V0}} \tag{24}$$

From equation (24), it is found that the inclination of the chain or the form of the chain catenary is invariable throughout the oscillating motion when the acceleration of motion is large.

The horizontal displacement of the chain catenary can be calculated with a similar equation to eq. (20) and it is found to be much smaller than the vertical one. Then, we can consider that the chain makes a heaving motion as a rigid body. We have the same expression for F_H as that for F_V.

$$F_H=(|F_H^{(S)}|-|F_{Hi}|)\cos\omega t-|F_{HD}|\sin\omega t \tag{25}$$

Substituting the eq. (22) and (25) into (24), we have

$$\frac{T_{h0}}{T_{V0}}=\sqrt{\frac{(|F_H^{(S)}|-|F_{Hi}|)^2+|F_{HD}|^2}{(|F_V^{(S)}|-|F_{Vi}|)^2+|F_{VD}|^2}}. \tag{26}$$

From a example of records Fig. 15, it is seen that the phases of F_V agree with those of F_H. Thus the following relation is obtained.

$$\frac{|F_H^{(S)}|-|F_{Hi}|}{|F_V^{(S)}|-|F_{Vi}|}=\frac{|F_{HD}|}{|F_{VD}|} \tag{27}$$

from eq. (26) and (27)

$$\frac{T_{h0}}{T_{V0}}=\frac{|F_H^{(S)}|-|F_{Hi}|}{|F_V^{(S)}|-|F_{Vi}|}=\frac{|F_{HD}|}{|F_{VD}|} \tag{28}$$

or

$$\left.\begin{aligned}|F_{Hi}|&=|F_H^{(S)}|-\frac{T_{h0}}{T_{V0}}(|F_V^{(S)}|-|F_{Vi}|)\\ |F_{HD}|&=\frac{T_{h0}}{T_{V0}}\times F_{VD}\end{aligned}\right\} \tag{29}$$

In case that the acceleration of the oscillating motion is small, $|F_{Vi}|$, $|F_{VD}|$, $|F_{Hi}|$ and $|F_{HD}|$ are smaller than $F_H^{(S)}$ and $F_V^{(S)}$. As ω approaches zero, $|F_{Vi}|$, $|F_{VD}|$, $|F_{Hi}|$ and $|F_{HD}|$ should tend to zero. From eq. (17), we have

$$\lim_{\omega\to 0}|F_{Vi}|,\ |F_{VD}|=0 \tag{30}$$

Then, we obtain following results from eq. (29) and eq. (30).

$$\left.\begin{aligned}&\lim_{\omega\to 0}|F_{Hi}|=|F_H^S|-\frac{T_{h0}}{T_{V0}}|F_V^S|\neq 0\\ &\lim_{\omega\to 0}|F_{HD}|=0\end{aligned}\right. \tag{31}$$

In this paper, it is assumed that the horizontal components of the dynamic tension are induced by coupling spring constant C_{yz} from the vertical dynamic force F_{VD}.

Then, we have following expressions in case of small ω,

$$|F_{Hi}|=\frac{C_{yz}}{C_{zz}}|F_{Vi}|$$
$$|F_{HD}|=\frac{C_{yz}}{C_{zz}}|F_{VD}| \tag{32}$$

When $|F_V^{(S)}|$ is greater than $|F_{Vi}|$, F_{Hi} and F_{HD} can be calculated using eq. (32). They can be calculated using eq. (29) in case of $|F_V^{(S)}|<|F_{Vi}|$.

The differences between the values obtained with eq. (29) and those with eq. (32) are very small on the boundary.

The basic calculating procedure can be described as follows.

1) The linear spring constants and $F_H^{(S)}$, $F_V^{(S)}$ are calculated, then h_t and h_c which are the height of a mooring point when $F_H^{(S)}$ become maximum or minimum in a cycle are obtained.
2) Using eq. (20), ΔZ_m is calculated.
3) $|F_{Vi}|$ and $|F_{VD}|$ is obtained by substituting ΔZ_m and ω into eq. (17), in which C_D and m are obtained from a experiment. For simplicity, θ_m is determined by the following equation.

$$\theta_m=\tan^{-1}\frac{h_0}{y_{w0}}$$

4) $|F_{Hi}|$ and $|F_{HD}|$ are calculated by substituting the quantities obtained in step (1)-(3) into eq. (32) in case of $|F_{Vi}|\leqq F_V^{(S)}$ or into eq. (29) in case of $|F_{Vi}|>F_V^{(S)}$.

The calculating results are compared with the values obtained from the forced oscillating tests in Fig. 16~22. They show fairly good agreement with the experimental ones except a case shown in Fig. 20. A fault of the calculation in the case is caused by the fact that the horizontal component of the initial tension is smaller than the vertical one. In case of $T_{h0}<T_{v0}$, the horizontal displacement of the chain catenary induced by the swaying motion of the upper end of the chain is larger than the vertical one. The proposed calculating method in this paper can not be utilzed for a slack mooring. The calculating results for the dynamic tension amplitude of the chain which moors a cylinder in the regular waves are compared with the experimental ones in Fig. 13 together with Fig. 24 through 30. The results of the dynamic calculation show fairly good agreement with the experimental ones except those in the vicinity of the rolling resonance frequency. The calculated results of the rolling motion in the vicinity of its resonance frequency (Fig. 9 and Fig. 14) are not in accord with the experiments. So it is natural that the calculated results of the mooring tension

using the theoretical values of the rooling motion are not in accord with the experimental ones in the vicinity of the rolling resonance frequency. In those figures, the dotted lines show the calculated results of the dynamic mooring tension using the experimental data of the rolling motions, and they show good agreement with the experiments.

4. Conclusions

The main conclusions obtained from the present study can be written as follows.

1. For the motions of a moored cylinder in regular waves, the amplitudes and phases of the heaving motions can be estimated precisely by the linear theory. The calculated results of the swaying and rolling motions by the linear theory using the experimental data for the damping coefficients show fairly good agreement. One exception is a case of the rolling motion in the vicinity of its resonance frequency.

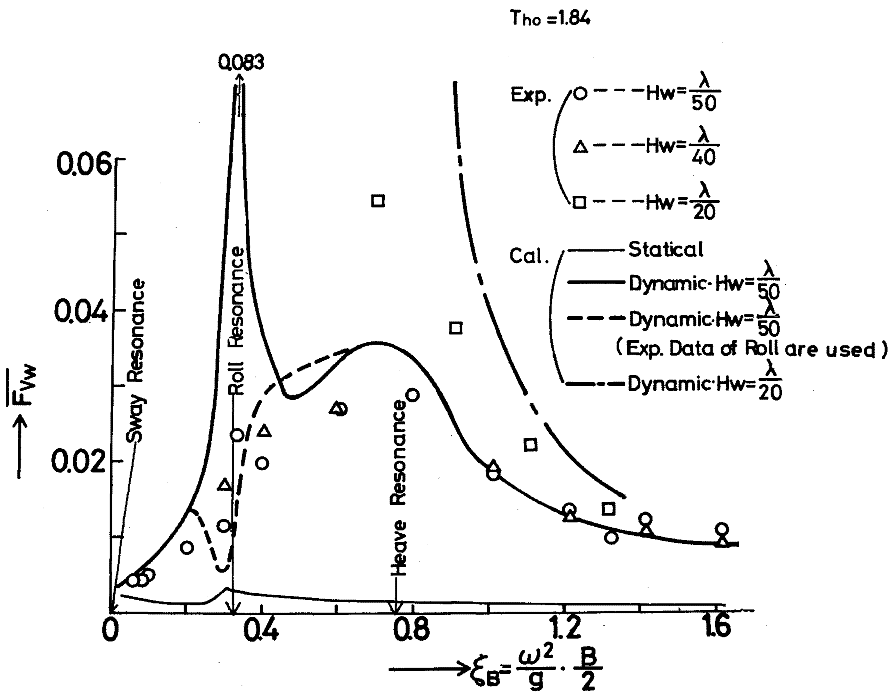

Fig. 24 Vertical component of tension amplitude of weather side chains (T_{h0}=1.8 kg).

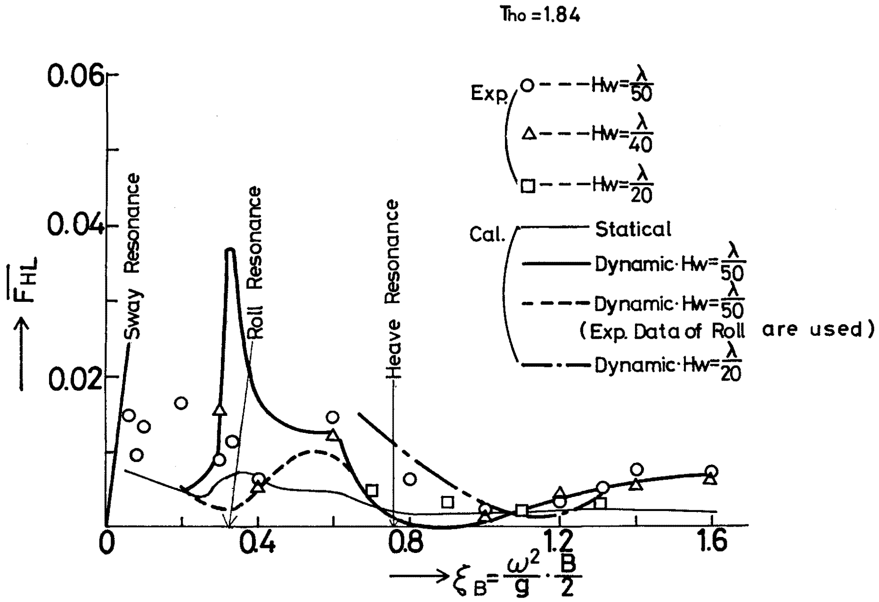

Fig. 25 Horizontal component of tension amplitude of lee side chains (T_{h0}=1.8 kg).

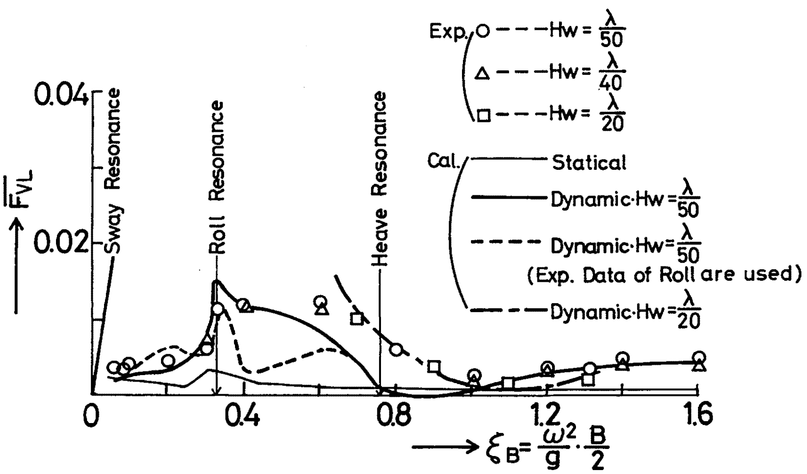

Fig. 26 Vertical component of tension amplitude of lee side chains (T_{h0}=1.8 kg).

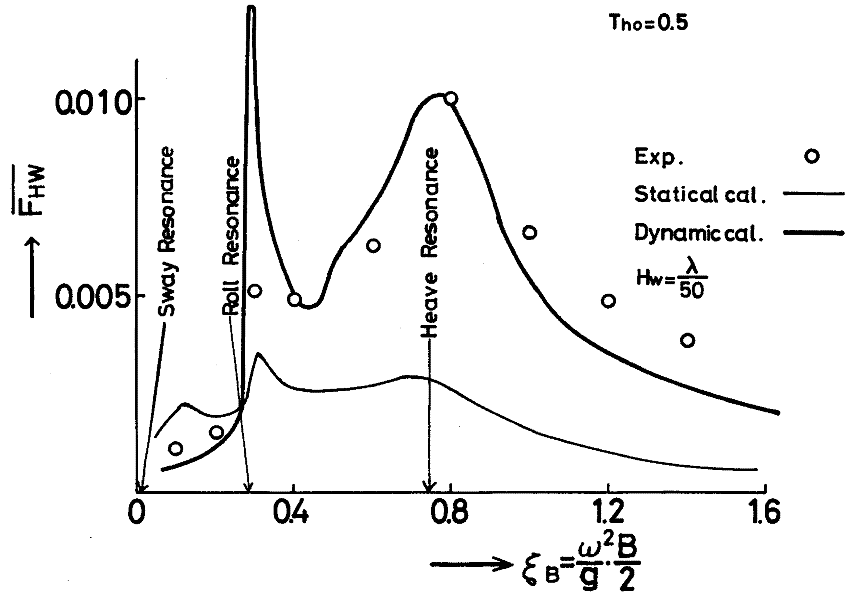

Fig. 27 Horizontal component of tension amplitude of weather side chains ($T_{h0}=0.5$ kg).

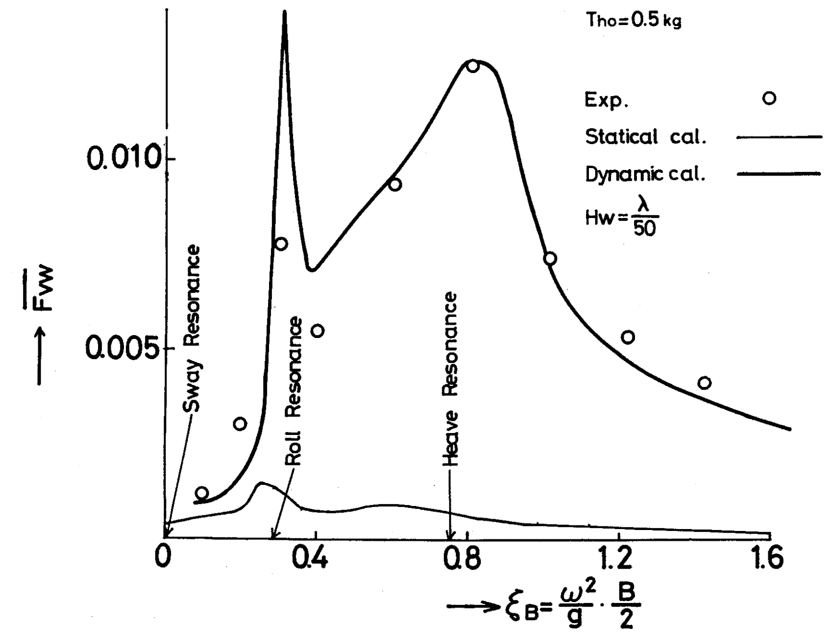

Fig. 28 Vertical component of tension amplitude of weather side chains ($T_{h0}=0.5$ kg).

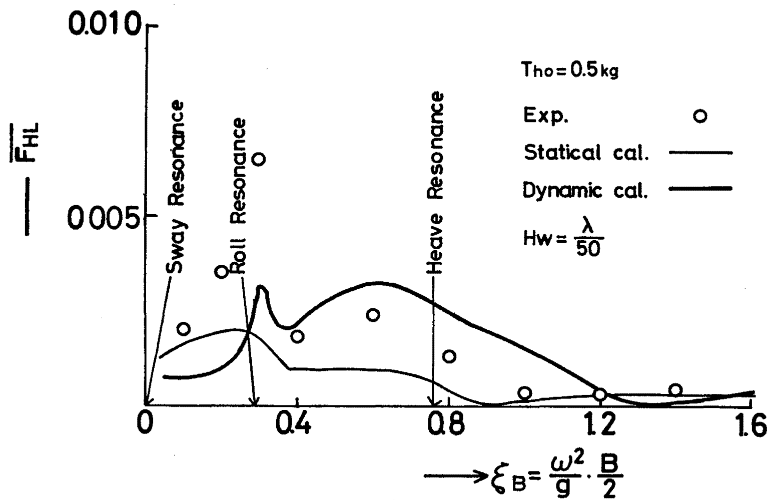

Fig. 29 Horizontal component of tension amplitude of lee side chains (T_{h0}=0.5 kg).

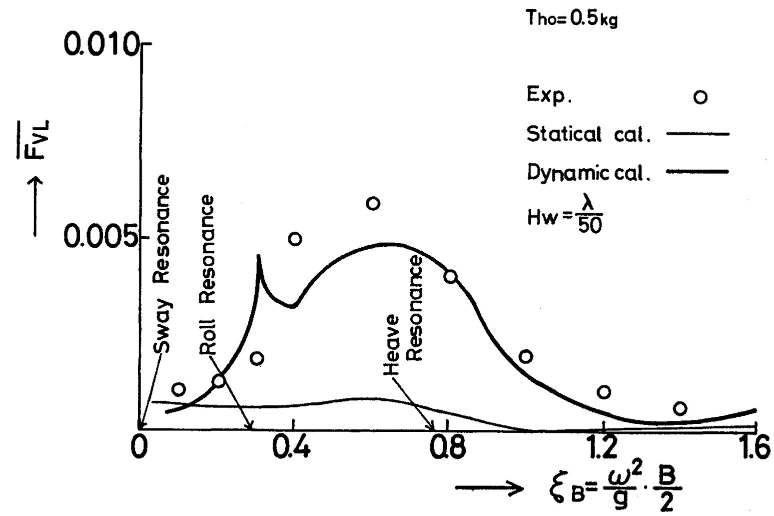

Fig 30 Vertical component of tension amplitude of lee side chains (T_{h0}=0.5 kg).

2. The calculated results of the drifting displacement and the average increase in mooring force using the linear spring constants and Maruo's theory show good agreement with the experiments.

3. The calculated results for the tension amplitude of the mooring by a statical calculation using the linear spring constants is not accord with the experiments except the cases in very low frequency range.

We have proposed an approximate calculation method in which both the inertia force and the hydrodynamic forces acting on the chain are taken into account. The calculated results agree with the experimental results in all frequency range.

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Appendix I

Let T_h, h and w denote respectively the horizontal component of a mooring tension, the height of mooring point and the weight of the mooring line of unit length in water. The length S_w of a catenary line is given by

$$s_w=\sqrt{h(h+2a)} \tag{A-1}$$

in which $a=\frac{T_h}{w}$

The horizontal length y_w of catenary line is expressed as

$$y_w=a\log\left[\frac{s_w}{a}+\sqrt{\left(\frac{s_w}{\mathrm{a}}\right)^2+1}\right] \tag{A-2}$$

we have the linear sping constant of a mooring line as

$$\left.\begin{aligned}C_{yy1}&=\frac{w\cdot\sinh(D)}{Q}\\C_{yz1}&=\frac{w[\cosh(D)-1]}{Q}\\C_{zz1}&=\frac{w[D\cdot\cosh(D)-\sinh(D)]}{Q}\end{aligned}\right\} \tag{A-3}$$

where

$$D=\frac{y_w}{a}$$

$$Q=D\cdot\sinh(D)-2[\cosh(D)\text{-}1]$$

For four mooring lines

$$\left.\begin{aligned}C_{yy}&=4C_{yy1}\\C_{yz}&=4C_{yz1}\\C_{zz}&=4C_{zz1}\\C_{y\varphi}&=bC_{yz}\end{aligned}\right\} \tag{A-4}$$

where b is the horizontal length between the center of gyration of a floating body and the mooring point.

As shown in Fig. A-1. The mooring point are defined in the coordinates

$$(y_0, z_0)=(b, 0),\ (y_1, z_1)=(b\cos\Phi, b\sin\Phi),\ (y_2, z_2)=(-b\cos\Phi, -b\sin\Phi)$$

Let T_{h0} and T_{v0} denote respectively the horizontal and vertical component of the mooring tension when the rolling angle Φ equals to zero, T_{hw}, T_{vw} and T_{hL}, T_{vL} denote respectively the tension of a weather side mooring chain and those of lee side mooring chain when Φ equals to Φ, then we have

$$\left.\begin{aligned}T_{hw}&=T_{h0}+(y_0-y_1)C_{yy1}+(z_0-z_1)C_{yz1}\\T_{Vw}&=T_{V0}+(y_0-y_1)C_{yz1}+(z_0-z_1)C_{zz1}\\T_{hL}&=T_{h0}-(-y_0-y_2)C_{yy1}+(z_0-z_2)C_{yz1}\\T_{VL}&=T_{V0}-(-y_0-y_2)C_{yz1}+(z_0-z_2)C_{zz1}\end{aligned}\right\} \tag{A-5}$$

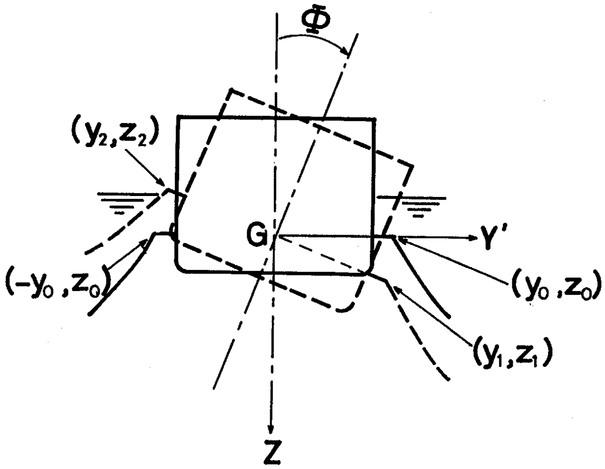

Fig. A-1 Displacement of mooring points due to roll.

The restoring moment M_ϕ by four mooring chains is expressed as

$$\begin{aligned} M_\varphi &= 2T_{hw}\cdot b\sin\Phi - 2T_{Vw}b\cos\Phi + 2T_{hL}b\sin\Phi + 2T_{VL}b\cos\Phi \\ &= 2(T_{hw}+T_{hL})b\sin\Phi + 2(T_{VL}-T_{Vw})b\cos\Phi \end{aligned} \qquad \text{(A-6)}$$

Substituting eq. (A-5) into eq. (A-6) leads to the following results:

$$\begin{aligned} M_\varphi &= [4T_{h0}+4b(1-\cos\Phi)C_{yy1}]b\cdot\sin\Phi + 4b\cdot\sin\Phi\cdot C_{zz1}b\cos\Phi \\ &\doteqdot 4T_{h0}b\Phi + 4b^2\Phi C_{zz1} \end{aligned} \qquad \text{(A-7)}$$

The linear spring constant $C_{\phi\phi}$ for the rolling motion and the restoring moment M_ϕ by four mooring chains can be written as follows:

$$M_\varphi \doteqdot 4T_{h0}b\Phi + b^2\Phi C_{zz} \qquad \text{(A-8)}$$

$$C_{\varphi\varphi} = \frac{\partial M_\varphi}{\partial \Phi} = 4T_{h0}b + b^2 C_{zz} \qquad \text{(A-9)}$$