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Finite Element Computation of Magnetic Field Problems with the Displacement Current

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Abstract

The effectiveness of the A-ϕ method in eddy current probelms is widely known. On the other hand, the demand of high frequency computations increases. In this paper, A-ϕ method is applied to finite element approximations for three-dimensional high frequency problems with the displacement current. Numerical results show that A-ϕ method is more appliable in wide range of frequencies.

Key words: A-ϕ method, high frequency problem, displacement cuurent

1 Introduction

Until now we have treated finite element computation of three-dimensional eddy current problems in lower frequencies, where we can neglect the displacement current. Mainly, we have used the A-ϕ method, where the magnetic vector potential (A) and the electric scalar potential (ϕ) are used as unknown complex valued functions and we have also used the A method, where the magnetic vector potential is used as an only unknown function. Both methods are well known as the formulation of problems; see [3]. In the resultant linear systems, the number of degrees of freedom in A-ϕ method is more than that in A method. However, recent several papers insist that the convergence of the iterative solver, Bi-Conjugate Gradient method (BiCG) in A-ϕ method

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is faster than that in A method; see [3], [4], [5], [6], [7]. On the other hand, the demand to analyze problems where the displacement current can not be neglected increases. In fact, there are several papers that research high frequency problems and their finite element approximations; see [1], [9], [10]. In this paper, we apply A-ϕ method and A method to magnetic field problems with the displacement current, examining which is more effective in finite element approximations in case of considering the displacement current.

Here, as the first step of finite element computation of high frequency problems, we consider the TEAM Problem 7 [8] in frequencies from 50Hz to 5MHz. The magnetic vector potential is discretized by the Nedelec elements of simplex type, and the electric scalar potential by the conventional piecewise linear tetrahedral elements.

The TEAM model is originally a benchmark model for eddy current problems. As in eddy current problems [4], we also choose BiCG for the complex symmetric systems. Numerical results show that A-ϕ method is more applicable than A method in wide range of frequencies.

This paper is organized as follows. In Section 2, we formulate A-ϕ method and A method for magnetic field problems with the displacement current. In Section 3, some numerical results are shown. Finally, concluding remarks are given in Section 4.

2 Formulation

2.1 A-ϕ method

Let Ω be a polyhedral domain with the boundary $\partial\Omega$. Assume that the domain Ω consists of two non-overlapping regions, a conducting part R and a non-conducting one S, with the interface Γ between two regions. In this section, for simplicity, assume that the conducting part R is also a polyhedral domain and strictly included in Ω.

We use the magnetic vector potential A [Wb/m] and the electric scalar potential ϕ [V] as unknown complex functions. Then, the three-dimensional time-harmonic magnetic field problem with the displacement current is derived from the Maxwell equations:

$$\begin{cases} \mathrm{rot}(\nu\,\mathrm{rot}\,A) - i\omega\sigma A + \sigma\,\mathrm{grad}\,\phi - \epsilon\omega^2 A - i\omega\epsilon\,\mathrm{grad}\,\phi = J & \text{in } \Omega, \quad \text{(1a)} \\ \mathrm{div}(i\omega\sigma A - \sigma\,\mathrm{grad}\,\phi + \epsilon\omega^2 A + i\omega\epsilon\,\mathrm{grad}\,\phi) = 0 & \text{in } \Omega, \quad \text{(1b)} \\ A \times n = 0 & \text{on } \partial\Omega, \text{(1c)} \\ \phi = 0 & \text{on } \partial\Omega, \text{(1d)} \end{cases}$$

where J denotes the excitation current density [A/m^2], ν the magnetic reluctivity [m/H], σ the conductivity [S/m], ω the angular frequency [rad/s],

ϵ the permittivity [F/m], n the unit normal to Γ and i the imaginary unit. Throughout this paper, assume that ν and ϵ are piecewise positive constants, that σ is a positive constant in R, while is equal to 0 in S and that ω is a positive constant. Moreover, we also assume

$$\operatorname{div} J = 0 \ \text{ in } \ \Omega. \tag{2}$$

As usual, let $L^2(\Omega)$ be the space of functions defined in Ω and square summable in Ω with its inner product $(\cdot,\cdot)$, and let $H^1(\Omega)$ be the space of functions in $L^2(\Omega)$ with derivatives up to the first order. Let us define some function spaces :

$$V \equiv \Big\{ v \in (L^2(\Omega))^3; \operatorname{rot} v \in (L^2(\Omega))^3, v \times n = 0 \ \text{ on } \ \partial\Omega \Big\},$$
$$U \equiv \Big\{ u \in H^1(\Omega); u = 0 \ \text{ on } \ \partial\Omega \Big\}.$$

The weak form of (1) is described as follows:
Find $(A,\phi) \in V \times U$ such that, for any $(A^*,\phi^*) \in V \times U$,

$$\begin{cases} (\nu \operatorname{rot} A, \operatorname{rot} A^*) - ((i\omega\sigma + \epsilon\omega^2)A, A^*) + \\ \qquad\qquad ((\sigma - i\omega\epsilon) \operatorname{grad} \phi, A^*) = (J, A^*), & \text{(3a)} \\ ((\sigma - i\omega\epsilon) \operatorname{grad} \phi, \operatorname{grad} \phi^*) - ((i\omega\sigma + \epsilon\omega^2)A, \operatorname{grad} \phi^*) = 0. & \text{(3b)} \end{cases}$$

The domain Ω is decomposed into a union of tetrahedra. The magnetic vector potential A is discretized by the Nedelec elements of simplex type, and the electric scalar potential ϕ by the conventional piecewise linear tetrahedral elements. Let V_h and U_h denote finite element spaces corresponding to V and U, respectively. A finite element approximation of (3) is as follows:
Find $(A_h,\phi_h) \in V_h \times U_h$ such that, for any $(A_h^*,\phi_h^*) \in V_h \times U_h$,

$$\begin{cases} (\nu \operatorname{rot} A_h, \operatorname{rot} A_h^*) - ((i\omega\sigma + \epsilon\omega^2)A_h, A_h^*) + \\ \qquad\qquad ((\sigma - i\omega\epsilon) \operatorname{grad} \phi_h, A_h^*) = (\tilde{J}_h, A_h^*), & \text{(4a)} \\ ((\sigma - i\omega\epsilon) \operatorname{grad} \phi_h, \operatorname{grad} \phi_h^*) - ((i\omega\sigma + \epsilon\omega^2)A_h, \operatorname{grad} \phi_h^*) = 0, & \text{(4b)} \end{cases}$$

where $\tilde{J}_h$ denotes a corrected excitation current density described in the following. As in eddy current problems [4], a corrected excitation current density $\tilde{J}_h$ is defined by

$$\tilde{J}_h \equiv J_h - \operatorname{grad} I_h, \tag{5}$$

where J_h denotes an approximated excitation current density by conventional piecewise linear tetrahedral elements, and $I_h \in U_h$ satisfies

$$(\operatorname{grad} I_h, \operatorname{grad} I_h^*) = (J_h, \operatorname{grad} I_h^*) \ \text{ for any } \ I_h^* \in U_h. \tag{6}$$

In partial computation, the excitation current density is corrected only in a part of Ω ; see [4].

2.2 A method

Here, we introduce another formulation of the magnetic field problem with the displacement current, that is A method where the magnetic vector potential A [Wb/m] is an only unknown function:

$$\begin{cases} \mathrm{rot}(\nu\,\mathrm{rot}\,A) - i\omega\sigma A - \epsilon\omega^2 A = J & \text{in } \Omega, \quad (7a) \\ A \times n = 0 & \text{on } \partial\Omega. \quad (7b) \end{cases}$$

The weak form of (7) is described as follows:
Find $A \in V$ such that, for any $A^* \in V$,

$$(\nu\,\mathrm{rot}\,A, \mathrm{rot}\,A^*) - ((i\omega\sigma + \epsilon\omega^2)A, A^*) = (J, A^*). \quad (8)$$

As in A-ϕ method, the magnetic vector potential is discretized by the Nedelec elements of simplex type, then a finite element approximation of (8) is as follows:
Find $A_h \in V_h$ such that, for any $A_h^* \in V_h$,

$$(\nu\,\mathrm{rot}\,A_h, \mathrm{rot}\,A_h^*) - ((i\omega\sigma + \epsilon\omega^2)A_h, A_h^*) = (\tilde{J}_h, A_h^*). \quad (9)$$

3 Numerical results

We consider Problem 7 in the TEAM workshop that is used as a benchmark problem of eddy current problems (the TEAM model); see Figure 1 and [8]. We apply frequencies over 50Hz to the model. The magnetic reluctivity ν is $1/(4\pi) \times 10^7$ [m/H], the conductivity σ is 3.256×10^7 [S/m], the permittivity in vacuum ϵ_0 is 8.85×10^{-12} [F/m],the relative permittivity of aluminum ϵ_r is 8.0, and the absolute value of the real (or immarginary) part of the excitation current density $|J_r|$ (or $|J_i|$) is 1.0968×10^6 (or 0) [A/m^2]. Moreover, frequencies are set to be 50Hz, 500Hz, 5kHz, 50kHz, 500kHz and 5MHz. As in Figure 2, the domain Ω is decomposed into a union of tetrahedra. The number of elements and complex degrees of freedom are 47,716 and 85,833, respectively; the number of complex degrees of freedom of the magnetic vector potential is 74,341, and that of the electric scalar potential is 11,492. As a solver for the resultant linear system, BiCG is used with the shifted incomplete Cholesky factorization [2]. Computation is stopped when the relative residual norm $\|\mathbf{M}^{-1}(\mathbf{A}\mathbf{x} - \mathbf{b})\|/\|\mathbf{M}^{-1}\mathbf{b}\|$ becomes smaller than 1.0×10^{-7}. Here $\mathbf{A}$ denotes the resultant coefficient matrix, $\mathbf{x}$ denotes the solution vector, $\mathbf{b}$ the resultant given vector, $\mathbf{M}$ the preconditioner, and $\|\cdot\|$ the Euclidean norm. The shifted value is set to be 1.08. The initial vector is set to be 0. Computation was performed on a Pentium 4 (2 GHz) with 1 CPU and double precision arithmetic.

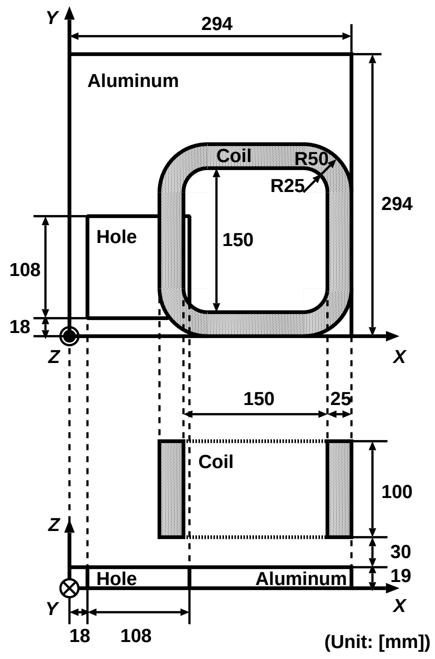

Fig. 1. The TEAM model.

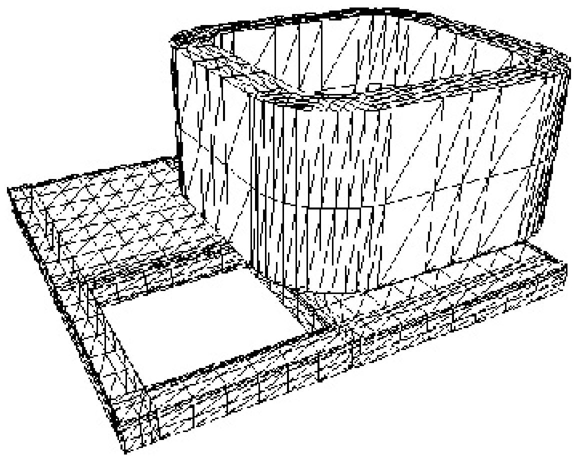

Fig. 2. A finite element mesh around the coil and the conductor for the TEAM model.

Figures 3-8 show the profiles of $\|\mathbf{M}^{-1}(\mathbf{A}\mathbf{x}-\mathbf{b})\|/\|\mathbf{M}^{-1}\mathbf{b}\|$ versus the number of iterations when the frequencies are 50Hz, 500Hz, 5kHz, 50kHz, 500kHz and 5MHz, respectively. Table 1 shows CPU time and number of iterations in each frequency. "Matrix" means the time of making the linear system, "Solver" means that of solving the system, "All" means that of whole computation. From 50Hz to 5kHz, BiCG converges in both methods. However, in over 50kHz, while BiCG convergs in A-ϕ method, BiCG does not converge within 100,000 iterations in A method. Moreover when BiCG converges in both methods for 50Hz, the CPU time and the number of iterations of A-ϕ method are less than those of A method. But, for 500Hz and 5kHz, CPU time of A-ϕ method is more than that of A method.

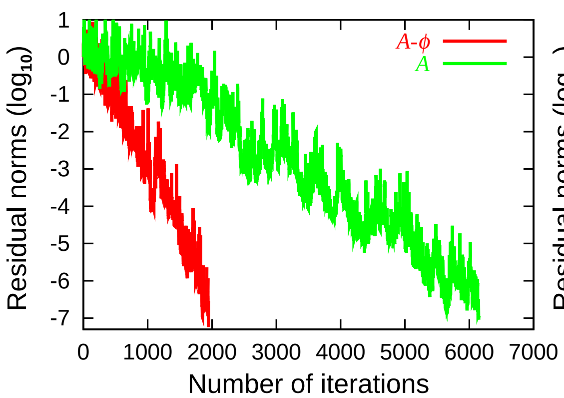

Fig. 3. Profiles of residual norms of 50Hz

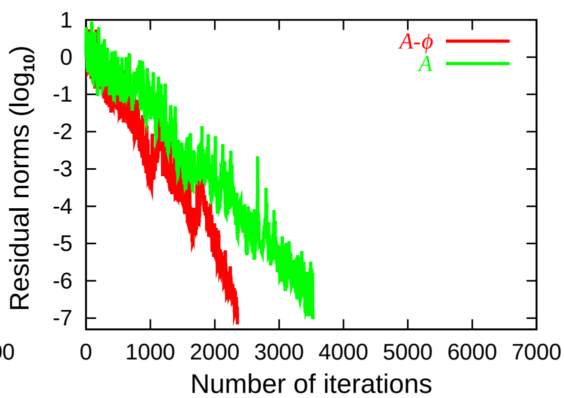

Fig. 4. Profiles of residual norms of 500Hz

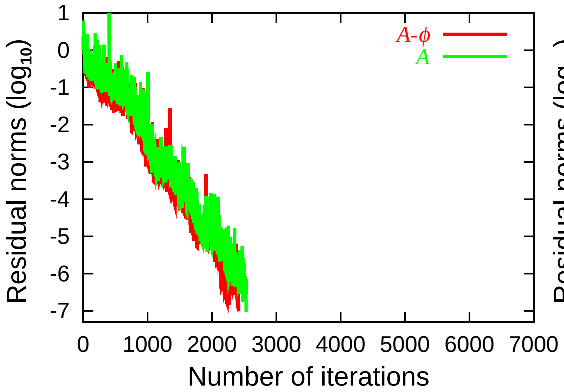

Fig. 5. Profiles of residual norms of 5kHz

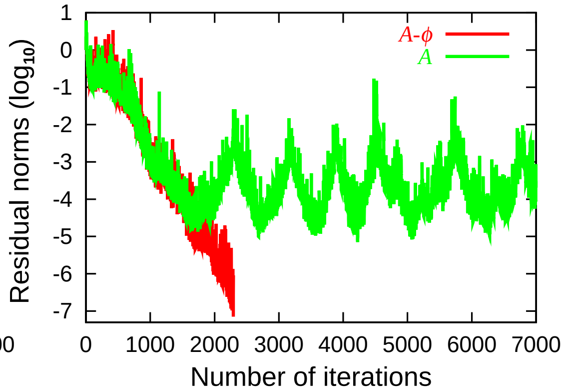

Fig. 6. Profiles of residual norms of 50kHz

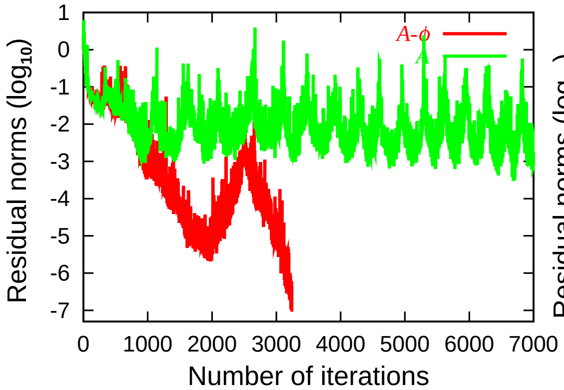

Fig. 7. Profiles of residual norms of 500kHz

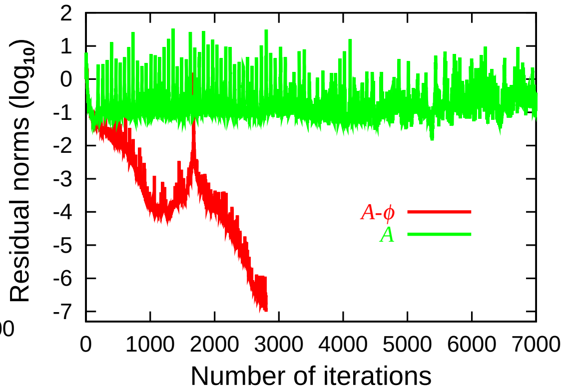

Fig. 8. Profiles of residual norms of 5MHz

4 Concluding remarks

We have applied the A-ϕ method and the A method to a three-dimensional magnetic field problem (the TEAM model) with the displacement current. For both formulations, we have considered finite element approximations.

The CPU time of A method is less than that of A-ϕ method for 500Hz and 5kHz. However, BiCG does not converge within 100,000 iterations in A method when the frequency is over 50kHz. Consequently, A-ϕ method is more applicable than A method in the wide range of frequencies. In this paper, we consider the TEAM model under the condition that $\epsilon_r \epsilon_0 \omega / \sigma$ is less than 6.83e-11. For those ranges, the formulations without the displacement current are also available, and CPU costs of the formulations without the displacement current are less than those of the formulations with the displacement current, because the numbers of degrees of freedom decrease.

We are planning to compute other models in wide range of frequencies.

Table 1
CPU time and number of iterations in 50Hz-5MHz

Frequency	Form.	Matrix(s)	Solver(s)	All(s)	Num. of iterations
50Hz	A-ϕ	5.83	1.49e+03	1.50e+03	1,945
	A	3.70	2.67e+03	2.67e+03	6,149
500Hz	A-ϕ	5.81	1.82e+03	1.82e+03	2,352
	A	3.66	1.53e+03	1.53e+03	3,530
5kHz	A-ϕ	5.79	1.87e+03	1.87e+03	2,418
	A	3.66	1.10e+03	1.10e+03	2,531
50kHz	A-ϕ	5.79	1.76e+03	1.77e+03	2,293
	A	3.68	4.96e+04	4.97e+04	114,688
500kHz	A-ϕ	5.78	2.49e+03	2.49e+03	3,242
	A	3.68	1.20e+05	1.20e+05	276,788
5MHz	A-ϕ	5.79	2.15e+03	2.16e+03	2,802
	A	3.68	1.85e+05	1.85e+05	427,331

References

[1] R.Dyczji-Edlinger and O.Biro, A joint vector and scalar potential formulation for driven high frequency problems using hybrid edge and nodal finite element, *IEEE Trans. Microwave Theory Tech.*, 44(1996)15-23.

[2] T.A.Manteuffel, An incomplete factorization technique for positive definite linear systems, *Math. Comp.*, 34(1980)473-497.

[3] O.Biro, Edge element formulation of eddy current problems, *Comput. Meth. Appl. Mech. Engrg.*, 169(1999)391-405.

[4] H.Kanayama, D.Tagami and K.Imoto, Effectiveness of A-ϕ method for 3-D eddy current problems, *Theor. Appl. Mech.*, (2002)411-417.

[5] K.Fujiwara, T.Nakata and H.Ohashi, Improvement of convergence characteristic of ICCG method for the A-ϕ method using edge elements, *IEEE Trans. Magn.*, 32(1996)804-807.

[6] Y.Kawase, Magnetic field analysis coupled with electric circuit and motion equation, *ICS Newsletter*, (2000)12-16.

[7] H.Igarashi, On convergence of ICCG applied to finite element equation for quasi-static fields, *IEEE Trans. Magn.*, 38(2002)565-568.

[8] K.Fujiwara and T.Nakata, Results for benchmark problem 7 (asymmetric conductor with a hole), *COMPEL*, 9(1990)137-154.

[9] K.Ise, K.Inoue and M.Koshiba, Three-dimensional finite-element solution of dielectric scattering obstacles in a rectangular waveguide, *IEEE Trans. Microwave Theory Tech.*, 38(1990)1352-1359.

[10] A.Alonso and A.Valli, Unique slovability for high-frequency heterogeneous time-harmonic Maxwell equations via the Fredholm alternative theory, *Math. Meth. Appl. Scien.*, 21(1998)463-477.